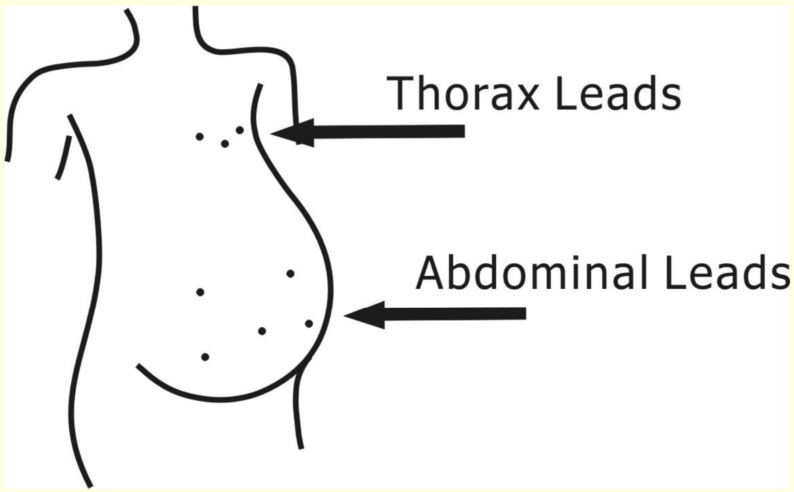
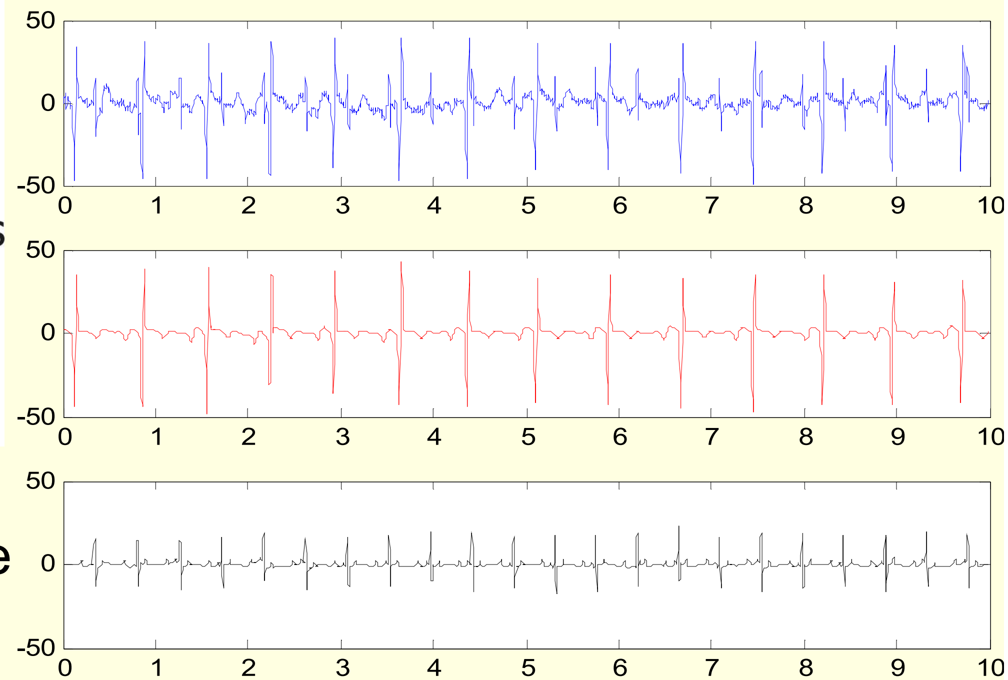
Robust 3-way Tensor Decomposition and Extended State Kalman Filtering to Extract Fetal ECG

Mohammad Niknazar, Hanna Becker, Bertrand Rivet, Christian Jutten, and Pierre Comon

GIPSA – lab, Grenoble, France

The Basic Problem





- Several sources of interference and noise
- Much lower amplitude of fECG compared with mECG

