Finance and Mathematics: Where is the Ethical Malaise?

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The Viewpoint *column offers readers of* The Mathematical Intelligencer *the opportunity to write about any issue of interest to the international mathematical community. Disagreement and controversy are welcome. The views and opinions expressed here, however, are exclusively those of the author, The publisher and editor-in-chief do not endorse them or accept responsibility for them. Articles for* Viewpoint *should be submitted to the Editor-in-Chief, Marjorie Senechal.*

Discussions of the role of mathematics in finance appearing in *The Mathematical Intelligencer* can be split into two classes. Rogalski [26] and Korman [18] capture a widespread fury at a collapse in commercial ethics, whereas Ekeland [6] and Haggstrom [13] describe the economy nonjudgmentally. In summary, Rogalski and Korman would have mathematicians spurn the financial world; Haggstrom and Ekeland point to technocratic solutions, characterised by better regulation.

I do not buy into the argument that the problems of finance can be solved by regulations; they are, as both the U.K. and U.S. governments have identified,[1] ethical in nature. But I also do not think it is virtuous for mathematicians to spurn finance, so I am not completely aligned with Rogalski or Korman. My position is that mathematicians should be forthright in presenting financial mathematics as a discipline centred on the concept of justice, making it explicit that successful finance must be moral finance.

During the Financial Crisis of 2007 through 2009 I was the U.K. Research Council's "Academic Fellow" in Financial Mathematics, so that my background is, like Ekeland's and Haggstrom's, that of a financial mathematics "insider." In this role I was expected to explain the discipline I represented to U.K. policy-makers, both in government and in the media. As I attempted to meet these expectations, I took an unconventional step for a mathematician and started looking into the origins of mathematical probability, both its technical side and its cultural context.

I noticed that in solving the Problem of Points in 1654, Pascal and Fermat were pricing a derivative contract on a binomial tree, and their solution would today be recognised as the Cox-Ross-Rubinstein (CRR) option-pricing model that was published in 1978. There is a difference between the 1654 and the 1978 models: CRR provide a methodology for identifying the branch probabilities on the tree, but Pascal and Fermat assume they are a half. This raised a question: how did Pascal and Fermat conceive the probabilities they use?

The answer came, initially, in some work that the historian Edith Dudley Sylla did in the process of translating the *Ars Conjectandi*. Sylla observes that

> equity among associates or partners rather than probabilities in the sense of relative frequencies provided the foundation for the earliest mathematical probability theory [28, p 13]

and that

> the foundations (...) [were] not chance (frequentist probability), but rather *sors* (expectation) in so far as it

[1]In the U.S. Financial Crisis Inquiry Commission's report of 2011 and the U.K. "Changing Banking for Good" report of 2013.

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was involved in implicit contracts and the just treatment of partners.[28, p 28]

Intrigued by this point, I followed the path of mathematical probability from the origins of Western mathematics in Fibonacci's text on financial mathematics, the *Liber Abaci*, to contemporary mathematics's *Fundamental Theorem of Asset Pricing*. The *Fundamental Theorem* is a consequence of Black and Scholes's paper on pricing options [2]. It is based on the arbitrage argument, which originates in Aristotle's discussion of justice in commercial exchange in *Nicomachean Ethics*, and features in the *Liber Abaci*. The novelty of contemporary financial mathematics is not in the techniques used, or the products traded,[2] but in the fact that mathematicians today approach the problem as one of "positive" science, not "normative" ethics. For example, Black and Scholes opens with the observation that "it should not be possible to make sure profits" (this is Ekeland's central point), appealing to a consequentialist argument: that if you get your price wrong[3] someone will bankrupt you. Quite otherwise for medieval merchants, who needed a moral argument and who were conscious of the Catholic Church's injunction that a riskless profit was *turpe lucrum* (filthy money).

Back in 2009, at the start of this journey, I took a position similar to Ekeland's: there are economic laws that "outweigh the puny might of mathematicians" and the solution is in the hands of regulators.

