

# Survey of stochastic models for wind and sea state time series

V. Monbet<sup>a,b,\*,1</sup>, P. Ailliot<sup>a</sup>, M. Prevosto<sup>b</sup>

<sup>a</sup>Department of Applied Statistics, University of South Brittany, BP 573, 56017 Vannes cedex, France

<sup>b</sup>Hydrodynamics and Metocean, IFREMER/DCB/ERT/HO, BP 70, 29280 Plouzané, France

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## Abstract

The knowledge of sea state and wind conditions is of central importance for many offshore and nearshore operations. In this paper, we make a complete survey of stochastic models for sea state and wind time series. We begin with methods based on Gaussian processes, then non-parametric resampling methods for time series are introduced followed by various parametric models. We also propose an original statistical method, based on Monte Carlo goodness-of-fit tests, for model validation and comparison and this method is illustrated on an example of multivariate sea state time series.

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## 1. Introduction

The knowledge of sea state and wind conditions is of central importance for many offshore and nearshore operations. For instance, wind time series permit us to evaluate the power values produced by wind turbines, or to investigate load matching and storage requirements [18,21]. The evolution of sea state and wind conditions is also determinant in coastal erosion [73]. And several questions concerning safety, reliability and feasibility of offshore activities [55,49] as well as maritime transport [11,2] and the drift of floating objects [5] or oil spills are directly related to wave and wind conditions.

Even if there is a huge amount of data collected on physical quantities related to wind and waves, they remain sparse relative to the size of oceans (simply, one cannot observe everywhere at the same time). Hence estimates of risks for undesired scenarios for ocean operations are usually computed by means of stochastic models. Often scenarios are defined as excursions above critical values of some responses of complicated systems

and Monte Carlo can be the only way to derive probabilities of interest.

In this paper, we make a survey of stochastic models for wind and sea state time series and we focus mainly on simulation. We have chosen to consider only time series at the scale of the sea state (i.e. with time step from 1 to 6 h) and at a given geographical point. As a consequence, events at the scale of waves are not modeled and no spatial information is taken into account. Several authors have proposed spatio-temporal models, see for instance [71,11,4] and references therein.

In Section 2, after a short description of the various sources of data, we discuss what a good model should be and then we briefly present various methods to deal with the non-stationarities (interannual variability, seasonal and daily components, ...). Then the most famous tools for time series modeling, i.e. the Box and Jenkins method [15] and some extensions of it are introduced in Section 3. Non-parametric resampling methods are described in Section 4. These methods were rarely proposed in the literature for sea state processes but we think that they could be of great interest for many applications. In Section 5, various parametric models are briefly presented. Then, in Section 6, a validation method is proposed to measure the ability of a model to simulate realistic artificial sequences. Finally, in Section 7 some of these methods are illustrated on a multivariate time series describing sea state conditions along a ferry line in the Aegean Sea.

\* Corresponding author at: Department of Applied Statistics, University of South Brittany, BP 573, 56017 Vannes cedex, France.

E-mail addresses: [valerie.monbet@univ-ubs.fr](mailto:valerie.monbet@univ-ubs.fr) (V. Monbet), [pierre.ailliot@univ-ubs.fr](mailto:pierre.ailliot@univ-ubs.fr) (P. Ailliot), [mpr@ifremer.fr](mailto:mpr@ifremer.fr) (M. Prevosto).

<sup>1</sup> Present address: UBS/SABRES, CERYC, BP 573, 56017 Vannes Cedex, France. Tel.: +33 2 97 017 225; fax: +33 2 97 017 165.

## 2. Generalities

### 2.1. Data

The data which describe wind and sea state conditions can be gathered in two categories:

In situ observations obtained from ships, buoys, satellites, ...

Outputs of meteorological models, i.e. hindcast, nowcast or forecast data.

Some authors have compared the quality of these different kinds of data, see for instance [76,20,41] and references therein. The main drawbacks of buoy data is that they often include missing and noisy data and the longer buoy time series are typically only about 10 years long. Satellite data are recorded with very small time steps along the track but it generates sparse data in time for a given location. Generally, outputs of numerical models are easier to use because these data are available on long period (up to 50 years) and there is no missing data. But their quality is not always good, in particular, they are known to be smoother as in situ data and also to underestimate extreme events.

Let us now introduce some notations for the synthetic parameters which are used throughout this paper. For more precise definitions, see [40].

$H_s$ : significant wave height (m). For  $H_s$  the most common definitions are the average of the highest one-third of wave heights or 4 times the standard deviation of the sea surface elevation process. These definitions are equivalent for seas with narrow band spectra. There is a third definition of  $H_s$ , namely  $H_s = 4\sqrt{m_0}$ , where  $m_0$  is the zeroth-order moment of the sea state spectrum.

$T$ : wave period (s). The most often used definitions are the spectral peak wave period, which is the inverse of the frequency at which the spectral density function of the elevation time series is maximum, and the zero crossing period which is the average time between successive zero downcrossing waves.

$\Theta_m$ : mean direction in which waves are traveling (degree).

$U$ : wind intensity ( $\text{m s}^{-1}$ ). It is the mean of the speed of the air particles at 10 m over a fixed time period (20 min in general).

$\Phi$ : wind direction (degree). It is the mean direction from which the wind is blowing at 10 m over a fixed time period (20 min in general).

In practice, for each sea state, the sea state parameters  $H_s$ ,  $T$  and  $\Theta_m$  can be deduced from the wave directional spectra, and this spectra may exhibit several peaks. It is the case for instance when several systems coexist, such as wind sea generated by local winds and swell radiated from distant storms. For some applications, it can be useful to separate the wave energy into different components (see [75]), but such separation is not considered in this paper.

### 2.2. What is a good model?

First of all, the response to this question depends on what the model is built for. The contexts of use considered in this paper are: explanation of a physical phenomena, time

series simulation, forecasting, and time series reconstruction. However, particular attention is paid to simulation and reconstruction. The word *reconstruction* can refer to two different problems: *missing data reconstruction* which uses the observed time series itself for missing values completion [65] and *cross-reconstruction* where a time series is reconstructed given another one. For instance a wave time series can be approximately reconstructed given a wind time series [2] or wave time series at other locations [7].

Once the context of use is specified, the choice of the model relies on a compromise between its ease of use and its accuracy in describing various features of the physical phenomenon.

The *ease of use* can be measured qualitatively according to several criteria:

Model robustness to the data source (ship, buoy, satellite, meteorological model). As mentioned above, there may exist missing or aberrant data, the data may not be recorded at a constant time step or only descriptive statistics may be available, such as marginal distributions or persistence durations, for instance [72,37].

Model robustness to the nature of the process. Each sea state parameter has specific characteristics, its evolution may also depend on the geographical area and on the season. Furthermore, depending on the considered application, the process may be univariate or multivariate, and in the second case a strong dependence may exist between the components such as for  $H_s$  and  $T$  or  $H_s$  and  $U$ . Finally, the state space of the considered parameters can be either finite, positive or the torus  $\mathbb{R}/2\pi\mathbb{Z}$  for the circular processes  $\Theta_m$  and  $\Phi$ .

Mathematical properties: amount of data needed to accurately estimate the model, asymptotic properties of the estimators ...

Necessary time for implementation of the algorithms, and running estimation, simulation or reconstruction.

The *accuracy of a model* can be evaluated by comparing statistics of the observed time series with those computed from artificial realizations of the model. Generally, only graphical comparisons are performed, and they do not permit us to measure the goodness of fit. We propose in Section 6 a formal method, based on Monte Carlo tests, to compare and validate the models. It quantifies, in particular, the ability to restore chosen features of the observed time series such as the marginal cumulative distribution functions (cdf), the distribution of annual extremes, the distribution of storm duration, the autocovariance functions, etc.

### 2.3. Modeling non-stationarity

Generally, several types of non-stationary components can be identified in meteorological time series. In particular, there may exist interannual components induced by natural cycles (ENSO, IPO, NAO ...) or human activities, seasonal components and also daily components.

