ABSTRACT
The curvature of a subducting lithosphere is chiefly controlled by the viscosity ratio between the slab and the surrounding mantle. On the basis of a semi-analytical flow model, we explore the rheological dependence of the geometrical response of a viscous slab subjected to toroidal mantle flow. Mantle flow is excited by slab retreat at a prescribed mean velocity and is iteratively solved for by using a stream function approach, in turn providing the stresses that bend the slab. Comparison between model predictions and geophysical observations of slab curvature gives an average slab-to-mantle viscosity ratio of 45.

INTRODUCTION
Much effort has been recently devoted to improve our understanding of the dynamics of subduction zones; surprisingly, the viscosity of the subducting lithosphere remains a poorly known yet fundamental parameter. Modeling subduction dynamics (e.g., Billen and Hirth, 2007; Capitanio et al., 2007; Wu et al., 2008) or the geoid (e.g., Zhong and Davies, 1999) has led to a wide range of values. We take advantage of the dependence of slab curvature on the viscosity ratio between the slab and the mantle to determine this ratio by studying the response of a retreating slab in a longitudinal plane (i.e., perpendicular to the slab plane at a given depth) to mantle flow around a rigid slab, while Husson (2006) reproduced observations of dynamic topography with an isoviscous rheology. Most oceanic trenches have a convex arc shape, a feature that is not limited to surface topography with an isoviscous rheology.

GEOMETRY AND RHEOLOGY OF SLABS
The interaction between a subducting lithosphere and the surrounding mantle is strongly affected by their relative strengths. For example, whether slabs penetrate the lower mantle is controlled by the strength of the lithosphere in comparison with the viscosity structure of the mantle at the 660 km discontinuity (Goes et al., 2008). Commonly used values for the viscosity ratio between the slab and the mantle to determine this ratio by studying the response of a retreating slab in a longitudinal plane (i.e., perpendicular to the slab plane at a given depth) to mantle flow around a rigid slab, while Husson (2006) reproduced observations of dynamic topography with an isoviscous rheology.

Geosciences Rennes, UMR 6118 CNRS, Université de Rennes 1, Rennes Cedex CS 35042, France

From longitudinal slab curvature to slab rheology
Christelle Loiselet*, Laurent Husson, and Jean Braun

© 2009 Geological Society of America. For permission to copy, contact Copyright Permissions, GSA, or editing@geosociety.org.

ABSTRACT
The curvature of a subducting lithosphere is chiefly controlled by the viscosity ratio between the slab and the surrounding mantle. On the basis of a semi-analytical flow model, we explore the rheological dependence of the geometrical response of a viscous slab subjected to toroidal mantle flow. Mantle flow is excited by slab retreat at a prescribed mean velocity and is iteratively solved for by using a stream function approach, in turn providing the stresses that bend the slab. Comparison between model predictions and geophysical observations of slab curvature gives an average slab-to-mantle viscosity ratio of 45.

INTRODUCTION
Much effort has been recently devoted to improve our understanding of the dynamics of subduction zones; surprisingly, the viscosity of the subducting lithosphere remains a poorly known yet fundamental parameter. Modeling subduction dynamics (e.g., Billen and Hirth, 2007; Capitanio et al., 2007; Wu et al., 2008) or the geoid (e.g., Zhong and Davies, 1999) has led to a wide range of values. We take advantage of the dependence of slab curvature on the viscosity ratio between the slab and the mantle to determine this ratio by studying the response of a retreating slab in a longitudinal plane (i.e., perpendicular to the slab plane at a given depth) to mantle flow around a rigid slab, while Husson (2006) reproduced observations of dynamic topography with an isoviscous rheology. Most oceanic trenches have a convex arc shape, a feature that is not limited to surface topography with an isoviscous rheology.

GEOMETRY AND RHEOLOGY OF SLABS
The interaction between a subducting lithosphere and the surrounding mantle is strongly affected by their relative strengths. For example, whether slabs penetrate the lower mantle is controlled by the strength of the lithosphere in comparison with the viscosity structure of the mantle at the 660 km discontinuity (Goes et al., 2008). Commonly used values for the viscosity ratio between the slab and the mantle to determine this ratio by studying the response of a retreating slab in a longitudinal plane (i.e., perpendicular to the slab plane at a given depth) to mantle flow around a rigid slab, while Husson (2006) reproduced observations of dynamic topography with an isoviscous rheology. Most oceanic trenches have a convex arc shape, a feature that is not limited to surface topography with an isoviscous rheology.

