ABSTRACTS

Francesca Acquistapace, *On Pfister's multiplicative formulae for the ring of real analytic functions* :

We present "infinite" multiplicative formulae for countable collections of sums of squares (of meromorphic functions on $\mathbb{R}^n$). Our formulae generalize the classical Pfister's ones concerning the representation as a sum of 2^r squares of the product of two elements of a field K which are sums of 2^r squares. As a main application, we reduce the representation of a positive semidefinite analytic function on $\mathbb{R}^n$ as a sum of squares to the representation as sums of squares of its irreducible factors.

Joint work with Fabrizio Broglia and José F. Fernando.

Janusz Adamus, *Tameness of complex dimensions in real algebraic sets* :

Given a real-algebraic (or more generaly, semialgebraic) set R in a complex ambient space, a natural question to ask is how much of the complex structure is inherited (locally) by R. One way of measuring this influence at a point $p \in R$ is to look at the minimal dimension of a complex germ containing R_p and, dually, the maximal dimension of a complex germ contained in R_p. We will consider the problem of tameness of these "outer" and "inner" complex dimensions along R.

Joint work with Serge Randriambololona and Rasul Shafikov.

Alexandre Bardet, *Diviseurs à support réel sur les courbes réelles* :

Dans un article sur les sommes de carrés, Scheiderer a montré que pour toute courbe réelle projective lisse, il existe un entier naturel N tel que tout diviseur de degré plus grand que N soit linéairement équivalent à un diviseur dont le support est totalement réel. Bien que la preuve laisse penser que l'entier N est grand, Huissman et Monnier ont montré qu'on pouvait prendre $N = g - 1 + s$ si le nombre de composantes connexes s est plus grand que g. On s'intéressera alors à étendre des résultats de Monnier sur un analogue concernant les courbes singulières.

Salvatore Barone, *Refined bounds on the number of connected components of sign conditions on a variety* :

Let $\mathbb{R}$ be a real closed field, $\mathcal{P}, \mathcal{Q} \subset \mathbb{R}[X_1, ..., X_k]$ finite subsets of polynomials, with the degrees of the polynomials in $\mathcal{P}$ (resp. $\mathcal{Q}$) bounded by d (resp. d_0). Let $V \subset \mathbb{R}^k$ be the real algebraic variety defined by the polynomials in $\mathcal{Q}$ and suppose that the real dimension of V is bounded by k'. We prove that the number of semi-algebraically connected components of the realizations of all realizable sign conditions of the family $\mathcal{P}$ on V is bounded by

$$2^{k-k'}(2^k+1)\, d_0^{k-k'} \sum_{j=0}^{k'} 4^j \binom{s+1}{j} d^j \;\max\{2d_0, d\}^{k'-j}.$$

In case $2d_0 \leq d$, the above bound can be written simply as

$$(\sum_{j=0}^{k'} \binom{s+1}{j}) d^{k'} d_0^{k-k'} O(1)^k = (sd)^{k'} d_0^{k-k'} O(1)^k.$$

This improves in certain cases (when $d_0 \ll d$) the best known bound of

$$\sum_{1 \leq j \leq k'} \binom{s}{j} 4^j d(2d-1)^{k-1}$$

on the same number proved in the case $d = d_0$. The distinction between the bound d_0 on the degrees of the polynomials defining the variety V and the bound d on the degrees of the polynomials in $\mathcal{P}$ that appears in the new bound seems to be significant in several applications in discrete geometry, especially in recent work on bounding incidences between points and algebraic varieties in fixed dimensional real affine spaces.

Frédéric Bihan, *Recent Fewnomial Bounds* :

Since the work of A. Khovanskii on Fewnomials, we know explicit bounds on the topology of real algebraic varieties, in particular on the number of real solutions to polynomial systems, which depend only on the number of monomials appearing in the defining equations. Such bounds are better than the classical ones when the number of monomials is small comparatively to the degrees. This talk will survey recent fewnomial bounds, including bounds which take into account additionnal structures on the supports of the equations.

Ludwig Broecker, *Stability indices over* $\mathbb{R}((X))$ *and p-adic fields* :

Basic sets are the generators for the lattice of all semialgebraic sets over $R((X))$ or p-adic fields. For the semialgebraic sets one has elimination of quantifiers. However, the basic sets are defined by n th- power predicates for arbitrary n. (Over the ordinary reals $n = 2$ is sufficient). As for the reals we present bounds, only depending on the dimension of the ambient space, for the number of polynomials wich is required to describe arbitrary basic sets.

Sabine Burgdorf, *Hilbert's question on trace-positive polynomials* :

Hilbert proved that a ternary quartic is nonnegative if and only if it can be written as a sum of squares of polynomials whereas this is false in general if the degree of the polynomial exceeds four. We will consider the question if the same holds true in the tracial context. Namely, a real polynomial in non-commuting variables is called trace-positive if its trace is nonnegative under all matrix evaluations of symmetric matrices or of a given semialgebraic set K of symmetric matrices. These polynomials are intimately connected to the embedding conjecture of Connes, which corresponds to Tsirelson's problem from Quantum Physics. A second connection to Quantum Physics is given via the BMV conjecture. We present results and examples concerning the question whether a trace-positive polynomial can be written as a sum of hermitian squares and commutators. If time permits we will also focus on the dual problem given by the tracial K-moment problem.

Joint work with Igor Klep.

Fabrizio Catenese, *Moduli spaces of Galois coverings of algebraic curves.*

Yann Cogan, *Minimal degree of affine algebraic surfaces of given genus* :

We present a study of the minimal degree of all smooth compact connected algebraic surfaces of $\mathbb{R}^3$ of given genus. We prove that this minimal degree is equal to

(i) 4 in genus $1, 2, 3, 4, 5$,
(ii) 6 in genus $10, 11, \ldots, 27, 28$,
(iii) 8 in genus $63, 64, \ldots, 80, 81$, and
(iv) 10 in genus $172, 173, 174, 175, 176$.

The result is based on level surfaces of sums of Tchebychev's polynomials of even degree d (Banchov-Chmutov surfaces). It's genus is $(\frac{d}{2} - 1)^2(d + 1)$. For example, the Banchov-Chmutov surface of degree 4 has genus equal to 5, and the one of degree 6 has genus equal to 28.

An explicit perturbation of the polynomial allows us to reduce the genus of the surface without losing connectedness, compactness and smoothness. For each even degree d, any genus less than $(\frac{d}{2} - 1)^2(d + 1)$ can be obtained in this way. It results in an upper bound of the minimal degree in terms of the genus.

