USE AND EVALUATION OF ELECTRE III/IV

Raimon Redondo Pena, Laura Prieto Rebollo
Karina Gibert Oliveras, Aïda Valls Mateu

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Electre III method

The starting point for most outranking methods is a decision matrix describing the performance of the alternatives to be evaluated with respect to identified criteria. The output of an analysis is an outranking relation on the set of alternatives. An alternative *a* is said to outrank another alternative *b* if, taking account of all available information regarding the problem and the decision maker's preferences, there is a strong enough argument to support a conclusion that *a* is at least as good as *b* and no strong argument against.

Considering two alternatives *a* and *b*, four situations may occur:
- aSb and not bSa, i.e., aPb (*a* is strictly preferred to *b*).
- bSa and not aSb, i.e., bPa (*b* is strictly preferred to *a*).
- aSb and bSa, i.e., aIb (*a* is indifferent to *b*).
- Not aSb and not bSa, i.e., aRb (*a* is incomparable to *b*).

Concordance principle: If *a* is demonstrably as good as or better than *b* according to a sufficiently large weight of criteria, then this is considered to be evidence in favor of *a* outranking *b*.

Discordance principle: If *b* is very strongly preferred to *a* on one or more criteria, then this is considered to be evidence against *a* outranking *b*.

Terms

- i: index labeling a criterion.

- $g_i(a)$: individual partial preference function of the alternative *a* with regard to the criterion *i*.

- w_i: weight of the criterion *i*.

- Preference threshold [p_i]: is a difference above which the decision maker strongly prefers a management alternative over all for the criterion *i*.
Alternative *b* is strictly preferred to alternative *a* in terms of criterion *i* if
$g_i(b) > g_i(a) + p(g_i(a))$.

- Indifference threshold [q_i]: is a difference beneath which the decision maker is indifferent between two management alternatives for the criterion *i*.
Alternative *b* is weakly preferred to alternative *a* in terms of criterion *i* if
$g_i(b) > g_i(a) + q(g_i(a))$.

- Veto threshold [v_i]: blocks the outranking relationship between alternatives for the criterion *i*.
Alternative *a* cannot outrank alternative *b* if the performance of *b* exceeds that of *a* by an amount greater than the veto threshold, i.e. if $g_i(b) \geq g_i(a) + v_i(g_i(a))$.

- Concordance index [C(a,b)]: measures the strength of support, given the available evidence, that *a* is at least as good as *b* considering all criteria.

$C_i(a,b)$: concordance index over alternative *a* and *b* with regard to the criterion *i*.

- Discordance index [D(a,b)]: measures the strength of the evidence against this hypothesis.

$D_i(a,b)$: discordance index over alternative *a* and *b* with regard to the criterion *i*.

- Credibility index [S(a,b)]: measures the strength of the claim that "alternative a is at least as good as alternative b".

Algorithm

1- The start point is the decision matrix. The parameters p_i, q_i and v_i have to be defined by the user.

2- Calculate concordance index for each criterion:

$$C_i(a,b) = \begin{cases} 0, & \text{if } g_i(b) \geq g_i(a) + p_i(g_i(a)) \\[2mm] 1, & \text{if } g_i(b) \leq g_i(a) + q_i(g_i(a)) \\[2mm] \dfrac{g_i(a) + p_i(g_i(a)) - g_i(b)}{p_i(g_i(a)) - q_i(g_i(a))}, & \text{otherwise} \end{cases}$$

3- Calculate overall concordance index.

$$C(a,b) = \frac{\sum w_i C_i(a,b)}{\sum w_i}$$

4- Calculate discordance index for each criterion:

$$D_i(a,b) = \begin{cases} 0, & \text{if } g_i(b) \leq g_i(a) + p_i(g_i(a)) \\[2mm] 1, & \text{if } g_i(b) \geq g_i(a) + v_i(g_i(a)) \\[2mm] \dfrac{g_i(b) - g_i(a) - p_i(g_i(a))}{v_i(g_i(a)) - p_i(g_i(a))}, & \text{otherwise} \end{cases}$$

If no veto threshold (v_i) is specified $D_i(a,b) = 0$ for all pairs of alternatives.

