Downscaling of extremes : empirical and theoretical issues. Application to severe precipitation.

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Motivation

- ≈ 30 % of the world economic activities are affected by the meteorological conditions (source: IPCC AR4)
- IPCC scenarios of climate change and GCMs :
 - Have a coarse spatial resolution (≈ 250 km)
 ⇒ Needs for *downscaling*
 - Smooth extreme values
 - → Needs for correction to recover *extreme events*



Downscaling for Spatial Extremes

- 1. Statistical downscaling
- 2. Region of interest and data
- 3. Max-stable processes
- 4. Conditional distribution according to a single condition
- 5. Spatial Hybrid Downscaling



Downscaling for Spatial Extremes

1. Statistical downscaling

3.

2.

- 4.
- 5.

What is downscaling ???

 $\approx 250 \text{ km}$

Coarse atmospheric data Precipitation, temperature, humidity, geopotential, wind, etc.

Definition:

Downscaling is the action of generating climatic or meteorological values and/or characteristics at a local scale, based on information (from GCM/reanalyses) given at a large scale.



Sol

Region, city, fields, station

Local variables (e.g., precip., temp.)

(small scale water cycle, impacts - crops, resources - etc.)

How to downscale?: The basics

≈ 250 km

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Dynamical downscaling (RCMs):

- GCMs to drive regional models (5-50km) determining atmosphere dynamics
- Requires a lot of computer time and resources => Limited applications

Statistical downscaling:

- Based on statistical relationships between large- and local-scale variables
- Low costs and rapid simulations applicable to any spatial resolution
- Uncertainties (results, propagation, etc)

Region, city, fields, station

Local variables (e.g., precip., temp.)

(small scale water cycle, impacts - crops, resources - etc.)

Main statistical approaches

Could also be RCM simulations...



Statistical downscaling : bias correction

CDF-Transform Method

Build a bias correction by comparing the CDF of the high-scale variable and the one of the low-scale variable (Michelangeli et al., 2009).



Statistical Downscaling for Spatial Extremes

Simulate spatial fields of extremes conditioning to large scale information



Simulate spatial fields of extremes conditioning to large scale information

- Model for spatial extremes
- Conditional simulation for this model



Downscaling for Spatial Extremes

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Low scale: *SAFRAN* data (Quintana-Segui et al., 2008)

Low scale daily precipitation data from SAFRAN analysis (1960–2007) on a grid with $8 \text{ km} \times 8 \text{ km}$ -resolution (Quintana-Seguí et al., '08)



Figure: Study area of SAFRAN data subset.

Description:

- Cévennes region in SE of France
- Autumnal maximum of daily precipitation for 1960-2007.
- 457 points uniformly distributed.
- Grid data from interpolation.

Large scale: MEDCORDEX-IPSL-WRF data



Figure: MEDCORDEX-IPSL-WRF maximum precipitation data for the year 1989 (in mm).

Description:

- RCM outputs of autumnal daily precipitation for 1989-2007.
- Grid resolution: about 50km.
- Area of interest is covered by 12 grid-cells.



Downscaling for Spatial Extremes

1.

2.

3. Max-stable processes

4.

5.

 $\{Z(s), s \in \mathbb{S}\}$ random field, Z is **max-stable** if there exist $a_n(s)$ and $b_n(s) > 0$, such that for all n, $(Z_i)_{i=1,n}$ ind copies Z,

$$\left\{\max_{i=1,n}\frac{Z_i(s)-a_n(s)}{b_n(s)}\right\}_{s\in\mathbb{S}}\stackrel{\mathcal{D}}{=} \{Z(s)\}_{s\in\mathbb{S}}$$

Asymptotic dependence at lag h

$$\chi(h) = \lim_{z \to z^*} P(Z(s+h) > z | Z(s) > z)$$

$$\chi(h) > 0$$

De Haan construction

- (ξ_k, x_k) Poisson point process on $(0, +\infty) \times \mathcal{X}$, with intensity $\xi^{-2} d\xi \mu(dx)$,
- $W(.;s): \mathcal{X} \mapsto [0, +\infty)$ functions $\int_{\mathcal{X}} W(x,s)\mu(dx) = 1$ for all $s \in \mathbb{S}$.

Then

$$Z(s) = \max_{k} \xi_k W(x_k, s)$$

is a max-stable process with Fréchet margins.

Models for maxima random fields.

