On singularities in direct and inverse scattering problems

Simon Chandler-Wilde, Monique Dauge and Roland Potthast

September 11, 2007
Collaborators

- Simon Chandler-Wilde, Research Group in Reading, UK
- Monique Dauge, Research Group in Rennes, France
- Roland Potthast, Research Group in Reading, UK / Göttingen, GER

Supported by EPSRC, Leverhulme Trust
**Introduction**

Direct and Inverse Scattering Problems

**Edge and Corner Singularities**

Corner Singularities around a Polygonal Obstacle (2d)

Edge Singularities around a Polyhedral Obstacle (3d)

Combined Corner and Edge Singularities

**High-Frequency Scattering**

Effect of Singularities - Far Field Behaviour

Rigorous High Frequency Asymptotics for Whole Bounded Scatterers

**Inverse Scattering**

Field Reconstructions via Point Source Method

Shape reconstruction via the singular sources method

Reduce need of data via the no response test (NRT)
Introduction
Setting for reconstruction problem

Partial Differential Equation (Acoustic, Electromagnetic, Elastic)

Boundary Condition on Object

Remote Measurements

Incident Wave
Acoustic Scattering

Given some smooth incident field $u^i$ and bounded obstacle $D$, find a scattered field $u^s$, governed by

- A Differential Equation, for example time-harmonic Helmholtz equation

$$\Delta u^s + \kappa^2 u^s = 0 \quad \text{in} \quad \Omega := \mathbb{R}^3 \setminus D$$

- A Boundary Condition, for example Dirichlet BC

$$u|_{\partial D} = 0$$

for the total field

$$u = u^i + u^s$$

- A Radiation Condition, for example Sommerfeld RC (3d)

$$\left( \frac{\partial}{\partial r} - i\kappa r \right) u^s \to 0, \quad r \to \infty.$$
Electromagnetic Scattering

- **Bounded scatterer** \( D \) in three dimensions with piecewise smooth boundary, *incident field* \( E^i \)
- **Scattered field** \( E^s \) solves **Maxwell equations**
  \[
  \text{curl} E^s - i\kappa H^s = 0 \quad \text{curl} H^s + i\kappa E^s = 0
  \]
  in \( \mathbb{R}^3 \setminus \overline{D} \) and satisfies the **Silver-Müller radiation condition**
  \[
  E^s \times x + rH^s \to 0, \quad r = |x| \to \infty.
  \]
- On the boundary \( \Gamma := \partial D \) the tangential component of the total field \( E = E^i + E^s \) vanishes, i.e. we have the **perfect conductor boundary condition**
  \[
  \nu \times E|_\Gamma = 0
  \]
Solution Techniques

- There are several techniques to prove uniqueness for the forward problem.
- Existence can be shown by variational methods, integral equation methods or several further special techniques.
- For numerical calculations groups work on FEM, BEM, FDM, Spectral Methods, FIT ...
- The scattered field has the asymptotic behaviour

\[ u^s(x) = \frac{e^{i\kappa|x|}}{|x|} \left\{ u^\infty(\hat{x}) + O\left(\frac{1}{x}\right) \right\}, \quad \hat{x} := x/|x| \]
Measured data

Measured data are either the scattered field $E^s$ on some surface $\Lambda$ or the far field pattern $E^\infty$ defined by

$$E^s(x) = \frac{e^{i\kappa|x|}}{|x|} \left\{ E^\infty(\hat{x}) + O\left(\frac{1}{x}\right) \right\}, \quad \hat{x} := x/|x|$$

uniformly on $\mathbb{S}$ for $|x| \to \infty$.

For the above scattering problem the far field pattern is calculated via integral equations of the second kind.
The inverse problem

Task: reconstruct the shape and properties of the unknown scatterer!
Edge and Corner Singularities
Corner Singularities in 2d

Polygonal Obstacle $D$: Corners $c$, apertures $\omega'_c$
Exterior Domain $\Omega = \mathbb{R}^2 \setminus D$: Corners $c$, apertures

$\omega_c = 2\pi - \omega'_c$

- Typical Laplace-Dirichlet singularity:

$$d \ r^{k\pi/\omega} \sin \left(\frac{k\pi\theta}{\omega}\right), \quad d \in \mathbb{R}, \quad k \in \mathbb{N},$$

with polar coordinates $(r, \theta)$ centered at the corner.

