Workshop on Stochastic Weather Generators

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Monthly average temperatures and precipitation stochastic simulation in the Lake Baikal region

Vannes, 2016

Plan

1. Real data analysis
2. Spatio model
3. Validation of the model + numerical experiments
4. Spatio-temporal model
5. Model of joint spatio fields
6. Conclusion

Real data analysis

Area: $1400\,\text{km} \times 1000\,\text{km}$, data from 33 weather stations, years of observation: 1978 – 2009

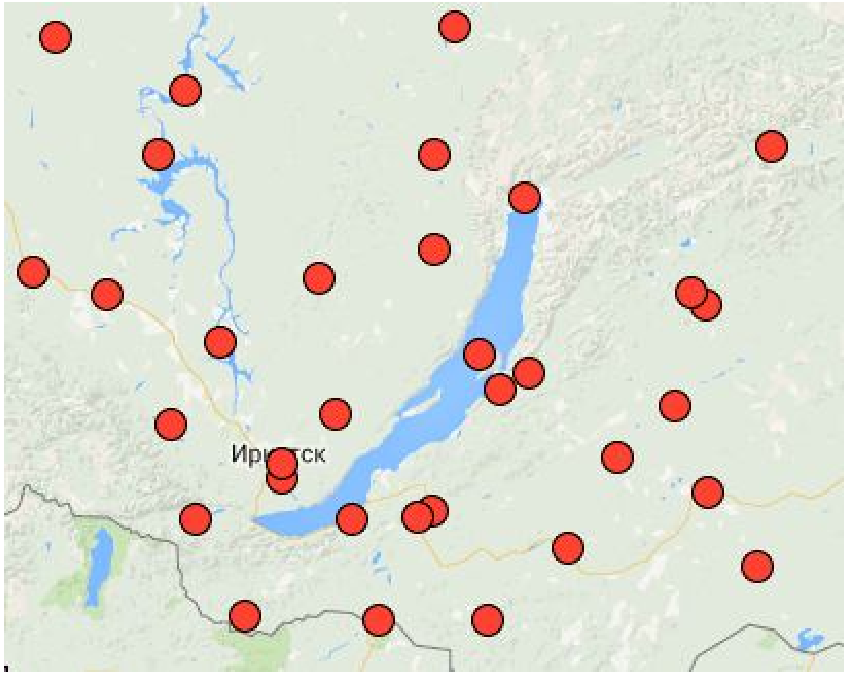


Figure 1: The Lake Baikal region. Red circles- weather stations.

One-dimensional distributions

Parameters of one-dimensional distributions depend on both spatial coordinates and time.