Max-stable models



Max-stable models

- Smith (storm) processes $W_k(x_k, s) = \varphi(s - x_k)$
- Schlather processes $W_k(x_k,s) = \max(0,Y_k(s)), Y_k$ Gaussian
- extremal-t processes $\xi^{-1+\alpha}d\xi\mu(dx)$, $W_k(x_k,s) = \max(0,Y_k(s))$, Y_k Gaussian
- Brown-Resnick processes $W_k(x_k,s) = \exp \left(Y_k(s) \sigma^2(s)/2\right) Y_k$ Gaussian stationary inc.

Data transformation

Max-stable model : *t*-extremal process with α -Fréchet margins.

Transformation of the data by modeling the GEV parameters:

$$\mu = \mu_0 + \mu_1 * LON + \mu_2 * LAT$$

$$\sigma = \sigma_0 + \sigma_1 * LON + \sigma_2 * LAT$$

$$\zeta = \zeta_0$$

Parameters $(\mu_0, \mu_1, \mu_2, \sigma_0, \sigma_1, \sigma_2, \zeta, \alpha, \rho)$ estimated by composite likelihood

$$\mu = \exp(-3.06 - 0.22LON + 0.18LAT)$$

$$\sigma = \exp(-4.34 - 0.37LON + 0.21LAT)$$

$$\zeta = 0.11$$

$$\alpha = 2.21$$

$$\rho = 1.23$$

Safran data year 2007

Safran data year 2005





simulation



simulation

simulation





Downscaling for Spatial Extremes

2.

1.

3.

4. Conditional distribution according to a single condition

5.

$$Z(s) = \max_{k \in \mathbb{N}} \xi_k W_k(s), \quad s \in K$$

Task: Simulate Z given some conditions on Z.

Condition **s**

•
$$\ell(Z) = z, \quad z > 0$$

for some positively homogeneous functional ℓ , i.e.

$$\ell(af) = a\ell(f), \quad a > 0, \ f \in C_+(K).$$

Examples for ℓ **:**

integral, point evaluation, maximum, minimum, ...

$$Z(s) = \max_{k \in \mathbb{N}} \xi_k W_k(s), \quad s \in K$$

condition: $\ell(Z) = z$, z > 0for some positively homogeneous functional ℓ

Difficulties:

- number of "active" functions may be arbitrarily large
- condition cannot be carried over to conditions on the Poisson point process
- condition cannot be expressed in terms of the exponent measure

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~> conditional distribution not tractable analytically

Our Approach: Sampling via MCMC techniques!

- $\{\xi_k\}_{k \in \mathbb{N}}$: Poisson point process with intensity $\xi^{-2} d\xi$
- $\{W_k(\cdot)\}_{k\in\mathbb{N}}$: i.i.d. sample-continuous processes

$$Z(s) = \max_{k \in \mathbb{N}} \quad \xi_k W_k(s), \quad s \in K$$

4

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What about the distribution $\{\xi_k\}_{k \in \mathbb{N}}$?

• $\{\xi_k\}_{k\in\mathbb{N}}$ can be numbered s.t. $\xi_1 > \xi_2 > \dots$

• then:
$$\xi_k =_d (E_1 + \ldots + E_k)^{-1}$$
 for $\{E_k\}_{k \in \mathbb{N}} \sim_{iid} \operatorname{Exp}(1)$



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Assumption: $\sup_{s \in K} W_k(s) < C$ a.s. for some C > 0 (*)

Then,

$$\begin{split} \xi_k \cdot C &< \inf_{s \in K} Z(s) \\ \Rightarrow \xi_k W_k(\cdot) \text{ cannot contribute to } Z(\cdot) \end{split}$$



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Define

$$N(\xi, \mathbf{W}) = \min \left\{ k \in \mathbb{N} : \ \xi_k \cdot C < \inf_{s \in K} Z(s) \right\}.$$

4

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(*)

Note: *W_k* can always be chosen s.t. (*) holds (de Haan & Ferreira, '06, Oesting *et al, '13*)

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Two-Step Procedure for Max-Stable Processes:

A Sample *n* from distribution of $N(\xi, \mathbf{W}) \mid \ell(Z) = z$

$$Z(s) = \max_{i=1}^{N(\mathbf{U},\mathbf{W})} \xi_k W_k(s)$$

Two-Step Procedure for Max-Stable Processes:

A Sample (\mathbf{w}, n) from distribution of $(\mathbf{W}, N(\xi, \mathbf{W})) \mid \ell(Z) = z$

$$Z(s) = \max_{i=1}^{N(\mathbf{U},\mathbf{W})} \xi_k W_k(s)$$

Two-Step Procedure for Max-Stable Processes:

