# A Dialogue on the Ethics of Mathematics

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**Absalom:** Welcome, Consuela, to this dialogue. You have the unenviable task of upholding your view that mathematics is ethical. How will you bridge the divide between ethics and pure mathematical knowledge?

**Consuela:** Good morning, Absalom. I suppose it is harder to argue that mathematics is ethical instead of the standard view of mathematics as ethically free, because the 'received' absolutist view so permeates the culture of mathematics. However, the idea that mathematics is ethical goes back to Plato or earlier.

**Abs.:** And will you use Plato's arguments to make your case?

**Con.:** No. In the *Republic* Plato argues that mathematics is ethical because it teaches unity, concord, order, and proportion, and these are central features of ethics as he understands it. In modern times we do not see these as the core of ethics, even if they can be virtues when applied to some behaviours and social situations.[1] By the way, we notice that the name Absalom happens to begin with the same three letters as *absolutism*, the position that you will uphold.

**Abs.:** Yes—just as your name begins the same as *constructivism*.[2]

**Con.:** From my point of view, mathematics is ethical, that is, pure mathematical knowledge has ethical implications. My claim is that there are intrinsic ethical elements within mathematics itself.

**Abs.:** Let's get this clear. I think it is important to distinguish: (1) the ethics of applications of mathematics, (2) the professional ethics of mathematicians, (3) the ethics of teaching mathematics, and (4) the ethics of pure mathematics itself, whatever that might be. We can clearly agree that ethical obligations are entered into within all professional and applicational work, as they are with all social practices. Any scientific application has an ethical dimension, there are professional codes of ethics for mathematicians, and clearly any teaching is bound by ethical codes.[3]

**Con.:** That's right, Absalom, we can both agree on issues 1, 2, and 3. These are not controversial. What I am claiming is your point 4, that mathematics itself is intrinsically ethical.

[1]Not everyone rejects Plato's (2000) position. Myles Burnyeat (2000), basing his arguments on Plato, proposes that mathematics is good for the soul by making it receptive to higher ideas including reasoning, truth, the good, and ethics.

[2]Note that, as emerges in the dialogue, Consuela espouses social constructivism, the view that mathematical knowledge and objects are constructed socially. This should be distinguished from the foundationalist philosophy named constructivism, related to intuitionism, which proposes that all mathematical objects are constructed by finite procedures, and rejects completed infinities as well as the validity of proofs of existence by contradiction. Social constructivism has, in the past few decades, won a growing number of philosophers and mathematicians as adherents. For example, see the collections Gold and Simons (2008) and Tymoczko (1986).

[3]A number of mathematicians have written on the need for ethics for applications of mathematics, including Chandler Davis (1988) and Philip Davis (2007); Reuben Hersh (1990, 2007) has written on mathematics and ethics and on ethics for mathematicians; and the American Mathematical Society (2005) has published Ethical Guidelines for mathematicians including research and teaching obligations. There is also an extensive literature on ethics in mathematics education, including Atweh (2013), Boylan (2013), and D'Ambrosio (1998).


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**Abs.:** You are claiming that pure mathematical knowledge itself has ethical implications? This is a radical claim that most people would find difficult to swallow. With all due respect, I think I will have no trouble refuting it.

**Con.:** Well I'm, not entirely on my own with this claim.[4] To make my case I have three arguments to offer.

## First Argument

**Con.:** My first argument comes from the warranting of mathematical knowledge. Do you agree that to establish the validity of a mathematical claim you must display the means of verification openly? Thus, the warrant for asserting a theorem is a proof, and the solution of a simple problem is a calculation, and these must be offered for inspection.

**Abs.:** I'm not clear why you bring up calculation. Mathematical problems are solved by deductive proofs. However, I agree that mathematical knowledge requires warranting through mathematical proof.

**Con.:** No, calculations are important because they are the means for warranting most mathematical knowledge. When a shopkeeper shows you a bill, you can verify the correctness of the total by checking the individual sums, the components of the calculation by which the total is derived.

**Abs.:** Your example is too trivial to have any place in this discussion.

**Con.:** I disagree. Compared to modern research mathematics, everyday calculations are trivial, but they still represent the most widespread mathematics in all societies, and were once the major part of all mathematics. Jens Høyrup finds that in ancient Mesopotamia and Egypt, where mathematics was invented, the reliability of calculation, measures, and numerical records was understood as part of the idea of justice, taking on an ethical value. It was the openness and democratic checkability of accounting, taxation, and trade calculations that enabled these activities to be trusted and relied upon by all parties involved.[5]

**Abs.:** Please let us focus on proper mathematics, such as theories, theorems, and proofs.

**Con.:** Technically every calculation can be represented as an arithmetic proof,[6] and conversely, so we can consider them together if we want to.