- Splitting in regular and singular parts

$$u = u_{\text{reg}} + \sum_{c \mid \omega'_c > \pi} d_c \ r_c^{\pi/\omega_c} \sin \left(\frac{\pi\theta_c}{\omega_c}\right) \chi(r_c),$$

- with regular part $u_{\text{reg}} \in H^2(B_R \cap \Omega), \quad \forall R > 0$
- coefficients $d_c$ depending on the incident field.
- and cut-off function $\chi$. 
**References for corner asymptotics**

- **V. A. Kondrat’ev.**
  Boundary-value problems for elliptic equations in domains with conical or angular points.

- **P. Grisvard.**
  *Boundary Value Problems in Non-Smooth Domains.*

- **S. Nicaise.**
  *Polygonal interface problems.*

- **V. A. Kozlov, V. G. Maz’ya, J. Rossmann.**
  *Elliptic boundary value problems in domains with point singularities.*
  Mathematical Surveys and Monographs **52** 1997.
**Edge Singularities in 3d – closed surface**

Obstacle $D \subset \mathbb{R}^3$ with edges: Edges $e$, apertures $e \ni z \mapsto \omega'_e(z)$

Domain $\Omega = \mathbb{R}^3 \setminus D$: Edges $e$, apertures $\omega_e(z) = 2\pi - \omega'_e(z)$

- Typical Laplace-Dirichlet edge singularity:
  
  $$d(z) \ r^{k\pi/\omega} \sin\left(\frac{k\pi\theta}{\omega}\right), \quad z \mapsto d(z) \in \mathbb{R}, \quad k \in \mathbb{N},$$

  cylindrical coordinates $(r, \theta, z)$ with axis on the edge.

- Splitting in regular and singular parts

  $$u = u_{\text{reg}} + \sum_{e \mid \omega_e > \pi} d_e(z_e) \ r_e^{\pi/\omega_e(z_e)} \sin\left(\frac{\pi\theta_e}{\omega_e(z_e)}\right) \chi(r_e),$$

  - with regular part $u_{\text{reg}} \in H^2(B_R \cap \Omega)$, $\forall R > 0$
  - smooth edge coefficients $e \ni z_e \mapsto d_e(z_e)$.
  - and cut-off function $\chi$. 
2d obstacle $D$ with smooth boundary (screen surface).
Exterior Domain $\Omega = \mathbb{R}^3 \setminus D$.
Edge $e$ of $\Omega = \text{connected component of } \partial D$.

- As before, with $\omega_e \equiv 2\pi$.
- Splitting in regular and singular parts

$$u = u_{\text{reg}} + \sum_{e} d_e(z_e) r_e^{1/2} \sin\left(\frac{\theta_e}{2}\right) \chi(r_e),$$

- with regular part $u_{\text{reg}} \in H^2(B_R \cap \Omega), \quad \forall R > 0$
- smooth edge coefficients $e \ni z_e \mapsto d_e(z_e)$.
- and cut-off function $\chi$.
References for edge asymptotics

V. G. Maz’ya, J. Rossmann.
Über die Asymptotik der Lösungen elliptischer Randwertaufgaben in der Umgebung von Kanten.

V. G. Maz’ya, J. Rossmann.
On a problem of Babuška...

M. Costabel, M. Dauge.
General edge asymptotics of solutions of second order elliptic boundary value problems I.

M. Costabel, M. Dauge, R. Duduchava.
Asymptotics without logarithmic terms...
Corner Singularities in 3d

Obstacle $D$ with conical points: Corners $c$, solid angles $G'_c$

Domain $\Omega = \mathbb{R}^3 \setminus D$: Corners $c$, solid angles $G_c = S^2 \setminus G'_c$

- Typical corner singularity: $\rho^\lambda \Phi(\vartheta)$, with

$$ (\Phi, \lambda(\lambda + 1)) \text{ Laplace-Beltrami eigenpair on } H^1_0(G_c) $$

and polar coordinates $(\rho, \vartheta)$ centered at the corner.

- Splitting in regular and singular parts

$$ u = u_{\text{reg}} + \sum_{c \mid \lambda_c \leq \frac{1}{2}} d_c \rho_c^\lambda \Phi_c(\vartheta_c) \chi(\rho_c), $$

- with regular part $u_{\text{reg}} \in H^2(B_R \cap \Omega), \forall R > 0$, provided

$$ \Phi_c \in H^2(G_c), \text{ for all occurrence in } (*) $$

- coefficients $d_c$ depending on the incident field.

