THE CENTRAL LIMIT THEOREM FOR HOLOMORPHIC ENDOMORPHISMS AND MODULAR CORRESPONDENCES.

SERGE CANTAT AND STÉPHANE LE BORGNE

Abstract. Here we summarize the corresponding french paper entitled "Théorème limite central pour les endomorphismes holomorphes et les correspondances modulaires"

This work deals with stochastic properties arising in two different dynamical systems.

1. Iteration of endomorphisms of $\mathbb{P}^k(\mathbf{C})$. Let $f : \mathbb{P}^k(\mathbf{C}) \to \mathbb{P}^k(\mathbf{C})$ be a holomorphic transformation of the k-dimensional complex projective space. Its topological entropy equals the logarithm of its topological degree $\mathsf{d_t}$,

$$\mathsf{h_{top}}(f) = \log(\mathsf{d_t}).$$

It has been shown that, if $\mathsf{d_t}$ is larger than 2, there exists a unique maximal entropy measure μ_f. Our first result extends a theorem of Denker, Przytycki and Urbański to the case of holomorphic transformations of complex projective spaces of arbitrary dimensions.

Théorème 0.1. *Let f be a holomorphic endomorphism of the projective space $\mathbb{P}^k(\mathbf{C})$ with topological degree larger than 1 and μ_f be the maximal entropy measure of f. If φ is a Hölder continuous function with mean 0 for μ_f, the sequence*

$$\sigma_N^2 = \int_{\mathbb{P}^k(\mathbf{C})} \varphi^2(m) d\mu_f(m) + \frac{1}{N} \sum_{k=0}^{N-1} \int_{\mathbb{P}^k(\mathbf{C})} \varphi(m)\ \varphi(f^k(m))\, d\mu_f(m)$$

tends toward a non negative number σ^2. This number is null if and only if φ is a coboundary. If it is positive, φ satisfy the central limit theorem with variance σ, the Donsker invariance principle and the Strassen invariance principle.

We also obtain a similar result for the Markov chain that corresponds to backward iteration of the endomorphism f: At each step, one chooses a point x_{n+1} at random among the $\mathsf{d_t}$ elements of $f^{-1}\{x_n\}$. When the post-critical set of f is a finite union of varieties we prove a central limit theorem for this

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Markov process for any hölder continuous function and for any initial probability measure which does not charge the post-critical set.

2. Markov processes associated to Hecke operators. Here the situation is completely different for Hecke operators are locally isometric correspondances.

Let G be a linear algebraic group defined over $\mathbf{Q}$, and assume that G is almost simple and connected and that the real part $G(\mathbf{R})$ is not compact. Let Γ be a congruence subgroup of $G(\mathbf{Q})$ and K a maximal compact subgroup of G. Let X be the locally symmetric space $\Gamma\backslash G/K$ and λ be the probability measure on X coming from the Haar measure on G. If a belongs to $G(\mathbf{Q})$, the group $\Gamma_a = a^{-1}\Gamma a \cap \Gamma$ has finite index in Γ and in $a^{-1}\Gamma a$. The manifold $Y_a = \Gamma_a\backslash G/K$ has two covers with a common degree over X, defined by

$$\pi_1(\Gamma_a y) = \Gamma y \quad \text{et} \quad \pi_2(\Gamma_a y) = \Gamma a y.$$

The transformation $\pi_2 \circ \pi_1^{-1} : X \to X$ defines a correspondence on X, named modular correspondence or Hecke correspondence. The second goal of our paper is to prove a central limit theorem for the Markov chain associated to this correspondence. Let T_a be the operator defined by the following property : $T_a\varphi(x)$ is the mean of φ on the set $\pi_2 \circ \pi_1^{-1}\{x\}$.

Théorème 0.2. *Let G be a linear algebraic group defined over $\mathbf{Q}$, almost simple and connected with non-compact real part $G(\mathbf{R})$. Let Γ be a congruence subgroup of $G(\mathbf{Q})$. Let a in $G(\mathbf{Q})$ defining a modular correspondence $\Gamma\backslash G(\mathbf{R})$. If the $\mathbf{Q}$-rank of G is positive, any square integrable measurable function φ on X with zero mean such that*

$$\sigma^2 = \int_X \varphi^2(m)d\lambda(m) + 2\sum_{k=1}^{\infty}\int_X \varphi(m)\ T_a^k\varphi(m)\,d\lambda(m)$$

is positive satisfies the central limit theorem with variance σ^2.

3. The proof. In both cases the proof relies on Gordin's method which consists in reducing the study of a process to the study of a sequence of a martingale or reversed martingale. This technique is briefly described in section 2 of the paper. In the introduction we give a description of the frames of our study and state our results. Proofs are given in section 3 for the endomorphisms of $\mathbb{P}^k(\mathbf{C})$, in section 4 for Hecke operators.

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IRMAR, UMR 6625 du CNRS, Université de Rennes I, 35042 Rennes, France
E-mail address: cantat@univ-rennes1.fr