5- Calculate credibility index:

$$
S(a,b) = \begin{cases} C(a,b), & \text{if } D_i(a,b) \leq C(a,b) \ \forall i \\[2em] C(a,b) \displaystyle\prod_{D_i(a,b)>C(a,b)} \dfrac{1-D_i(a,b)}{1-C(a,b)}, & \text{otherwise} \end{cases}
$$

If no veto thresholds (v_i) are specified $S(a,b) = C(a,b)$ for all pairs of alternatives.

6- Determine rank order:

- Descending distillation:

6.1- Determine the maximum value of the credibility index: $\lambda_{max} = \max S(a,b)$.

6.2- Calculate $\lambda = \lambda_{max} - (0.3 - 0.15\,\lambda_{max})$. Where -0.15 and 0.3 are the preset up values of distillation coefficients, α and β.

6.3- For each alternative a determine its λ-strength, i.e. the number of alternatives b with $S(a,b) > \lambda$

6.4- For each alternative a determine its λ-weakness, i.e. the number of alternatives b with $(1 - (0.3 - 0.15\lambda)) * S(a,b) > S(b,a)$

6.5- For each alternative determine its qualification, i.e. the difference between λ-strength and λ-weakness.

6.6- The set of alternatives with largest qualification is called the first distillate (D1).

6.7- If D1 has more than one alternative, repeat the process on the set D1 until all alternatives have been classified. If there is a single alternative, than this is the most preferred one. Then continue with the original set of alternatives minus the set D1, repeating until all alternatives have been classified.

- Ascending distillation:

This is obtained in the same way as the descending distillation but at step 6.6, the set of alternatives having the lowest qualification forms the first distillate.

- Final ranking:

There are several ways how to combine both orders. The most frequent is the intersection of two outranking relations: *aRb* (*a* outranks *b* according to *R*) if and only if *a* outranks or is in the same class as *b* according to the orders corresponding to both relationships.

Electre IV method

The Electre IV exploiting procedure is the same as in Electre III. Moreover, in this case there are 5 possible binary relations:

Quasi-dominance: The couple (b, a) verifies the relation of quasidominance if and only if:
- for every criterion, b is either preferred or indifferent to a,
- and if the number of criterion for which the performance of a is better than the one of b (a staying indifferent to b) is strictly inferior to the number of criteria for which the performance of b is better than the one of a.

Canonic-dominance: The couple (b, a) verifies the relation of canonic-dominance if and only if:
- for no criterion, a is strictly preferred to b,
- and if the number of criteria for which a is weakly preferred to b is inferior or equal to the number of criteria for which b is strictly preferred to a,
- and if the number of criteria for which the performance of a is better than the one of b is strictly inferior to the number of criteria for which the performance of b is better than the one of a.

Pseudo-dominance: The couple (b, a) verifies the relation of pseudo-dominance if and only if:
- for no criterion, a is strictly preferred to b,
- and if the number of criteria for which a is weakly preferred to b is inferior or equal to the number of criteria for which b is strictly or weakly preferred to a.

Sub-dominance: The couple (b, a) verifies the relation of subdominance if and only if:
- for no criterion, a is strictly preferred to b.

Veto-dominance: The couple (b, a) verifies the relation of vetodominance if and only if:
- either for no criterion, a is strictly preferred to b,
- or a is strictly preferred to b for only one criterion but this criterion not vetoing the outranking of a by b and furthermore, b is strictly preferred to a for at least half of the criteria.

Electre III/IV software

The Electre III/IV software has been developed between 1992 and 1994 by the Dauphine University of Paris and the Institute of Computazionali Sciences of the University of Poznan in Polonia. It implements the decision support models Electre III and IV.