**Abs.:** Go on then, what is the point you wish to make about proofs?

**Con.:** I said that any kind of mathematical claim could become mathematical knowledge only when it is warranted. The role of proof is to persuade its audience of the correctness of the claim via a convincing chain of reasoning. Agreed so far?

**Abs.:** Yes, a mathematical proof needs to be verified by mathematicians as a correct chain of reasoning.

**Con.:** My point is that this makes mathematics open and democratic. Mathematical knowledge cannot be established by fiat or authoritative pronouncement. Its claims must be made publicly and be openly available for challenge. Thus mathematics is not a secret or occult knowledge. To qualify as knowledge at all, it needs openness and democratic access. Whereas for difficult abstract theorems only a limited audience can read and validate them, for simpler mathematical claims, such as an elementary calculation, a much broader audience can validate them. Thus mathematics embodies openness and democracy. And these are ethical values.

**Abs.:** Ah! … I now see why you were keen to include calculations as well as proofs in your claim. But aren't the shopkeeper's calculations applied mathematics? Didn't we concede the ethical nature of applications and dismiss it from your main claim that mathematical knowledge itself is ethical?

**Con.:** We did agree that, but calculations come into my argument not as applications, but as pure mathematics. In checking a calculation we are not seeing how well a theory fits with the physical world, but looking within pure mathematics to check the correctness of an internal mathematical procedure. If I spend 17 Euros, my change from a 50-Euro note is 33 Euros because $50 - 17 = 33$ is correct within arithmetic, not because of any empirical observations or practices.

**Abs.:** Okay, I'll concede that point. But you have opened another can of worms. Because if calculation is not a mathematical application but pure mathematics, then it is free of value and ethics, since these are necessarily excluded from epistemology and epistemological concerns.[7]

**Con.:** To be sure, I can't agree to that. You are presupposing the very thing I am trying to disprove. Yes, I recognize mathematical knowledge as knowledge, an epistemological issue, but that does not of necessity exclude values or ethics. This is precisely the point I am arguing.

**Abs.:** Whilst I cannot agree with you, I will suspend this reservation while I hear your case. But I have another criticism to your argument so far. A proof is presented openly not because mathematics is intrinsically open or democratic, but just because a warrant is needed. Mathematical knowledge is not open or democratic; it is a body of absolute truths. Such truths need logical warranting, and the process opens mathematics up to the human gaze, but that is quite incidental. The proofs in themselves are purely epistemic or epistemological.

**Con.:** Now here I disagree with you. If mathematical theorems need proofs to warrant them as mathematical knowledge, who is the intended reader of the mathematical proof? Whom else but humans can the reasoning persuade?

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[4]A few authors have argued that mathematical knowledge is itself ethical or has ethical implications, including Ernest (1998) and Johnson (2012).

[5]See Høyrup (1980) and Høyrup (1994). To be sure, he goes on to say that precise accounting also served to disguise exploitation by giving the illusion of justice.

[6]It is easy to show that all calculations can be represented as deductive proofs (and vice versa), for example, see Ernest (2009). Gödel's Incompleteness theorems depend on the arithmetization of Russell-Whitehead's first-order logic and proof, so that if $t$ is the arithmetic representation of a theorem $T$, and $p$ encodes its proof $P$, there is a primitive recursive (decidable) function $f$ such that $f(p,t) = 1$ iff $P$ proves $T$, for any $P$ and $T$. Thus any proof in the system is in fact verifiable by the calculation of the value of $f(p,t)$, which is decidable.

[7]Epistemology is the branch of philosophy that studies knowledge and its foundations.

Who is the knower of mathematical knowledge? How can knowledge exist without a knower? If the knower were god, why would she—all-seeing and all-knowing—need a proof to warrant mathematical assertions? Mathematical knowledge needs open warrants in the form of proofs, and this makes openness and hence democracy an intrinsic part of the epistemology of mathematics. To deny this is to deny that the purpose of proof is to let any listeners or readers come to know the certainty of its conclusions.