The Inequality of Milnor-Olienik-Petrovski-Thom, and Kharlamov's Theorem on projective quartic surfaces give rise to a lower bound of the minimal degree of a surface of genus g. Finally, we obtain the exact values given above, and tight bounds in other cases. The question of the minimal degree of surfaces of genus 6, 7, 8 and 9, in particular, remains open. It is equal to 4 or 6.

Marc Coppens, *Separating pencils on* $(M-2)$*-curves* :

A smooth real curve X of genus g is called an $(M-2)$-curve in case the real locus has exactly $g-1$ components. A separating pencil on X is a base point free real linear system g^1_k on X such that the support of each real divisor in g^1_k only contains real points. If X has such pencil then $X(\mathbb{C}) \setminus X(\mathbb{R})$ is disconnected (X is called separating). A result of Gabart implies a separating $(M-2)$-curve has a separating pencil of degree at most g. We show the existence of separating $(M-2)$-curves having no separating pencil of degree $g-1$. As a corollary, although for a separating $(M-1)$-curve X the scheme W^1_{g-1} parameterizing linear systems g^1_{g-1} on X has dimension $g-4$, there exist such curves having an isolated point in the real locus $W^1_{g-1}(\mathbb{R})$.

Felipe Cucker, *On a Problem posed by Steve Smale* :

At the request of the International Mathematical Union, in 1999, Steve Smale proposed a list of 18 problems for the mathematicians of the 21st century. The 17th of these problems asks for the existence of a deterministic algorithm computing an approximate solution of a system of n complex polynomials in n unknowns in time polynomial, on the average, in the size N of the input system. The talk gives fundamental advances in this problem including the smoothed analysis of a randomized algorithm and a deterministic algorithm working in near-polynomial (i.e., $N^{O(\log\log N)}$) average time.

Joint work with Peter Buergisser.

Alex Degtyarev, *On real determinantal quartics* :

We describe all possible arrangements of the ten nodes of a generic real determinantal quartic surface in $\mathbb{CP}^3$ with nonempty spectrahedral region.

Joint work with I. Itenberg.

Charles N. Delzell, *Eliminating nontransversal zeros in the Finiteness Theorem for open semialgebraic sets* :

Let R be a real closed field (e.g., $\mathbb{R}$), with the usual, order topology. Let K be a subfield (e.g., $\mathbb{Q}$). Below, f and g_{ij} will denote elements of $K[X] := K[X_1, \ldots, X_n]$, and $\{f > 0\}$ will denote $\{\, x \in R^n \mid f(x) > 0 \,\}$ (a "subbasic open s.a. set"). A subset S of R^n is called (K-R-)*semialgebraic* (or "s.a.") if it is a Boolean combination of sets of the form $\{f > 0\}$. The "Finiteness Theorem for open s.a. sets" asserts that if S is *open* s.a., then there are finitely many g_{ij} such that S can be written as

$$\bigcup_i \bigcap_j \{g_{ij} > 0\} \tag{1}$$

(the converse is obvious).

We call a zero $x \in R^n$ of f *transversal* if f changes sign in every neighborhood of x; we write $Z_{\mathrm{t}}(f)$ for the set of all transversal zeros of f, and $Z_{\mathrm{nt}}(f)$ for the set $Z(f) \setminus Z_{\mathrm{t}}(f)$.

Theorem: *For every f, there are finitely many g_{ij} such that $\{f > 0\}$ can be written in the form* (1) *with $Z_{\mathrm{nt}}(g_{ij}) = \emptyset$ for each i, j.*

The Theorem is proved by induction on n, and then by induction on $\max \dim W$, where W ranges over the "strata" of $Z_{\mathrm{nt}}(f)$.

An open set V in a topological space is called *regular open* if $V = \overset{\circ}{\overline{V}}$.

For any open set V, $\overset{\circ}{\overline{V}}$ is regular open. A finite intersection of regular open sets is regular open.

Corollary 1: *If S is open s.a., then there are finitely many g_{ij} satisfying* (1) *with $\{g_{ij} > 0\}$ regular open for each i, j.*

This answers a question of Brumfiel (1991), who had proved:

Corollary 2: *If S is open s.a., then there are finitely many g_{ij} such that*

$$S = \bigcup_i \Bigl(\overset{\circ}{\overline{\bigcap_j \{g_{ij} > 0\}}} \Bigr).$$

I.e., S is a finite union of "basically" (*not necessarily basic*) regular *open s.a. sets.*

Corollary 2 follows easily from Corollary 1 and the paragraph before it. And Corollary 1, in turn, follows immediately from the Theorem using the Finiteness Theorem and either of the following two (obvious) propositions:

Proposition 1: *For every f, $Z_{\mathrm{nt}}(f) = \emptyset$ iff $\{f > 0\}$ and $\{-f > 0\}$ are regular open.*

Proposition 2: *For every f, $\{f > 0\}$ is regular open iff $\overline{\{f > 0\}} \setminus \{f > 0\} \subseteq Z_t(f)$.*

Example: Let $n = 2$, and write (X, Y) instead of (X_1, X_2). Then $Z_{\mathrm{nt}}(Y^2 - X^2(X-1)) = \{(0,0)\} \neq \emptyset$, and hence $\{Y^2 - X^2(X-1) > 0\}$ is not regular open. But the latter equals

$$\begin{aligned} &(\{Y(Y^2 - X^2(X-1)) > 0\} \cap \{Y > 0\}) \\ &\cup (\{Y(Y^2 - X^2(X-1)) < 0\} \cap \{Y < 0\}) \cup (\{X > 0\} \cap \{1 - X < 0\}) \cup \{X < 0\}, \end{aligned}$$

where each polynomial on the righthand side has only transversal zeros, as in the Theorem.

Paweł Domański, *Extension properties of real analytic sets and composition operators* :
Some properties of real analytic sets turned out to be crucial in the study of composition operators C_φ, $C_\varphi(f) = f \circ \varphi$, acting on spaces $\mathscr{A}(\Omega)$ of real analytic functions, where $\varphi : \Omega \to \Omega$ is a fixed analytic map and Ω is a real analytic manifold.

For instance, we prove that for semi-proper φ the image of φ is an analytic set with an extension property (i.e., every real analytic function on $\varphi(\Omega)$ extends to the whole space) if and only if C_φ has closed range. Analogously, C_φ is open onto its image if and only if $\varphi(\Omega)$ has some "semi-local" extension property and φ is semi-proper.

We compare these two extension properties, explain which real analytic sets satisfy these extension properties and which do not satisfy. We present some open problems and compare results for C_φ on spaces of real analytic functions with analogous results for operators on spaces of smooth functions.

Based on a joint work with M. Goliński (Poznań) and M. Langenbruch (Oldenburg).