Electre III starts with a finite set of actions evaluated on a consistent family of pseudo-criteria and aggregates these partial preferences into a fuzzy outranking relation. Electre IV builds several non-fuzzy outranking relations when criteria cannot be weighted.
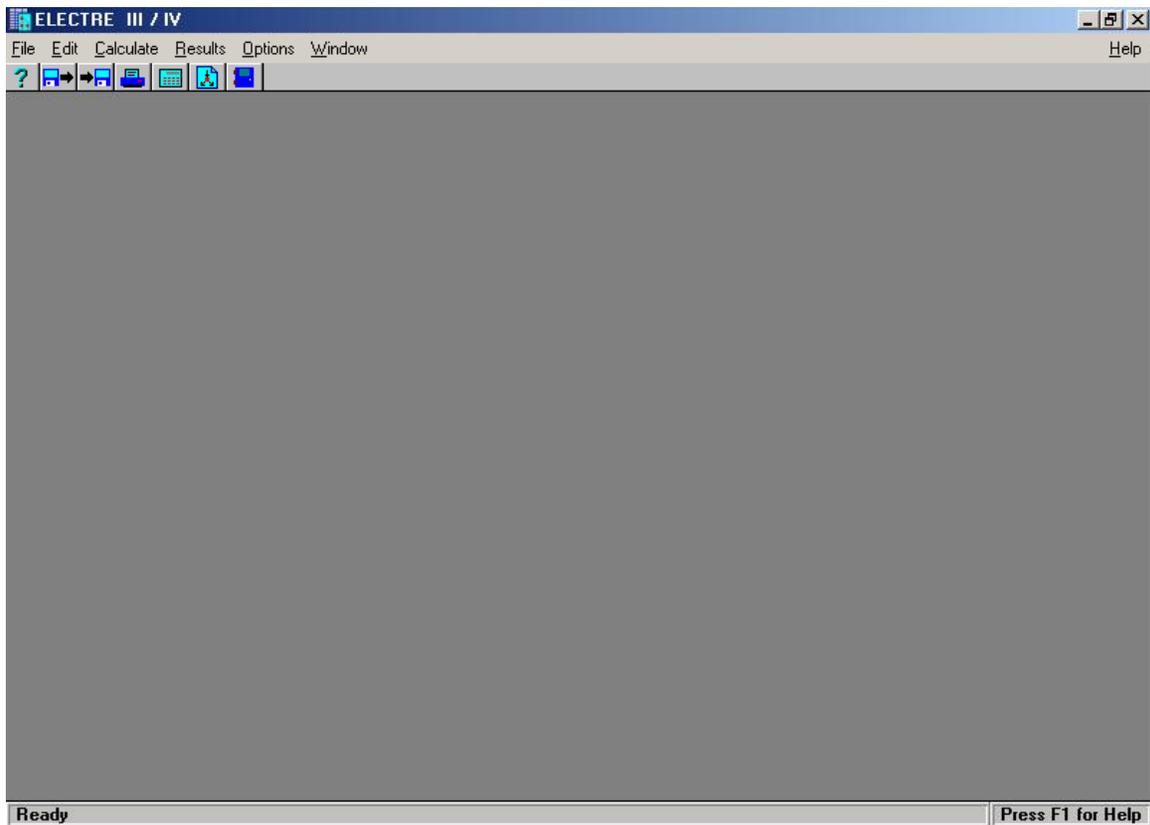
User manual

Edit menu

Project Reference

To visualize or modify some project data (name, description, owner and chosen algorithm for the problem solution).

Criteria

To visualize, insert, modify or delete the project criteria and their information (name, code, preferences direction and weight).

Alternatives

To visualize insert, modify or delete the project alternatives and their information (name and code).

Performances

To visualize or modify the performances table of the alternatives in relation to the criteria. This command is not available if the project type is "Matrix of degrees of credibility".

Thresholds

To visualize or modify the coefficients values of the threshold functions on every criterion. This command is not available if the project type is "Matrix of degrees of credibility".

The software Electre III/IV allows to modulate the thresholds by means of the coefficients α and β of a generic linear function $\alpha*g(a)+\beta$, where $g(a)$ means that g is the function of the performances of the alternative a. This means that it is possible to insert the thresholds as percentages of the performances of the alternatives (α) and/or as absolute values (β).

If you have entered alternatives into the project, there will be for each criterion three different values:

- Min (gj): indicates the smallest value that has been entered for the selected criterion.
- Max (gj): indicates the biggest value that has been entered for the selected criterion.
- Min (Δgj): indicates the smallest difference in values for the selected criterion.