- and cut-off function $\chi$. 
Polyhedral Edge-Corner Singularities

Obstacle $D = $ polyhedron: edges $e$, corners $c$. Exterior Domain $\Omega = \mathbb{R}^3 \setminus D$: same edges and corners. Splitting in regular and singular parts

$$u = u_{\text{reg}} + \sum_{c \mid \lambda_c \leq \frac{1}{2}} u_c + \sum_{e \mid \omega_e > \pi} u_e$$

- with regular part $u_{\text{reg}} \in H^2(B_R \cap \Omega), \forall R > 0$,
- corner singularities $u_c = d_c \rho_c^{\lambda_c} \Phi_c(\vartheta_c) \chi(\rho_c)$ as in ($\star$)
- residual edge singularities $u_e = d_e(z_e) \rho_e^{\pi/\omega_e} \sin \left( \frac{\pi \theta_e}{\omega_e} \right) \chi(\rho_e)$ with
  - $\rho_e = r_e/\delta_e$, with a smooth distance function $\delta_e$ to both ends of $e$,
  - Edge coeff. $d_e$ satisfies: $\forall \alpha \in \mathbb{N}, \delta_e^{\alpha-1} \partial_{z_e}^\alpha d_e \in L^2(e)$. 

Comments on Edge & Corner Singularities

- We assume that *incident field* $u_i$ *is smooth*, which makes the edge coefficients $d_e$ smooth inside each edge $e$. We do not need any *smoothing* (or lifting) operator for edge coefficients.

- The results above are also valid for the exterior domain $\Omega$ of a *polygonal screen* $D$. The edge exponents are then $\frac{1}{2}$.

- We look for a *regular part in* $H^2$. Then
  - There is one edge singularity per edge, and at most one corner singularity per corner
  - The *wave number* $\kappa$ has no influence at this level
  - The possible *curvature* of boundary has not much influence on the structure of singularities.
References for edge-corner asymptotics

M. Dauge.
Elliptic Boundary Value Problems in Corner Domains – Smoothness and Asymptotics of Solutions.
Lecture Notes in Mathematics, Vol. 1341.

M. Costabel, M. Dauge.
Singularities of electromagnetic fields in polyhedral domains.

M. Costabel, M. Dauge, S. Nicaise.
Singularities of Maxwell interface problems.
High-Frequency Scattering
The above theory of edge/corner singularities tells us what happens near the corner for Helmholtz/Maxwell, e.g. within one wavelength.

The edges/corners also influence the field strongly globally, especially at high frequency, by generating diffracted wave fields.
see Keller and Lewis, 1995. It is a partly heuristic, semi-rigorous theory, whose principles are:

- At high frequency a **ray model** is appropriate
- The paths of rays are determined by Fermat’s **principle**, i.e. rays take the quickest route
- **Phase of the field** on a ray is determined by distance along the ray, i.e. \( u(x) = A(x)e^{iks} \), \( s \) distance along ray, where \( A \) is not oscillatory or oscillates only slowly.
- **Localization**: interaction with obstacles depends only on the geometry local to the point where the ray hits the obstacle, and so can be determined by solving canonical scattering problems
Example 1: Polygon.

If obstacle has corners then rays are reflected from sides but also diffracted from corners. Each diffracted ray (in 2D) has the form:

$$u_{\text{diff}}(x) = u^i(x_c)D(\theta, \theta_0)\frac{e^{ikr}}{\sqrt{kr}},$$

as $kr \to \infty$, where $x_c$ is the corner, $(r, \theta)$ are polar coordinates of $x$ relative to the corner (i.e. of $x - x_c$), $\theta_0$ is the angle of incidence and $D(\theta, \theta_0)$ is a diffraction coefficient which depends on the local geometry.
Example 2: Polyhedron.

The local problems are:
(i) reflection by an infinite plane (for reflection at a side);
(ii) reflection by an infinite 2D wedge (for diffraction at each edge);
(iii) conical diffraction problems (for diffraction at the corners).
**Example 2: The Conical Diffraction Problem.**

The total field consists of waves specularly reflected from the surface plus a tip diffracted wave which behaves like

\[
\mathbf{u}^{\text{diff}}(x) = \frac{e^{ikr}}{kr} f(\omega, \omega_0) + O((kr)^{-2}),
\]

as \( kr \to \infty \). Here \((r, \omega)\) are the spherical coordinates of \( x \) and \( \omega_0 \) is the incident direction. The function \( f(\omega, \omega_0) \) is computed by solving BVPs for the Laplace-Beltrami operator on that part of the surface of the unit sphere lying outside the cone. See Bonner, Graham Smyshlyaev, 2005.
Rigorous High Frequency Asymptotics for Whole Bounded Scatterers

