Silo Collapse: An Experimental Study

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Summary. The purpose of this experimental work is to develop some basic insight into the pre-buckling behavior and the buckling transition toward plastic collapse of a granular silo by studying different patterns of deformation generated on thin paper cylindrical shells during granular discharge. We study the collapse threshold considering the influence of the bed heights, flow rates and grain sizes. We compare the patterns that appear during the discharge of spherical beads, with those obtained in the axially compressed cylindrical shells. When the height of the granular column is close to the collapse threshold, we observe a ladder like pattern that rises around the cylinder surface in a spiral path of diamond shaped localizations, and develops into a plastic collapsing fold that grows around the collapsing silo.

1 Introduction

Cylindrical shells are used in many industrial processes to store fluids or grains; they are also used in chimneys, aircrafts and rockets. Thin shells are prone to buckle leading to their collapse. There is a vast engineering literature on empty shells [1–4], rigorous studies of cylindrical shells filled with fluids have been carried to some extent [5], and there are fewer investigations of the physics of shells filled with grains. Janssen and Vereins made a continuum model for a silo in 1895 [6]. They addressed the observation that the mean pressure at the bottom of the silo is generally significantly smaller than the hydrostatic pressure that a liquid would produce. This implies that an important part of the load is taken by the walls. This important formulation is still addressed, and has caught the attention of the physics community [7–13]. The problem of silos has gone to the physics laboratories posing new challenges to physicist. We are interested in the way thin walls of a cylindrical silo are affected by the load produced during the discharge of the grains. Thin walls become unstable and deform, sometimes globally and other times locally. These localizations and the different patterns that grow during the buckling and collapse are interesting from a fundamental point of view. We focus on the diamond shaped defects that appear on the surface of cylindrical paper shells when the silo is under discharge of the grains. This localized buckling grow into patterns similar to the ones observed in cylindrical empty shells subjected to axial compression. In our work the compression of the silo is due to the friction produced by the grains sliding down the internal wall of the shell, during the discharge by gravity through a central hole at the bottom. In section 2 we describe the experimental setup. In section 3 we indicate the parameters chosen to quantify the experimental observations. In section 4 we show our results. In section 5 we compare the patterns observed in our experiments with those due to axial compression of cylindrical shells. In section 6 we state the conclusion.

2 Experimental Set Up

In Fig. 1(a) we show a photograph of the experimental setup. We make the silos with tissue paper of thickness $t = 0.04$mm and nominal surface density $\rho_s = 20$g/m^2. The paper is wrapped around a long metal tube and glued along a narrow band to form the paper tube of diameter $D = 4$cm. The shell,

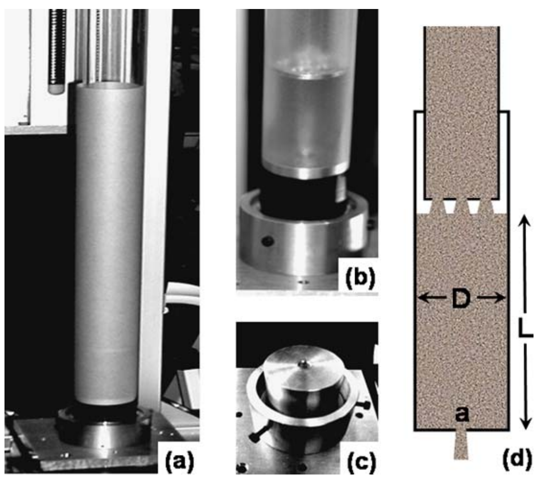

Fig. 1. (a) The photograph shows a paper silo with diameter $D = 4$cm, taped to a metal base. The silo is open at the top and a Plexiglas cylinder with an external diameter less than D is introduced from above to feed the grains. (b) A transparent plastic silo is shown; a perforated metal plate at the bottom of the Plexiglas internal tube can be seen through the walls of the silo. The level of this plate is varied to fix the effective height of the grain column. (c) The metal base shown has a center hole of depth $l = 3.8$cm. (d) Sketch of the experimental apparatus introducing the height of the granular bed acting on the silo walls, when the internal Plexiglas tube is used.