Most oceanic trenches have a convex arc shape, a feature that is not limited to surface level. Seismic tomography studies as well as earthquake occurrences provide information on the geometry of subducted slabs (Isacks and Molnar, 1971; Van der Hilst and Karason, 1999; Wortel and Spakman, 2000; Fukao et al., 2001). The shape of ocean trenches was first explained by the ping-pong ball analogy (Frank, 1968; Tovish and Schubert, 1978), which suggested that the curvature of the trenches is naturally imposed by the intersection of a slab and a spherical Earth. Alternatively, numerical and analogue simulations have shown that this curvature is more likely the response of the slab in a toroidal flow. This curvature seems to decrease when slab width increases (Dvorkin et al., 1993; Morra et al., 2006, 2009; Stegman et al., 2006; Schellart et al., 2007), as a response to the balance between the forces driving slab motion and the viscous resistance of the mantle (Schellart, 2004; Funiciello et al., 2006), although internal heterogeneities may have a strong impact (Morra et al., 2006).

Because the flow associated with a migrating slab is predominantly toroidal (e.g., Schellart, 2004; Funiciello et al., 2006; Piromallo et al., 2006), we assume that the time evolution of the shape of a vertical slab that retreats at a given rate can be directly compared to the depth evolution of the longitudinal shape of a slab; this operation can be performed via the conversion of depth along slab into a residence time into the mantle, calculated as the depth divided by the sinking velocity (assuming that retreat velocity, and therefore slab dip, is constant with depth).

MODELING APPROACH
The retreating slab is approximated by a uniform infinite vertical viscous layer of variable width and thickness h moving in a square domain of constant dimensions throughout (Fig. 1A). Our approach is based on a two-dimensional (2-D) numerical solution of the Stokes equation that requires the computation of (1) a stream function to describe the toroidal flow of a Newtonian, isoviscous, and incompressible mantle around a rigid retreating plate, infinite in the direction perpendicular to the model (2-D), and (2) from the resulting stress field, the deflection of a viscous slab of uniform yet different viscosity. The velocity of the fluid is obtained by solving the biharmonic equation for the stream function \( \Psi \), \( \nabla^2 \Psi = 0 \), with \( u = -\frac{\partial \Psi}{\partial y} \) and \( v = \frac{\partial \Psi}{\partial x} \) and \( u \) and \( v \) being the x and y velocities, respectively. To simulate the presence of the retreating slab, we prescribe a velocity \( V_t \) (slab retreat rate) in...
a rectangular region of width \( L \) and thickness \( h \) in the center of the model (Fig. 1A), while the velocities at the boundaries of the domain are set to zero. We then calculate \( \Psi \) at the nodes of a regular grid using a centered finite difference operator to approximate the spatial derivatives. In the first step of the model, we assume that the slab is rigid, and iteratively solve for the velocity field that minimizes work while satisfying the incompressibility condition.

The deflection of the slab is obtained in a second step by solving for the viscous bending of a half-plate embedded at its origin (and by symmetry, for the entire plate), of thickness \( h \) and length \( L/2 \), and subjected to a nonuniform load \( q(\chi) \) (or spatially integrated deviatoric stress):

\[
D \frac{\partial^3 w}{\partial x^2 \partial \tau} = q(\chi), \tag{1}
\]

where \( D = \mu h^3 / 3 \) is the viscous rigidity of the slab, \( \mu \) the slab viscosity, and \( q(\chi) = \mu_\omega \frac{\partial \mu}{\partial x} \) is the deviatoric stress induced by the mantle flow. The approximation is made that \( \sigma_\omega = 0 \), which only holds when the slab viscosity is larger than the mantle viscosity \( \mu_\omega \). The deflection \( w \) of the half-plate as a function of time \( t \) and \( x \) is given by the integration of Equation 1. Standard boundary conditions are \( w = 0 \) and \( \frac{\partial w}{\partial x} = 0 \) at \( x = 0 \); \( \frac{\partial^2 w}{\partial x^2} = 0 \) and \( \frac{\partial^3 w}{\partial x^3} = 0 \) at \( x = L/2 \) (Turcotte and Schubert, 1982). The slab geometry is modified according to the computed final displacement after each time increment (small enough for the solution to become independent of its value). The coupled equations are iteratively solved through time in order to predict the evolving plate curvature.