Credibility Matrix

To visualize or modify the values of the credibility matrix on which the customers would apply the ordering algorithm Electre III. This command is only available if the project type is "Matrix of the credibility degree".

Calculate menu

Calculate

To execute the project calculations.

Method

To choose the solution algorithm of the problem (Electre III or IV), and to choose the relations that must be used for Electre IV or to modify the distillation coefficients of Electre III.

Information

To visualize some project information (number of alternatives and criteria, project state, available memory).

Statistics

To visualize statistic information concerning to the result of the Calculate command.

Results menu

Distillations

To visualize the project result of the two distillations (ascending and descending). Electre III/IV processes the data every time this command is accessed if the Calculate command has been previously launch.

In distillation procedure, the ranking is based on the number of alternatives outranked by each alternative minus the number of alternatives which outrank it.

Descending distillation: begins with the alternatives with greater qualification.

Ascending distillation: begins with the alternatives with lesser qualification.

In both cases, the effect of the selected alternatives is annulled on the remaining ones.

Ranks in Final Preorder

To visualize the alternatives classified through the intersection of the preorders coming from the two distillations. This command allows classifying the position of every alternative.

Median Preorder

To visualize the alternatives classified from the Median Preorder. This preorder is complete and is built in the following way: the alternatives are classified following the orderings of the Final Preorder and two incomparable alternatives in the same position are classified according to the differences in their positions in the two distillations.

Additional Results

To visualize three types of complementary results: another way to visualize the final partial preorder (using a matrix: Ranking matrix), results from immediate calculations such as the Concordance Matrix (only for projects executed with Electre III) or such as the Credibility Matrix.

The **ranking matrix** explains only the nature of the relationship in terms of preference, indifference or incomparability among the alternatives.

The **credibility matrix** is a measure of the strength of the claim that "alternative a is at least as good as alternative b".

The **concordance matrix** is equal to the credibility matrix if no veto thresholds are assigned, so the discordance matrices are all zero matrices.

Final Graph

To visualize the graph that represents the partial preorder obtained using the classification algorithm of Electre.

Example

In this section it is described the followed procedure to resolve a problem between chosen alternatives with the Electre III/IV software; it comes from the insertion of the data to the visualization and printing of the final results. Electre III/IV allows to charge the data manually or to import them from rows in ASCII format. In order to import or to export the data it is necessary to respect the very precise text formatting rules. If someone wants to execute this operation, it is recommendable to read carefully the content of README.ASC file.

During all the procedure is available a contextual guide which you can access pushing the Help button situated in most of the dialogue windows. With the F1 key it is always possible to recall the general index of the guide.

Creating a new project

Once the Electre III/IV program it is started select "New Project" from the "File" menu. It will appear the "Created New Project" window, from which it is possible to choose "Define new data set", in order to insert a new one with your data, or "Browse existing data sets" in order to recall of the data already stored ("eld" file extension); we select "Define new data set".

The window "Edit Project Reference" will appear, on that window we can choose the algorithm to solve the problem and it is also possible to insert the name of the customer and one short description of the project. With the mouse select Electre III.
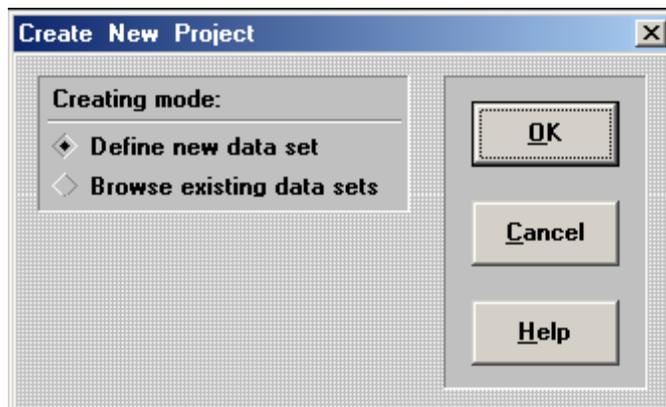
Inserting the criteria

On the main window select "Criteria" from the "Edit" menu to insert the criteria to evaluate the alternatives: the window "Edit Criteria Table" will appear. Click on Insert and proceed to the insertion of the name and the weight of the first criterion.