Athanassoulis and Stefanakos [10] identified year-to-year variability in sea state time series by comparing mean annual values. Different methods have been proposed to describe trends in time series (see [17]). For example, they can be

approximated by a polynomial function and then eliminated in order to obtain a trend-free time series. However, such an approach is difficult to implement here because of the few data with respect to the temporal scale of the events.

Seasonal components are generally easy to observe on meteorological time series and several methods have been proposed to describe these components (see [22], for a recent review). Two methods are commonly used:

Let  $\{Y_t\}$  denote the process under consideration. It can be assumed, in a first approximation, that the following decomposition holds [74,48,67]:

$$Y_t = m(t) + \sigma(t)Y_t^{\text{stat}} \quad (1)$$

where  $m$  and  $\sigma$  are deterministic periodic functions with period one year which represent respectively the seasonal mean and standard deviation of the process. And  $\{Y_t^{\text{stat}}\}$  is assumed to be a stationary process. Methods for estimating the deterministic components  $m(t)$  and  $\sigma(t)$  and for checking the stationarity of  $Y_t^{\text{stat}}$  have been discussed by Athanassoulis et al. [10]. When  $\{Y_t\}$  is an univariate process,  $m$  and  $\sigma$  can be approximated by a low-order trigonometric polynomial. Another common approach consists in computing the estimates of the monthly mean and the monthly standard deviation as a seasonal pattern. The periodic functions  $m$  and  $\sigma$  are then deduced by repeating the same estimated pattern over successive years [67,49]. This last method is easily generalized to multivariate processes. In this case,  $\sigma_t$  is a matrix which must describe interactions between the different components.

In [14,66], the assumption that  $m(t)$  and  $\sigma(t)$  are deterministic is relaxed in order to introduce random variation, and it is assumed that

$$Y_t = m(t) + \sigma(t)(Y_t^{\text{stat}} + \epsilon_t) \quad (2)$$

where  $\epsilon_t$  is a white noise process evolving at a monthly time scale.

Another approach consists in supposing that the process is piecewise stationary and to fit separate models for each month or for each season of the year [18,13].

In the first method, it is assumed that the standardized process  $Y_t^{\text{stat}}$  is stationary, and it may be too strong an assumption in some cases. However, its main advantage is that the estimation of  $m$  and  $\sigma$  does not require a lot of data, typically 3 or 4 years of data provide accurate estimates. On the contrary, in order to apply the second method, more data  $i$  is generally required since a different model is fitted each month. Furthermore, artificial ruptures of the model are induced between successive months. But a great advantage of this method over the first one is that the stationarity assumptions seem less restrictive.

Some wind and sea state time series also exhibit daily components. The most common method to remove this component is to use the decomposition (1), with  $m$  and  $\sigma$  periodic functions with period one day (see [18,23]).

In the following, we suppose that all the studied processes are stationary.

### 3. Models based on Gaussian processes

In general, wind and sea state time series cannot be assumed to be Gaussian. For instance, the marginal distribution of these processes are often asymmetric with positive support and positive skewness. However, when they have a continuous state space, it is possible to transform these time series into time series with Gaussian marginal distributions. If the transformed time series is supposed to be Gaussian, we can then use one of the existing techniques to simulate Gaussian processes (ARMA models, exact simulation methods, ...). Let us describe more precisely the simulation method in a general framework.

Let  $\{Y_t\}$  be a stationary process with values in  $\mathbf{R}^d$ . We assume that there exists a transformation  $f : \mathbf{R}^d \rightarrow \mathbf{R}^d$  and a stationary Gaussian process  $\{X_t\}$  such that  $Y_t = f(X_t)$ . The procedure consists of three main steps:

*Model calibration*, which consists in determining the function  $f$  and the second order structure of the process  $\{X_t\}$ . In practice,  $f$  is chosen such that the marginal distributions and the second order structures of the processes  $\{f(X_t)\}$  and  $\{Y_t\}$  match. Transformations of different nature can be applied: Box–Cox transformation which directly operates on the process [22], Rozenblatt transformation which is based on the marginal distribution of the process [49] and the  $g$ -transformation described in [61] which preserves the crossing level of the process.

*Sample generation* in which realizations of the process  $\{X_t\}$  are generated given the second order structure estimated in the previous step.

*Mapping*. In this step, the generated samples of  $\{X_t\}$  are transformed into samples of  $\{Y_t\}$  using the transformation  $f$ .

This general method includes in particular the method of Box and Jenkins [15] and the Translated Gaussian Process method which are described more precisely hereafter.

#### 3.1. Box and Jenkins method

The method of Box and Jenkins [15] is undoubtedly the most usual model for wind time series [18,23,54] and sea state time series [55,64,22,77]. It is also used in many other application fields.

Let us first consider the univariate case for simplicity. The transformation  $g = f^{-1}$  is selected in the family of the Box–Cox transformations [22]:

$$g(y) = (y^\lambda - 1)/\lambda \quad \text{for } 0 < \lambda \leq 1 \quad \text{and} \\ g(y) = \ln(y) \quad \text{for } \lambda = 0.$$

Parameter  $\lambda$  is selected in such a way that the marginal distribution of  $X_t = g(Y_t)$  is roughly Gaussian. Various methods can be used to estimate the parameter  $\lambda$  [18,23]. Generally, for  $H_s$ , the transformation  $g(y) = \ln(y)$  is used [55, 64], the marginal distribution of this process being generally well approximated by a lognormal distribution. For the wind intensity, one generally applies a Box–Cox transformation with  $0.5 < \lambda < 1$  [18,54]. When the process is multivariate, a Box–Cox transformation is usually applied independently on each component.

Then, once the transformation  $f$  has been selected, an ARMA model is adjusted to the transformed time series. For the process  $H_s$ , the following models have been proposed: AR(1) [55], ARMA(2,2) [64], AR(20) [22] and ARMA(4,4) [77]. For the bivariate time series  $(H_s, T)$ , Guedes Soares et al. [33] select AR(4) or AR(5) models depending on the location. For the wind intensity  $U$ , models AR(1) [69], AR(2) [23,54,53] or AR(4) [57] have been used, more complex models giving no improvement.

### 3.2. Translated Gaussian process

For wind and sea state parameters, another approach, based on the same principle, is also popular. It is a non-parametric method, in which the function  $f$  is selected on the basis of the normal score transformation and the Gaussian process is simulated by using exact simulation algorithms. This approach was initially proposed by Watton et al. [74] in order to simulate realizations of the process  $H_s$  and then extended by Borgman et al. [13] to multivariate time series  $(H_s, T, \Theta_m)$  and it was also applied, for example, to simulate the wind pressure on buildings (see [30] and references therein) and by DelBalzo et al. [24] to simulate  $(H_s, T, \Theta_m)$  using buoy and ship observations. In the following, it will be denoted TGP (Translated Gaussian Process).

The normal score transformation, which permits us to transform a continuous random variable  $Y$  into a Gaussian variable  $X$ , is defined as

$$x = N^{-1}F_Y(y)$$

where  $F_Y$  is the cumulative distribution function (cdf) of  $Y$  and  $N$  is the standard normal cdf. For multivariate variables  $Y = (Y_1, \dots, Y_n)$ , a first generalization consists in applying independently the transformation on the various components:

$$\begin{aligned} (x_1, \dots, x_d) &= g(y_1, \dots, y_d) \\ &= (N^{-1}F_{Y^{(1)}}(y_1), \dots, N^{-1}F_{Y^{(d)}}(y_d)) \end{aligned}$$

with  $F_{Y^{(i)}}$  the cdf of  $\{Y^{(i)}\}$ . This method is used for example in [13] and in [30]. However, it was shown in [49] that this transformation does not allow us to restore the marginal joint distribution when a strong relation exists between the components of the process. The Rozenblatt transformation (3) can then be used:

$$g : (y_1, \dots, y_d) \rightarrow \left( N^{-1}F_{Y^{(1)}}(y_1), N^{-1}F_{Y^{(2)}|Y^{(1)}=y_1}(y_2), \dots, N^{-1}F_{Y^{(d)}|Y^{(1)}=y_1, \dots, Y^{(d-1)}=y_{d-1}}(y_d) \right) \quad (3)$$