**Abs.:** No, not at all. The purpose of proofs is logical, to provide an ironclad demonstration of the certainty of mathematical theorems. Proof is purely epistemological, and although its existence potentially opens up the reasoning to scrutiny, it is not itself democratic or open. Human access is a fortuitous by-product of epistemology.

**Con.:** Here we have to differ. I do not believe there can be knowledge without some knower, or proof without some demonstration, warrant, or persuasive reasoning. Furthermore, it is always the case that you demonstrate something to someone, you warrant something to someone, and you persuade someone of something. These are all transitive verbs, and the idea that there can be knowledge without a knowing subject or persuasion without a persuadee does not make sense to me. To accept it you must, by nominalizing knowing, reasoning, demonstrating, warranting, or persuading, believe in free-floating ideas with no connections to their originating authors or agents.

**Abs.:** You are making the fallacy of slipping from the context of justification to what Popper and Reichenbach termed the context of discovery. These are wholly separate; one is in the domain of logic and one in the domain of psychology.[8]

**Con.:** I am aware of this distinction promoted by Popper and Reichenbach. Both of these philosophers of science were very keen to demarcate science from other domains of knowledge. But following the philosophers Quine and Putnam, I think it is more accurate to see all of human knowledge and knowing as a single connected web encompassing both a priori knowledge, such as mathematics and logic, and a posteriori knowledge including the physical and human sciences, to use Kant's distinction.[9] Of course some aspects of discovery in mathematics and science fall outside the domains of logic and epistemology. But as Lakatos asserts, there are rational aspects to scientific and mathematical discovery, which are the business of epistemology, just as there are elements of sociology and psychology in warranting scientific and mathematical knowledge.[10] Thus the contexts of discovery and justification are not as philosophically distinct as Popper says.

**Abs.:** Of course I disagree with you about the context of discovery. As Feyerabend says, in mathematical or scientific discovery anything goes,[11] whereas the justification of knowledge is purely about logic and epistemology. Mathematical proofs are purely logical, and scientific testing of hypotheses is just about deducing logical consequences from theories and testing them by observation.

**Con.:** Sure, that's the official story: mathematical proofs are purely logical. In practice, no actual proof in mathematics follows the rules of formal mathematical logic. Published proofs are always abbreviated deductions that make claims that are persuasive to mathematicians, compelling even, but rarely if ever fully rigorous.

**Abs.:** It may be that in practice almost all published mathematical proofs are abbreviated, but they could always be written out in full as formal logical proofs.

**Con.:** This is a potential promise that you will never see fulfilled. Even if it were true, the complete proofs would be too long for humans to check them. Even in the relatively primitive system of *Principia Mathematica,* it took more than 300 pages to derive the fact that $1 + 1 = 2$.[12] Imagine the size of the proof of a nontrivial theorem: it would be too large for a single human to comprehend. We would need to check the proof with some aids such as computer verification of proofs. Then we would need to check the computer verification procedures with some other safeguard. This would send us into an infinite vicious spiral of verification.

**Abs.:** Nevertheless, the brevity of proofs in practice is a matter of convenience, not of principle.

**Con.:** I disagree. Mathematicians' tacit knowledge is brought to bear in deciding that a published proof is adequate, and this tacit knowledge is acquired experientially, by immersion in the practices of professional mathematics.[13] Thus mathematical proofs cannot be said to exist purely in the domain of logic. We do not look solely to the rules of logic to warrant them. We also depend on the tacit knowledge of mathematicians, knowledge that I claim cannot be made fully explicit. The important thing about proofs and calculations is that they must be understandable and readable to their intended audiences, and that they are persuasive to them. To serve their purposes as epistemological warrants, they have to be open and democratically accessible. Hence my claim that mathematics embodies these values.

**Abs.:** I agree that mathematics needs to be open and accessible to mathematicians to provide epistemological warrants. But I say proofs are primarily epistemological, and these properties are necessary only incidentally, not as values of mathematics.

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[8]The distinction between the contexts of discovery and justification was introduced by Hans Reichenbach (1951) and taken up by Karl Popper (1959).

[9]By a priori knowledge, Kant (1781/1961) means that which we can know and justify by reasoning alone without drawing on any evidence from, or experience of, the world. In contrast, a posteriori knowledge draws on experience of the world for its justification. Willard V. O. Quine (1953, 1960) and Hilary Putnam (1975) both reject a clear-cut a priori/a posteriori distinction. Although not overtly excluding the distinction between the contexts of discovery and justification, this makes it more difficult to sustain in mathematics, where justification is traditionally viewed as a priori and the context of discovery is seen as drawing on the experience of mathematical practice.

