Random field models for certain environmental applications

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Outline

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  - Motion!

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Collaborators: Krys Podgórski, Igor Rychlik, Jorg Wegener, Jan Lennartsson

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- But we want stochasticl fields that exhibit certain spatial variability and with non trivial temporal variability.
- We suggest to construct spatio-temporal fields driven by a deterministic flow

Static Model

The starting point is the spectral representation of a non-stationary process

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with covariance

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Which can easily be extended to spatio-temporal fields

$$\eta(\mathbf{s},t) \stackrel{d}{=} \int_{\mathbb{R}^{n+1}} e^{i(\mathbf{s},t)\cdot(\omega,\tau)}\sqrt{S_{\mathbf{s}}(\omega)S_{\mathbf{s}}^T(\tau)}\, dB(\omega,\tau).$$

Alternatively define

$$X(\mathbf{s}, t) = \int f(t, \tau; \mathbf{s})\ \Phi(\mathbf{s}; d\tau),$$

for $\Phi(\cdot; d\tau)$ a Gaussian field-valued measure uniquely characterized by the time dependent spatial covariances $r_S(\mathbf{s}, \mathbf{s}'; \tau)$ and f some deterministic kernel

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$$r(\mathbf{s}, \mathbf{s}'; t, t') = \int f(t, \tau; \mathbf{s}) f(t', \tau; \mathbf{s}') \cdot r_S(\mathbf{s}, \mathbf{s}'; \tau)\ d\tau$$

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For $f(t) = e^{-\lambda t} \mathbf{1}_{[0,\infty)}(t)$ we get the temporal Ornstein- Uhlenbeck field which corresponds to autoregression model of order one

$$X(\mathbf{s}, t) = \rho X(\mathbf{s}, t - \Delta t) + \sqrt{1 - \rho^2}\ \Phi_t(\mathbf{s}).$$

An alternative method?

The two fields $\eta(\mathbf{s}, t)$ and $X(\mathbf{s}, t)$ are equal in distribution if the covariances in time at a fixed location $\mathbf{s}$ are the same and the kernels $f_{\mathbf{s}}$ are symmetric with non-negative Fourier transform.

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The two fields $\eta(\mathbf{s}, t)$ and $X(\mathbf{s}, t)$ are equal in distribution if the covariances in time at a fixed location $\mathbf{s}$ are the same and the kernels $f_{\mathbf{s}}$ are symmetric with non-negative Fourier transform.
Symmetry of the kernels cannot be relaxed, for example for exponential function (the Ornstein-Uhlenbeck case) the two approaches lead to different processes!

Stochastic Velocity Field

One of the many possible definitions of velocity on random surfaces

- $$v(\mathbf{s}, t) = \left(-\frac{X_t(\mathbf{s}, t)}{X_x(\mathbf{s}, t)}, -\frac{X_t(\mathbf{s}, t)}{X_y(\mathbf{s}, t)} \right)$$

  is the x and y - coordinate of the slope of the tangent plane to the upcrossing contour attached to the point (x, y, t)

- $$\mathbf{v} \sim Cauchy(\cdot, \cdot)$$

Is there non-trivial dynamics?

- If

$$r(\mathbf{s}, \mathbf{s}'; t, t') = \int_{-\infty}^{\infty} f(t, \tau) \cdot f(t', \tau) \cdot r_S(\mathbf{s} - \mathbf{s}'; \tau)\, d\tau.$$

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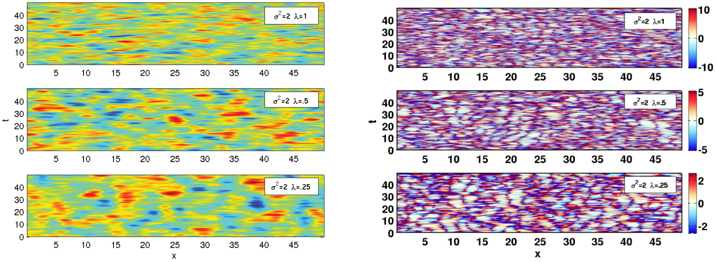
then the dynamics of the field is trivial (i.e. velocities are centered at zero).
- Thus the field
$$X(\mathbf{s},t) = \int f(t,\tau)\,\Phi(\mathbf{s};d\tau),$$
with $\Phi(\mathbf{s};d\tau)$ governed by a stationary spatial covariance, does not exhibit any organized motion.