We dimensioned the model by assigning a mean slab retreat rate \( V_r = 50 \text{ mm/a} \) in a 10,000-km-wide square box filled with a linear viscous fluid of viscosity \( 10^{20} \text{ Pa s} \). Note that in a Newtonian viscous fluid, the flow pattern does not depend on the absolute value of the viscosity, but rather on the viscosity ratios. Thus, if the imposed velocity is twice as large, we obtain the same velocity field but at twice the amplitude. Equation 1 breaks down when the viscosity ratio is too low; we must ensure that the slab rigidity is sufficiently large for the deflection to be inferior or equal to the slab displacement, which puts a minimum bound on the allowable viscosity ratio.

**RESULTS**

The resulting flow pattern (Fig. 1A) shows a pair of symmetrical toroidal cells, one on each side of the slab, focused close to the plate edges. Velocities are maximum just upstream and downstream of the plate (most positive values) and close to the slab edges (most negative values). On the sides of the slab, the negative \( y \) velocity is the return flow. Slab retreat results in compression along the upstream side of the slab and extension along the downstream side of the slab (Fig. 1B). The stress pattern also shows two opposite-sign ear-shape anomalies, on either side of the slab, of amplitudes proportional to the slab width. It is interesting that maximum stresses are found close to the slab edges (Fig. 1C); away from the slab edges, the stress field abruptly reverts and becomes extensional (negative) on the upstream side and compressive (positive) on the downstream side. Near the center of the slab, the deviatoric stresses diminish and tend toward zero (Fig. 1C). In all experiments, slab deformation is concentrated along its edges, where stresses are maximum. This surprising stress distribution (see also Morra et al., 2006) is responsible for slab curvature along their edges. As time progresses, the slab geometry evolves at a rate that is dictated by the slab to mantle viscosity ratio.

We performed a series of experiments in which the slab was alternatively 500, 1000, and 2000 km wide and 50, 100, and 200 km thick. For each experiment, the viscosity ratio between the slab and surrounding mantle was systematically changed to encompass the desired range. The system is bounded by the no-velocity conditions along the edges of the box, and the behavior of the slab depends on the box width to slab dimension ratio. This condition mimics real Earth conditions, where subduction zones are embedded in the finite terrestrial system.

We characterize slab curvature from a quadratic regression on the slab shape. After a mean retreat of 1000 km, the coefficient of curvature \( C \) of the slab (defined as the quadratic coefficient of a quadratic polynomial function fit) varies between \( 10^{-4} \) and \( 6 \times 10^{-5} \text{ km}^{-1} \) (Fig. 2). In all cases, curvature is inversely proportional to the viscosity ratio \( [C = 1/(\mu_s/\mu)] \), with the lowest ratio leading to the highest curvature, as imposed by the above equation for flexure. As slabs widen, the stresses in the fluid during slab retreat become high because free space around the slab decreases. Consequently, wide slabs get highly loaded and slab curvature increases accordingly. Slab rigidity varies as the cube of slab thickness, the thicker the slab the smaller the curvature, and our models accordingly yield \( C \sim h^{-3} \).

**COMPARISON TO REAL EARTH**

The curvature of narrow slabs (e.g., Scotia, Calabria, Hellenic) is larger than that of wider slabs (e.g., Aleutian, South America, Java, Tonga). In order to determine the shape of slabs at depth, we use the results of P-wave seismic tomography (Li et al., 2008) and catalogues of earthquake locations (International Seismological Centre, 2008; Engdahl et al., 1998) for four subduction systems (Aleutians, Scotia, Hellenic, and Calabria) selected for the variety of width, thickness, and retreat velocity they present. We also compare our interpretation to the slab geometries derived from the RUM (regionalized upper mantle) model (Gudmundsson and Sambridge, 1998).