Fill up the name of the criterion, its weight and the direction of the preference. The weight represents the importance of the criterion on the object regarding the others criterions. It is possible to insert weights between 0 and 100, being allowed the use of float number too. The direction of the preference specify if the criterion is increasing, like for example a price (greater is the score of the criterion, better is the satisfaction of the decision), or decreasing, like for example a cost (smaller is the score of the criterion, smaller is the satisfaction of the decision). To every criterion it comes associated a identification code of 4 characters (Cr01, Cr02, …) that it can be modified by the customer.

Once it is finished the insertion, the inserted criteria can be modified or cancelled selecting them from the list of the window "Edit Criteria Table".


Inserting the alternatives

Select "Alternatives" from the "Edit" menu. The window "Edit Alternatives Table" will appear from which it is possible to insert new alternatives. To each alternative it comes associated a identification code of 5 characters (A0001, A0002, …). When the insertion it is finished, it is possible to modify or to cancel the inserted alternatives selecting them from the list of the window "Edit Table Alternatives".
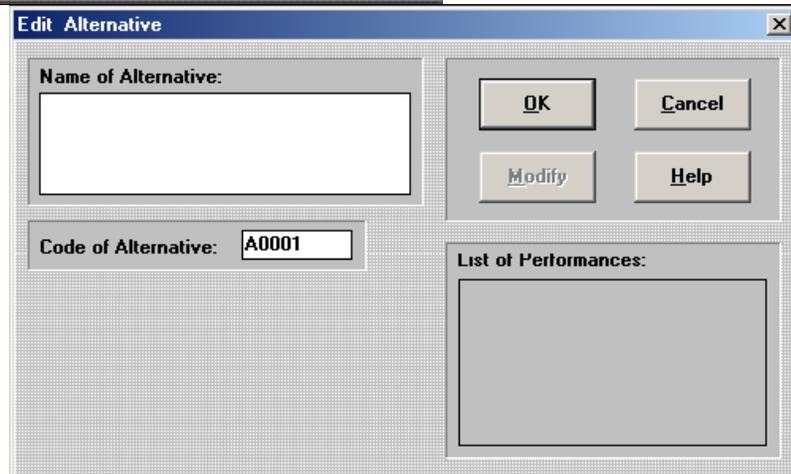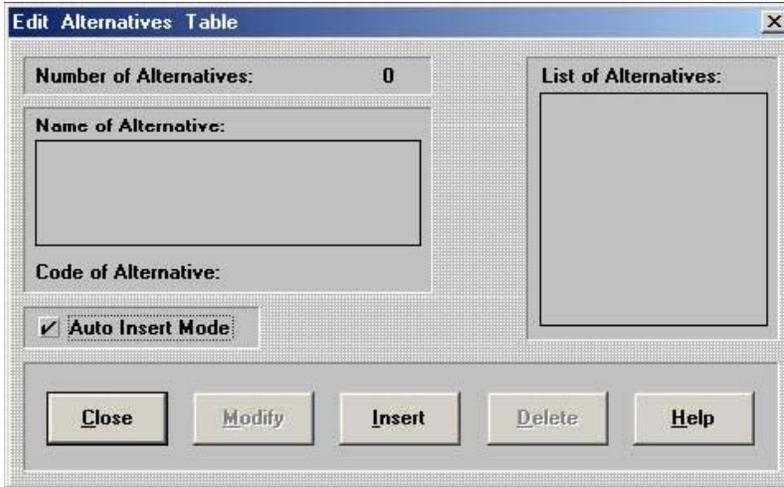
Modifying the performances