Examples of fields



Spatio-temporal fields $X(x,t)$ with covariance $r_X(x,t) = \sigma^2 e^{-x^2/2} e^{-\lambda|t|}$ for various values of λ and σ^2 *(Left)* and their corresponding velocities *(Right)*.

Feeding deterministic flow into stochastic field

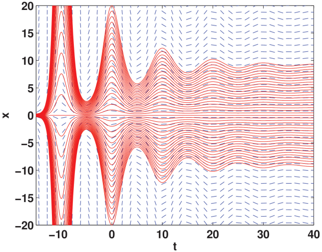
- Flow $\psi_{t,h}(\mathbf{s})$ obtained from a velocity field $\mathbf{v}(\mathbf{s}, t)$ satisfying the transport equation

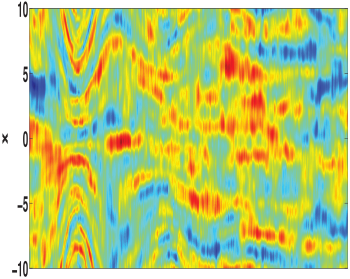
$$\psi_{t,h}(\mathbf{s}) = \mathbf{s} + \int_t^{t+h} \mathbf{v}(\psi_{t,u-t}(\mathbf{s}), u)\, du = \mathbf{s} + \int_0^h \mathbf{v}(\psi_{t,s}(\mathbf{s}), t+\tau)\, d\tau,$$

Construction of the stochastic field at fixed location $\mathbf{p}$ and fixed time t:

$$Y(\mathbf{s}, t) = \int_{-\infty}^{\infty} f(t,\tau)\, \Phi(\psi_{t,\tau-t}(\mathbf{s}); d\tau)$$

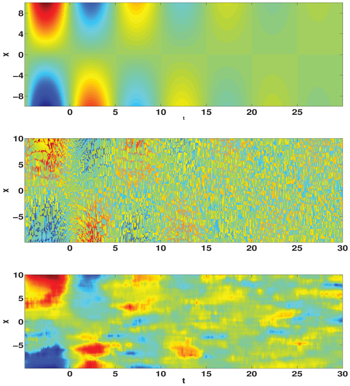
Dynamic Flow





Trajectories and directional field of a simple dynamical flow representing damped oscillations *(Left)*. Dynamic spatio-temporal random fields $X(x, t)$ for $\lambda = 0.5$ and $\sigma^2 = 0.5$ *(Right)*.

Estimated Velocities



Velocities of dynamical flow *(Top)* vs. velocities of stochastic field: unfiltered *(Middle)* estimated by the median filter *(Bottom)*. $\lambda = 0.5$ and $\sigma^2 = 0.5$.

There is a method in the madness

Theorem

If the spatial covariance r_S of the innovations $\Phi(\mathbf{s}; dt)$ are isotropic, then the distribution of random velocities on the surface of $Y(\mathbf{s}, t)$ has its center at the value of the deterministic velocity field $\mathbf{v}(\mathbf{s}, t)$.

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In other words, stochastic dynamics follows the one represented by the underlying deterministic field. So we have managed to construct spatio-temporal fields (Gaussian) that can move.

Highlights

- We have constructed spatio-temporal Gaussian as convolutions of Gaussian spatial noise with deterministic kernels
- We can obtain almost all known spatio-temporal covariances for specific cases of spatial covariances and kernels
- We have shown that when spatial correlations are stationary the resulting fields do not move
- We have embedded deterministic flow in the innovations resulting in moving fields
- We have shown that for isotropic spatial covariances the distribution of the random velocities has its center at the deterministic velocity