Nicolas Dutertre, *On the topology of semi-algebraic functions on closed semi-algebraic sets* :
We consider a closed semi-algebraic set $X \subset \mathbb{R}^n$ and a C^2 semi-algebraic function $f : \mathbb{R}^n \to \mathbb{R}$ such that $f_{|X}$ has a finite number of critical points. We relate the topology of X to the topology of the sets $X \cap \{f * \alpha\}$, where $* \in \{\leq, =, \geq\}$ and $\alpha \in \mathbb{R}$, and the indices of the critical points of $f_{|X}$ and $-f_{|X}$. We also relate the topology of X to the topology of the links at infinity of the sets $X \cap \{f * \alpha\}$ and the indices of these critical points. We give applications when $X = \mathbb{R}^n$ and when f is a generic linear function.

Ido Efrat, *Topological spaces as spaces of* $\mathbb{R}$*-places* :
The set of $\mathbb{R}$-places of a field of characteristic not 2 carries a natural topology, induced from its space of orderings. Becker and Gondard asked which topological spaces are realizable in this way. We report on recent progress made on this open problem. In particular, we show that this class of spaces is closed under various topological constructions.

Joint work with Katarzyna Osiak.

Abdelhafed Elkhadiri, *Link between noetherianity and weierstrass division theorem on some quasianalytic local rings* :
In the setting of well behaved quasianalytic differentiable system, we prove that Weierstrass Division Theorem holds if, and only if, the system is Noetherian.

José Fernando, *On the polynomial and regular images of* $\mathbb{R}^n$:
The first part of this talk is devoted to present a panoramic view of the main results concerning the study of the polynomial and regular images of $\mathbb{R}^n$ developed during the last 20 years. As far as we know the problem of determining the semialgebraic sets which are either polynomial or regular images of $\mathbb{R}^n$ was firstly proposed by Gamboa in the 1990 Oberwolfach week "Reelle Algebraische Geometrie".

In the second part of the talk we present some new results developed during the last year. We prove first that the set of points at infinite of a semialgebraic set $S \subset \mathbb{R}^m$ which is the image of a polynomial map $f : \mathbb{R}^n \to \mathbb{R}^m$ is connected. This result is no further true in general if f is a regular map, although it still works for a large family of regular maps that we call "quasi-polynomial maps". We also provide new obstructions for a semialgebraic set $S \subset \mathbb{R}^m$ to be the image of an either polynomial or regular map $f : \mathbb{R}^n \to \mathbb{R}^m$. Finally, we present a full geometric characterization of the 1-dimensional polynomial and regular images of $\mathbb{R}^n$.

Séverine Fiedler-Le Touzé, *M-curves of degree 9* :

The first part of Hilbert's sixteenth problem deals with the classification of the isotopy types realizable by real plane algebraic curves of a given degree m. For $m = 9$ the classification of the M-curves is still wide open. After systematic constructions, Korchagin formulated three conjectures predicting that some lists of isotopy types shouldn't be realizable. We will present the current state of knowledge about these conjectures, and expose some restrictions. The method, inspired from the classical one, combines Bezout's theorem with rational curves or pencils of curves, and all of the existing results on complex orientation. The novelty here is the involvement of auxiliary cubics and quartics, and of Orevkov's complex orientation formulas.

Sergey Finashin, *Real cubics and their varieties of lines* :

I will mainly focus on the Fano variety of lines on a real cubic threefold and the related spectral curves (plane real quintics with a Spin structure).

Andrei Gabrielov, *Semi-monotone sets and triangulation of tame monotone families* :

Let $S(t)$, for $t > 0$, be a monotone (decreasing) family of compact sets in a compact subset K of $\mathbb{R}^n$. Both $S(t)$ and K are assumed to be definable in an o-minimal structure (for example, real semialgebraic). The following problem emerges from a conjecture formulated by Gabrielov and Vorobjov (2009) in connection with their work on approximation of a definable set by homotopy equivalent compact sets: Construct a triangulation of K so that restriction of $S(t)$ to each open simplex is equivalent to one of the $1 + 2^n$ "standard" families. The list of standard families is based on lex-monotone Boolean functions in n Boolean variables. This can be done for $n < 4$. A weaker conjecture claims that K admits a regular cell decomposition such that restriction of $S(t)$ to each k-cell is a family of regular k-cells, and its boundary is a family of regular $(k-1)$-cells. To prove this conjecture, Basu, Gabrielov and Vorobjov (2010) introduced semi-monotone sets, a generalization of convex sets. Definable semi-monotone sets are PL-regular cells. They are related to regular Boolean functions, for which the result of any quantifier elimination does not depend on the order of quantifiers.

Joint work with S. Basu and N. Vorobjov.

Riccardo Ghiloni, *The principle of moduli flexibility in Real Algebraic Geometry* :

This talk deals with deformations of algebraic structures in the *purely real* setting.

The notion of deformation of the complex analytic structure of a given compact complex analytic manifold M has been studied since the time of Riemann, who considered the 1–dimensional case. In simplest terms, a deformation of M is a family $\{M_t\}_{t \in B}$ of compact complex analytic manifolds, parametrized by a domain B of some $\mathbb{C}^n$, depending analytically on $t \in B$, such that $M_{t_0} = M$ for some $t_0 \in B$. In this context, a first basic problem is to compute the maximum number of effective parameters on which a deformation of M can depend. As discovered by Kodaira, Spencer and Kuranishi, such a maximum number is *finite* for each compact complex analytic manifolds.

In the setting of complex algebraic geometry, the notion of deformation has a different, more algebraic, nature. In fact, it is deeply connected with the moduli problem; that is, the problem of finding spaces, called moduli spaces, that classify, up to complex biregular isomorphism, all the projective complex algebraic manifolds with assigned numerical invariants or additional structures as the polarizations. Anyway, we can assert again that the complex algebraic structure of every projective complex algebraic manifold can be deformed by an at most *finite* number of effective parameters.

In complex algebraic geometry, a real manifold is usually defined as a pair (X, σ) in which X is a projective complex algebraic manifold and $\sigma : X \longrightarrow X$ is an anti–holomorphic involution. We call (X, σ) *real–complex* algebraic manifold and σ *real–complex* algebraic structure on X. In this real–complex setting, we do not know if the real–complex algebraic structure of *every* real–complex algebraic manifold can be deformed by an at most finite number of effective parameters. However, if the real isomorphic class of a given real–complex algebraic manifold (X, σ) belongs to a (coarse) real moduli space $\mathcal{R}$, then the maximum number of effective parameters on which a deformation of (X, σ) can depend is $\leq \dim \mathcal{R}$ and hence is finite. A shining example is the one of real–complex curves.