Select "Performances" in the "Edit" menu. The window "Edit Performances Table" will appear, that window shows the spreadsheet of alternatives (lines) and criteria (columns) said appraisal matrix. To type (modify) the value of a cell, you may either click on this cell or move to it with the arrow keys.
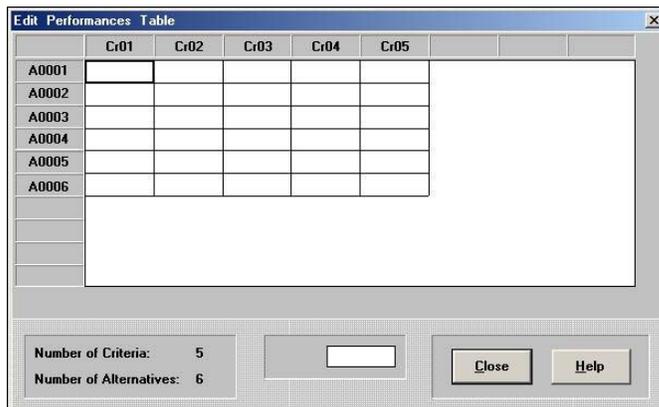
It is possible to insert numbers six digits long, decimals included. Therefore it is allowed the values between -999,999 and +999.999. The Close button stores the inserted data and closes the dialogue window.

Inserting the thresholds

Select the command "Thresholds" in the "Edit" menu, then, either double-click on the criterion whose parameters you wish to modify, or activate this criterion and click on Modify. It is possible to insert the coefficients of indifference, preference and, optionally, veto thresholds.

In the example, if for the criterion *Cr01* the alternative *A0001* has the performance 100, the alternative *A0002* has the performance 110 and the indifference threshold q = $0.1*g(a)+5$, then $q(g)$ will be equal to 15 for the alternative *A0001* and 16 for *A0002*. In other words, a difference of 15 points regarding the criterion *Cr01* is not meaningful in order to judge *A0001* indifferent to the other alternatives (in this single case *A0002*) and a difference of 16 points is not meaningful in order to judge *A0002* indifferent regarding the other alternatives (in this case *A0001*). With the same procedure we can fill up the preference p and veto v thresholds.
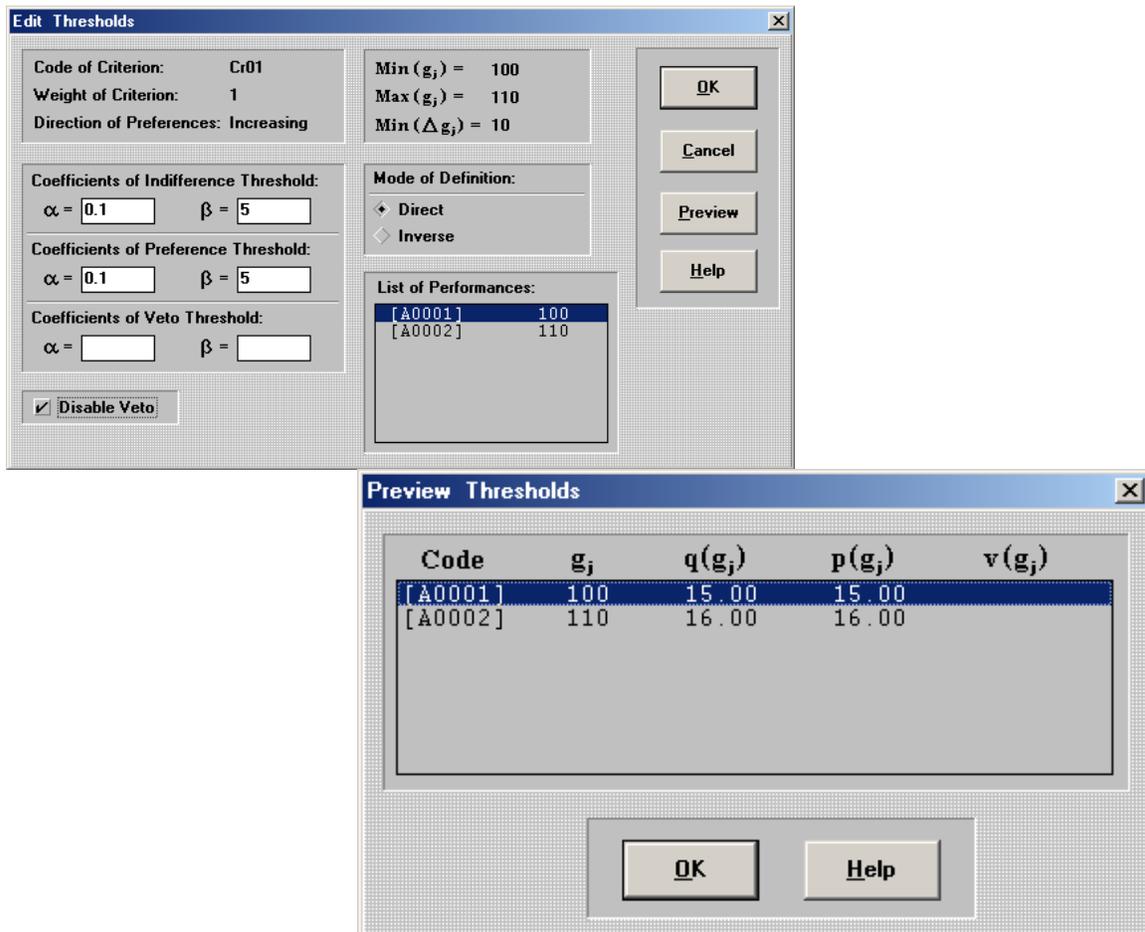
Calculating the data