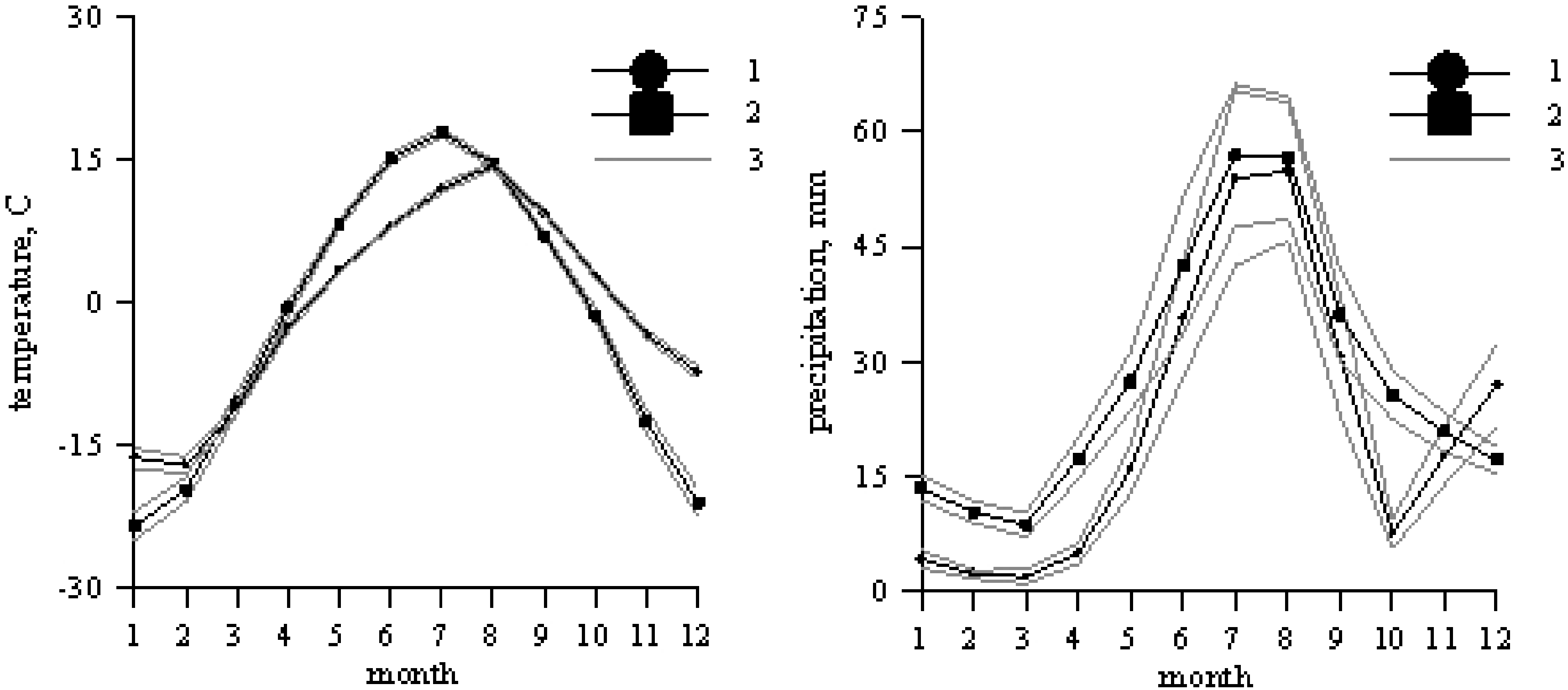


Figure 2. Sample means of monthly average temperature and month total precipitation amount.
Curve 1: weather station on Ushkaniy Island; curve 2: weather station in Chervyanka; curve 3: 90% confidence intervals.

Temporal correlation structure

Assumption: temporal sample correlation coefficients do not depend on spatial coordinates.

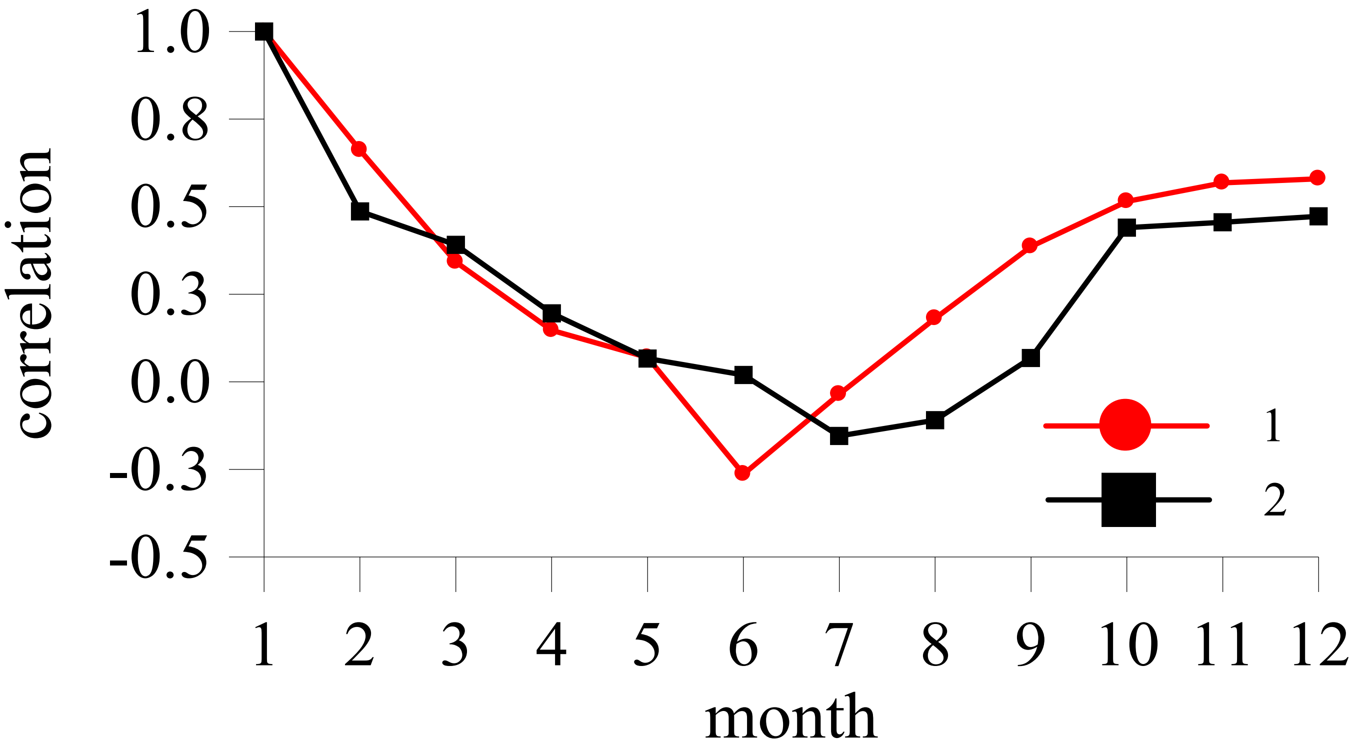


Figure 3. Temporal sample correlation coefficients. Curve 1: temperature, curve 2: precipitation.