In this talk, we treat the notion of deformation of real algebraic structures from the point of view of *purely real* algebraic geometry; that is, of the real algebraic geometry systematically studied, as an independent discipline, in the foundational book "Real Algebraic Geometry" of Bochnak, Coste and Roy. As far as we know, this is the first time that such a treatment has been done.

The main purpose of this talk is to make rigorous the following informal principle, which is in sharp contrast with the complex analytic, complex algebraic and real–complex algebraic cases.

Principle of real algebraic moduli flexibility. The algebraic structure of every real algebraic manifold of positive dimension can be deformed by an arbitrarily large number of effective parameters.

Joint work with Edoardo Ballico.

Dima Grigoriev, *Complexity of resolution of singularities* :

We estimate the complexity of Hironaka's desingularization algorithm in terms of Grzegorczyk's classes (the latter being a hierarchy of primitive-recursive functions). The main conclusion is that the dimension of a variety brings the principal contribution into the complexity bound.

Joint work with E.Bierstone, P.Milman, J.Wlodarczyk.

Viatcheslav Kharlamov, *Anti-symplectic involutions on rational symplectic 4-manifolds.*

Igor Klep, *The Convex Nichtnegativstellensatz in a free algebra* :

Given linear matrix inequalities (LMIs) L_1 and L_2 it is natural to ask: does one dominate the other? That is,

$$\text{(Q)} \qquad \text{does } L_1(x) \succeq 0 \text{ imply } L_2(x) \succeq 0?$$

In this talk we describe a natural relaxation of an LMI, based on substituting matrices for the variables x_j. With this relaxation, the domination question (Q) has an elegant answer. Indeed, for our "matricial" relaxation, a positive answer to (Q) is equivalent to the existence of matrices V_j such that

$$\text{(A)} \qquad L_2(x) = V_1^T L_1(x) V_1 + \cdots + V_r^T L_1(x) V_r.$$

The relaxed LMI domination problem is equivalent to a classical problem in operator algebras. Namely, the problem of determining if a linear map is *completely positive.*

Algebraic certificates for positivity, such as (A) for LMIs, are typically called Positivstellensätze. We shall also give a positivity certificate for polynomials: p is *positive semidefinite* on the matricial LMI domain $L(X) \succeq 0$ if and only if it has a weighted sum of squares representation with optimal degree bounds:

$$\text{(B)} \qquad p(x) = s(x)^T s(x) + \sum_j f_j(x)^T \, L(x) \, f_j(x),$$

where $s(x), f_j(x)$ are vectors of polynomials of degree no greater than $\deg(p)/2$.

A main ingredient of the proof is an analysis of extensions of *Hankel matrices.*

Based on joint papers with J.W. Helton and S. McCullough.

Manfred Knebusch, *Tropical and supertropical degenerations of a commutative ring* :

If R is a commutative ring, then degeneration of R to a "simpler" commutative ring usually means moding out the congruence relation by an ideal. In particular a field does not have such a degeneration. Things become more interesting if we allow degeneration to semirings. The simplest such degenerations are provided by m-valuations (= monoid valuations). They can be interpreted as a modest generalization of the valuations on R in the sense of Bourbaki (Alg. comm. Chap. 6). An m-valuation is a multiplicative and subadditive map $v \ : R \to M$ to a totally ordered semiring of very special kind, a so-called "bipotent" semiring.
An m-valuation $v \ : R \to M$ can be "covered" by a supervaluation $\varphi \ : R \to U$ in various ways. This means degenerating R to a multiplicative submonoid of a "supertropical" semiring U. Applying a supervaluation φ to the coordinates of R-valued points of an affine scheme V over R means degenerating $V(R)$ in a less coarse way than by applying v. The various supertropical degenerations of $V(R)$ provide a refinement of tropical geometry. If time allows I will give natural examples of m-valuations and supervaluations in the talk.

Joint work with Zur Izhakian and Louis Rowen.

Wojciech Kucharz, *Transcendental submanifolds of projective space* :

A smooth submanifold M of real projective n-space $\mathbb{P}^n(\mathbb{R})$ is said to be of algebraic type if it is isotopic in $\mathbb{P}^n(\mathbb{R})$ to the set of real points of a nonsingular complex algebraic subset of $\mathbb{P}^n(\mathbb{C})$ defined over $\mathbb{R}$; otherwise M is said to be transcendental. If codim $M = 1$ or $2 \dim M < n$, then M is of algebraic type. In particular, every submanifold of dimension 1 is of algebraic type. It is not at all obvious that transcendental submanifolds exist. I will give an explicit construction of transcendental submanifolds of any dimension greater than 3 and of any codimension greater than 1. The result is particularly nice for submanifolds of codimension 2.

Aaron Kunert, *Faces of cones of nonnegative quartics* :

We study the facial structure of cones of positive semidefinite quartics. For certain types of faces we will name an ambient vector space and give a criterion of fulldimensionality in this vector space. This leads to an estimate of occurring dimensions of faces. We will compare them to dimensions of corresponding faces of the cone of sums of squares and point out explicit dimensional differences between these cones. As an application we can list all faces of the cone of ternary quartics and thereby we obtain a complete description of the facial structure of this cone. In particular this will give us an alternative proof of Hilbert's theorem.

Noa Lavi, *Some possitivstellensatz in real closed valued fields* :

The purpose of this talk is to give a generalization of Hilbert's seventeenth problem in real closed valued fields, that is, to give an algebraic characterization, for a definable set, of the set of polynomials which get only non-negative values on it. We give a general characterization of the positive semi-definite polynomials for any definable set with a Ganzstellensatz, and we also give a representation of those polynomials in the sense of Hilbert 17th problem (that is, in terms of sums of squares) for definable sets from a certain kind.

Antonio Lerario, *Systems of Quadratic Inequalities* :

Systems of quadratic inequalities are very flexible objects in mathematics, e.g any system of polynomial equations can be reduced to a system of quadratic equations by substitutions. Thus the set X of the solutions of a system of quadratic inequalities can describe a very large class of semi-algebraic sets (the complexity of X is hidden in the number of linearly independent inequalities). To study such a system we focus on the dual object: the convex hull, in the space of all real quadratic forms on $\mathbb{R}^n$, of those quadratic forms involved in the system (n is the number of variables in the system). It turns out that the homology of X is determined by the arrangement of this convex hull with respect to the cone of degenerate forms. This approach allows to efficiently compute homology for a very big number of variables n as long as the number of linearly independent inequalities is limited. Moreover, it works also for systems of integral quadratic inequalities, i.e. in the infinite dimension, beyond the semi-algebraic context. The calculations are organized in a spectral sequence whose member E_2 and the differential d_2 have a simple clear geometric interpretation.

Joint work with A. Agrachev.