Once inserted the data (criteria, alternatives, performances and thresholds) select "Calculate" in the "Calculate" menu in order to calculate the data. A dialogue window will appear with statistics information about the calculations carried out. Selecting "Method" in the "Calculate" menu and after "Advanced" it is possible to modify the coefficients of distillation of the Electre III algorithm. It is advised not to modify the preset up coefficients ($\alpha = -0,15$ and $\beta = 0,3$).
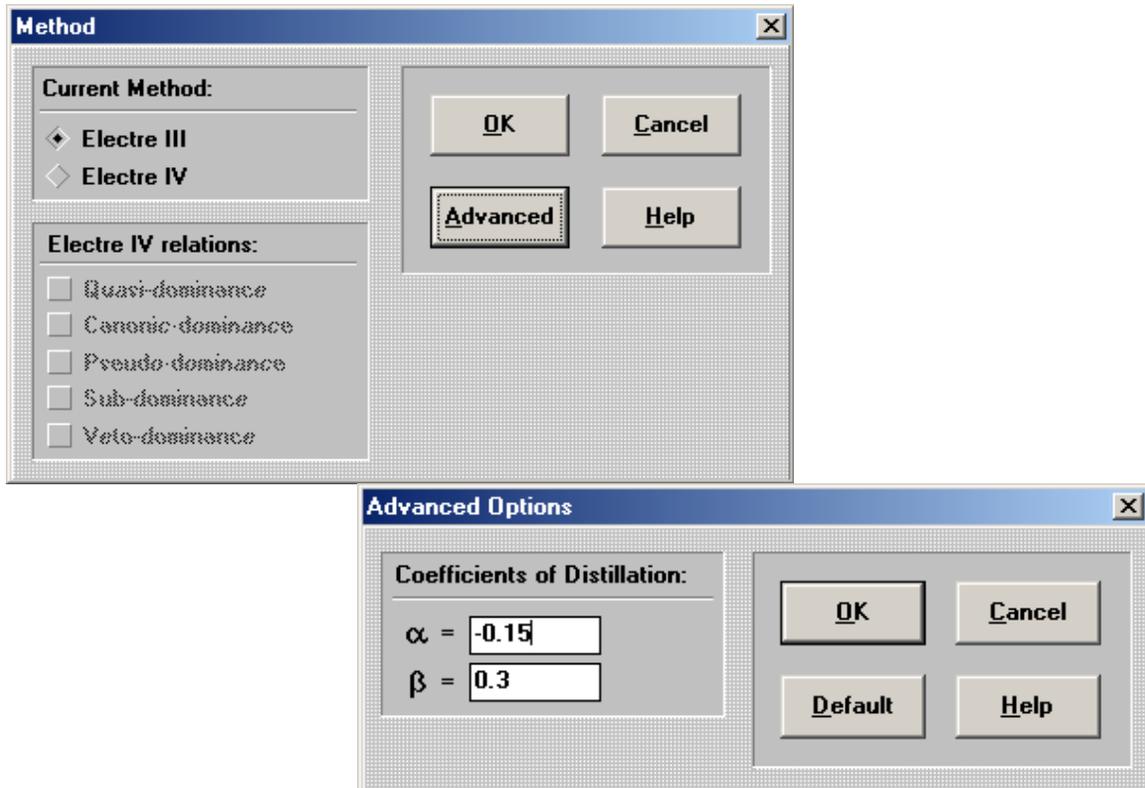
Selecting the commands "Information" and "Statistics" in the "Calculate" menu it is also possible to visualize other information relative to the running project and the memory available.