where  $F_{Y^{(1)}}$  denotes the cdf of  $Y^{(1)}$  and  $F_{Y^{(k)}|Y^{(1)}=y_1, \dots, Y^{(k-1)}=y_{k-1}}$  the one of the conditional distribution  $P(Y^{(k)}|Y^{(1)}=y_1, \dots, Y^{(k-1)}=y_{k-1})$ . In [49], the case of  $(H_s, T)$  was given as an example. In [1] a transformation which takes into account the specificity of the circular parameters  $\Theta_m$  is proposed for the process  $(H_s, \Theta_m)$ . Using the same idea, in Section 7 the transformation (4) is used for  $(U, \Phi)$ :

$$g : (u, \phi) \rightarrow (R^{-1}F_{U|\Phi=\phi}(u), F_{\Phi}(\phi)) \quad (4)$$

where  $R$  denotes the cdf of a Rayleigh distribution. If we denote  $(L, \Theta) = g(U, \Phi)$ , with  $g$  defined by (4), then it can be shown that the bivariate marginal of  $(L \cos(\Theta), L \sin(\Theta))$  is Gaussian.

Generally, the cdf used to defined the transformation  $f$  are estimated using non-parametric methods (see [9] and references therein). Parametric models have also been used, in particular to approximate the tails of the distribution [74]. Several authors have also proposed parametric models for the multivariate distribution of metocean parameters (see [8] and references therein, for example). [27] compare parametric models and kernel density estimates for the joint probability of  $H_s$  and  $T$ . And some papers concern more general approaches. For instance [28] describe two general methods in order to derive an approximate joint distribution from the marginals. The first one matches the correlation matrix only, whereas the second one, which is based on a multivariate Hermite polynomial expansion of the normal distribution, is able to match joint moments of orders higher than two.

Once the initial process is transformed into a process which is assumed to be Gaussian, the second order structure can be estimated as in [13,49]. It may occur that the autocorrelation functions of the generated time series are significantly different from those of the initial sequence (see Section 7). Gioffre et al. [30] have proposed a solution to this problem in the particular case where the normal score transformation is applied independently on the different components.

The final step consists in simulating realizations of the stationary Gaussian process  $\{X_t\}$  whose marginal distribution is the standard Gaussian distribution and whose autocorrelation function is known. Various exact simulation methods have been proposed [13,31,58].

### 3.3. General discussion

The Box and Jenkins method is convenient to make simulation, forecasting and reconstruction (see for instance [64,32,36]). Until now, the TGP method has been only used for simulation, however it could also be applied for reconstruction and forecast since the underlying Gaussian process is completely characterized.

Both Box and Jenkins and TGP methods seem robust to noisy data [49]. These methods are easy to adapt and they have been used for various metocean parameters, as is shown in the references above. Their main limitation is the dimension of the time series: it is difficult to apply these methods to time series with several components, in particular when the relation between the different components is complex. Indeed, in this case it may be hard to transform the original time series into Gaussian ones.

For the Box and Jenkins method, statistical criteria exist to help in the choice of the order of ARMA models and to validate the model (tests on the residuals). Moreover statistical properties of these models are well known (see [17]). As far as we know, convergence properties of TGP have not been studied. When the underlying models are parametric, they do not need a large amount of data for the estimation. In TGP, the autocorrelation function and eventually the marginal



distributions are estimated using non-parametric methods and it may require larger data sets than in the case of parametric models.

The methods discussed in this section are extensively used in many application fields, so that they are implemented in quite a lot of software. And they run quickly.

It has been found that these methods provide a good description of the marginal distribution and the second order structure of the time series. However, they cannot restore some nonlinearities which exist in many natural phenomena. For example, for a monovariate stationary Gaussian process with 0 mean, the time durations of the sojourns above a threshold  $s_0$  has the same distribution as the time durations of the sojourns below the threshold ( $-s_0$ ). The transformed time series has similar characteristics and thus cannot succeed in reproducing both calm and storm durations if they have different characteristics, as is generally the case for sea state time series.

#### 4. Resampling methods

Resampling methods have only seldom been used for meteorological time series. The principle is simple since it consists in randomly sampling in the data. These methods are generally used for bootstrap estimation and it was proved that it allows us to estimate the distribution of a wide range of estimators in the case of independent observations [26] and also of time series [35].

We describe here briefly standard methods for time series resampling. More details can be found in [35] and references therein.

##### 4.1. Block resampling

Resampling by block is a well known method for implementing bootstrap for time series. For a time series  $\{y_t\}$ , blocks are defined as follows:

$$B_i = \{y_{t_i}, y_{t_i+1}, \dots, y_{t_i+l_i}\}.$$

Times  $t_i$  and block lengths  $l_i$  are sampled randomly. The resampled time series is the sequence of blocks:

$$\{B_1, \dots, B_N\}.$$

The blocks may be overlapping or not. The length of the blocks can for instance be sampled from the geometric distribution. Refer to [43] for a survey and exhaustive references.

Block bootstrap has been used to derive the statistical properties of many estimators (mean, variance, spectral density, etc.) for time series under quite low assumptions. For instance, several authors have shown that the block bootstrap is an efficient method to estimate confidence intervals for any function of the empirical mean (see [35] and references therein). But this method may not be appropriate for applications which involve persistence statistics. Indeed, if the blocks are small the probabilities of sojourns in a set as well as autocorrelation functions may not be well reproduced. If the blocks are long the resampled time series tends to be an

exact replication of the observed time series and no innovation is brought by the simulated time series.

As far as we know this method has never been used to resample sea state time series. But Hogben et al. [37] proposed a sampling method where observed time durations of sojourn over given levels are arranged at random. The main advantage of this method is that it can be applied when only persistence data are available. A discussion on duration statistics for sea state time series can be found for example in [63,42].

##### 4.2. Resampling Markov chains

This method consists in assuming that the observed time series is Markovian and a non-parametric method is used to estimate transition kernels. Finally, realizations of the chain are simulated with this empirical kernel. Generally, the transition kernel is estimated locally using nearest-neighbor estimators.

As concerns meteorological applications, nearest-neighbor resampling was first proposed by Young [79] to simulate daily minimum and maximum temperatures and precipitations. Independently, Lall et al. [44] used an analog method to generate hydrological time series, and Buishand et al. [19] used basically the same method for multi-site generation of artificial meteorological time series.

Nearest-neighbor resampling for time series  $\{y_t\}$  is based on a very simple idea. Let us assume that we have already generated the  $t - 1$  first values  $y_1^{\text{sim}}, \dots, y_{t-1}^{\text{sim}}$ . We search in the data the  $k$  nearest neighbors of  $y_{t-1}^{\text{sim}}$ . One of these neighbors is randomly selected and the observed value for the date subsequent to the selected point is adopted as the simulated values at time  $t$  [19].

Various discrete probability distributions or kernels may be used to select randomly 1 of the  $k$  nearest neighbors. Lall et al. [44] suggested to use a kernel which assigns a higher probability to the closer points, as for instance  $p_j = \frac{1/j}{\sum_{i=1}^k 1/i}$ ,  $j = 1, \dots, k$  where the point  $j = 1$  is the closest point and  $j = k$  the most distant. Monbet et al. [51] have suggested a more sophisticated method which permits us to sample points which have not been observed, as in smoothed bootstrap methods [26]. This method, named Local Grid Bootstrap (LGB), has also been used to simulate realizations of the multivariate sea state processes  $(H_s, T)$ ,  $(H_s, T, U)$  [50] and  $(U, \Phi)$  [3]. It was shown that this method restores most of the characteristics of the data, such as the marginal distribution, the distribution of storm durations and inter-arrivals as well as the autocovariance functions. This method is illustrated on wind data in Section 7.