Thierry Limoges, *Products of real weight filtrations* :

The weight filtration for real algebraic varieties has been developed by McCrory and Parusiński, by analogy with Deligne's weight filtration for complex algebreaic varieties. They associate to each variety X a filtered chain complex $\mathcal{G}_\bullet C_*(X)$ which computes the Borel-Moore homology $H_*^{BM}(X)$, and a spectral sequence $E^r(X)$, functorial and additive for closed inclusions. The filtration is build using semi-algebraically constructible functions. We explain how the operation of cross product allows us to compare the respective filtered complexes and spectral sequences for varieties X, Y and their product $X \times Y$. We have a dual cohomological theory on $H_c^*(X)$, which gives informations about cup and cap products of X.

Henri Lombardi, *Effective Positivstellensatz* :

In this talk, we speak about a work in progress with D. Perrucci and M.-F. Roy about bounds on the real Positivstellensatz. A Positivstellensatz is a rational algebraic certificate of impossibility for a system of polynomial equalities and inequalities in a real closed field. This certificate can also be seen as a very elementary proof of impossibility within the theory of ordered domains. Previous algorithmic proofs of the Positivstellensatz are based upon very long elementary proofs of impossibility and lead to very large bounds on the degree of the certificate. More geometric proofs of impossibility lead to testing emptiness through algorithms of simple exponential size. But these "short" algorithmic proofs are very unlikely transformed in algebraic certificates. Finding a "good" algorithm for the Positivstellensatz is similar to finding a short and elementary proof for impossibilities within the theory of ordered domains. It happens that this could be achieved through a convenient variation on proofs through CAD (cylindric algebraic decomposition). But usual proofs that CAD work rely on the subtle notion of semialgebraic connectedness, which is not defined as a first order concept. So we have to replace connectedness arguments by a convenient use of Hermite theory of signature of real quadratic forms, which is based on the existence of complex roots. From the point of view of algebraic certificates, existence has to be replaced by a kind of dual notion, called "weak existence". It happens that weak existence of complex roots lead to triple exponential bounds. Combined with the usual double exponential bounds for CAD, we obtain, hopefully, 5-exponential bounds for the Positivstellensatz.

Frédéric Mangolte, *Topologie des variétés algébriques réelles de dimension* 3 :
La topologie des variétés algébriques réelles de dimension 3 est de mieux en mieux connue. Depuis que Kollar, il y a une dizaine d'années, a ouvert une voie d'étude grâce une solution du MMP sur $\mathbb{R}$, les avancées ont été nombreuses. Dans cet exposé, je parlerai de plusieurs conjectures de Kollar résolues depuis. On se rapproche d'une classification des 3-variétés uniréglées et des 3-variétés rationnellement connexes. Je décrirai l'état de l'art concernant ce problème, et en particulier mes contributions obtenues en collaboration avec J. Huisman, F. Catanese, et J.-Y. Welschinger.

Arnaud Moncet, *Real versus complex volumes on real algebraic surfaces* :
Let X be a real algebraic surface. The comparison between the volume of $D(\mathbb{R})$ and $D(\mathbb{C})$ for ample divisors D brings us to define the concordance $\alpha(X)$, which is a number between 0 and 1. This number equals 1 when the Picard number $\rho(X_\mathbb{R})$ is 1, and for some surfaces with a "quite simple" nef cone, e.g. Del Pezzo surfaces. For abelian surfaces, $\alpha(X)$ is $1/2$ or 1, depending on the existence or not of positive entropy automorphisms on X. In the general case, the existence of such an automorphism gives an upper bound for $\alpha(X)$, namely the ratio of entropies $\mathrm{h}_{top}(f|X(\mathbb{R}))/\mathrm{h}_{top}(f|X(\mathbb{C}))$. Moreover $\alpha(X)$ is equal to this ratio when the Picard number is 2. An interesting consequence of the inequality is the non-density of $\mathrm{Aut}(X_\mathbb{R})$ in $\mathrm{Diff}(X(\mathbb{R}))$ as soon as $\alpha(X) > 0$. Finally we show, thanks to this upper bound, that there exist K3 surfaces with arbitrary small concordance, considering a deformation of a singular surface of tridegree $(2,2,2)$ in $\mathbb{P}^1 \times \mathbb{P}^1 \times \mathbb{P}^1$.

Seydou Moussa, *Singularités des robots* 6-$U\underline{P}S$:
L'exposé a pour but de présenter une étude géométrique des singularités des robots parallèles 6 degrés de liberté de type 6-$U\underline{P}S$. Ce dernier présente beaucoup de similarités avec les robots parallèles plans 3-$R\underline{P}R$. En utilisant un espace de travail modifié et une paramétrisation de la surface des configurations singulières, le Professeur Michel Coste a donné en particulier une preuve géométrique simple du fait qu'un robot 3-$R\underline{P}R$ générique a deux aspects (deux composantes connexes). Nous nous proposons d'utiliser cette méthode dans l'étude du robot 6-$U\underline{P}S$.

Tim Netzer, *Polynomials with and without determinantal representations* :

The problem of writing real zero polynomials as determinants of linear matrix polynomials has recently attained a lot of attention. It is in fact the algebraic question behind the geometric problem to characterize spectrahedra. Spectrahedra are the feasible sets of semidefinite optimization problems, and thus of great importance. I will discuss positive and negative results concerning the problem of finding determinantal representations of polynomials.

Ha Nguyen, *Polynomials Non-negative on Strips and Half-strips* :

Recently, M. Marshall answered a long-standing question in real algebraic geometry by showing that if $f \in \mathbb{R}[x, y]$ and $f \geq 0$ on the strip $[0, 1] \times \mathbb{R}$, then f has a representation $f = \sigma_0 + \sigma_1 x(1 - x)$, where $\sigma_0, \sigma_1 \in \mathbb{R}[x, y]$ are sums of squares.

Representation theorems of this type have a rich and remarkable history, going back at least to Hilbert. In this talk, we give the background to Marshall's result, and our generalizations to other non-compact semialgebraic subsets of $\mathbb{R}^2$. Our results give many new examples of non-compact semialgebraic sets in $\mathbb{R}^2$ for which all polynomials that are non-negative on the set can be characterized.

Andreea Nicoara, *The Non-Noetherianity of the Denjoy-Carleman Rings of Germs* :

A ring R of Denjoy-Carleman quasianalytic germs of functions that is stable under derivation and strictly contains the ring of analytic germs is not Noetherian in dimension 2 or higher. This result settles a question open since 1976 when Childress proved such rings fail Weierstrass Division. The argument uses a stronger version of Artin Approximation.

Joint work with Liat Kessler (Technion).