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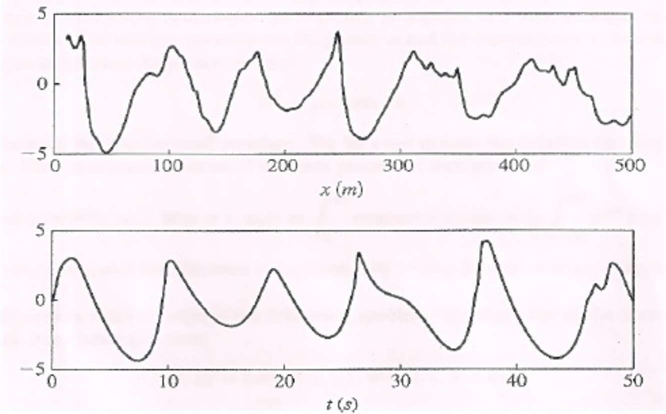
- Interesting alternative to the Gaussian models are the Laplace moving average models, which are formulated as convolutions of Laplace noise and some deterministic kernel.
- These models constitute a rich class that is capable of modeling a variety of geometrical asymmetries in the records, and simultaneously can efficiently acount for occasional highly extreme events.

Asymmetries in random records

The following is an example of space and time records from a non-linear model for sea surface



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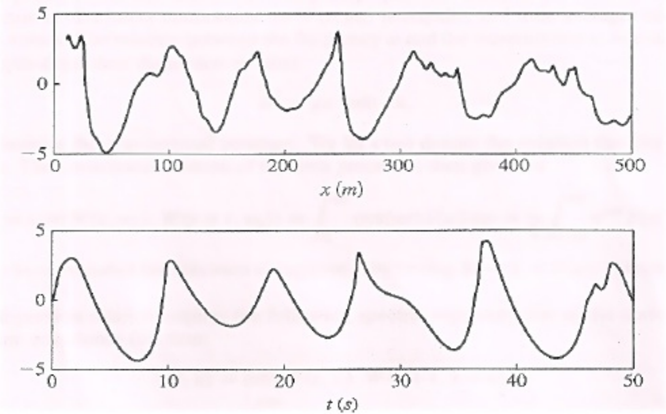
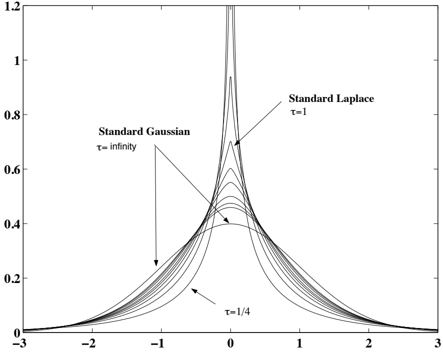
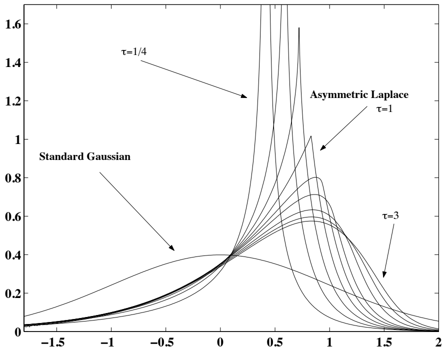


Visually, the asymmetries are clear but how to model and quantify them?

We could start from some asymmetric densities





Laplace densities. *Left:* $\mu = 0$ and *Right:* $\mu = -\frac{3\sigma}{2^{3/2}}$, $\quad \tau = 1/\nu$, mean =0 and variance=1

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$$\sigma\sqrt{W}Z + \mu W + \delta,$$

  Z – standard normal, W – gamma variable with the shape ν.

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  Z – standard normal, W – gamma variable with the shape ν.
- Density in terms of the modified Bessel function of the third kind

$$\frac{2e^{\mu/\sigma^2(y-\delta)}}{\sqrt{2\pi}\sigma\Gamma(\nu)}\left(\frac{(y-\delta)/\sigma}{\sqrt{2+\mu^2/\sigma^2}}\right)^{\nu-1/2} K_{\nu-1/2}((y-\delta)/\sqrt{2+\mu^2/\sigma^2}),$$

Multivariate extension

- Gamma variance models

$$\mathbf{Y} = \boldsymbol{\delta} + \Gamma \cdot \boldsymbol{\mu} + \Gamma^{1/2} \cdot \boldsymbol{\Sigma}^{1/2}\mathbf{Z}$$

where Γ is a gamma variable with the shape parameter ν.