Spatial correlation structure

Assumptions: temperature – isotropic field;
precipitation – homogeneous field.

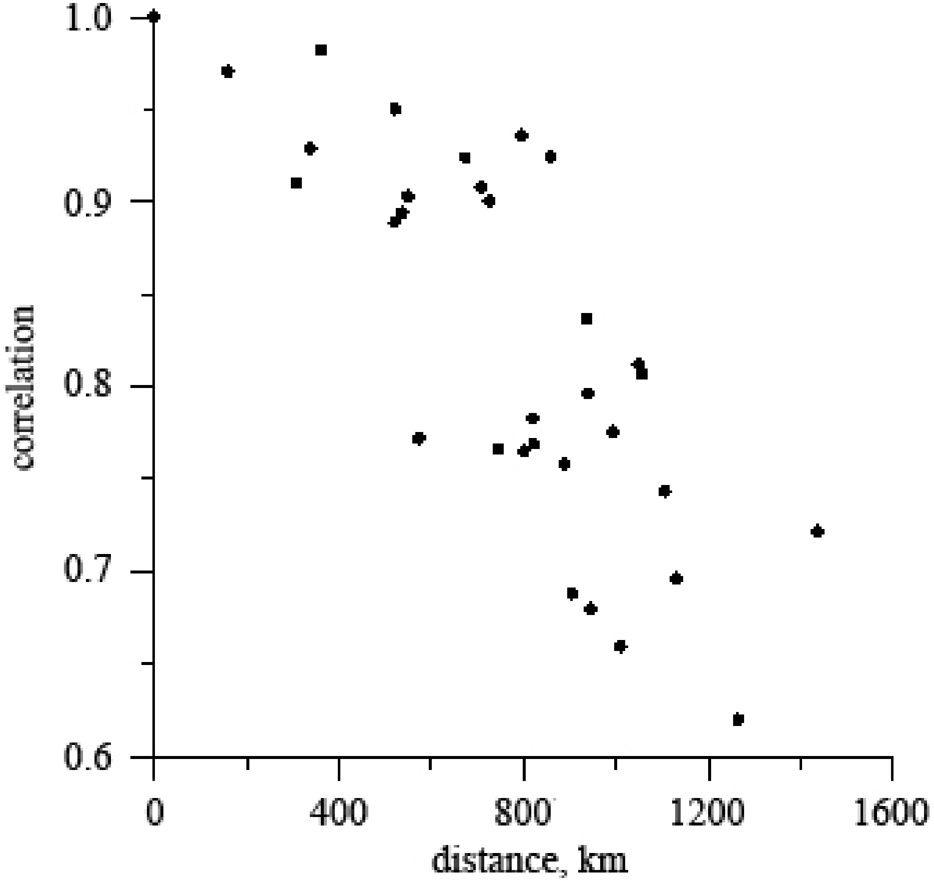


Figure 4. Spatial sample correlation coefficients between monthly average temperature at the Irkutsk weather station and at all other weather stations. January.

Spatio model

Temperature:

one-dimensional distributions – Gaussian
(parameters depend on spatial coordinates)

correlation structure – isotropic field

Precipitation:

one-dimensional distributions – gamma-distribution
(parameters depend on spatial coordinates)

correlation structure – homogeneous field

Simulation:

in nods of a regular rectangular grid with 35 km $\times 25$ km sells

Problem:

how to define parameters of one-dimensional distributions in arbitrary nodes and correlation matrixes?

<u>Parameters of one-dimensional distributions:</u>

interpolation of sample mean and sample variance from weather stations to nodes by inverse weighted distance method

$$X = \sum_{i=1}^{n} \lambda_i X_i, \ \lambda_i = \frac{d^{-\alpha}\left(A, A_i\right)}{\sum_{i=1}^{n} d^{-\alpha}\left(A, A_i\right)},$$

$$\alpha = 1, n = 2$$

+

method of moments

Correlation matrixes:

approximation of sample correlation coefficients with specific correlation function $corr(x_1, y_1, x_2, y_2)$