European science is often distinguished from other cultures' science by the fact that it is mathematicised. There is an argument, first offered in 1934,[4] but developed more recently [12, 17], that this came out of Aristotle's examination of ethics in commerce. Justice in exchange is distinguished from distributive and restorative justice by Aristotle as being characterised by equality: "there is no giving in exchange," it is a reciprocal arrangement essential for binding society together [17, p 51]; [3, 1133a15–30]. It is notable that Aristotle approached this ethical problem mathematically, for he rarely applied mathematics to the physical world elsewhere [12, p 75]; [4, p 13]; [3, 1094b15–28]. On this basis, the medieval Scholastic scholars regarded money as a universal measure. Until then, Hellenic thought (including the Islamic scholarship of the time) had considered different physical properties, such as time and space, to be "incommensurable"; and it was accepting the universality of money that enabled the development of modern physics based on mathematics [4, 17]. To appreciate this point, observe that Copernicus wrote on money before he wrote on the planets; that Stevin, founder of the influential Dutch Mathematical School, was a financier; and that the financier Gresham endowed the first chair of mathematics in England and laid the foundations for the Royal Society. Bernard Bru has explained the significance of Bachelier's experience of stock markets in the development of Kolmogorov's ideas on probability [30, pp 20–21]. The close relationship between mathematics and finance is born out of the fact that finance is concerned with relations, measured as prices, between objects. Finance informs mathematics on measurement and uncertainty, whereas mathematics is critical to finance because we cannot perform experiments in the economy. It might not be possible to divorce the two disciplines, even if we wanted to.

The classicist Richard Seaford offers some insight into this account when he goes into the roots of Western thought and argues that Greek philosophy, including democracy and mathematics, is a consequence of Archaic Greece's use of money [27]. He notes that other ancient civilisations were based on centralised redistribution, whereas pre-Socratic Greek society was based on exchange, reliant on a conception of equality and reciprocity. He suggests that when the Pythagoreans assigned a number to every object, they were pricing the object.

The view that finance is socially corrosive is more novel than the practices of finance. One way of approaching Shakespeare's *The Merchant of Venice* is as a study of the four natures of love—erotic, familial, friendship, and the highest form of love—charity/*caritas*/*αγαπη*. Shakespeare personifies charity in Antonio, the merchant of Venice. Throughout the seventeenth and eighteenth centuries, commerce was considered a civilising influence. In *The Rights of Man* (1792) Thomas Paine writes that "commerce is a pacific system, operating to cordialise mankind." He is following a path laid by Montesquieu, Hume, Condorcet, and Adam Smith [8, 15]. After the Industrial Revolution, these attitudes all but disappeared: today one would scarcely choose to personify Christian love in the form of a merchant.

An explanation for this cultural shift can be found in *Dialectic of Enlightenment* [1], where it is argued that the Enlightenment led to the objectification of nature and its mathematisation, which in turn leads to "instrumental mindsets": to seek optimal means to predetermined ends and to an underlying need to control external events. Whereas during the seventeenth and eighteenth centuries public spaces emerged—the public sphere that facilitated rational discussion and sought the truth in support of the public good—in the nineteenth century mass circulation mechanisms came to dominate the public sphere and these were controlled by private interests. As a consequence, the public became consumers of information rather than creators of a consensus through engagement with information [11].

One aspect of this process of alienation for the public is the attitude that mathematics is an almost mystical pursuit that can show hidden truths, but only for the initiated,which is a recurring theme in the presentation of

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[2]Most of these products existed in medieval times. The "Triple Contract" shares the features of "structured products" prominent in the crisis. "Mortgage Backed Securities" were introduced in the United States in the late nineteenth century; see [19, Ch 5] for an enlightening account. It is not in the interests of well-dressed bankers to tell their clients that the products for which they are charging fat fees existed before Columbus.

[3]Ramsey's "Dutch book" argument, which has been described as a modern version of the "Golden Rule," "Do unto others as you would have them do unto you," Luke 6:31.

[4]By the Marxist theoretician Borkenau in *The Transition from the Feudal to the Bourgeois World View*.

mathematics in popular culture. This is nicely captured in a documentary film on the development of the Black-Scholes-Merton equation, where the economist Paul Samuelson describes how he "discovered" Bachelier's thesis, much as Indiana Jones might discover a magical artefact:

> In the early 1950s I was able to locate by chance this unknown book by a French graduate student in 1900 rotting in the library of the University of Paris and when I opened it up it was as if a whole new world was laid out before me.[5]

This trope might seem benign in the context of popularising mathematics, but when combined with the idea that mathematics is immutable and indubitable, as is done in traditional histories and philosophies of mathematics, we receive the impression, to paraphrase William Tait, that

> A mathematical proposition is about a certain structure, such as financial markets. It refers to prices and relations among them. If it is true, it is so in virtue of a certain fact about markets. And this fact may obtain even if we do not or cannot know that it does. [29, p 341]