A Sample (\mathbf{w}, n) from distribution of $(\mathbf{W}, N(\xi, \mathbf{W})) \mid \ell(Z) = z$

B Sample ξ from distribution of $\xi \mid \mathbf{W} = \mathbf{w}, N(\xi, \mathbf{W}) = n, \ell(Z) = z$

$$Z(s) = \max_{i=1}^{N(\mathbf{U},\mathbf{W})} \xi_k W_k(s)$$

Two-Step Procedure for Max-Stable Processes:

- A Sample (\mathbf{w}, n) from distribution of $(\mathbf{W}, N(\xi, \mathbf{W})) \mid \ell(Z) = z$
 - by Metropolis-Hastings algorithm
 - proposal distribution: unconditional distribution of $(\mathbf{W}, N(\xi, \mathbf{W}))$
- B Sample ξ from distribution of $\xi \mid \mathbf{W} = \mathbf{w}, N(\xi, \mathbf{W}) = n, \ell(Z) = z$

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- **B** Sample ξ from distribution of $\xi \mid \mathbf{W} = \mathbf{w}, N(\xi, \mathbf{W}) = n, \ell(Z) = z$
 - Metropolis-Hastings algorithm
 - conditional sampling for max-linear model

$$Z(s) = \max_{k=1}^{n} a_k(s) U_k = \mathbf{A} \odot \mathbf{U}$$

MCMC Algorithm for Max-Linear Models

$$Z(s) = \max_{k=1}^{n} a_k(s) U_k = \mathbf{A} \odot \mathbf{U}, \quad s \in K$$

Aim: Simulate
$$\mathbf{Y} = \left(rac{U_2}{U_1}, \dots, rac{U_n}{U_1}
ight) \mid \ell(Z) = z.$$

Metropolis-Hastings Algorithm:

- 1. Choose starting value $\mathbf{y} \in (0, \infty)^{n-1}$.
- Draw y* from non-conditional distribution of Y as proposal for y.
 Set y = y* with probability

$$\min\left\{1, \qquad \frac{\mathbb{P}(\mathbf{Y} \in \mathrm{d}\mathbf{y})\mathbb{P}(\mathbf{Y} \in \mathrm{d}\mathbf{y}^* \mid \ell(Z) = z)}{\mathbb{P}(\mathbf{Y} \in \mathrm{d}\mathbf{y}^*)\mathbb{P}(\mathbf{Y} \in \mathrm{d}\mathbf{y} \mid \ell(Z) = z)}\right\}.$$

4. Goto 2.

MCMC Algorithm for Max-Linear Models

$$Z(s) = \max_{k=1}^{n} a_k(s) U_k = \mathbf{A} \odot \mathbf{U}, \quad s \in K$$

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Metropolis-Hastings Algorithm:

- 1. Choose starting value $\mathbf{y} \in (0, \infty)^{n-1}$.
- Draw y* from non-conditional distribution of Y as proposal for y.
 Cat * with probability
- 3. Set $y = y^*$ with probability

$$\min\left\{1, \frac{\left(\sum_{k=1}^{n} \frac{\ell(A \odot \mathbf{y}^{*})}{y_{k}^{*}}\right)^{n} \exp\left(-\frac{1}{z} \sum_{k=1}^{n} \frac{\ell(A \odot \mathbf{y}^{*})}{y_{k}^{*}}\right)}{\left(\sum_{k=1}^{n} \frac{\ell(A \odot \mathbf{y})}{y_{k}}\right)^{n} \exp\left(-\frac{1}{z} \sum_{k=1}^{n} \frac{\ell(A \odot \mathbf{y})}{y_{k}}\right)}\right\}.$$

Goto 2.

Simulation conditional to each RCM grid-cell

- Standard downscaling (cdf-transform) for each Safran grid point covered by the RCM grid cell
- the averaged of the downscaled values is the conditioning value
- perform conditional simulations with the MCMC algorithm

Results

Year 2007





Averaged simulations



Safran data



Simulation 3



Simulation 42





Downscaling for Spatial Extremes



1.

3.

Spatial Hybrid Downscaling

From conditional simulations to downscaling

Downscaling of the regional climate models (RCM) outputs

Climate model : cells M_1, \ldots, M_n

Objective: Simulate $(\mathbf{Z})_s | M_1 = m_1, \dots, M_n = m_n$

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Climate model : cells M_1, \ldots, M_n

Objective: Simulate $(\mathbf{Z})_s | M_1 = m_1, \dots, M_n = m_n$

Method: conditional simulations using downscaled values as conditioning points.