Precipitation: $corr(x_1, y_1, x_2, y_2) = corr(x_1 - x_2, y_1 - y_2) =$

$$= corr(x, y) = \exp\left(-\left[ax^2 + bxy + cy^2\right]^\theta\right)$$

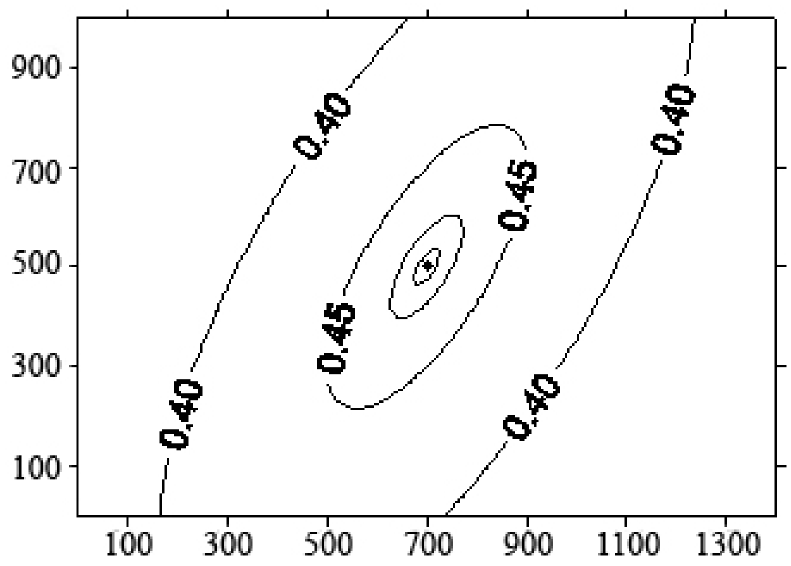


Figure 5. Isolines of approximating correlation function. May.

Month	a	b	c	θ
January	0.01	0.04	0.04	0.04
April	0.01	0.01	0.01	0.05
May	0.02	-0.02	0.01	0.07
December	0.01	0.08	0.16	0.02

Table 1. Parameters of approximating correlation function.

$$\underline{\text{Temperature:}}\; corr_1(x_1,y_1,x_2,y_2) = corr\left(\sqrt{(x_1-x_2)^2+(y_1-y_2)^2}\right) =$$

$$= corr(r) = \exp\left(-\alpha r^2\right)$$

or

$$corr_2(x_1,y_1,x_2,y_2) = corr(r) = \lambda/\left(\lambda + r^2\right)$$

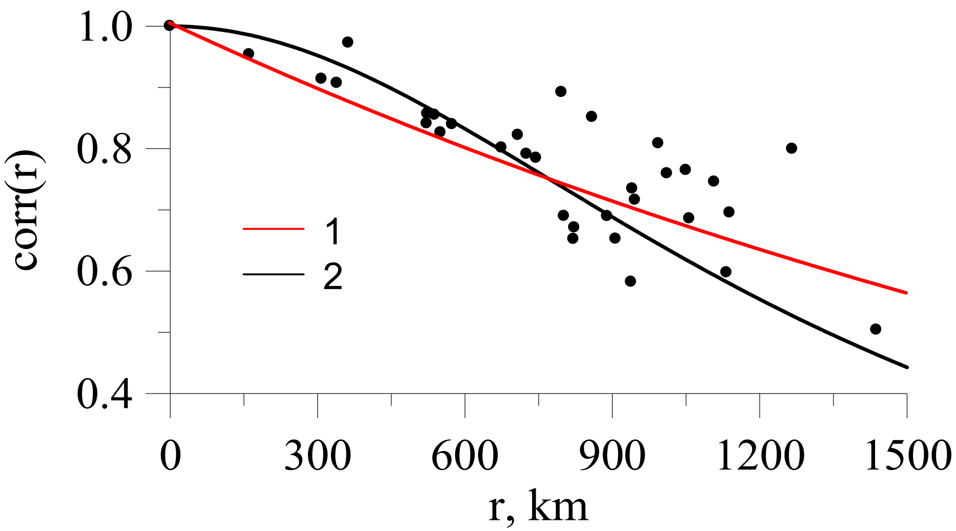


Figure 6. Vertical profiles of approximating correlation functions (1 – $corr_1$, 2 – $corr_2$) and sample correlation coefficients (•). August.

Month	α	λ
February	0.00029	2339810
April	0.00016	4836330
July	0.00050	1144200

Table 2. Parameters of approximating correlation functions.

Simulation

Field of month total precipitation amount

Inverse distribution function method

$+$

Method of conditional distributions (with regularization if necessary)

Field of monthly average temperature

Method of conditional distributions (with regularization if necessary)

OR

Randomized algorithm “by rows and by columns”

$$\text{Simulation Time}_{\text{method of conditional distributions}} \approx$$

$$\approx 100 \cdot \text{Simulation Time}_{\text{randomized algorithm "by rows and columns"}}$$

Randomized algorithm "by rows and by columns" –

efficient algorithm for simulation of isotropic random fields with correlation function

$$R(r) = \int_a^b \exp\left(-xr^2\right) f(x)\,dx$$

where $f(x),\ x \in [a;b],\ a > 0$ – arbitrary distribution density.