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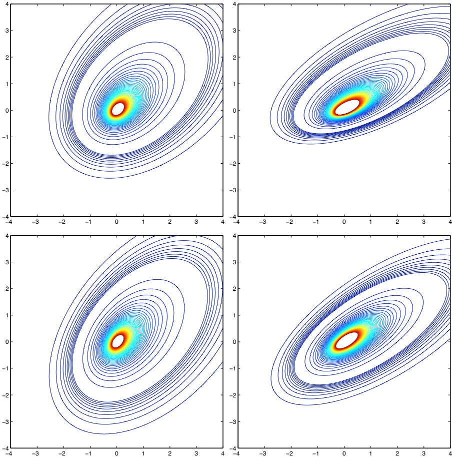
- Density
$$\frac{2\exp(\boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}(\mathbf{y}-\boldsymbol{\delta}))}{(2\pi)^{d/2}\Gamma(\nu)|\boldsymbol{\Sigma}|^{1/2}} \left(\frac{Q(\mathbf{y}-\boldsymbol{\delta})}{C(\boldsymbol{\Sigma},\boldsymbol{\mu})} \right)^{\nu-d/2} K_{\nu-d/2}(Q(\mathbf{y}-\boldsymbol{\delta})C(\boldsymbol{\Sigma},\boldsymbol{\mu})),$$
where $Q(\mathbf{x}) = \sqrt{\mathbf{x}'\boldsymbol{\Sigma}^{-1}\mathbf{x}}$ and $C(\boldsymbol{\Sigma},\boldsymbol{\mu}) = \sqrt{2 + \boldsymbol{\mu}'\boldsymbol{\Sigma}^{-1}\boldsymbol{\mu}}$.

Multivariate extension - Examples of densities



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$$L(\lambda) = B(\Gamma(\lambda))$$
- Some properties are inherited from the distribution (stochastic self-similarity):
$$\sqrt{p} L(\lambda) = L(NB_p(\lambda)),$$
where NB_p is binomial Lévy motion.

Moving averages through stochastic integration

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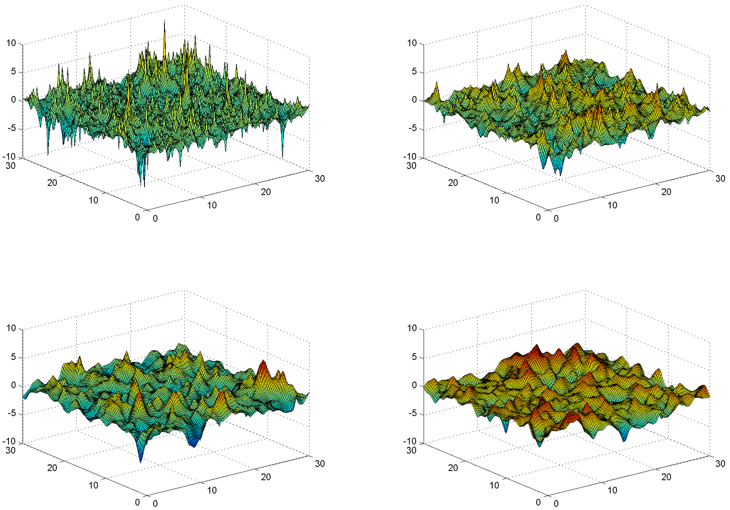
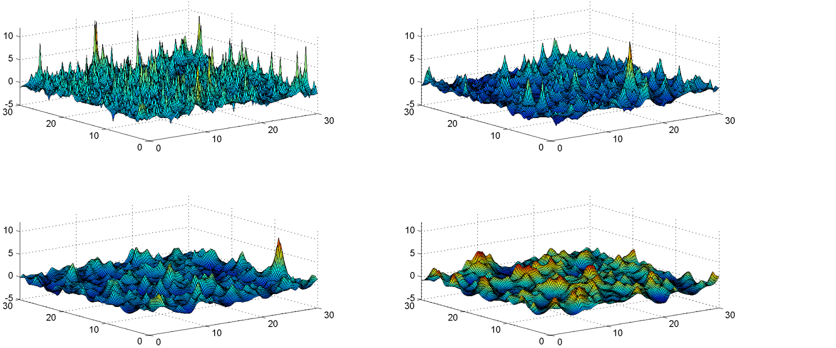
- If $d = 1$, then $\Lambda(-\infty, x] = B(\Gamma(x))$, where B is a Brownian motion with drift and Γ is a gamma process.
- Conditionally on Γ the process X can be viewed as a non-stationary Gaussian process.