Although mathematicians themselves might not make this claim explicitly, mathematics has been used by many to obscure and legitimise financial activity, passing over any consideration of the ethical implications of those activities. Ekeland might see mathematics as "puny," but others value its authority, and there are all too many examples of how mathematics has been used to prevent democratic oversight of the markets. In its submission to the Parliamentary Commission on Banking Standards in 2013, the Bank of England was highly critical of how some firms have used advanced mathematical techniques to "pull the wool" over the eyes of the regulator [22, v. II, para. 89]; likewise, U.S. authorities saw this type of mathematical sleight of hand as playing a part in the "London whale" episode [23, p 14]. The existence of the Gaussian copula as a "truth-teller" of the value of complex debt portfolios played a central role in the Crisis of 2007 to 2009, justifying the actions of banks, despite its shortcomings being known to mathematicians [9, 21, 31]. In the early 1970s, the Black-Scholes-Merton framework played an important part in legitimising the re-emergence of financial derivatives markets [20, p 158]. As long ago as 1877, a large corporate insurer defended to legislators its actions in undermining fraternal/mutual insurers with the argument that

> There are certain fundamental rules ...which can only be understood by actuaries, and it is impossible for me to go into here [19, p 198]

An antidote to the causes and consequences of "instrumental mindsets" identified earlier is to turn away from the philosophical paradigm of Foundationalism, which sees language as being made up of statements that are either true or false, and sees complex statements as valid if they can be deduced from true primitive statements. The foundationalist approach is exemplified in the standard mathematical technique of axiom-theorem-proof. An alternative approach is to shift the focus from what language says (which then may be true or false) to what it does.

Specifically, the function of language is to enable different people to come to a shared understanding and to achieve a consensus; this is defined as discourse[6] [10]. Because discourse is based on making a claim, which may be challenged and then justified, discourse needs to be governed by rules or norms to be successful.The most basic rules are logical and semantic; on top of these are norms governing procedure, such as sincerity; and finally there are norms to ensure that discourse is not subject to coercion or skewed by inequality. This is why reciprocity is central to financial mathematics: it is a norm of market discourse, embedded in the language of mathematics.

Mathematics has not been passive in recent financial crises, and I would argue that if mathematicians are not part of the solution, they are part of the problem. For me, the correct response of mathematicians to the financial crises is to join those who wish to redirect finance away from regarding markets as competitive arenas toward seeing them as centres of cooperative, democratic discourse.[7] In this vein I have developed the argument [16] that reciprocity is the central message of financial mathematics, and it is one of three norms of market discourse, the others being sincerity and charity. For this case to be coherent I have followed Putnam [25] and have abandoned the idea of mathematics as a value-neutral truth-teller; rather it is a means of discourse. This is a significant step if you perceive mathematics as being monogamously wedded to the natural and physical sciences, or even celibate. I believe certain twentieth-century mathematicians, such as Poincaré,[8] [14] Ramsey, [5] and Putnam, would have sympathy with the approach I take, particularly in the cases in which mathematics is used in the social and human sciences.

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[5]The programme is "The Midas Formula," also known as "The Trillion-Dollar Bet," and it is available on YouTube. The relevant section is around 12:20/48:53 minutes. A transcript is available at http://www.bbc.co.uk/science/horizon/1999/midas_script.shtml.

[6]According to a translator of Fibonacci, a key feature of the techniques provided in the *Liber Abaci* was that they enabled ideas to be transmitted and improved on [7, Introduction].

[7]An I.M.F. paper on the crisis, *Resolution of Banking Crises: The Good the Bad and the Ugly* (WP/10/146) shows that countries with a significant proportion of "not for profit" mutual banks (e.g., Germany, France, Italy) did not require the public bailouts needed in the United Kingdom and United States—finance is not necessarily capitalist.

[8]Poincaré's approach is captured in his observation that "these two propositions 'the earth turns round,' and 'it is more convenient to suppose that the earth turns round,' have one and the same meaning" [24, p 91].

REFERENCES

[1] T. W. Adorno and M. Horkheimer. *Dialectic of Enlightenment*. Stanford University Press, 1944 (2002).

[2] F. Black and M. Scholes. The pricing of options and corporate liabilities. *Journal of Political Economy*, 81(3):637–654, 1973.