Ogorodnikov V.A. Simulation of a class of isotropic Gaussian fields // Theory and applications of stochastic simulation, Novosibirsk, 1988, p. 25 – 30. (in Russian)

If $f(x) = \lambda \exp(-\lambda x),\ x \geq 0$ then $R(r) = corr_2(r) = \lambda / \left(\lambda + r^2\right)$.

Modification of this algorithm may be used for simulation of homogeneous random fields.

Babicheva G.A., Kargapolova N.A., Ogorodnikov V.A. Special algoritm for simulation of homogeneous random fields // Numerical Analysis and Applications (2016), Vol. 19, No 2, p. 125-138.

Validation of the model + numerical experiments

Validation:

data from 2 additional weather stations + comparison of characteristics that are not model input parameters.

Level, °C	Share of territory	
	Real data	Simulated data
-34	0.00	0.02
-30	0.00	0.10
-26	0.18	0.32
-22	0.59	0.68
-18	0.90	0.92
-14	1.00	0.99
-10	1.00	1.00

Table 3. Share of territory, where monthly mean temperature is below given level, and this level. February.

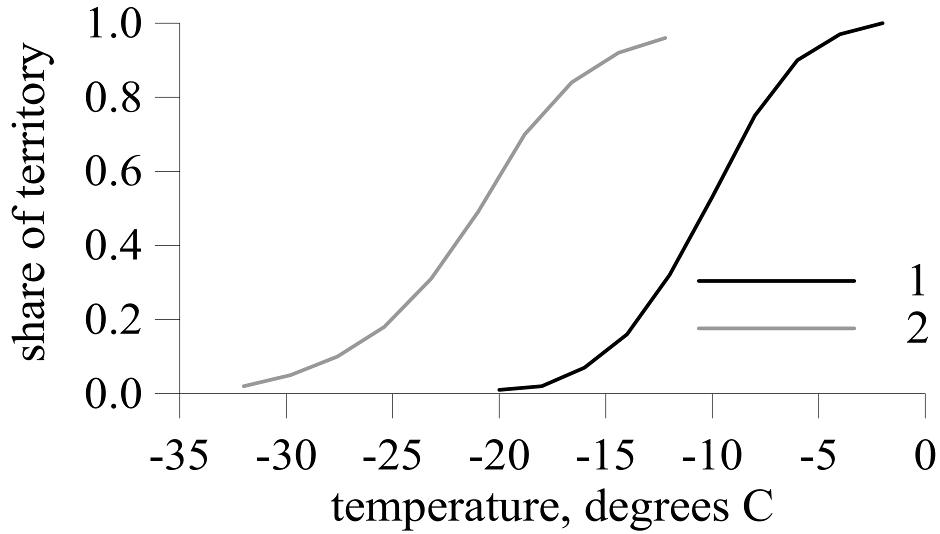


Figure 7. Share of territory, where monthly mean temperature is below level. Curve 1- March, curve 2 – December. Simulated data.

Month	Probability of very cold month	Probability of very warm month
January	0.005	0.122
April	0.013	0.016
December	0.020	0.049

Table 4. Probability of very cold/warm month on $875\,km^2$ -area around Irkutsk. Simulated data.