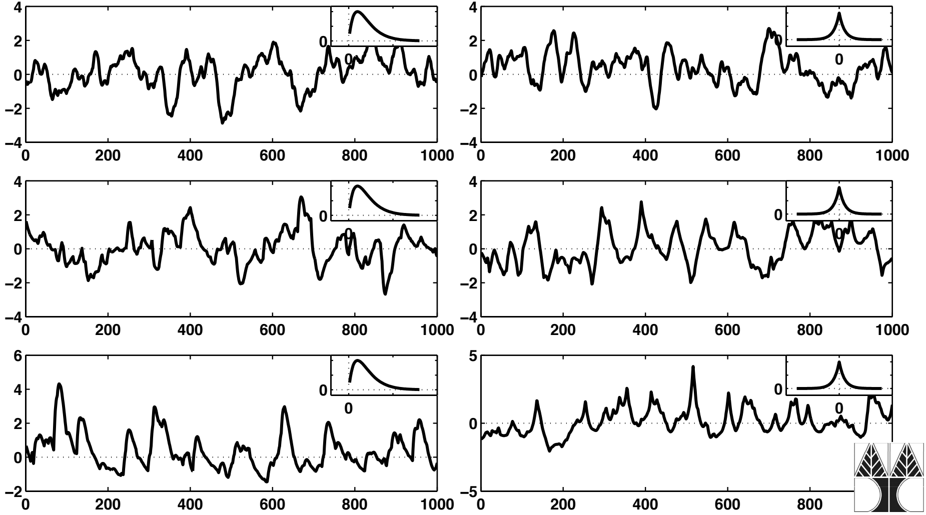
Symmetric spatial models - realizations



Asymmetric spatial models



Tilting of trajectories



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$$\lim_{T\to\infty} \frac{N(T, A)}{N(T)} = \frac{\mathbb{E}\left[\{X \in A\}|\dot{X}(0)|\middle|X(0) = 0\right]}{\mathbb{E}\left[|\dot{X}(0)|\middle|X(0) = 0\right]},$$

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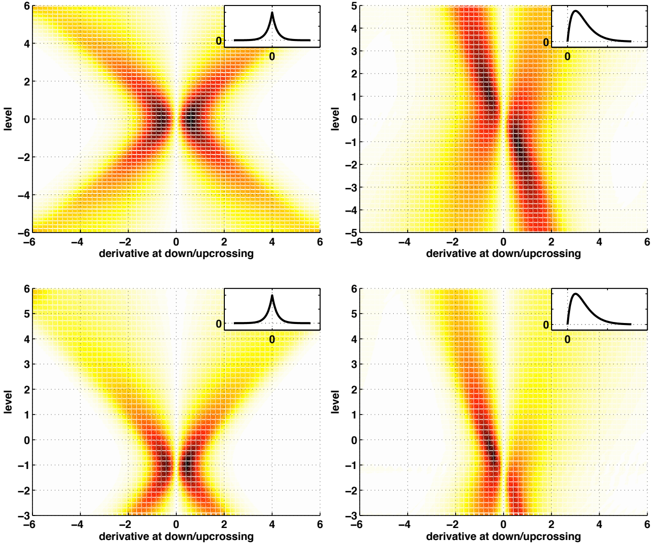
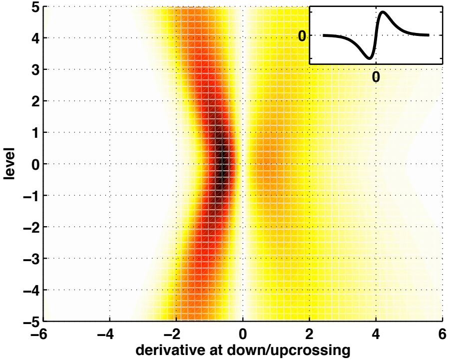
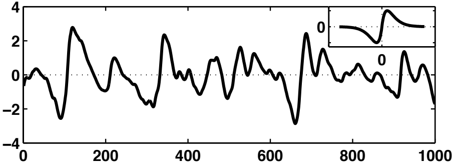
- the right hand side represents the biased sampling distribution when sampling is made over the 0-level contour $\mathcal{C}_0 = \{\tau : X(\tau) = 0\}$

