CSP TD n°7

Solutions

1) Notez que :

i) $R \subset T \Rightarrow \text{Aire}(R) < \text{Aire}(T)$

ii) $\text{Aire}(T) = \alpha + \beta + \gamma - \pi$

iii) $\text{Aire}(R) = \int_0^{\pi/4}\int_0^{\pi/4} \cos u \, du \, dv = \frac{\pi}{4}\left(\frac{1}{\sqrt{2}}\right) = \frac{\sqrt{2}\,\pi}{8}$

Donc

$$\alpha + \beta + \gamma - \pi > \frac{\sqrt{2}\,\pi}{8} \Rightarrow \alpha + \beta + \gamma > \frac{(8+\sqrt{2})\,\pi}{8}$$

d'où

Un des angles α, β, γ de T doit être $> \frac{(8+\sqrt{2})\,\pi}{24}$

2) i) $z = 2xy^2 + 3y^2 + x^6$

$2z = 6y^2$ + Termes de degré ≥ 3

Donc $2z = Lx^2 + 2Mxy + Ny^2 \; (+ O(\|(x,y)\|^3))$ $\quad L = M = 0$, $N = 6$

$$A = \begin{pmatrix} 0 & 0 \\ 0 & 6 \end{pmatrix}$$

$\Rightarrow k_1 = 0, \; k_2 = 6, \; H = 3, \; K = 0$ en $\bar{0}$

ii) $z = 3x^2 - 2xy + xy^2$

$2z = 6x^2 - 4xy$ + terme de degré ≥ 3

Donc $2z = Lx^2 + 2Mxy + Nz^2 \; (+ O(\|(x,y)\|^3))$ $\quad L = 6, \; M = -2$, $N = 0$

$$A = \begin{pmatrix} 6 & -2 \\ -2 & 0 \end{pmatrix}$$

$\Rightarrow H = 3, \; K = -4, \; k_1, k_2 = 3 \pm \sqrt{9+4} = 3 \pm \sqrt{13}$

3) $z = xy \Rightarrow 2z = 0 \cdot x^2 + 2 \cdot xy + 0 \cdot y^2 \Rightarrow A = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}$

$\Rightarrow H = 0, \; K = -1, \; k_1, k_2 = \pm 1$

Autrement, $A = \begin{pmatrix} z_{xx} & z_{xy} \\ z_{xy} & z_{yy} \end{pmatrix}_{|(0,0)} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \Rightarrow \dots$

4) $\gamma_\theta(t) = \left(t\cos\theta,\ t\sin\theta,\ t^2 \underbrace{(\cos^2\theta - \sin^2\theta)}_{=\cos 2\theta} \right)$

$\gamma'_\theta(0) = (\cos\theta,\ \sin\theta,\ 0)$

$\gamma''_\theta(0) = (0,\ 0,\ 2\cos 2\theta)$

$$K_\theta(0) = \frac{\| \gamma'_\theta(0) \wedge \gamma''_\theta(0) \|}{\| \gamma'_\theta(0) \|^3} = |2\cos 2\theta|$$

$\Rightarrow k_\theta = 2\cos 2\theta$ $\qquad k_\theta = \begin{cases} 2 \text{ maximale} & \theta = 0, \pi \\ -2 \text{ minimale} & \theta = \frac{\pi}{2}, \frac{3\pi}{2} \end{cases}$

Donc $k_1 = 2$, $k_2 = -2$

d'où $H = 0$, $K = -4$