[10]Lakatos (1976, 1978).

[11]Paul Feyerabend (1975) argues against any presupposition that there can be a fixed method of discovery in science.

[12]The proof is on page 379 of Whitehead and Russell (1910). Of course part of the work that precedes it is devoted to setting up and defining the logical system employed.

[13]Polanyi (1958).

**Con.:** But you concede that openness and democratic accessibility are needed, and they are ethical values, so to this extent you have conceded my point.

## Second Argument

**Con.:** My second argument for the ethical nature of pure mathematics is as follows. We agree that pure mathematics is that field of knowledge in which mathematical concepts, methods, proofs, problem solutions, and theories are refined, developed, and extended, often without any thought of applications outside of mathematics.

**Abs.:** Yes, I agree, pure mathematics is pure and is thus unrelated to the world, to applications, human purposes, or their everyday or philosophical concerns, like ethics.

**Con.:** Not so fast, please, I agree with your premise but not all of your conclusions. Newton's developments in the calculus are regarded as a triumph of pure mathematics even though they were at least partly motivated by the need to solve problems for Newton's gravitational theory.

**Abs.:** Are you trying to argue that physical problems and applications drive the development of pure mathematics and they bring an ethical dimension with them?

**Con.:** No, for I acknowledge that much of pure mathematics is developed with no inkling of any real-world applications in mind, not even on the horizon. Instead I want to point out that it represents virtuosity both in the working mathematician and in the refinement and generalisation of the discipline.

**Abs.:** I acknowledge that great mathematicians exhibit virtuosity in their inventive practices, but aren't you changing the subject? We're talking about mathematical knowledge, not about mathematical practice.

**Con.:** It's not a change of subject. I want to look at the growth of mathematical knowledge. We don't say that the forces that cause plant growth are not the business of botany, or that the forces fostering growth of science are not the business of the philosophy of science. Likewise I want to say that the intrinsic forces that drive the growth of mathematics are also relevant and central to mathematics and its philosophy. Mathematical knowledge is not a once-given and forever-fixed body of knowledge. Any philosophy must acknowledge its growth, whether we say it grows by invention or by discovery. Mathematicians find new theorems and theories and verify them as sound. What drives this growth process?

**Abs.:** Aren't you going back to the context of discovery, which I excluded from epistemology?

**Con.:** I certainly am. I argued earlier that it is wrong to exclude it. I want to argue that there are two levels of impulse that drive pure mathematics forward. First, there are the personal impulses of mathematicians to extend knowledge, to expand their skills for their own sake, and to improve their virtuosity as mathematicians. Now my argument is that the development of virtuosity for its own sake is a move toward improvement, betterment. After all, the term 'virtuosity' is based on virtue and the virtuous, and these are facets of the good. Human virtuosity is part of the good life, of human flourishing, and is thus intrinsically good and ethical.

**Abs.:** You surely can't maintain that all human virtuosity in the pursuit of knowledge is good. What about Dr Mengele and the atrocious experiments on living humans? Or the hurtful and harmful uses of animals in science during earlier times? Even if great virtuosity were developed and important advances in knowledge made, which I am not conceding, could you really call this an unqualified good? Can you call this an ethical pursuit of knowledge?

**Con.:** You make a good point. Not all skills developed to the point of virtuosity are necessarily ethical or good. I have to accept this constraint, that we are discussing pure theoretical knowledge whose content and development have no immediate impact on people, animals, or the world. I believe that theoretical blue-sky research, and the development of virtuosity that goes with it, are good for humans. Human virtuosity contributes to the flourishing of humanity as a whole.

**Abs.:** Are you claiming that any and all virtuosity, no matter whether it has any impact on humanity or not, is a force for good and against evil?

**Con.:** I do concede that I should be claiming only that human virtuosity under the constraint I put on it is *potentially* rather than always good for humankind; but I am also claiming it is never bad or evil.

**Abs.:** What about a brilliant painter who expresses his virtuosity in painting a refugee ship sinking with all souls lost as he stands before it on the shore with his easel, without helping anybody?

**Con.:** I would argue that the painter's past development of virtuosity is a good thing even if when he applied his skills during a crisis he did not do the right thing.