Where $s$ is distance along $\gamma$,

$$\frac{\partial u}{\partial \nu}(s) = 2\frac{\partial u^i}{\partial \nu}(s) + e^{iks}v_+(s) + e^{-iks}v_-(s)$$

where (rigorous h.f. asymptotics)

$$k^{-n}|v_+^{(n)}(s)| \leq C_n(ks)^{-1/2-n}, \quad |ks| \geq 1,$$

and (from the corner singularity theory),

$$k^{-n}|v_+^{(n)}(s)| \leq C_n(ks)^{-\alpha-n}, \quad 0 < ks \leq 1.$$

where $\alpha < 1/2$ depends on the exterior corner angle $\omega_c$ ($\alpha = 1 - \pi/\omega_c$).
A Numerical Scheme for the Convex Polygon

... which uses this precise understanding of solution behaviour

\[
\frac{\partial u}{\partial \nu}(s) = 2 \frac{\partial u^i}{\partial \nu}(s) + e^{iks} v_+(s) + e^{-iks} v_-(s)
\]

where

\[
k^{-n}|v_+^{(n)}(s)| \leq C_n(ks)^{-1/2-n}, \quad |ks| \geq 1,
\]

and

\[
k^{-n}|v_+^{(n)}(s)| \leq C_n(ks)^{-\alpha-n}, \quad 0 < ks \leq 1.
\]

where \(\alpha < 1/2\) depends on the exterior corner angle.
Thus approximate

\[ \frac{\partial u}{\partial \nu}(s) \approx 2 \frac{\partial u^i}{\partial \nu}(s) + e^{iks} V_+(s) + e^{-iks} V_-(s), \]

where \( V_+ \) and \( V_- \) are finite element functions,

using meshes optimally designed given this behaviour!
\[ k^{-n} |\nu_{+}^{(n)}(s)| \leq \begin{cases} 
C_n(ks)^{-\alpha-n}, & 0 < ks < 1, \\
C_n(ks)^{-1/2-n}, & ks \geq 1,
\end{cases} \]

\[ s = 0 \quad t_m = \left( \frac{m}{N} \right)^q \lambda \quad t_m = cr^m \]
\[ \frac{\partial u}{\partial \nu}(s) \approx 2 \frac{\partial u_i}{\partial \nu}(s) + e^{iks} V_+(s) + e^{-iks} V_-(s), \]

where \( V_+ \) and \( V_- \) are \( p \)th order finite elements on graded meshes.

**Theorem (C-W & Langdon, 2007)**

Where \( \phi = \frac{\partial u}{\partial \nu} \), \( \phi_M \) is the best \( L_2 \) approximation to \( \phi \) from the approximation space, \( J \) is the number of sides, \( M \) the degrees of freedom, \( p \) the polynomial degree, and \( L \) the total arc-length,

\[ k^{-1/2} \| \phi - \phi_M \|_2 \leq C \sup_{x \in D} |u(x)| \frac{[J(1 + \log(kL/J))]^{p+3/2}}{M^{p+1}}, \]

where \( C \) depends (only) on the corner angles and \( p \).
Some References


Inverse Scattering
Iterative Methods

1. Least Squares Method
2. Newton’s Method, Local Newton
3. Gradient or Landweber Methods
4. Level Set Methods
5. Reciprocity Gap Methods
6. Evolutionary Method, 
   Evolutionary Newton Method, 
   Evolutionary Gradient Method
7. Successive Iteration Schemes (Kress-Rundell)
8. Hybrid Method
1. Series Expansion Methods: Spherical Waves, Bessel functions
2. Fourier - Plane Wave Expansion Methods
3. Kirsch-Kress Method or Potential Method
4. Point Source Method: Greens Representation and Point Source Approximation

When the field is reconstructed, singularities can be used to find corners and reconstruct polygonal scatterers.
Sampling and Probe Methods

1. Linear sampling method (Colton-Kirsch)
2. Factorization method (Kirsch)
3. Singular sources method (P.)
4. Probe method (Ikehata)
5. Enclosure method (Ikehata)
7. Range test (Kusiak/P./Sylvester)
8. Orthogonality Sampling (P./Nakamura)
**Decomposition Methods: Point source method**

1. **Green’s formula** (using Dirichlet boundary condition)

   \[ u^s(x) = \int_\Gamma \Phi(x, y) \frac{\partial u}{\partial \nu}(y) ds(y) \]  
   \[ (5) \]