##### 4.3. General discussion

As mentioned above, resampling methods can be used for simulation. Some of them can also be adapted to perform forecast and reconstruction. For instance, Ailliot et al. [2] used a nearest-neighbor method to simulate time series of  $(H_s, T, \Theta_m)$  at several locations  $(x_0, \dots, x_L)$  along a line to study the profitability of a maritime line in the Aegean Sea (see also Section 7). Caires et al. [20] proposed a non-parametric

regression method to correct outputs of meteorological models. And another method, based on a non-parametric Hidden Markov model is described in [52] to reconstruct  $H_s$  time series given wind time series (see also [47]). However, one of the main drawbacks of non-parametric methods is that their descriptive power is almost null.

All the methods introduced in this section can be used with noisy and missing data. Indeed, missing data do not affect the nearest-neighbor search, and aberrant data may have a positive probability to be resampled but this probability can be estimated. One of their main advantages is that they can be easily adapted for various time series (wind, wave, circular time series, ...) because of the few assumptions required. For circular data, particular attention has to be paid to the choice of the distance used for nearest-neighbor search and kernel estimation [9]. If simulation is combined with cross-reconstruction, the dimension of the time series is not difficult. Indeed, when the dimension is high, simulation can be performed in two steps: the time series of a subset of components are generated first, then the other components are simulated given the first time series (see [2] and Section 7).

Some asymptotic convergence properties of resampling and reconstruction methods have been studied (see [51,52] and references therein). In practice, it is known that a large amount of data is needed for estimation in non-parametric methods. Another well known drawback is the difficulty to choose the smoothing parameters such as the bandwidth parameters which appear in the probability density function estimators.

As concerns the computational aspects, algorithms for resampling are generally very simple to implement but they can be time consuming. A solution consists in working with algorithms based on tree structures for the nearest-neighbor search [50].

## 5. Parametric models

In this section, we discuss various parametric models which have been proposed for wind and sea state time series. Linear autoregressive models are discussed in Section 3, and we focus now on finite state space Markov chain models and on nonlinear autoregressive models. A section is also devoted to circular time series.

### 5.1. Finite state space Markov chains

Let us briefly describe the general principle of the methods considered in this section.

When the process has a continuous state space, it is discretized in a finite number of classes. This allows to use a representation as a finite state space process.

This finite state space process is then supposed to be a Markov chain whose transition matrix is estimated from the observed sequence.

New realizations of the process are finally simulated given the estimated transition matrix.

The success of this method is undoubtedly mainly due to its simplicity, and the main limitation is probably the number

of parameters which have to be estimated in the case where no assumption is made on the shape of the transition matrix. Various models have been proposed to reduce the number of parameters. For instance, Vik [72] assumed that  $H_s$  is a first-order Markov chain with tridiagonal transition matrix, which means that only the transitions to the closest states are allowed. The transition matrix is estimated in such a way that the stationary distribution of the Markov chain and the mean durations of persistence above certain levels match with those of the observed time series. A more general approach is proposed in [59] who introduced the Mixture Transition Distribution model. If  $r$  is the order of the Markov chain, it is assumed that

$$P(Y_t = j | Y_{t-1} = i_1, \dots, Y_{t-r} = i_r) = \lambda_1 q_{i_1, j} + \dots + \lambda_r q_{i_r, j}$$

with  $Q = (q_{i, j})$  is a stochastic matrix and  $(\lambda_1, \dots, \lambda_r)$  are positive parameters such that  $\lambda_1 + \dots + \lambda_r = 1$ . This model has been fitted to time series of wind speed [59] and wind direction [60,45]. See also [12] for a recent review on Mixture Transition Distribution models.

In [1] another approach is proposed for  $(H_s, \Theta_m)$  where the data are preprocessed before using a first-order Markov chain model. The preprocessing consists in detecting the slope change times by fitting a linear spline curve  $H_{lin}$  to the  $H_s$  time series. A marked point process  $(\tau, H)$  is then associated to  $H_{lin}$ , where  $\tau$  denotes the dates when the slope changes and  $H$  the significant wave height at these dates. Then a Markov model is proposed for the trivariate process  $(H, \tau, \Theta_m)$ . It was found that this model permits us to reproduce the cdf of  $H_s$  and  $\Theta_m$ , as well as the mean duration of the storms.

### 5.2. Nonlinear autoregressive models

In this part, we describe various autoregressive models which have been introduced recently to model some nonlinearities which can be observed on wind and sea state time series.

*Artificial neural networks*—During the last decades, artificial neural networks (ANN) were successfully used to solve problems in ocean, coastal and environmental engineering applications. ANN can be seen as particular regression models in which the link function simulates the circulation of the information in a biological neural network. The parameters of the regression model are generally estimated by maximum likelihood or least square methods. See for instance [34] for an introduction to ANN. ANN was used in order to model the evolution of  $U$  [68,53] and  $(H_s, T)$  [25,46]. The results obtained by these authors show that ANN models permit us to obtain better short term forecasts than those obtained with linear autoregressive models. Arena et al. [7] illustrate how multivariate ANN can be used to reconstruct  $H_s$  time series in buoy networks.

*Time-varying autoregressive models*—Huang et al. [39] use a time-varying autoregressive model of order 2 in order to forecast the wind speed. The coefficients of the model are estimated in real time following the idea of Young et al. [78].

The autoregressive model of order  $r$  is given by:

$$U_t = \sum_{i=1}^r a_{t,i} U_{t-i} + \epsilon_t \quad (5)$$

Let  $\Psi_t = (a_{t,1}, \dots, a_{t,r})$ , then

$$\begin{cases} \Psi_t = \psi_{t-1} + \Gamma_{t-1} \\ \Gamma_t = H\Gamma_{t-1} + \Omega_t \end{cases} \quad (6)$$

where  $I$  is the identity matrix,  $H$  is a diagonal matrix and  $\Omega$  is a zero white noise vector.  $\Gamma$  is a vector of dummy parameters which is either a white noise process or a random walk process. The unknown parameter vector  $\Psi$  is either a random walk process or a smoothed integrated random walk process depending on the matrix  $H$ . The parameters of model (5) and (6) are estimated by a Kalman filter with state vector  $\Psi$  given observation vector  $\{u_1, \dots, u_t\}$ .

Scotto et al. [62] have proposed a Self-Exciting Threshold Autoregressive model for the process  $Y = H_s$ . It is assumed that:

$$Y_t = \sum_{i=1}^r a_i^{(S_t)} Y_{t-i} + b^{(S_t)} + \sigma^{(S_t)} \epsilon_t$$

with  $S_t = i$  if and only if  $Y_{t-d} \in [r_i, r_{i+1}]$  for a fixed integer  $d$ . Here,  $r_1 < r_2 < \dots < r_M$  are parameters of the model and  $\epsilon_t$  denotes a Gaussian white noise. It is a switching autoregressive model, in which the regime at time  $t$  only depends on the last values of the process. In practice, the identified model has 2 regimes, the evolution in the different regimes being described by AR(10) models and  $d = 7$ , which corresponds to a delay of 21 h. The authors compare the results obtained with this model and those corresponding to an AR(22) model. The sequences simulated with the Self-Exciting Threshold Autoregressive model was shown to have features closer to the data than those simulated with the model AR(22), in particular as concern the marginal distribution and the autocorrelation function.

Ailliot et al. [2] proposed a Markov Switching Autoregressive model (MS-AR) for the wind intensity. In MS-AR models the observed proposed  $\{Y_t\}$  is represented by an autoregressive model of order  $r$  with parameters depending on a non-observable Markov chain  $\{S_t\}$ . This hidden variable represents the “weather type”. More precisely, a bivariate process  $\{S_t, Y_t\}$  follows a MS-AR model if  $\{S_t\}$  is a Markov chain on a finite space  $\{1, \dots, M\}$  with  $M > 0$  the number of regimes. This process is supposed to be hidden.