Slope distributions at level crossings



Another way of tilting





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- Prediction and estimation are performed using the covariance function.
- Although Gaussian models are the most popular ones in spatial statistics, mainly because of their simplicity, they have difficulty in efficiently accounting for asymmetries and unusually extreme values in the data. Frequently a transformed Gaussian model is considered, while the choice of transformation is cattering toward a particular application.

Truncated and transformed Gaussian models

- Latent and transformed Gaussian model

$$Y(\mathbf{s},t) = \begin{cases} \psi_{\mathbf{s},t}(Z(\mathbf{s},t)), & \text{if } (Z\mathbf{s},t) > 0 \quad (u), \\ 0 & \text{if } Z(\mathbf{s},t) \leq 0 \quad (u), \end{cases}$$

- which can be also writen as

$$Y(\mathbf{s},t) = \phi_{\mathbf{s},t}(Z(\mathbf{s},t))$$

with

$$\phi_{\mathbf{s},t}(z) = 0 \cdot \{z \leq 0\} + \psi_{\mathbf{s},t}(z) \cdot \{z > 0\}$$

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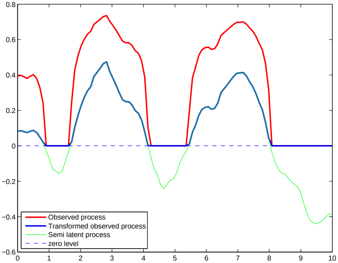

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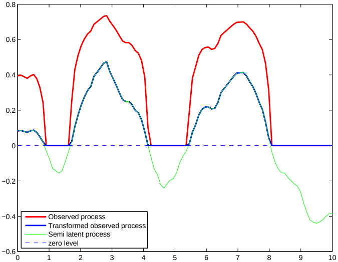

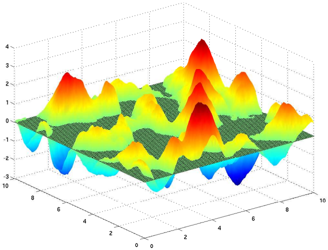

Idea:

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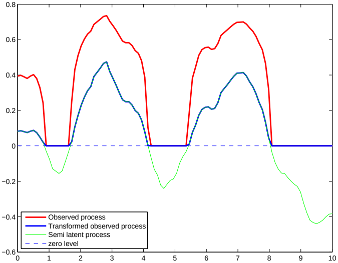

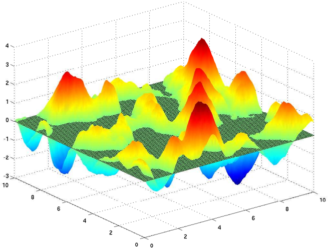

So what is needed?

- Model the transformation
- Model the mean function
- Model the covariance structure

This gives you a stationary (or not) Gaussian model that you can marginally transform to give you distributions that resemble the data. If additionally want to extrapolate to locations with no observations then we need

- stochastic models for the parameters
- regres the parameters on covariates

Highlights

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- Model fitting can be obtained by utilizing: a) Method of Moments (simpler) or b) Maximum Likelihood (more accurate)
- Markov Random Fields can be used to localize efficiently the fitting problem and provide a way to account for non-stationarity in the data.

A very incomplete and terribly biased bibliography

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Final Slide

What does Mother Earth think of our methods?

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We asked...

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