[3] S. Broadie and C. Rowe. *Aristotle: Nicomachean Ethics: Translation, Introduction, Commentary*. Oxford University Press, 2011.

[4] A. W. Crosby. *The Measure of Reality*. Cambridge University Press, 1997.

[5] D. Edgington. Ramsey and Pragmatism: Probability, Conditionals and Truth. Cambridge Workshop on Pragmatism, May 2012.

[6] I. Ekeland. Response to Rogalski. *The Mathematical Intelligencer*, 32(2):9–10, 2010.

[7] L. Fibonacci and L. Sigler. *Fibonacci's Liber Abaci: A Translation into Modern English of Leonardo Pisano's Book of Calculation*. Springer, 2003.

[8] M. Fourcade and K. Healy. Moral views of market society. *Annual Review of Sociology*, 33:285–311, 2007.

[9] M. B. Gordy. A risk–factor model foundation for ratings-based bank capital rules. *Journal of Financial Intermediation*, 12(3):199–232, 2003.

[10] J. Habermas. *The Theory of Communicative Action: Reason and the Rationalization of Society*. Beacon Press, 1985.

[11] J. Habermas. *The Structural Transformation of the Public Sphere: An Inquiry Into a Category of Bourgeois Society*. MIT Press, 1991.

[12] R. W. Hadden. *On the Shoulders of Merchants: Exchange and the Mathematical Conception of Nature in Early Modern Europe*. State University of New York Press, 1994.

[13] P. Haggstrom. Mathematicians in the world of finance. *The Mathematical Intelligencer*, 34(1):6–7, 2012.

[14] G. Heinzmann. Henri Poincaré and his thoughts on the philosophy of science. In E. Charpentier, E. Ghys, and A. Lesne, editors, *The Scientific Legacy of Poincaré*. American Mathematical Society/London Mathematical Society, 2010.

[15] A. O. Hirschman. Rival interpretations of market society: civilizing, destructive, or feeble? *Journal of Economic Literature*, 20(4):1463–1484, 1982.

[16] T. C. Johnson. Reciprocity as a foundation of Financial Economics. *The Journal of Business Ethics*, Forthcoming, doi: 10.1007/s10551-014-2257-x.

[17] J. Kaye. *Economy and Nature in the Fourteenth Century*. Cambridge University Press, 1998.

[18] J. Korman. Finance and mathematics: a lack of debate. *The Mathematical Intelligencer*, 33(2):4–6, 2011.

[19] J. Levy. *Freaks of Fortune: The Emerging World of Capitalism and Risk in America*. Harvard University Press, 2012.

[20] D. MacKenzie. *An Engine, Not a Camera: How Financial Models Shape Markets*. The MIT Press, 2008.

[21] D. MacKenzie. The credit crisis as a problem in the sociology of knowledge. *American Journal of Sociology*, 116(6):1778–1841, 2011.

[22] PCBS. Changing Banking for Good. Technical report, The Parliamentary Commission on Banking Standards, 2013.

[23] Permanent Subcommittee on Investigations. JPMorgan Chase Whale Trades: A Case History of Derivatives Risks and Abuses. Technical report, Committee on Homeland Security and Governmental Affairs, 2013.

[24] H. Poincaré. Science and Hypothesis. In S. J. Gould, editor, *The Value of Science: Essential Writings of Henri Poincaré*. Modern Library, 1902 (2001).

[25] H. Putnam. *The Collapse of the Fact/Value Dichotomy and Other Essays*. Harvard University Press, 2002.

[26] M. Rogalski. Mathematics and finance: an ethical malaise. *The Mathematical Intelligencer*, 32(2):6–8, 2010.

[27] R. Seaford. *Money and the Early Greek Mind: Homer, Philosophy, Tragedy*. Cambridge University Press, 2004.

[28] E. D. Sylla. Commercial arithmetic, theology and the intellectual foundations of Jacob Bernoulli's *Art of Conjecturing*. In G. Poitras, editor, *Pioneers of Financial Economics: contributions prior to Irving Fisher*, pages 11–45. Edward Elgar, 2006.

[29] W. W. Tait. Truth and proof: The Platonism of mathematics. *Synthese*, 69(3):341–370, 1986.

[30] M. S. Taqqu. Bachelier and his times: a conversation with Bernard Bru. *Finance and Stochastics*, 5(1):3–32, 2001.

[31] G. Tett. *Fools' Gold*. Little Brown, 2009.