2. **Approximation of the point source** (on test domain)

   \[ \Phi(x, y) \approx \int_S e^{i\kappa y \cdot d} g_x(d) ds(d) \]  
   \[ (6) \]

3. Insert and exchange order of integration:

   \[ u^s(x) \approx \int_\Gamma \left( \int_S e^{i\kappa y \cdot d} g_x(d) ds(d) \right) \frac{\partial u}{\partial \nu}(y) ds(y) \]

   \[ = \int_S \left( \int_\Gamma e^{i\kappa y \cdot d} \frac{\partial u}{\partial \nu}(y) ds(y) \right) g_x(d) ds(d) \]

   \[ = 4\pi \int_S u^\infty(-d) g_x(d) ds(d). \]  
   \[ (7) \]
PSM: acoustics 3d, field reconstruction

PSM: Field reconstruction by Ben Hassen, Erhard and P. 2005
PSM: Shape reconstruction by Ben Hassen, Erhard and P. 2005
PSM: electromagnetics 3d, field reconstruction

Point source method, electromagnetics
(Stratton-Chu formula and approximation of the point source.)

\[
E^S(x) = \text{curl} \int_{\Gamma} \nu(y) \times E^S(y) \Phi(x, y) \, ds(y) \\
- \frac{1}{ik} \text{curl} \text{curl} \int_{\Gamma} \nu(y) \times H^S(y) \Phi(x, y) \, ds(y), \quad x \in \mathbb{R}^3 \setminus \overline{D}, \tag{8}
\]

Approximation of Point source by superposition, exchange of the order of integration \(\Rightarrow\) Reconstruction formula

\[
E^S(x) \approx 4\pi \int_{S} E^\infty(-d)g_x(d) \, ds(d), \quad x \in \mathbb{R}^3 \setminus \overline{D} \tag{9}
\]
PSM: electromagnetics 3d, field reconstruction

Original and reconstruction of the modulus of the total electromagnetic field.
PSM: electromagnetics 3d, field reconstruction

Reconstruction with data error of 1%.
PSM: electromagnetics 3d, field reconstruction

Reconstruction with data error of 10% and 20%.
PSM: electromagnetics 3d, field reconstruction

Reconstruction with data error of 33% and 53%.
PSM Literature 1

Potthast, R.
A fast new method to solve inverse scattering problems.

Potthast, R.
A point-source method for inverse acoustic and electromagnetic obstacle scattering problems.

Potthast, R.
Point sources and multipoles in inverse scattering theory
PSM Literature 2

Chandler-Wilde, S. and Lines, C.
The point source method for reconstruction of rough surfaces
Computing (75) 2005.

Ben Hassen, F., Erhard, K. and Potthast, R.
The point source method for 3d reconstructions for the Helmholtz and Maxwell equations.
Inverse Problems 22 (2006), 331-353.

Luke, R. and Potthast, R.
The point source method for inverse scattering in the time domain.
PSM Literature 3

Liu, J., Nakamura, G. and Potthast, R.
A new approach and improved error analysis for reconstructing the scattered wave by the point source method.
Journal of Computational Mathematics, accepted for publication.

Potthast, R.
A survey on sampling and probe methods for inverse problems.
Topical Review for Inverse Problems 22 (2006), R1-R47.
**Singular sources method, idea**

scatterers

measurements

singular source
**Singular sources method, idea**

Basic property used for the singular sources method

\[ |E^s(z, z)| \to \infty \text{ for } z \to \Gamma. \quad (10) \]

1. Use PSM to reconstruct \( E^s(z, z) \) on a sampling grid from the knowledge of \( E^\infty(\hat{x}, d) \) for \( \hat{x}, d \in S \).

2. Use the blow-up (10) to find the unknown shape as level curves of \( |E^s(z, z)| \).
Singular sources method, derivation

Approximation of the point source (on test domain)

\[ \Phi(x, y) \approx \int_S e^{i\kappa \cdot d} g_y(d) ds(d) \] (11)

leads to the far field approximation

\[ E^\infty(\hat{x}, y) \approx \int_S E^\infty(\hat{x}, d) g_y(d) ds(d) \] (12)

Using the point source method to reconstruct \( E^s(\cdot, y) \) from \( E^\infty(\cdot, y) \) leads to the formula

\[ E^s(x, y) \approx 4\pi \int_S \int_S E^\infty(-\hat{x}, d) g_y(d) ds(d) g_x(\hat{x}) ds(\hat{x}) \] (13)
Singular sources method