**Abs.:** What if most of the time he was developing his virtuosity he ignored urgent ethical demands, or if he persistently behaved unethically to sustain his own development? Can you still say his virtuosity is a force for good?

**Con.:** Yes I can, because my argument does not depend on judging the ethics of individual humans, but on seeing human virtuosity, especially that of mathematicians, as contributing to positive cultural growth and human flourishing.

**Abs.:** I do agree that development of mathematicians' virtuosity is intrinsically good, along with the development of mathematics, I don't agree that this makes mathematics ethical. As objective knowledge, it stands aloof from ethical judgements. It just *is,* irrespective of our opinions or applications.

**Con.:** So, only partial agreement on this part of my claims. The second level of impulse that drives pure mathematics forward is the force within mathematical knowledge itself to expand.[14] This is of course a figure of speech, to say that an artefact should 'cry out' for the development and improvement of itself through analogy, abstraction, extension, unification, simplification, and so

[14]According to Imre Lakatos (1976) ''Mathematics, this product of human activity, 'alienates itself' from the human activity which has been producing it. It becomes a living, growing organism, that acquires a certain autonomy from the activity which has produced it; it develops its own autonomous laws of growth'' (p. 146).

on. But an artefact is a tool, in the widest sense, it is imbued with human intention. It is *for* something. Even in the case of art objects and classical music, the purest of human creations, the larger purpose is human understanding, enjoyment, and flourishing. All artefacts including mathematics have human intentionality inscribed within them. Thus the discipline of mathematics has development and progress inscribed within it. It embodies the drive toward making our understanding better, toward the good. According to Habermas, the aim of extending pure knowledge, even for its own sake, can never transcend the underlying interest of predicting and controlling the world.[15] Thus even the purest of pure mathematics, as an expansion and improvement of knowledge, represents the desire to improve human understanding, technology, and hence the place of humans in the world. But by striving for the good, it is inescapably ethical.

**Abs.:** You will not be surprised to hear that I cannot accept this argument. First, I do not accept that the virtuosity of mathematicians somehow makes mathematics good. They may strive to make themselves good or even excellent mathematicians, but such human features are immaterial to their discoveries. For if a particular individual did not make a discovery, then another would make it. Second, I reject the claim that mathematical knowledge is just another human artefact. It is a domain of objective knowledge that transcends the human-made world. Third, I cannot accept that mathematics is imbued with any drives, impulses, or intentions. These belong to persons, mathematicians and others, and not to the subject. Fourth, the fact that mathematics proves useful in predicting and controlling the world is an after-the-fact application, and not a property of mathematics. Mathematics might wish to claim credit for a valuable real-world application, but it could not do so legitimately, because it exists in a separate domain, unconnected to the world of its application except through the act of application itself.

**Con.:** We differ fundamentally; your philosophy disallows the premises and assumptions I build on for my arguments. It is hard to get you to reexamine your assumptions because your absolutist philosophy of mathematics has two and a half thousand years of tradition behind it, and is taken for granted by many mathematicians and philosophers. In contrast, the 'maverick' philosophy of mathematics that I espouse has less than a century of existence.[16] So we 'maverick' philosophers are still struggling to expose the implicit assumptions within 'received' philosophies of mathematics, as well as clarifying the epistemology and ontology of how mathematics is and can be socially constructed. You list four differences between you and me, but they centre on one: we differ in what we mean by saying that mathematics constitutes objective knowledge.

**Abs.:** Isn't it clear what that means? Objective knowledge comprises brute facts verifiable by the senses in the physical world, or in the domain of knowledge, by dint of logical necessity. Knowledge that is established objectively holds independently of any knower, and holds universally in all contexts, at all times and for all knowing beings, both real and potential.

**Con.:** I want to reject or at least critique much of this. First of all, there can never be total agreement on brute facts verifiable by the senses in the physical world. The senses may be deceptive, and besides, I believe that the fact-value dichotomy cannot be totally upheld.[17] Indeed, my main argument that mathematics is ethical is an example of the failure of fact-value dichotomy. I have already argued that mathematical knowledge is not established conclusively by logical necessity, but by persuasive proofs that fall short of absolute logical criteria. Furthermore, I think it is hubris to claim that objective knowledge can hold independently of any knower, universally, in all contexts. What evidence do we have, or could we possibly have, of such universality? How could it ever be established objectively by the senses or logic?