Conditionally to  $\{S_t\}$ ,  $\{Y_t\}$  is a non-homogeneous Markov chain of order  $r \geq 0$  on  $\mathbf{Y} \subset \mathbf{R}^d$ . More precisely, we assume that the conditional distribution of  $Y_t$  given  $\{Y_{t'}\}_{t' < t}$  and  $\{S_{t'}\}_{t' \leq t}$  only depends on  $S_t$  and  $\bar{Y}_{t-1} = (Y_{t-1}, \dots, Y_{t-r})$ . Then, it is assumed that for each  $s_t \in \{1, \dots, M\}$  and  $\bar{y}_{t-1} \in \mathbf{Y}^r$ , the conditional distribution  $P(Y_t | \bar{Y}_{t-1} = \bar{y}_{t-1}, S_t = s_t)$  is a gamma distribution with mean  $\sum_{i=1}^r a_i^{(s_t)} y_{t-i} + b^{(s_t)}$  and standard deviation  $\sigma^{(s_t)}$ .

MS-AR models are simple generalizations of Hidden Markov models (HMM), which correspond to the case  $r = 0$ . In [2], a non-homogeneous MS-AR models is also proposed

for the process  $(U, \Phi)$ , where the transition probabilities of the hidden Markov chain  $\{S_t\}$  depend on the wind direction. More precisely, it is assumed that  $\{S_t\}$  is a non-homogeneous Markov chain on  $\{1, \dots, M\}$  with a transition matrix such that

$$P(S_t = j | S_{t-1} = i) \simeq \pi_{i,j} \exp(\kappa^{(j)} \cos(\Phi_t - \phi^{(j)}))$$

where  $\Pi = (\pi_{i,j})_{i,j \in \{1, \dots, M\}}$  is a stochastic matrix,  $(\kappa^{(j)})_{j \in \{1, \dots, M\}}$  are positive parameters and  $(\phi^{(j)})_{j \in \{1, \dots, M\}}$  denote parameters in  $[0, 2\pi[$ . This model is validated on wind data in the North Atlantic in [3], and it is shown that the model restores the marginal distributions, the autocorrelation functions and the distribution of the durations of storms as well as their inter-arrivals. This model is also validated on wind data in the Aegean Sea in Section 7.

*GARCH model*—GARCH models have been proposed in [69] for the process  $Y = U$ . In this paper, it is supposed that  $U$  is Markovian of order  $r$ , the conditional distribution of  $Y_t$  given  $(Y_{t-1}, \dots, Y_{t-r})$  being described by a gamma distribution with mean

$$\mu_t = \sum_{i=1}^r a_i Y_{t-i} + b$$

and variance

$$\sigma_t^2 = \alpha + \sum_{i=1}^p \lambda_i (Y_{t-i} - \mu_{t-i})^2 + \sum_{i=1}^q \kappa_i \sigma_{t-i}^2.$$

The identified model is of order  $r = 2$  and it is shown that it makes it possible to predict the heteroscedasticity present in the wind time series.

### 5.3. Models for circular data

In Section 5, we have introduced a first family of model for directional time series. Circular time series can also be described by autoregressive models. Let  $\{\Phi_t\}$  be a stationary process with values in  $\mathbb{R}/2\pi\mathbb{Z}$ . Mainly three types of autoregressive models have been proposed in the literature for such time series, and the use of these models for time series of wind direction is discussed in [16].

Models are obtained by “wrapping” a real valued process (Wrapped Autoregressive model). One supposes that  $\Phi_t = Y_t$  modulo  $2\pi$  with  $\{Y_t\}$  a real valued process which follows an autoregressive model.

Models are obtained by using a “link function”,  $g : \mathbb{R} \rightarrow (-\pi, \pi)$  strictly monotonous and checking  $g(0) = 0$ . This function is used to transform a real valued autoregressive process  $\{Y_t\}$  into a process  $\Phi_t = g(Y_t)$  with values on the torus  $\mathbb{R}/2\pi\mathbb{Z}$ .

Models by specifying directly the density of the conditional distribution  $P(\Phi_t | \Phi_{t-1}, \dots, \Phi_{t-k})$ . It is for example the case of the autoregressive model of Von Mises proposed initially in [16]. The Von Mises distribution with parameters  $(\theta_0, k)$  is defined by its density:

$$f(\theta) = \frac{1}{2\pi I_0(k)} e^{\kappa \cos(\theta - \theta_0)}$$

for  $\phi \in R/2\pi\mathbb{Z}$  with  $k > 0$  the concentration and  $\theta_0 \in R/2\pi\mathbb{Z}$  the mean direction. The Von Mises autoregressive model is then defined in the following way: it is supposed that  $P(\Phi_t | \Phi_{t-1}, \dots, \Phi_{t-k})$  follows a Von Mises distribution with parameters  $(\theta_t, k_t)$  given by

$$\kappa_t e^{i\theta_t} = \kappa_0 e^{i\phi_0} + \kappa_1 e^{i\phi_{t-1}} + \dots + \kappa_p e^{i\phi_{t-p}}$$

with  $\kappa_0, \kappa_1, \dots, \kappa_p \in \mathbb{R}^+$  and  $\phi_0 \in R/2\pi\mathbb{Z}$ . In this model, the concentration  $k_t$  changes in time (heteroscedastic model) but it can also be assumed to be constant.

HMM have also been proposed for time series of wind direction. In [45], discrete (multinomial) distributions are used whereas in [38] continuous (wrapped-normal) distributions are used.

### 5.4. General discussion

A parametric model is generally build for a particular task and describes particular features of the data. For instance, ANN are currently used for short term forecasts (for time lag varying from 3 to 48 h) and reconstruction. Pittalis et al. [56] and Tsai et al. [70] used ANN to reconstruct  $H_s$  and  $(H_s, T)$  missing data in a buoy network. ANN have also been extensively applied to correct wind outputs of meteorological models (see [29] and references therein). One drawback of ANN is the large number of parameters that they involve and their lack of interpretability. MS-AR and GARCH models are more parsimonious and physically interpretable. They can be used to generate artificial sequences which restore specific features of the data (heteroscedasticity, existence of several meteorological regimes, ...) as is illustrated in Section 7.

The form of the model has to be specified and this is generally time expensive because it requires a precise study of the data before the construction of the model. It also limits the possibilities to export the model to other time series. However, once the model is chosen, the estimation of the parameters can be performed automatically and should require less data than for non-parametric models. In general, the parameters are estimated using the methods of maximum likelihood or least square. When maximum likelihood is used the existence of missing values may complicate the statistical inference. Statistical criteria such as Akaike (AIC) or Schwartz Bayesian (BIC) criteria can be used to select the best model.

Most of the parametric models discussed in this section are usual for time series, and software is available to fit these models.

## 6. Validation and comparison method

In the previous sections, various stochastic models are proposed for wind and sea state time series. And we need criteria to choose among these models. In this section, we present a general validation method, based on Monte Carlo tests, which can be used to measure the ability of a model to simulate realistic synthetic time series.

The most widespread method for model validation consists in comparing certain statistics calculated from the observations with those corresponding to the considered model. In general,

several criteria are used, such as the matching of the mean and the variance of the marginal distributions, or more generally its cdf. When the temporal dependence is important for the applications, other features are also considered, like the autocorrelation functions or the distribution of the time duration of sojourns below or above given levels.

Meanwhile, the authors often perform only visual comparisons. Such an approach remains not entirely satisfactory because it does not make it possible to decide whether the observed differences are significant or not. A more formal method, based on Monte Carlo tests, is proposed below. For the sake of simplicity, it is presented in the simple case of comparing means, but its generalization to other statistics is straightforward.

Let  $\{y_t\}_{t=1, \dots, T}$  be an observed sequence of a real valued process  $\{Y_t\}$  with mean  $m$  and  $\{Z_t\}$  a process corresponding to the model which has to be validated. The mean of the marginal distribution of  $\{Z_t\}$  is denoted  $m_0$ . We want to test

$$H_0 : m = m_0 \quad \text{versus} \quad H_1 : m \neq m_0. \tag{7}$$

The considered test statistic is the empirical mean  $\bar{Y} = \frac{1}{T} \sum_{t=1}^T Y_t$ , and the associated decision rule is given by

$$H_0 \text{ is rejected if } \bar{y} = \frac{1}{T} \sum_{t=1}^T y_t \in R(\alpha)$$

where  $\alpha$  is the level of the test. The critical region  $R(\alpha)$  is such that  $P_{H_0}(\bar{Y} \in R(\alpha)) = \alpha$ .