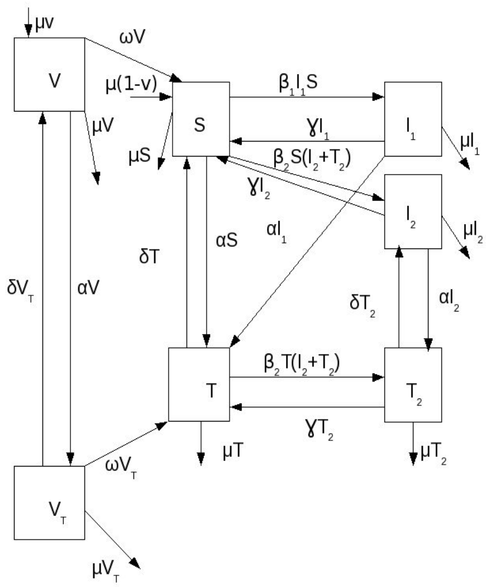
Krzysztof Nowak, *On the singular locus of sets definable in a quasianalytic structure* :

Given a quasianalytic structure, I prove that the singular locus of a quasi-subanalytic set E is a closed quasi-subanalytic subset of E. I rely on some stabilization effects linked to Gateaux differentiability and formally composite functions. An essential ingredient of the proof is a quasianalytic version of Glaeser's composite function theorem, presented in my earlier paper.

Adamou Otto, *Analysis of a two-strain transmission model with vaccination using computer algebra* :

We present a typical example of a compartmental transmission model that can be dealt with algebraically. We use exact methods from real algebraic geometry and computer algebra to find all the equilibria of the ODE system describing the model and to study their stability as well as their bifurcations.

The model concerns a host population, a part of its individuals are under antibiotic (Ab) treatment against a two-strain bacterial pathogen. Individuals who are not under Ab treatment can be colonized by an antibiotic-susceptible (Ab-S) strain or by an antibiotic-resistant (Ab-R) strain of a bacterial pathogen, but not by both at the same time (i.e., maximal competition), while those under antibiotic treatment can only be colonized by the Ab-R strain. We assume that there is a fitness cost for resistance such that the Ab-R strain is somewhat less transmissible than the Ab-S strain. The host population is subdivided into 7 compartments representing the fractions of the population in each state, 4 states for individuals not under Ab treatment: susceptible (S), colonized by the Ab-S strain (I_1) and colonized by the Ab-R strain (I_2), the individuals who are in the vaccinated state not under antibiotic treatment. those individuals are assumed to have a temporary complete immunity to infection by the 2 strains (V), and 3 states for individuals under Ab treatment: susceptible (T), and colonized by the Ab-R strain (T_2), the individuals who are in the vaccinated state currently under antibiotic treatment. Those individuals are assumed to have a temporary complete immunity to infection by the 2 strains (V_T) . We give below the transfer diagram of the model.



The model has a unique disease free equilibria and three other equilibrias. We give a complete characterization of their existence and stability. We also show that all codimension-one bifurcations are transcritical.

Joint work with M. EL Kahoui, M.-F. Roy and T. Van Effelterre.

Franklin Vera Pacheco, *Resolution of singularities of pairs preserving semi-simple normal crossings* :

A partial desingularization consists in removing all singularities, except for those of certain class S, with a proper birational map that is an isomorphism over the points already in S. For example, if S consists only of the smooth *singularities*, then a partial desingularization in this sense corresponds to the usual (strong) resolution of singularities. For other classes of singularities this problem has also been studied, solved or proved impossible, e.g. simple normal crossings, normal crossings, normal singularities, rational singularities... It was asked by János Kollár the existence of a partial desingularization preserving the semi simple normal crossings singularities of a pair. This is the analogous of simple normal crossings singularities in a non-normal ambient space. We show how to produce this partial desingularization by using a general philosophy applicable to some of these problems.

Joint work with Edward Bierstone.

Daniel Pecker, *On the minimum degree of a polynomial knot representing a given knot* :

A polynomial knot is a polynomial embedding of the real line into Euclidean space. The study of the space of polynomial knots of a given degree d is achieved only for $d \leq 4$. We study a converse problem: given a knot, what is the minimum degree of a polynomial knot representing it ? We give answers for the simplest knots, and for some infinite families of knots.

Albrecht Pfister, *An elementary and constructive proof of Hilbert's theorem on ternary quartics* :

We present a new proof of Hilbert's theorem which is elementary in the sense that it uses only undergraduate methods from algebra, analysis and topology including the implicit function theorem. The proof is also essentially constructive and shows that there are at most 8 inequivalent representations as a sum of 3 squares for a given positive semidefinite quartic over the reals.

Joint work with Claus Scheiderer.

Daniel Plaumann, *Quartic curves and their bitangents* :

We will consider two types of representations of real ternary quartics: 1) As determinants of symmetric 4x4-matrices with linear entries. 2) As sums of three (signed) squares. We will discuss explicit constructions of these representations, using the classical theory of bitangents. We also address more recent connections to spectrahedra and semidefinite programming.

Joint work with Bernd Sturmfels and Cynthia Vinzant.

Lucas Prelli, *O-minimal sheaf theory* :

O-minimal sheaf theory generalize Delfs semi-algebraic and real algebraic sheaf theory as well as Kashiwara-Schapira sheaf theory on the (globally) sub-analytic site. The formalism of Grothendiecks six operations on o-minimal sheaves that we propose to develop generalizes also similar works in the topological context (Verdier) and the subanalytic context (Kashiwara-Schapira and Prelli). Besides the interest that this theory has on its own, it will provide the main tools for the cohomological approach to problems related to algebraic analysis one one side and o-minimal geometry on the other.

Joint work with M. Edmundo.

Armin Rainer, *Quasianalytic and Lipschitz perturbation theory for normal operators* :

We study the regularity of the eigenvalues and the eigenvectors of families of normal operators. This is evidently connected to the regularity of the roots of complex polynomials. Surprisingly though, the eigenvalues and eigenvectors possess much better regularity properties than the roots. For instance, we shall see that the eigenvalues and eigenvectors of a real analytic (or, even quasianalytic) family of normal matrices may be desingularized by means of local blow-ups; for the roots, in addition, we must substitute powers. Moreover, any continuous eigenvalue of a Lipschitz family of normal operators is Lipschitz; the roots need not be Lipschitz even if the coefficients are polynomial. We will also state infinite dimensional versions of our results, i.e., for normal operators in a Hilbert space with compact resolvents and common domain of definition.