**Abs.:** My definition of objective knowledge is purely a definition, and its consequences follow by logic.

**Con.:** I am not saying that your definition of objective knowledge is inconsistent. I am saying that it is empty, that no knowledge, especially significant mathematical knowledge, falls under it.

**Abs.:** What about '$1 + 1 = 2$' or '$2 + 2 = 4$'?

**Con.:** I don't regard these facts as significant mathematical knowledge, since the first is a definition of '2' and the second is a trivial consequence of definitions like this and of Peano's axioms without even using the axiom of induction. Because of this extreme simplicity these facts are trivial.

**Abs.:** Then how do you define objective knowledge?

**Con.:** I wish to contrast two different meanings of objectivity. First, there is your definition, which I term traditional or absolute objectivity. In contrast, I take the meaning of objectivity as opposite to that of subjectivity. Objectivity in this sense means having an existence that goes beyond any individual knower's beliefs. I term this second, broader definition as cultural objectivity. Laws, bank account balances, and indeed language are objective in this cultural sense, because their existence is independent of any particular person or small groups of persons, though not of humankind as a whole. They are part of what is termed social or cultural reality.[18] Absolute and cultural objectivity evidently have different meanings, because

[15]Jurgen Habermas (1972) argues that all knowledge is based on human interests. The technical interest underpins scientific and mathematical knowledge, which means that it is about prediction, control, and certainty, rather than understanding (Ernest 1994c). This characterises pure mathematical knowledge even before it is applied.

[16]Philip Kitcher and William Aspray (1988) describe philosophies of mathematics that are antifoundationalist and focus on mathematical practice as forming a maverick tradition outside of the dominant tendency in the philosophy of mathematics.

[17]Many philosophers of science, such as Hilary Putnam (2002) and Harold Kincaid and colleagues (2007), reject the fact-value distinction.

[18]There is an upsurge of philosophical interest in social reality as evidenced in the work of John Searle (1995). Building on this, Julian Cole's (2008 & 2013) social constructivist philosophy of mathematics proposes that that mathematical facts stand on the basis of collective agreement and are part of what he terms institutional reality.

mathematical objects might exist in the social and cultural realm beyond any individual beliefs, thus having cultural objectivity without having independent physical existence or existence due to logical necessity, that is without having absolute objectivity.

**Abs.:** I don't accept your example of mathematical objects here, because I deny that they could exist only in culture.

**Con.:** I could stick to nonmathematical examples in making the point.

**Abs.:** But doesn't your definition of objectivity include mine?

**Con.:** Yes, I do say that absolutely objective knowledge is a proper subset of culturally objective knowledge; it's just that I say it is an empty subset.

**Abs.:** But you are saying that all mathematical knowledge and more generally that all objective knowledge is something constructed and accepted by people, and that it is human acceptance that warrants it as knowledge!

**Con.:** Yes, that is precisely what I claim. New mathematical knowledge is that which is accepted as warranted by mathematicians belonging to the social institution of mathematics, that is, those who have mastered the well-entrenched tacit and explicit criteria for knowledge acceptance.[19] Any accepted mathematical knowledge has a warrant, that is, a proof that persuades mathematicians of its certainty. Objectivity and certainty in their cultural sense are redefined as social, as I, and others, argue elsewhere.[20] This is how social constructivism views mathematical knowledge and objects. Such a perspective has a strong bearing on the discussion of values in mathematics, because it posits that at least some of mathematics is contingent on human history and culture, and thus mathematics itself can be imbued with the values of the culture of its human makers.

**Abs.:** Of course you know that such claims are anathema to me and to the majority of philosophers of mathematics.

## Third Argument

**Con.:** Yes, I do know this, but nevertheless it is the basis for my third argument for the ethical nature of mathematics. My philosophy of mathematics is social constructivism and I propose conversation as the underlying epistemological unit of this philosophy.[21]

**Abs.:** I cannot understand how this is relevant. Also, in what way can a mathematician stuck on a desert island for 30 years writing and proving theorems on her own be said to be in conversation?

**Con.:** Surely we acquire the language and its extensions that make up mathematical language conversationally from others, and internalize conversation as our mode of thinking.[22] So even when we are thinking on our own we are engaging in a form of internal conversation, taking different conversational roles such as proponent and critic, which we internalized during our apprenticeship. Furthermore, the desert island theorems do not become part of the objective corpus of mathematical knowledge until they are read and accepted by others. Thus their warranting results from a further conversational act, their reception by listeners/readers.