In order to compute  $R(\alpha)$ , we need to know the distribution of the test statistic  $\bar{Y}$  when  $H_0$  is true. When the model is complex, it is not always possible to derive the exact distribution of the test statistic. In this case, we can use the Monte Carlo method described hereafter to approximate this distribution:

Simulate  $B$  time series of length  $T$  with the model:

$$\begin{aligned} \{z_1^{(1)}, \dots, z_T^{(1)}\} \\ \vdots \\ \{z_1^{(B)}, \dots, z_T^{(B)}\}. \end{aligned}$$

Compute the empirical mean  $\bar{z}^{(i)} = \frac{1}{T} \sum_{t=1}^T z_t^{(i)}$  for each simulated sample  $i = 1, \dots, B$ .

Approximate the distribution of  $\bar{Y}$  under  $H_0$  by the empirical distribution of  $\{\bar{y}^{(1)}, \dots, \bar{y}^{(B)}\}$ . This allows us to compute an approximation of  $P_{H_0}(\bar{Y} \in R)$  for any region  $R \subset \mathbb{R}$  or equivalently deduce an approximative critical region  $\tilde{R}(\alpha)$  such that  $\frac{1}{B} \text{card}(\{i \in \{1, \dots, B\} | \bar{z}^{(i)} \in \tilde{R}(\alpha)\}) = \alpha$ .

Finally,  $H_0$  will be accepted if and only if  $\bar{y} \in \tilde{R}(\alpha)$ .

This framework can be applied to compare other features like cdf or autocorrelation functions. For this, a test statistic has to be chosen. As concern cdf, the most popular test statistic is probably the Kolmogorov–Smirnov distance. However, it is well known that it is more sensitive near the center of the distribution than at the tails. Due to this limitation, many analysts prefer to use the Anderson–Darling statistic which gives more weight to the tails. But, as this is an integrated deviation, like the Cramer–Von Mises or chi-square distance,



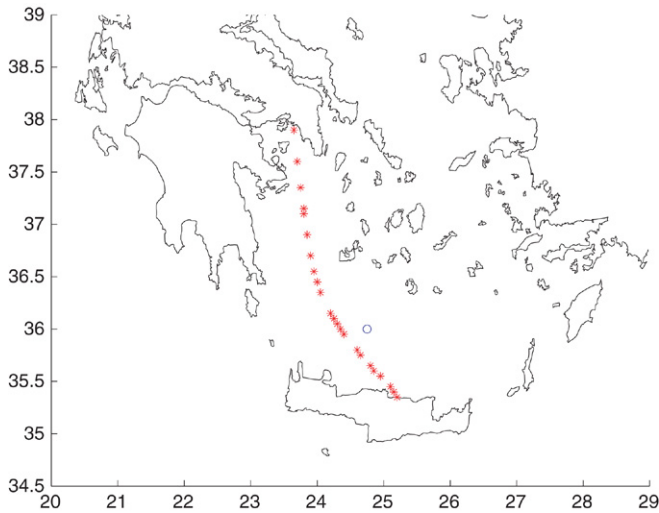


Fig. 1. Map of the Aegean sea. Maritime line ( $x_1, \dots, x_N$ ) (stars) and point  $x_0$  (circle).

it can mask local differences. In [6], a more sensitive test statistic was proposed, which permits us to measure locally the goodness-of-fit. This method is used in the next subsection to validate and compare three models.

## 7. Example: Simulation of multivariate sea state time series along a maritime line

In this section, an example is proposed in order to illustrate some of the models discussed in the previous sections. The data sets considered in this part have also been used in [2]. The initial objective of this study was to estimate the profitability of the maritime line Piraeus–Heraklion (Aegean Sea) for a given passenger ship. For that, we had at our disposal two data sets:

3 years of data (from October 1999 to September 2002) describing the sea state conditions (i.e.  $(H_s, T, \Theta_m)$ ) at  $N = 17$  points along the maritime line. These points will be denoted  $(x_1, \dots, x_N)$ .

11 years (from January 1992 to December 2002) of wind data at a point denoted  $x_0$ , located near the middle of the maritime line (see 1).

The sea state data have been produced using the WAM model by the National Center for Marine Research (NCMR), and the wind data are reanalysis data produced by ECMWF. The amount of sea state data is clearly too small in order to get reliable estimates of the variability of the sea state conditions, and thus we have proposed to use a stochastic generator. In [2], this sea state generator is combined with a crossing simulator. This crossing simulator is based on polar diagrams which describe the response of the passenger ship under consideration in different conditions. These diagrams permit us to compute, according to the sea state conditions on the maritime line, if the crossing can be done in normal conditions, or has to be delayed or canceled. And as a final result, we can deduce estimates of the distribution of canceled and delayed crossings. In this paper, we will only focus on the sea state generator.

The Piraeus–Heraklion line is located in the Aegean Sea, which is a relatively closed sea. Thus we can consider that the

Table 1

Results of the Monte Carlo tests for the wind time series

	TGP	LGB	MS-AR
$F_U$	0.282 [0.003]	0.000 [0.001]	0.078 [0.005]
$F_\Phi$	0.003 [0.000]	0.001 [0.000]	0.004 [0.000]
$F_{(U, \Phi)}$	0.210 [0.001]	0.000 [0.001]	0.000 [0.001]
$F_{\text{extr}}$	0.012 [0.012]	0.025 [0.009]	0.137 [0.008]
$C_U$	0.000 [0.001]	0.006 [0.003]	0.001 [0.000]
$F_{[U > 1/2]}$	0.000 [0.007]	0.046 [0.006]	0.015 [0.002]
$F_{[U < 1/3]}$	0.000 [0.007]	0.068 [0.002]	0.042 [0.003]

The first value is the observed statistic  $w^{\text{obs}}$  and the value in bracket the cut-off value  $w_\alpha$  with  $\alpha = 0.05$ . The null hypothesis is rejected at the level  $\alpha$  if  $w^{\text{obs}} < w_\alpha$ .

waves are essentially generated by local winds and it seems natural to perform the simulation in two steps. At first, a stochastic model is used to simulate artificial wind conditions at  $x_0$ , and then the sea state conditions corresponding to these artificial wind conditions are cross-reconstructed. These two steps are described more precisely hereafter.

### Simulation of artificial wind conditions

In order to simulate artificial wind conditions at  $x_0$ , we have tried three different methods:

The TGP method discussed in Section 3. The transformation (4) is used to transform the original time series and an exact simulation method is used to simulate the underlying Gaussian process.

The LGB method discussed in Section 4.

The non-homogeneous MS-AR model introduced in Section 5. In order to simulate the wind direction, we have used a simple finite state Markov chain of order 1 (see Section 5.1).

These different methods have been fitted to the wind data for the months of August. Then, they have been validated using the method described in Section 6. In order to check the realism of the simulated sequences, the list of criteria just below has been used.

$F_U$ : cdf of the marginal distribution of  $U$ .

$F_\Phi$ : cdf of the marginal distribution of  $\Phi$ .

$F_{(U, \Phi)}$ : cdf of the bivariate distribution of  $\{U, \Phi\}$ .

$F_{\text{extr}}$ : cdf of the monthly maxima of  $U$ .

$C_U$ : autocorrelation function of  $U$ .

$F_{[U > 1/2]}$ : cdf of sojourn durations above level  $1/2 \max(u)$ , with  $\max(u)$  the largest wind speed in the observed time series.

$F_{[U < 1/3]}$ : cdf of sojourn durations below level  $1/3 \max(u)$ .

The cdf  $F_U$ ,  $F_\Phi$  and  $F_{(U, \Phi)}$  are important criteria since the distribution of the wind is strongly related to the distribution of the sea state parameters.  $F_{\text{extr}}$  describes the interannual variability of the process at a monthly scale. The autocorrelation function  $C_U$  is a usual measure of linear dependence in time. Finally, the cdf of sojourn durations  $F_{[Y > 1/2]}$  and  $F_{[Y < 1/3]}$  describe the time duration of the stormy and calm conditions. These durations are also strongly related to the severity of the sea state conditions.