Tomas Recio, *Generalizing circles over algebraic extensions* :

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This contribution deals with a family of spatial rational curves that were introduced by Carlos Andradas, Tomás Recio, and J. Rafael Sendra at *Base field restriction techniques for parametric curves*, (Proceedings of the 1999 International Symposium on Symbolic and Algebraic Computation (Vancouver, BC), ACM, 1999, pp. 17–22), under the name of *hypercircles*, as an algorithmic tool in the context of simplifying, if possible, the coefficients of the rational functions of a given parametrization of an algebraic variety.

The simplest hypercircles should be the circles themselves. Circles live on a real plane. We can think of the real plane as the field of complex numbers $\mathbb{C}$, an algebraic extension of the reals $\mathbb{R}$ of degree 2. Analogously, we can consider a characteristic zero base field $\mathbb{K}$ and an algebraic extension of degree n, $\mathbb{K}(\alpha)$. Let us identify $\mathbb{K}(\alpha)$ as the vector space $\mathbb{K}^n$, via the choice of a suitable base, such as the one given by the powers of α. This is the framework in which hypercircles are defined.

Circles are real rational curves. This means that there are two real rational functions $\phi(t) = (\phi_1(t), \phi_2(t))$ whose image cover almost all the points of the circle. Every proper (almost one-to-one) rational parametrization of a circle verifies that $\phi_1(t)+i\phi_2(t) = \frac{at+b}{ct+d} \in \mathbb{C}(t) \setminus \mathbb{C}$, which defines a conformal mapping $u : \mathbb{C} \to \mathbb{C}$. Moreover, if we identify $\mathbb{C}$ with $\mathbb{R}^2$, the image of the real axis $(t, 0)$ under u is exactly the circle parametrized by $\phi(t)$. Conversely, let $u(t) = \frac{at+b}{ct+d} \in \mathbb{C}(t)$ be a unit of the near-ring $\mathbb{C}(t)$ under the composition operator. If $c \neq 0$ and $d/c \notin \mathbb{R}$ then, the closure of the image by u of the real axis is a circle. Otherwise, it is a line.

This method to construct circles generalizes easily to algebraic extensions. Namely, let $u(t) = \frac{at+b}{ct+d}$ be a unit of $\mathbb{K}(\alpha)(t)$ (i.e. verifying that $ad - bc \neq 0$). Let us identify $\mathbb{K}(\alpha)$ with $\mathbb{K}^n$ and let u be the map

$$u: \begin{array}{ccc} \mathbb{K}(\alpha) \approx \mathbb{K}^n & \to & \mathbb{K}(\alpha) \approx \mathbb{K}^n \\ t & \mapsto & u(t) \end{array}.$$

Then, the Zariski-closure of the image of the axis $(t, 0, \dots, 0)$ under the map u is a rational curve in $\mathbb{K}^n$. These curves are, by definition, our hypercircles.

Example 0.1. Let us consider the algebraic extension $\mathbb{Q} \subseteq \mathbb{Q}(\alpha)$, where $\alpha^3 + 2\alpha + 2 = 0$. The unit $\frac{t-\alpha}{t+\alpha}$ has an associated hypercircle parametrized by

$$\phi(t) = \left(\frac{t^3 + 2t + 2}{t^3 + 2t - 2}, \frac{-2t^2}{t^3 + 2t - 2}, \frac{2t}{t^3 + 2t - 2} \right)$$

Let $\mathbb{K}$ be a field of characteristic zero, $\mathbb{K} \subseteq \mathbb{L}$ a finite algebraic extension of degree n and $\mathbb{F}$ the algebraic closure of $\mathbb{K}$ and α be a primitive element of $\mathbb{L}$ over $\mathbb{K}$. As it stands, the definition of a hypercircle $\mathcal{U}$ depends on a given unit $u(t)$ and on a primitive generator α of an algebraic extension $\mathbb{K} \subseteq \mathbb{L}$. But notice that, given a unit $u(t) \in \mathbb{L}(t)$ and two different primitive elements α and β of the extension $\mathbb{K} \subseteq \mathbb{L}$, we can expand the unit in two different ways $u(t) = \sum_{i=0}^{n-1} \alpha^i \phi_i(t) = \sum_{i=0}^{n-1} \beta^i \psi_i(t)$. The hypercircles $\mathcal{U}_\alpha \simeq (\phi_0(t), \dots, \phi_{n-1}(t))$ and $\mathcal{U}_\beta \simeq (\psi_0(t), \dots, \psi_{n-1}(t))$ generated by $u(t)$ are different curves in $\mathbb{F}^n$, see Example 0.2.

Example 0.2. Let us consider the algebraic extension $\mathbb{Q} \subseteq \mathbb{Q}(\alpha)$, where $\alpha^4 + 1 = 0$. Let us take the unit $u(t) = \frac{t-\alpha}{t+\alpha}$. By normalizing $u(t)$, we obtain the parametrization $\phi(t)$ associated to $u(t)$:

$$\phi(t) = \left(\frac{t^4 - 1}{t^4 + 1}, \frac{-2t^3}{t^4 + 1}, \frac{2t^2}{t^4 + 1}, \frac{-2t}{t^4 + 1} \right).$$

This hypercircle $\mathcal{U}_\alpha$ is the zero set of $\{X_1X_2 - X_3X_0 - X_3, X_1^2 + X_3^2 - 2X_2, X_1X_0 + X_2X_3 - X_1, X_0^2 + X_3X_1 - 1\}$. Now, we take $\beta = \alpha^3 + 1$, instead of α, as the primitive element of $\mathbb{Q}(\alpha) = \mathbb{Q}(\beta)$. The same unit $u(t)$ generates the β-hypercircle $\mathcal{U}_\beta$ parametrized by

$$\psi(t) = \left(\frac{t^4 + 2t^3 - 2t^2 + 2t - 1}{t^4 + 1}, \frac{-6t^3 + 4t^2 - 2t}{t^4 + 1}, \frac{6t^3 - 2t^2}{t^4 + 1}, \frac{-2t^3}{t^4 + 1} \right),$$

which is different to $\mathcal{U}_\alpha$; note that $\psi(1) = (1, -2, 2, -1)$ that does not satisfy the equation $X_0^2 + X_3X_1 - 1 = 0$ of $\mathcal{U}_\alpha$.

Nevertheless, let $\mathcal{A} \in \mathcal{M}_{n\times n}(\mathbb{K})$ be the matrix of change of basis from $\{1, \alpha, \ldots, \alpha^{n-1}\}$ to $\{1, \beta, \ldots, \beta^{n-1}\}$. Then, $\mathcal{A}(\phi_0(t), \ldots, \phi_{n-1}(t))^t = (\psi_0(t), \ldots, \psi_{n-1}(t))^t$. That is, it carries one of the curve onto the other. Thus, $\mathcal{U}_\alpha$ and $\mathcal{U}_\beta$ are related by the affine transformation induced by the change of basis and, so, they share many important geometric properties.