**Abs.:** To me this is a bizarre way to talk, and I don't see its relevance to mathematics.

**Con.:** Just bear with me while I explain how mathematics is based on conversation. I say that conversation, consisting of symbolically mediated exchanges between persons, underpins mathematics, and that it does so in five ways. First, the ancient origins as well as various modern systems of proof use dialectical or dialogical reasoning, involving the persuasion of others. These are conversational exchanges. Second, mathematics is a symbolic activity using written inscriptions and language; it inevitably addresses a reader, real or imagined, so mathematical knowledge representations are conversational. Third, many mathematical concepts have an internal conversational structure.[23] Fourth, the epistemological foundations of mathematical knowledge, including the nature and mechanisms of mathematical knowledge genesis and warranting, utilise the deployment of conversation in an explicitly and constitutively dialectical way.[24] Fifth, following Julian Cole, I wish to assert that mathematical facts stand on the basis of collective agreement and are part of institutional reality. Now these social dimensions are built on interpersonal communicative interactions, that is, through conversation. Overall, the very content of mathematical knowledge—its concepts, methods, proofs, as well as its genesis and justification—are conversational.

**Abs.:** My previous objections seem again to apply. The activities that you have described, apart from the last two, take place in the context of discovery, not justification, and hence have no bearing on the nature of mathematical knowledge. The last two reasons draw on a sociological notion of warrants which I reject in favour of traditional objective criteria for certifying belief. Any analogy between the interior structure of some mathematical concepts and conversation or dialogue is entirely fortuitous, as far as I am concerned. Any conceivable structure can be found within mathematics, for it is the preeminent science of structure.[25] Conversation is firmly embedded in human activity and thereby relevant to mathematicians, but that does not make it relevant to mathematical knowledge.

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[19]See, for example, Ernest (1999).

[20]Bloor (1984), Ernest (1998), Fuller (1988), Harding (1986), and others, propose a social theory of objectivity.

[21]A version of social constructivism as a philosophy of mathematics is developed in Ernest (1991, 1998); its account of the conversational basis of mathematics draws on the work of Wittgenstein (1953) and Lakatos's (1976) *Logic of Mathematical Discovery*. Other authors such as Hersh (1997) and Cole (2008, 2013) offer social constructivist philosophies of mathematics, without explicitly drawing on conversation as a basic notion.

[22]This is the model of the origins of thought and language acquisition of the influential social constructivist psychologist Lev Vygotsky (1986).

[23]These include epsilon-delta definitions of limit in analysis, hypothesis testing in statistics, and many other concepts (Ernest 1994a).

[24]The conversational genesis and warranting mechanisms are described in Lakatos's (1976) *Logic of Mathematical Discovery* and in Ernest's (1998) *Generalised Logic of Mathematical Discovery*.

[25]Lynn Arthur Steen (1988), among others, characterises mathematics as the science of patterns and structure.

**Con.:** I am not trying to persuade you, for I know you have fixed views on this. I am trying to develop my argument from social constructivist premises. And I have not yet reached my argument concerning ethics.

**Abs.:** All right, let's hear the ethical argument.

**Con.:** My claim is that conversation, in a number of ways, lies at the heart of mathematics, providing it with a human foundation. But conversation as an interpersonal activity is inescapably ethical, it is not just about exchanging information.[26] It entails engaging with a speaker or listener with mutual respect and trust, attending to another's proposals and responding relevantly, and being aware of reactions to one's own contributions. Every participant in conversation has an ethical obligation to the others. In mathematics, putting one's proposals in an appropriate and accessible format following received norms of acceptability is part of one's ethical responsibility throughout pure, applied, and educational mathematics.

**Abs.:** Ah, so from the premise that mathematics is conversational, you want to infer that the ethical status of conversation must be shared by mathematics. Since I reject the premise, I need not worry about the conclusion, and need not test the strength or weakness of your logic.

**Con.:** Perhaps we have reached some understanding. If you start with an absolutist philosophy of mathematics, then mathematical knowledge exists in some domain independent of us, which does not admit ethics.

**Abs.:** Yes, this is my position and to me it is self-evident.

**Con.:** However, if you start from a humanistic or social constructivist philosophy of mathematics, then, knowing that mathematics is humanly made and warranted, you may well admit the relevance of ethics to mathematics and mathematical knowledge, as I do.[27] Indeed, you do concede that ethical obligations are entered into within all social practices. Since social constructivism sees mathematics as a social practice and mathematical knowledge as an artefact of social practice, it follows that it is ethical.