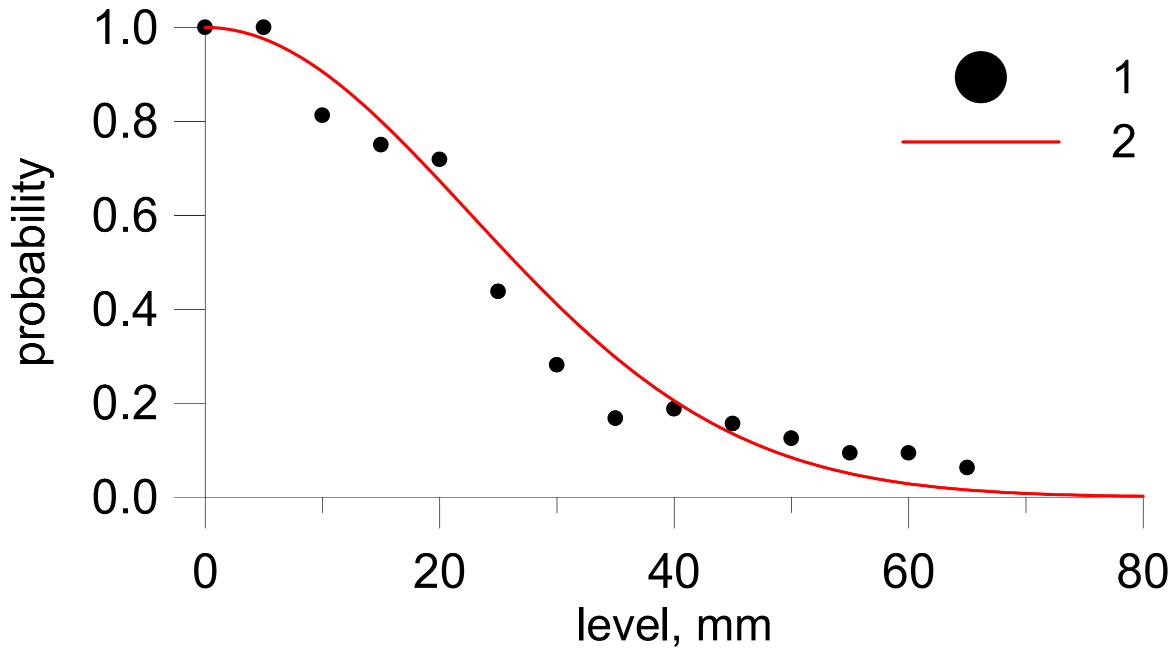


Figure 8. Probability of the event “sum of precipitations in Barguzin and Ust’Barguzin is not less than a given level (mm)”. Curve 1 – real data, curve 2 – simulated data. January.

Spatio-temporal model

Reminder:

spatial correlation structure depends on time.

Assumption:

temporal structure do not depend on spatial coordinates.

$R_i,\ i = 1, 2, \ldots, 12$ – spatial correlation matrixes

$$\begin{pmatrix} 1 & t_2^1 & t_3^1 & \ldots & t_{12}^1 \\ t_2^1 & 1 & t_3^2 & \ldots & t_{12}^2 \\ t_3^1 & \ldots & \ldots & \ldots & \ldots \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ t_{12}^1 & t_{12}^2 & \ldots & \ldots & 1 \end{pmatrix} -$$ temporal correlation matrix

$\Rightarrow$

$$\begin{pmatrix} R_1 & t_2^1 R_1 & t_3^1 R_1 & \ldots & t_{12}^1 R_1 \\ t_2^1 R_1^T & R_2 & t_3^2 R_2 & \ldots & t_{12}^2 R_2 \\ t_3^1 R_1^T & t_3^2 R_2^T & R_3 & \ldots & t_{12}^3 R_3 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ t_{12}^1 R_1^T & t_{12}^2 R_2^T & t_{12}^3 R_3^T & \ldots & R_{12} \end{pmatrix} -$$ spatio-temporal correlation matrix