For each criteria, Monte Carlo tests have been run on the basis of  $N = 1000$  synthetic time series, each of them having the same length as the initial wind time series. The results are given in Table 1. According to this table, the TGP method successfully reproduces the bivariate marginal distribution of

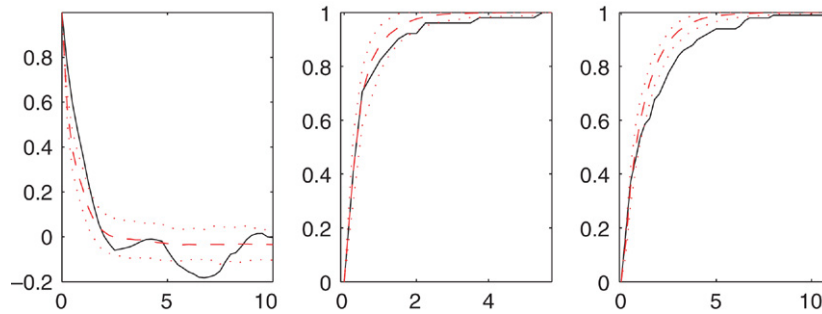


Fig. 2. Autocorrelation function of the wind intensity  $U$  (left), cumulative distribution functions of sojourn durations over  $1/2 \max(U) = 8.05 \text{ m s}^{-1}$  (middle) and below  $1/3 \max(U) = 5.35 \text{ m s}^{-1}$  (right). The time in the abscissa is expressed in days. Solid: observation, dashed: TGP model, dotted 95% interquartile interval for TGP model.

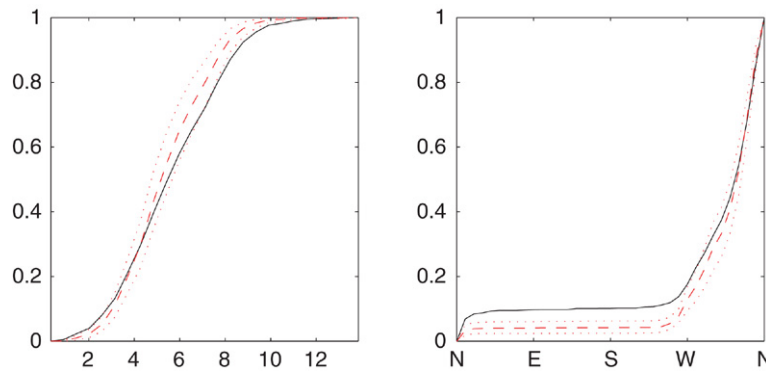


Fig. 3. cdf of the marginal distribution of the wind intensity  $U$  (left) and the wind direction  $\Phi$  (right). Solid: observation, dashed: LBG model, dotted 95% interquartile interval for LBG model.

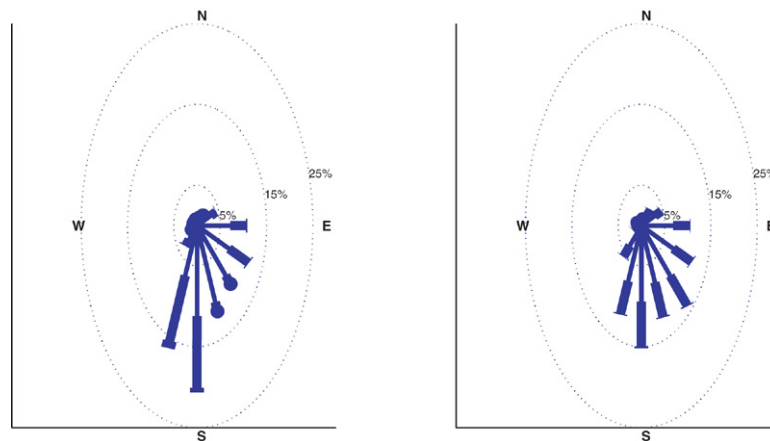


Fig. 4. Wind roses for the data (left) and the data simulated with the non-homogeneous MS-AR model (right).

the process but fails to reproduce its dynamics. This is also illustrated on Fig. 2. According to this figure, the model underestimates the first coefficients of the autocorrelation function and also the durations of the calm and stormy periods. It indicates that the transformed time series is not Gaussian, although its bivariate marginal distribution is Gaussian. On the opposite, the LGB method successfully describes the dynamics of the process but cannot restore the marginal distribution of  $U$  and  $(U, \Phi)$ . According to Fig. 3 this method simulates too many data close to the mode of the distributions. It is a well known problem of this kind of algorithm, and better results

could perhaps be obtained with another choice of the bandwidth parameters. However, the search of appropriate parameters is fastidious since there is no automatic criterion. As concerns the non-homogeneous MS-AR model, the model with  $M = 2$  regimes and autoregressive models of order  $p = 1$  has been selected with BIC, and the results obtained with this model are better than the ones corresponding to LGB and TGP. Indeed, this model reproduces all the criteria, except the bivariate marginal distribution. According to Fig. 4, the model simulates too many wind from the north-west and not enough from the north, but the distribution of the wind intensity in each

Table 2  
Maximum likelihood estimates for the non-homogeneous MS-AR model

	$a_1^{(i)}$	$b^{(i)}$	$\sigma^{(i)}$	$\pi_{i,i}$	$\phi^{(i)}$	$\kappa^{(i)}$
Regime 1 ( $i = 1$ )	0.85	0.82	1.27	0.96	1.64	1.47
Regime 1 ( $i = 2$ )	0.53	2.11	2.24	0.95	1.59	2.71

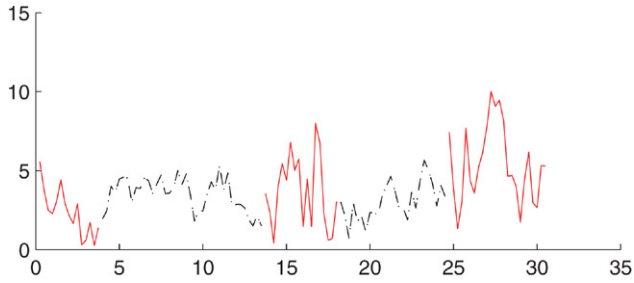


Fig. 5. Evolution of the wind speed at  $x_0$  in August 2002. The dash-dotted [resp. solid] line represents the date when the first [resp. second] regime is the most likely. The regimes have been identified using the Viterbi algorithm.

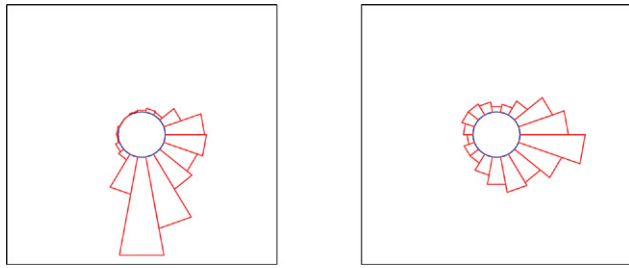


Fig. 6. Distribution of the wind direction in the two regimes (identified using the Viterbi algorithm). Regime 1 on the left and regime 2 on the right.

sector seems to be well reproduced. A better description of this bivariate distribution could probably be obtained by using a more sophisticated model for the wind direction and a higher number of regimes.

Another advantage of this model over the other models considered in this section is its physical interpretability. The maximum likelihood estimates are given in Table 2. According to this table, one important difference between the two regimes is the value of the conditional standard deviation  $\sigma$ , which is higher in the second regime. It indicates that the second regime is associated to weather conditions in which the wind speed evolves quickly, whereas the first regime corresponds to steady wind speed conditions (low volatility). This is illustrated on Fig. 5. And the two regimes are associated to different wind directions (see Fig. 6), the first one corresponding mainly to Northerlies and the second one to Westerlies. This relation is described through the parameters  $\phi$  and  $\kappa$ .