**Abs.:** But in what sense is it ethical? Does this mean that mathematical knowledge carries ethical obligations with it? This sounds like a non sequitur to me. It's nonsensical.

**Con.:** When I say that mathematical knowledge is ethical, I mean that it is *for* something and that its purpose is for good or ill. As a human artefact, mathematics is the purposive product of human activity. As such it both benefits its human makers and contributes to overall human well-being. Furthermore, like any human knowledge, mathematical knowledge must be open to human understanding, both in revealing its warrants and in acknowledging its human audience and participants.

**Abs.:** To me, mathematics is more than simply a human artefact. Irrespective of its human origins, mathematical knowledge, after it has been correctly formulated and warranted, is absolutely and universally certain. Thus it stands apart from the material world and from human beliefs, psychology, and ethics. These have become irrelevant.

**Con.:** I believe that your absolutist philosophy is self-consistent and defensible, even if I cannot fathom where you think mathematical knowledge and mathematical objects might exist nor how we can possibly interact with them. Needless to say I also believe that my social constructivism is both self-consistent and defensible. Apparently we have incompatible philosophical foundations, neither of them falsifiable. At least my philosophy is exempt from the need to bridge two seemingly incompatible ontological domains: on one hand objective knowledge and ideal objects, and on the other hand the physical world including all human activity. I believe that multiplying ontologies to accommodate the mathematical and the physical worlds separately is a categorical mistake. Just as a religious believer and a secular thinker may respect each other's worldviews, I respect absolutism even though I'm unable to understand your theology.

**Abs.:** That's magnanimous of you (although I don't know if 'theology' is the right word). Unfortunately I cannot reciprocate. I am convinced that a social constructivist perspective can be refuted, even if I have not done so in this debate. I think it is you who make a categorical mistake. Objective knowledge including mathematics transcends human activity and knowing, and all other earthly, transitory things. Thus I reject the idea that mathematics of itself can be ethical. I apply ethical considerations only to its worldly uses and applications.

**Referee:** You still might tell us, Absalom, your reaction to this point from Plato, which Consuela mentioned but does not rely on: that learning geometry, say from Euclid's *Elements*, teaches us what a sound and valid argument looks like. It provides us with an important standard against which to evaluate the cogency of all kinds of arguments, including the fit between evidence and conclusions. And that's certainly an important part of ethics: offering fair and cogent arguments.

**Abs.:** No, I reject that claim too. I regard truth, and logical argument used in warranting truth, as a part of epistemology. When you say 'fair reasoning' I think you simply mean correct reasoning. My interpretation puts this outside of ethics.

**Con.:** Thank you, Referee, for reminding us of this further way that mathematics is intrinsically ethical. However, it is evident that Absalom remains unconvinced.

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[26]Many authors stress the ethical nature of conversation, including Ernest (1994b), Johannesen (1996), Gadamer (1986), and Rorty (1979), for example. Habermas's (1981) idea of universal pragmatics is based on humans as communicative beings; and effective communication, as it requires truthfulness and some kind of equality, is intrinsically ethical.

[27]Reuben Hersh (1997) terms his social constructivist philosophy of mathematics *humanistic*.

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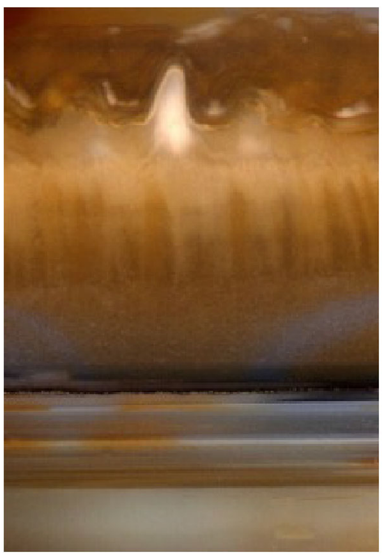

Ciurlionis by Eugene Plotkin

It seems to me that a misty army has risen over the sea, that fairy tales are revived, that legends are materialized, that this stone is just a previously unknown painting of "Sonata of the Sea" created by the ingenious Čiurlionis. It seems to me that the music sounds. But the author slips from memory. Definitely this is not Čiurlionis himself. Then who? Perhaps the composer has not been born yet?

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