The aim of this talk is to extend, to the case of hypercircles, some of the specific properties of circles. We will show that hypercircles are precisely, via $\mathbb{K}$-projective transformations, the rational normal curve of a suitable degree. We will also obtain a complete description of the points at infinity of these curves (generalizing the cyclic structure at infinity of circles). We will characterize hypercircles as those curves of degree equal to the dimension of the ambient affine space and with infinitely many $\mathbb{K}$-rational points, passing through these points at infinity. If time permits, we will give explicit formulae for the parametrization and implicitation of hypercircles.

Besides the intrinsic interest of this very special family of curves, we think the understanding of its properties has a direct application to the simplification of parametrizations problem.

The talk will be based on a recent paper by R. Sendra, L.F. Tabera, C. Villarino and the author: *Generalizing circles over algebraic extensions*, Mathematics of Computation, Volume 79, Number 270, April 2010, Pages 1067–1089.

Bruce Reznick, *Sums of fourth powers of real polynomials* :

What are necessary and sufficient conditions on a real polynomial $p(x)$ in one variable so that there exist polynomials $h_k(x)$ so that $p = \sum_k h_k^4$? We will present some partial results and conjectures.

Jean-Jacques Risler, *On the curvature of the Real Amoeba*

For a real smooth algebraic curve $A \subset (\mathbb{C}^*)^2$, the amoeba $\mathcal{A} \subset \mathbb{R}^2$ is the image of A under the map Log : $(x, y) \mapsto (\log|x|, \log|y|)$. We describe an universal bound for the total curvature of the real amoeba $\mathcal{A}_{\mathbb{R}A}$ and we prove that this bound is reached if and only if the curve A is a simple Harnack curve in the sense of Mikhalkin.

Joint work with Mikael Passare.

Claus Scheiderer, *Positive polynomials and sums of hermitian squares* :

In 1968, Quillen proved that every real polynomial that is strictly positive on a Euclidean sphere in complex n-space $\mathbb{C}^n$ coincides with a sum of hermitian squares on that sphere. We give an abstract characterization of all real algebraic subsets X of $\mathbb{C}^n$ on which every strictly positive polynomial is a hermitian sum of squares and discuss the relation with commuting subnormal tuples of operators. We also plan to discuss extensions of the results to semi-algebraic subsets of $\mathbb{C}^n$.

Joint work with Mihai Putinar.

Marco Schlichting, *The Mayer-Vietoris principle for Grothendieck-Witt groups of schemes* :

Extending Knebusch's definition of the Grothendieck-Witt group of a scheme to categories of chain complexes, and in analogy with algebraic K-theory, we define higher Grothendieck-Witt groups of a scheme (or category of chain complexes) as the homotopy groups of an explicitly defined topological space. We show that an open covering of a scheme with an ample family of line-bundles gives rise to a Mayer-Vietoris long exact of the corresponding higher Grothendieck-Witt groups. The main point here is that all this works even when the scheme is singular and 2 is not invertible in the ring of regular functions.

Tamara Servi, *Preparation theorem and quantifier elimination for quasi-analytic classes* :

We consider the structures generated by a family of quasi-analytic algebras of functions which have asymptotic expansion as generalised power series. We show that these structures are o-minimal and polynomially bounded. Furthermore, we prove that van den Dries' and Speissegger's preparation theorem for definable functions admits in these structures an "explicit" form, from which we deduce a quantifier elimination result (in a reasonable language).

Joint work with J.-P. Rolin.

Masahiro Shiota, *By replacement of real closed fields* :

By replacement of real closed fields we can sometimes prove globally problems on real algebraic geometry when they are already proved locally by algorithm. This is the case for the second Łojasiewicz inequality. Let f be a polynomial function on $\mathbb{R}^n$. Then there exist a semialgebraic neighborhood V of $f^{-1}(0)$ in $\mathbb{R}^n$ and a number θ such that $0 < \theta < 1$ and $|f(x)|^\theta \leq \sum_{i=1}^n |\frac{\partial f}{\partial x_i}(x)|$ for $x \in V$.

Rainer Sinn, *SO(2)-Orbitopes* :

An SO(2)-orbitope is the convex hull of an orbit under some linear action of SO(2) on a finite dimensional real vector space. Such a set is always a compact convex semi-algebraic set. We will study the question whether or not it is basic closed, i.e. defined by a finite number of simultaneous polynomial inequalities. We will be particularly interessted in the case of the so-called Barvinok-Novik orbitopes. This work is in progress.

Ahmed Srhir, *Lojasiewicz's exponents in o-minimal structures* :

We prove the rationality of the Łojasiewicz's exponent for definable functions in polynomially bounded *o*-minimal structures with certain conditions. In the parametric case, we show that the parameter space can be splitting into finitely many definable subsets on each of which the Łojasiewicz's exponent is constant.

Zbigniew Szafraniec, *Quadratic forms and the intersection number for polynomial immersions* :

Several important invariants associated with polynomial mappings and real algebraic sets may be expressed in terms of signatures of appropriate quadratic forms.

There will be presented such methods of computing the intersection number for polynomial immersions.

Carlos Ueno, *On convex polyhedra as regular images of* $\mathbb{R}^n$. :

We show that convex polyhedra in $\mathbb{R}^n$ and their interiors are images of regular maps $\mathbb{R}^n \to \mathbb{R}^n$. As a main ingredient in the proof we construct, given an n-dimensional, bounded, convex polyhedron $K \subset \mathbb{R}^n$ and a point $p \in \mathbb{R}^n \setminus K$, a suitable partition of the boundary ∂K of K determined by p and compatible with the interiors of the faces of K. Finally, we also prove that closed balls in $\mathbb{R}^n$ and their interiors are images of regular maps $\mathbb{R}^n \to \mathbb{R}^n$.

Guillaume Valette, *De Rham theorems on singular varieties* :

I will explain how some recent results of mine on the Lipschitz geometry of subanalytic sets make it possible to extend some theorems of differential geometry, such as the de Rham theorem, to the framework of (possibly singular) subanalytic varieties.

Nicolai Vorobjov, *Approximation of definable sets by compact families* :

We suggest a construction for approximating a large class of sets, definable in an o-minimal structure over the reals, by compact sets. The class includes sets defined by arbitrary Boolean combinations of equations and inequalities, and images of such sets under a large class of definable maps, e.g., projections. Based on this construction, we prove k-equivalence of definable sets to compact definable sets. This leads to a refinement of the known upper bounds on Betti numbers, and a proof of similar upper bounds, individually for different Betti numbers, for images under arbitrary continuous definable maps.

Joint work with Andrei Gabrielov.