inserted into a small aluminum cylindrical base, shown in Fig. 1(c), is taped so that the thin shell is fixed at the base and open at the top. The aluminum base has a center hole of diameter a (3mm,4.5mm,6mm,8mm and 10mm). A rubber plug is inserted in the bottom hole in order to be able to fill the silo with the grains. For the initial measurements the internal tube was left inside the shell until the grains had been poured from the top and then the metal tube was carefully extracted so as to fill the silo without deforming the shell. For later measurements the long tube was extracted before pouring the grains and then a Plexiglas tube of diameter smaller than D, attached to a movable support, was inserted from above down to the desired height, as can be seen in the transparent silo shown in Fig. 1(b). The bottom of this Plexiglas tube has a metal circular plate with holes of diameter greater than a. The grains were fed from the top of the Plexiglas cylinder until it was completely filled. The plug is removed to produce the granular discharge. With the internal tube the grains can flow from it to the silo during the discharge, increasing the time the silo wall is subjected to the force produced by the grains during the discharge. The position of the internal Plexiglas tube determines the effective height of the granular bed (see fig. 1(d)). We used glass beads of different diameters d (0.2–0.3mm, 0.3–0.4mm, $\leq$ 0.63mm, 1mm, 1.5mm, 1.6mm). A CCD camera registered the transition to buckling and the collapse of the silo.

3 Control Parameters for the Column Collapse

When the plug is released and the granular flow starts, the solid friction forces on the grains are essentially polarized upward, due to the sliding on the wall. This corresponds to the condition for pressure screening as derived classically in Janssen's theory. It means that the confining pressure inside the granular bed saturates with the column height instead of growing linearly as for a fluid. The physical reason is that the walls carry the remaining load and the effective force acting on the walls has a vertical component directed downward that grows with the column height. Therefore, if we consider the silo walls as a solid cylindrical shell, we may experience buckling of the structure if the force is large enough. Along those lines, the height of the granular bed would be a simple candidate for a collapse control parameter. However the situation may not be as "simple" since we have a possible non-trivial interaction between the granular flow and the material structure. Note that the outward pressure may eventually help to stabilize the shell; the effective granular stiffness next to the wall could also change the effective rigidity of the structure. And moreover, the stress fluctuations due to the granular flow could play an important role in changing the position of the collapse onset. In this paper, which is a preliminary study, we only consider the influence of the column height, mean flow rate and granular size as possible control parameters.

From the work of Beverloo [14] and simple scaling argument, we know that when the opening radius a at the bottom of a silo is changed, the mean flow J varies as: $J = \alpha g^{1/2}(a - \kappa d)^{5/2}$ where α and κ are constants that only depend on the actual coupling between the grains and the opening. For non cohesive grains κ is about 4. Thus, we decided to change the opening size and the grain size to control the granular flux and the fluctuations.

4 Experimental Results

We performed experiments in two ways. First we filled the granular column up to a given initial height, and then we started the flow abruptly by taking the plug off at the center hole on the bottom of the silo. Second, we kept a constant height using the device described in fig. 1 (b) and (d). The existence of a threshold height L_c was recorded. The second method has the advantage that we can fix an important parameter, but in terms of the actual force on the wall the situation is less simple since we do not have a well define free surface, and we must consider an extra offset pressure to account for the junction with the inner tube.

For spherical beads with diameter $d < 0.63$mm changing the size of the bottom hole from $a = 4$mm to $a = 8$mm, keeping everything else the same, did not cause a change in the collapse threshold L_c. This occurred at $L_c \approx 20$cm. For the larger diameter a, the discharge is just faster and the collapse occurs sooner. We note that small fluctuations on the shell surface are observed when L is close to the collapse threshold. With the same conditions used above and for smaller particles (0.2–0.3mm) and a diameter of the hole $a = 3$mm, we obtained a threshold $L_c \approx 21$cm. For these smaller particles precursor fluctuations could not be observed when L was close to the height for which buckling occur. For particles of diameter $d = 1.5$mm and hole size $a = 10$mm, the threshold also occurs close to $L_c = 20$cm. For the above cases, the cylindrical shell was free at the top and no internal Plexiglas tube was used. In Fig. 2 we

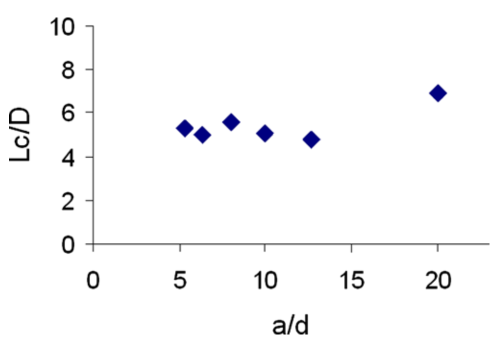

Fig. 2. Ratio of the collapse threshold to the diameter of the silo, as a function of the diameter of the bottom hole divided by the size of the glass beads.

plot the threshold value L_c divided by the diameter D of the silo as a function of the diameter a of the bottom hole divided by the grain size. For this graph we took the particle diameters d, by considering their highest nominal values, and the threshold L_c is obtained for the granular beds measured without using the internal Plexiglas tube.