Visualizing the results

Selecting "Distillation Result" in the "Results" menu it is possible to visualize the ascending and descending graphs of the alternatives.
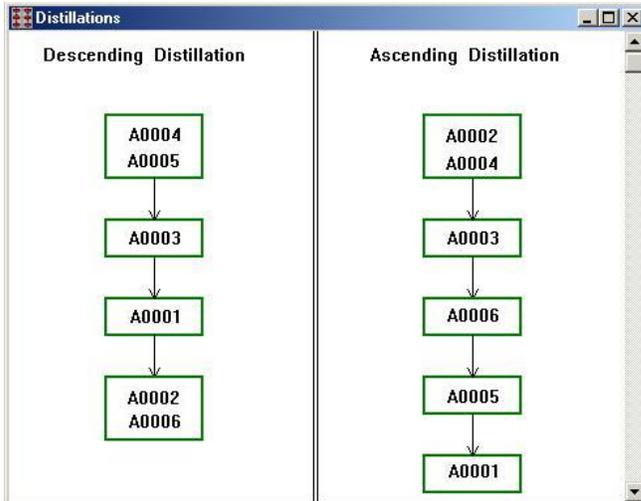
Selecting "Ranks in Final Preorder" in the "Results" menu it is possible to visualize an intersection of the two previous graphs.

Selecting "Final Graph" in the "Results" menu it is possible to visualize the final graph.
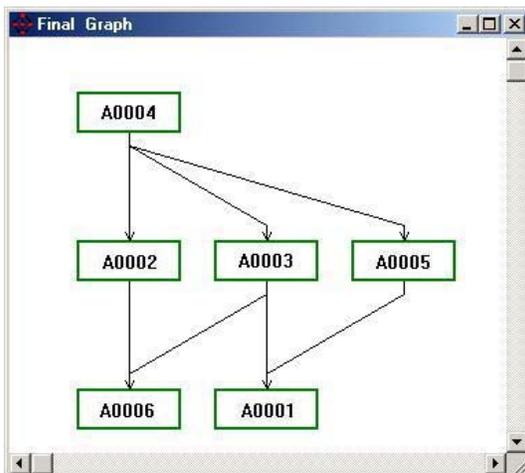
This example means that the alternative *A0004* is preferred to *A0002* and *A0003*, and therefore *A0004* is preferred to *A0001* and *A0006*.

Conclusions and future work

At first we had problems to understand how Electre III/IV works due to the lack of related information, but finally we have found out the whole algorithm joining information from several sources.

We have found some restrictions in Electre software, for example it only accepts numeric values for the criteria and it is limited to datasets with the extension ".eld", so we have not been able to test our own dataset. Moreover, we have had to use the demo version, and it only accepts up to 5 criteria and 6 alternatives.

Following the work of the other group we have seen the differences between Electre III/IV and Electre Tri software. The main difference is that Electre Tri uses profiles to compare with the alternatives and evaluate them, and Electre III/IV does not have this option.

As future work, once we know how Electre III/IV works, it would be possible to create an alternative software package with the same functionalities and with an improved graphic user interface. We recommend implementing it with Java programming language to allow the execution of the software in any platform supporting the Java Virtual Machine. This software would be licensed under GPL, allowing everybody to view and modify the source code and therefore understand the procedure of the algorithms. This type of license will protect this software against future copyrights and shareware restrictions (demo programs).

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