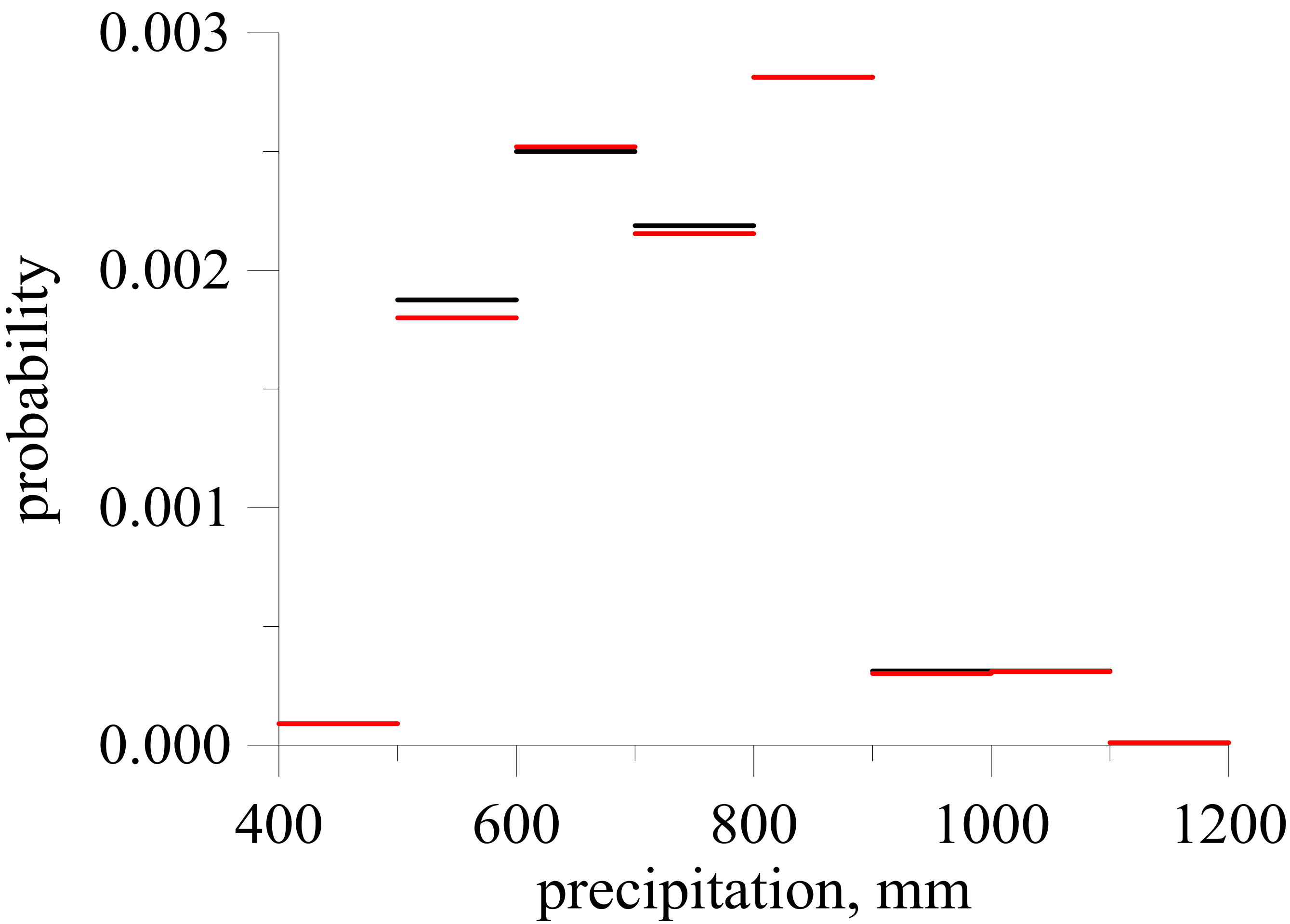


Figure 9. Histogram of sum of year total precipitation in Barguzin and Ust'Barguzin.
Black – real data, red – simulated data.

Model of joint spatio fields

Standard approach:

$$\begin{pmatrix} R_{TT} & R_{TP} \\ R_{PT} & R_{PP} \end{pmatrix}$$

Problems:

huge correlation matrix (3362×3362 elements),

time & memory consuming simulation,

computational mistakes

Idea:

simulate field of temperature,

simulate precipitation as a function of temperature

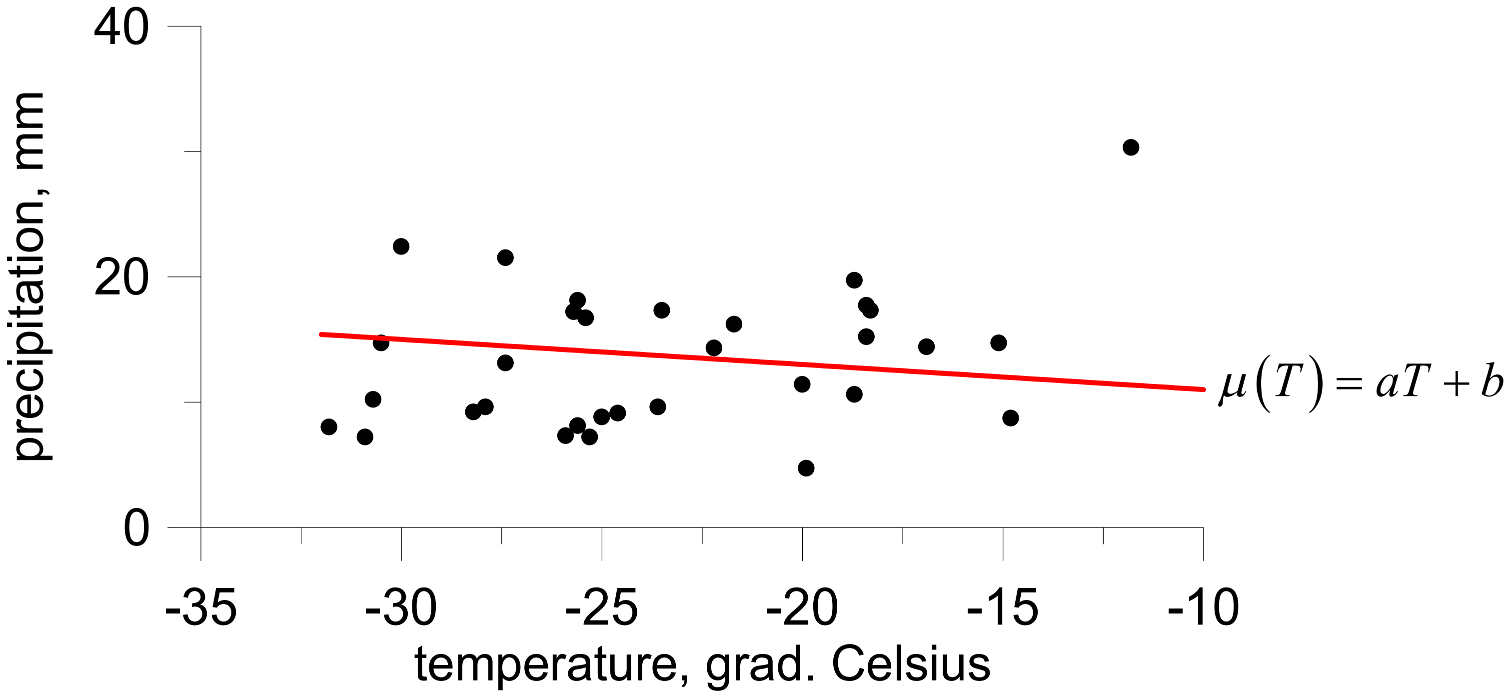


Figure 10. Precipitation-temperature dependence. Black – real data, red – approximating function. January. Chervyanka.