For each experiment, the viscosity ratio between the slab and surrounding mantle was systematically changed to encompass the desired range. The system is bounded by the no-velocity conditions along the edges of the box, and the behavior of the slab depends on the box width to slab dimension ratio. This condition mimics real Earth conditions, where subduction zones are embedded in the finite terrestrial system.

We characterize slab curvature from a quadratic regression on the slab shape. After a mean retreat of 1000 km, the coefficient of curvature \( C \) of the slab (defined as the quadratic coefficient of a quadratic polynomial function fit) varies between \( 10^{-4} \) and \( 6 \times 10^{-5} \text{ km}^{-1} \) (Fig. 2). In all cases, curvature is inversely proportional to the viscosity ratio \( [C = 1/(\mu_s/\mu)] \), with the lowest ratio leading to the highest curvature, as imposed by the above equation for flexure. As slabs widen, the stresses in the fluid during slab retreat become high because free space around the slab decreases. Consequently, wide slabs get highly loaded and slab curvature increases accordingly. Slab rigidity varies as the cube of slab thickness, the thicker the slab the smaller the curvature, and our models accordingly yield \( C \sim h^{-3} \).

**Figure 2. Slab coefficient of curvature (see text) as function of lithosphere to surrounding mantle viscosity ratio. Slab is alternatively 50 (dotted), 100 (solid), and 200 km (dashed) thick (h) and 500 (A), 1000 (B), and 2000 (C) km wide, respectively. Some curves are truncated when initial model assumptions no longer hold (see text).**
the modeled and observed curvatures match (Fig. 3). In all cases the domain is 10,000 km wide, which may induce a systematic bias, because we do not know what the most appropriate dimension would be for Earth. This recurrent issue relates to the Stokes paradox (Lamb, 1932). Nonetheless, the slab to box edge distance (5000 km) is comparable to (Lamb, 1932). Nonetheless, the slab to box edge distance (5000 km) is comparable to

Increasing slab width confines the toroidal flow into a constant size domain. It decreases trench velocity and the vigor of the mantle flow, and ultimately a higher rate of viscous energy dissipation (Fig. 4B). Consequently, slab width also tends to inhibit trench retreat (Schellart, 2004; Stegman et al., 2006) (Fig. 4A), and, at a given stage in the temporal evolution of the slab, slab curvature will be lower for a wide slab than for a narrow one. Although the buoyancy of a slab is linearly proportional to its width, because the viscous dissipation increases as a power law of slab width in a mantle of finite dimension (Fig. 4B), the resisting force will ultimately dominate and large slabs will retreat at slower rates than small ones as an indirect consequence of the Stokes paradox (Lamb, 1932). Thus, these results in turn explain why narrow slabs (e.g., Calabria or Scotia) retreat faster (Fig. 4A) than wider ones. Our assumption that the surface is implicitly stress free (2-D approximation) affects our results (e.g., Jarvis and Lowman, 2005), as does the no-slip lateral boundary condition that will influence the flow pattern (e.g., Piromallo et al., 2006) and accentuate the Stokes paradox effect.

The comparison of theoretical computations with natural slab curvature independently delimits the viscosity ratio between the subducted lithosphere and the surrounding mantle to range from 1 to 100, with an average of ~45,
smaller by a factor of 2 (Capitanio et al., 2007) to more than 10\(^4\) (Billen and Hirth, 2007) compared to previous estimates. In other words, if the upper mantle viscosity is assumed to be 10\(^{20}\) Pa s, slab viscosity is in the range 10\(^{20}\)–10\(^{22}\) Pa s, with a mean value of 4.5 \times 10^{21} Pa s. The choice of the reference frame may change the values of the viscosity ratio by a factor of 2 (Table 1; comparable to Funiciello et al., 2008). However, because the azimuthal distribution of trenches on Earth is approximately even, mean retreat rates, and therefore the mean viscosity ratios, should be independent of the reference frame.

ACKNOWLEDGMENTS

We thank W. Royden, F. Chambat, P. Yamato, and B. Huet for stimulating discussions. Loiselet and Braun acknowledge funding from the Agence Nationale de la Recherche (ANR), and Husson from the Institut National de la Recherche Scientifique (INSU/CNRS) program “SEDE.” We thank editor S. Wylde, and F. Funiciello, W. Schellart, and an anonymous reviewer for constructive reviews.

REFERENCES CITED