Singular sources method: T shape

Dirichlet BC. Image by Erhard, Ben Hassen, P. 2006.
Note:

The singular sources method (SSM) can also be used to detect piecewise constant inhomogeneous media! The strength of the blow-up of the scattered field of multipoles is proportional to the jump in the refractive index.
SSM Literature

Potthast, R.
*Stability estimates and reconstructions in inverse acoustic scattering using singular sources.*

Potthast, R.
*A new non-iterative singular sources method for the reconstruction of piecewise constant media.*

Potthast, R. and Stratis, I.:
*The singular sources method for an inverse transmission problem.*
Computing 75 (2005), 237-255.
SSM Literature

Honda, W., Nakamura, R., Potthast, R. and Sini, M.
Unification of the probe and singular sources methods for the inverse boundary value problem by the no response test
Comm. PDE, accepted for publication.

Potthast, R.
A survey on sampling and probe methods for inverse problems.
Topical Review for Inverse Problems 22 (2006), R1-R47.

Colten, D. and Kress, R.:
Using fundamental solutions in inverse scattering.

Potthast, R.
Point-sources and Multipoles in Inverse Scattering.
PSM and SSM revisited

- PSM uses measurements for one wave, but needs to know the boundary condition for shape reconstruction (only impenetrable scatterers)
- SSM uses measurements for many incident waves, does not need to know the boundary condition

Is there a one-wave method which does not need to know the boundary condition?
The idea of the no response test

**Setting.** Consider scattering of time-harmonic acoustic waves by some sound-soft obstacle (Dirichlet boundary condition).

**Preparation step I:** The far field pattern can be expressed as:

\[
u^\infty(\hat{x}) = \frac{1}{4\pi} \int_{\Gamma} e^{-i\kappa \hat{x} \cdot y} \frac{\partial u}{\partial \nu}(y) ds(y)\]  

(14)

for \( \hat{x} \in \mathbb{S} \).
The idea of the no response test

Preparation step II:
Consider a superposition of plane waves (a Herglotz wave function)

\[ v[g](y) := \int_{S} e^{-i\kappa y \cdot \hat{x}} g(\hat{x}) ds(\hat{x}), \quad y \in \mathbb{R}^m \] (15)

with density \( g \in L^2(S) \).

Choosing appropriate densities \( g \) we can make \( v[g] \) arbitrarily small on some given test domain \( \overline{G} \) where on compact subsets of \( \mathbb{R}^m \setminus \overline{G} \) the function \( v[g] \) becomes arbitrarily large.
The idea of the no response test

Basic derivation step:
Multiply \( u^\infty \) by \( g \in L^2(\mathbb{S}) \), integrate over \( \mathbb{S} \) and exchange the order of integration to obtain

\[
\mu(g) := \int_{\mathbb{S}} g(\hat{x}) u^\infty(\hat{x}) \, ds(\hat{x})
\]

\[
= \int_{\mathbb{S}} g(\hat{x}) \left(4\pi \int_{\Gamma} e^{-i\kappa \hat{x} \cdot y} \frac{\partial u}{\partial \nu}(y) \, ds(y)\right) \, ds(\hat{x})
\]

\[
= 4\pi \int_{\Gamma} \left( \int_{\mathbb{S}} e^{-i\kappa \hat{x} \cdot y} g(\hat{x}) \, ds(\hat{x}) \right) \frac{\partial u}{\partial \nu}(y) \, ds(y)
\]

\[
= 4\pi \int_{\Gamma} \nu[g](y) \frac{\partial u}{\partial \nu}(y) \, ds(y).
\]

response
NRT sampling

Sampling for the no response test
Reconstruction example from inverse scattering

Boat-like sound-soft obstacle:
Numerical proof of concept

Boat-like sound-soft obstacle2:
Numerical proof of concept

Boat-like sound-hard obstacle:
Numerical proof of concept

Rectangular penetrable obstacle:
Magnetic tomography

Images by L. Kühn, Dissertation Thesis Göttingen 2005
NRT Literature

Potthast, R.
*On the convergence of the no response test.*
SIAM J. Math. Anal, accepted for publication.

Nakamura, R., Potthast, R. and Sini, M.
*The no-response approach and its relation to non-iterative methods in inverse scattering.*
Annali di Matematica Pura ed Aplicata, accepted for publication.

Potthast, R.
*A survey on sampling and probe methods for inverse problems.*
Topical Review for Inverse Problems 22 (2006), R1-R47.
Literature