Simulation algorithm:

field of temperature - as described above,

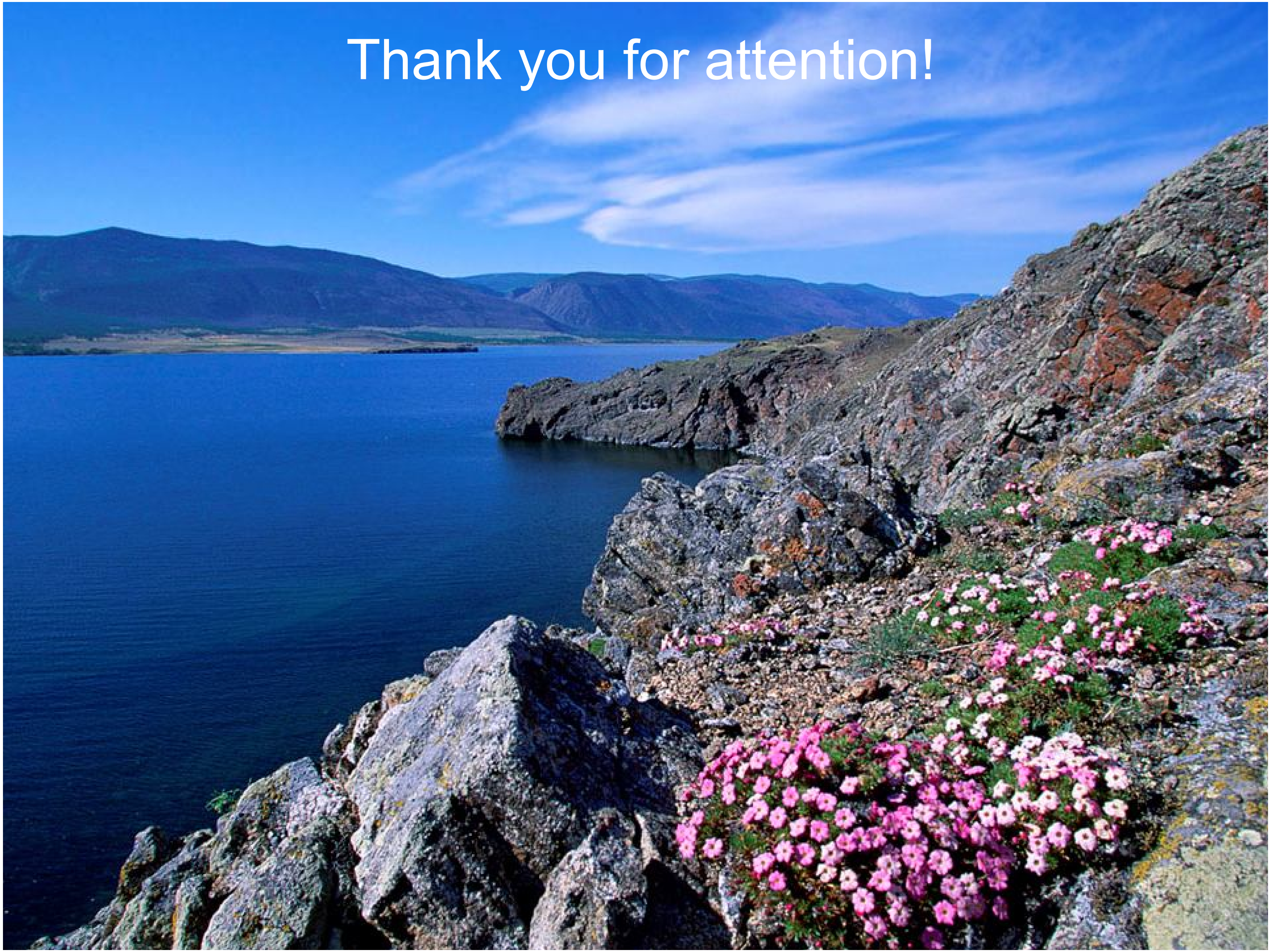
field of precipitation - homogeneous field ($\exp\left(-\left[ax^2 + bxy + cy^2\right]^\theta\right)$) with one-dimension gamma-distribution with mean $\mu(T)$ and variance $v(T) = cT + d$

Perspectives

-Better description of precipitation distribution and temperature-precipitation dependence;

- Station-temporal model of joint (temperature and precipitation) fields;

- Application of considered model for solution of hydrological problems.


Thank you for attention!