 \Rightarrow hybrid method



General Methodology:

- Choose a given number of points in the low scale dataset.
- Establish a statistical link (**transfer function**) between the high scale information (from GCM outputs) and these points (calibrated on the past)

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- Build with this transfer function some pseudo-observations at these given locations
- Perform a **conditional simulation algorithm** with the pseudo-observations as conditioning values.

Suppose that Z has been measured at a number of data points.

$$z_1 = Z(s_1), \dots, z_n = Z(s_n)$$

Objective: Simulate $(\mathbf{Z})_s | Z(s_1) = z_1 \dots, Z(s_n) = z_n$

$$Z(s) = \max_{k \ge 1} \xi_k W_k(s) = \max_{k \ge 1} \phi_k(s) = \max\left(\max_{k=1,\ell} \phi_k^+(s), \max_{k \ge 1} \phi_k^-(s)\right)$$

Algorithm

Dombry and Eyi-Minko (2012) propose:

(i) generate a partition \mathcal{J} given z_1, \ldots, z_n ; (ii) generate extremal random fields ϕ_i^+ given \mathcal{J} ; (iii) generate sub-extremal random fields ϕ_i^- . - Step (*i*): Gibbs sampler, *Dombry et al, 2013*) rests on the computation of **multidimensional** integrals, that may be not numerically tractable.

- Step (*ii*): Each extremal random field must be simulated under **equality** and **inequality** constraints. A rejection approach, may take very long time when the number of conditioning points is large.

Gathering steps 1 and 2

Instead of drawing a partition, draw directly the extremal random fields at **all conditioning points**

- avoid the calculation of integrals
- enables handling up to hundreds conditioning points instead of less than 30.

Basic choice: near the center of each grid-cell

Alternative choices

• choose a representative of the low-scale dataset with clustering algorithm.

 \Rightarrow Partitioning Around Medoids (PAM)

 choose randomly one conditioning point into each grid-cell and change it for each simulation

 \Rightarrow Stochastic Hybrid Method

Methods	TF	Cond. Sim.	Description
Interpolation	NO	NO	Bilinear interpolation of the RCM
			Madal the law scale veriable by the
Linear downsc.	YES	NO	Model the low-scale variable by the
			mean of the high-scale variable over
			the 9 pixels around the point of interest.
Raw	NO	YES	Use directly RCM outputs as
			conditioning values.
CDF-t	YES	YES	Build a bias correction by comparing the
			CDF of the high-scale variable and
			the one of the low-scale variable.
Linear reg.	YES	YES	Same as Linear downscaling but only
			at the conditioning points.
Optimal	NO	YES	Use directly the real observations
			as conditioning points

 Table: Different methods for building the pseudo-observations from the Medcordex and Safran datasets.

Skill-Score: % of improvement compared to a reference method.

	Models	CRPSS	QSS ₉₅	K-S SS	RMSE _v SS
No simulations	Interpolation	-20.1%	-54.8%	-19.8%	-0.4%
	Linear downscaling	0%	0%	0%	0%
Conditional	Raw	14.0%	44.4%	10.4%	20.2%
	CDF-t	14.1%	60.6%	12.3%	40.4%
Simulations	Linear Regression	20.1%	53.4%	15.0%	22.5%
	Optimal	24.2%	70.7%	18.7%	52.9%

Table: Skill-Scores (with *Linear downscaling* as reference) of the hybrid algorithm with the different methods for building the pseudo-obs. from the MEDCORDEX dataset.

Results



Figure: Annual means obtained by the 6 methods. The observed in blue and each trajectory in red is one conditional simulation.

Choice of the conditioning points

PAM

- Choice of conditioning points independent of the RCM model.
- 57 conditioning points
- Same order between the different methods (≈ same skill-scores)
- Results not improved (higher scores).

<u>Reasons:</u> the typicality of some conditioning points (alone in their clustering classes)

Stochastic Hybrid Method

- Choice of conditioning points depend on the RCM model
- No more case-by-case choice of conditioning points.
- Same order between the different methods (≈ same skill-scores)
- Improved results (Scores)

Perspectives

Downscaling of MSP, theoretical distribution

- several conditions ?
- if not reachable, smoothing to avoid discontinuities when changing large scale grid-cell

Hybrid downscaling

• choice of the points (and number)

Both

- transfer functions
- other RCM

Describe the future evolution of the extreme precipitation according to different high-scale scenarios.

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