For the small particles with diameters between 0.2–0.3mm, and hole size $a = 8$mm, and for big particles of size $d = 1.5$mm, we used a Plexiglas internal tube to feed more grains from above while the silo discharges (see Fig. 1(b)). With the internal tube, the threshold occurs when the tube is at a height $L \approx$ 16cm. If the threshold is taken to be around $20cm$, it is possible that the tube contributes for an effective offset of $4cm$. However, for particles of diameter $d = 1.6$mm the threshold obtained was $L_c \approx 11$cm, significantly lower than for all the other particles. This case was anomalous in the sense that huge fluctuations of the deformations on the surface of the shell appeared for L as low as $9cm$. These huge fluctuations can cause local curvatures that surpasses a local threshold value so that effectively a sufficiently large deformation is created, producing a large local defect that triggers the buckling and causes a collapse at a lower value of L. Further measurements have to be made in order to investigate the effect of these large fluctuations.

Therefore, these preliminary measurements suggest that for spherical particles and smooth walls in the ranges of particle diameters used, the threshold for collapse, for a silo which is free at the top, seems to be weakly dependent on particle diameter and the flux. The larger particles exhibit observable fluctuations of the silo surface when L is close to the collapse threshold. The precursor fluctuations in the elastic deformation of the surface of the cylindrical shell, was observed to increase with the particle size. The precursor fluctuations will be discussed in a future work. The results displayed in Fig. 2 indicate that the main factors determining the buckling and subsequent collapse of the silo is dominated should correspond essentially to a threshold in applied vertical force on the shell and should be determined at the first order by its mechanical characteristics. The presence of a granular material flowing inside could also have an influence on the value of the threshold via the effective elastic coupling or via other parameter still to be discovered. This could be explored by changing the geometrical and elastic characteristic of the silo.

5 Buckling Patterns

The focus of this section is on the different buckling patterns that were experimentally observed just prior or during collapse (see Fig. 3).

The type of buckling localizations that developed close to the threshold of collapse was common to all the cases that we measured. We observed that when the initial height L of the granular bed is sufficiently small, no change was observed at the lateral surface of the silo. For L sufficiently large some elastic fluctuations of the surface appear. Close to the collapse threshold some

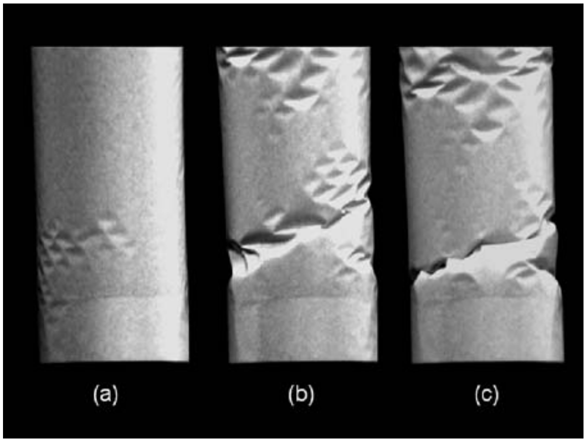

Fig. 3. The photographs show successive pictures of a collapsing silo during the discharge of small grains. (a) The buckling appears as a pattern of small diamond shaped elastic deformations occurring just before the collapse; (b) We can see the growth of bigger plastic deformations in a sequence like a spiral ladder around the cylinder; (c) During the collapse the paper folds in along a spiral that grows out of the chain of diamond shaped localizations.

persistent diamond shaped buckling occurs. This diamond shaped localized deformations are similar to the ones observed in real steel silos after damage. Some of the diamonds appear isolated and others form clusters similar to those observed in Fig. 3. During the discharge, when the Plexiglas column runs out of grains and the granular bed becomes sufficiently small these elastic deformations disappear so they seem to display some reversibility. When L is at the threshold value L_c and the collapse occur, the deformations propagate at an angle forming an "anticrack" that grows around the cylinder surface and eventually folds producing a catastrophic collapse. Very often (but not always), the first deformation appeared close to the silo bottom, in the vicinity of the boundary between the grains that remain fixed in the stagnant zone (forming a cone close to the bottom hole) and the grains that move down the wall.

6 Conclusion

We have observed different buckling patterns during the discharge of different size grains, from a central hole at the bottom of a cylindrical paper silo. These patterns are similar to the ones observed in axially symmetric compressed cylindrical empty shells. The height of the granular bed was changed to approach the threshold height L_c for different size particles and for different diameters of the discharging hole. It was found that the threshold was weakly dependent on the diameter of the discharge hole divided by the particle size. Precursor fluctuations were observed below the collapse threshold. These were found to increase with particle diameters. More experiments need to be made to fully characterize the above observations.

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