#### Reconstruction of the sea state conditions

The sea state conditions  $\{H_s^{(\text{sim})}, T^{(\text{sim})}, \Theta_m^{(\text{sim})}\}$  corresponding to the simulated wind  $\{U^{(\text{sim})}, \Phi^{(\text{sim})}\}$  are reconstructed by a nearest-neighbor resampling method. In practice, at each time  $t$  and each location  $x_i$ ,  $(H_s^{(\text{sim})}(x_i, t), T^{(\text{sim})}(x_i, t), \Theta_m^{(\text{sim})}(x_i, t))$

Table 3  
Results of the Monte Carlo tests for the sea state time series at location 16

$F_{H_s}$	$F_{\Theta_m}$	$F_T$	$F_{(H_s, \Theta_m)}$	$F_{(H_s, T)}$
0.007 [0.004]	0.000 [0.002]	0.017 [0.011]	0.000 [0.001]	0.017 [0.011]
$F_{(U, H_s)}$	$C_{H_s}$	$F_{[H_s > 1/2]}$	$F_{[H_s < 1/3]}$	
0.027 [0.005]	0.024 [0.001]	0.011 [0.001]	0.031 [0.005]	

The first value is the observed statistic  $w^{\text{obs}}$  and the value in brackets the cut-off value  $w_\alpha$  with  $\alpha = 0.05$ . The null hypothesis is rejected at the level  $\alpha$  if  $w^{\text{obs}} < w_\alpha$ .

$= (H_s(x_i, s^*), T(x_i, s^*), \Theta_m(x_i, s^*))$  where  $s^*$  is a solution of

$$\begin{aligned} \min_s \{ & \omega_1 \|U^{(\text{sim})}(t)e^{\sqrt{-1}\Phi^{(\text{sim})}(t)} - U(s)e^{\sqrt{-1}\Phi(s)}\| \\ & + \omega_2 \|H_s^{(\text{sim})}(x_{i_0}, t-1)e^{\sqrt{-1}\Theta_m^{(\text{sim})}(x_{i_0}, t-1)} \\ & - H_s(x_{i_0}, s-1)e^{\sqrt{-1}\Theta_m(x_{i_0}, s-1)}\| \\ & + \omega_3 \|T^{(\text{sim})}(x_{i_0}, t-1) - T(x_{i_0}, s-1)\| \} \end{aligned}$$

where  $\omega_1, \omega_2, \omega_3$  denote fixed weights,  $\|\cdot\|$  denotes the euclidean norm and  $x_{i_0}$  is the point of the line  $\{x_1, \dots, x_N\}$  which is the closest to  $x_0$  ( $i_0 = 16$ ). Here,  $\omega_1, \omega_2, \omega_3$  are chosen empirically.

We have used this simple method to compute the sea state conditions corresponding to the artificial wind conditions simulated with the non-homogeneous MS-AR model. Then, in order to check the realism of these artificial sea state conditions, we have performed Monte Carlo tests, and the list of criteria just below has been used.

$F_{H_s}$ : cdf of the marginal distribution of  $H_s$ .

$F_{\Theta_m}$ : cdf of the marginal distribution of  $\Theta_m$ .

$F_T$ : cdf of the marginal distribution of  $T$ .

$F_{(H_s, \Theta_m)}$ : cdf of the bivariate marginal distribution of  $(H_s, \Theta_m)$ .

$F_{(H_s, T)}$ : cdf of the bivariate marginal distribution of  $(H_s, T)$ .

$F_{(U, H_s)}$ : cdf of the bivariate marginal distribution of  $(U, H_s)$ .

$C_{H_s}$ : autocorrelation function of  $H_s$ .

$F_{[H_s > 1/2]}$ : cdf of sojourn durations above level  $1/2 \max(H_s)$ .

$F_{[H_s < 1/3]}$ : cdf of sojourn durations below level  $1/3 \max(u)$ .

Table 3 shows that all the considered statistics are well reproduced at location 16 (located near the middle of the maritime line), except the marginal distributions of  $\Theta_m$  and  $(H_s, \Theta_m)$ . This bivariate distribution is shown on Fig. 7 and the lack of fit is in accordance to the one identified on the simulated wind time series: the proportion of wave coming from the north is underestimated. The joint distribution of  $U$  and  $H_s$  is shown on Fig. 8.

## 8. Concluding remarks

In this paper, we make a review of stochastic models for metocean time series. The models are classified in three groups: non-parametric models, models based on Gaussian approximations, other parametric models.

For each group of models, we discuss the possible uses of the models, their advantages and drawbacks, etc. Then a quantitative method is proposed to measure the ability of

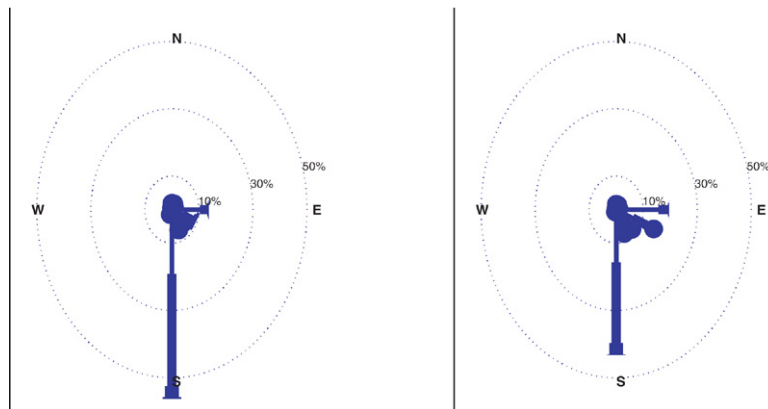


Fig. 7. Wave roses for the data at location 16 (left) and for the simulated sequences (right).

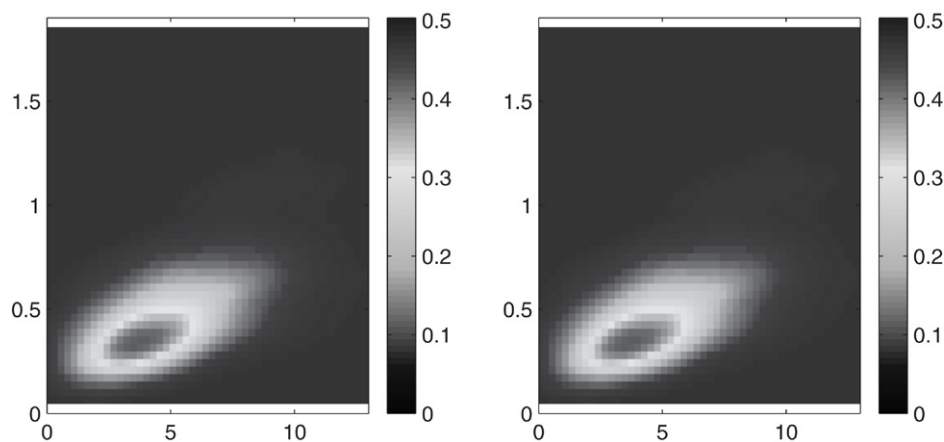


Fig. 8. Joint distribution of  $U$  ( $x$ -axis) and  $H_s$  ( $y$ -axis). Data on the left and model on the right.

a model to restore chosen statistical features like marginal distribution, covariance functions, durations, etc., and this method allows us to validate or compare models for given data. And finally an example is discussed where a stochastic model is used to generate a multivariate time series  $\{U, \Phi, H_s, T, \Theta_m\}$  at several locations along a ferry line in the Aegean Sea (Greece).

Finally, a lot of tools and methods are available for modeling metocean time series and the choice of a model depends on the nature of the studied process (univariate or bivariate, intensity and/or direction, . . .), of the considered location and also on the objectives of the users.

The review focus on models for time series at the scale of the sea state. And, as a consequence, we have neglected many usual and interesting aspects of metocean studies, such as, for instance, linear and nonlinear models for waves, extremes, spatial process, . . .

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