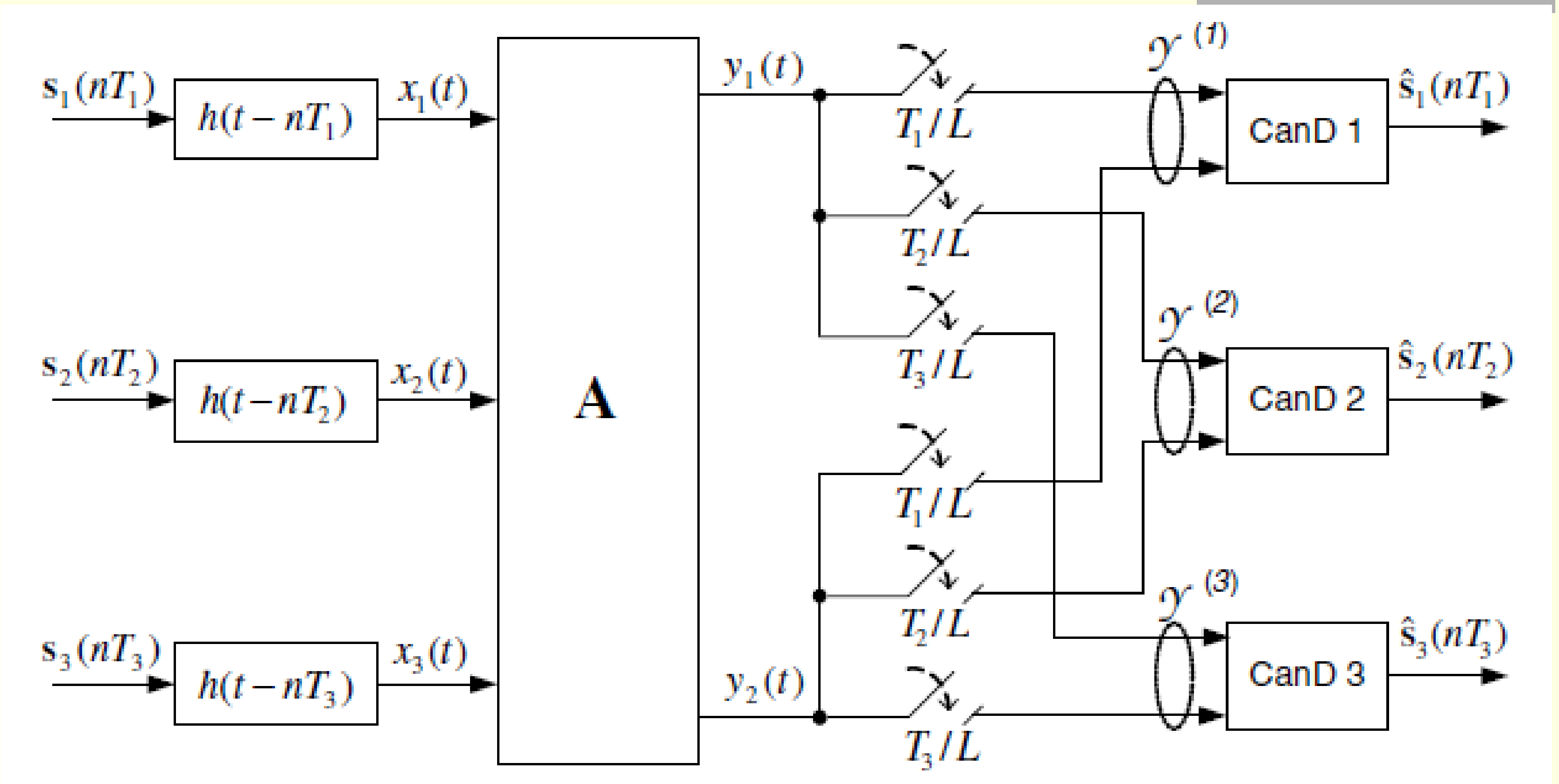
Multichannel Fetal ECG Extraction

- Current methods
  - Adaptive filtering [Widrow et al. 1975]
  - Independent component analysis (ICA) [De Lathauwer et al. 1995, Zarzoso & Nandi 2001]
  - Periodic component analysis (ΠCA) [Tsalaile et al. 2009]
- Basic idea: Exploitation of the redundancy of the multichannel ECG to reduce mECG
- Drawback
  - Exogenous noise cannot be totally canceled in this way
  - Demand several channels

Outline of Fetal ECG Extraction and Denoising Using Only Two Electrodes

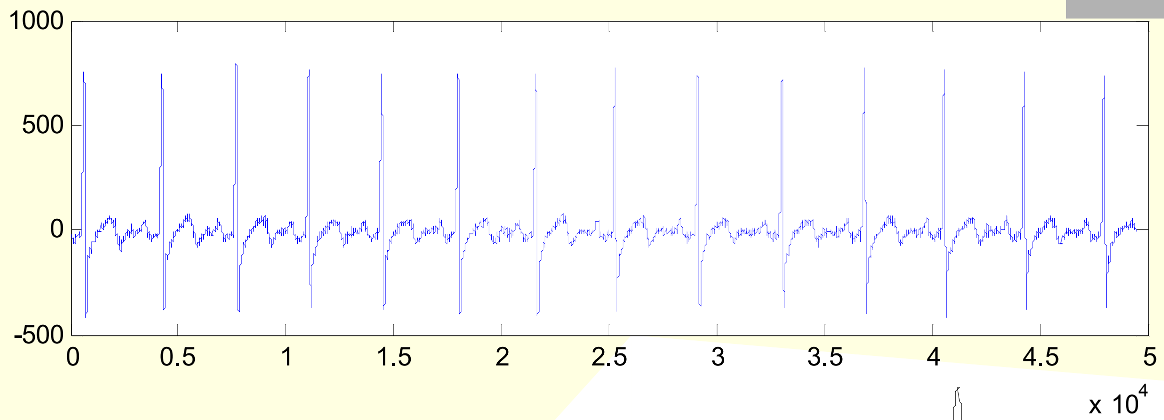
- ECGs estimation based on modifications of a tensor based parallel deflation procedure
  - Weighted Canonical Polyadic decomposition
  - Gaussian-shaped cost function
- Improvement of fECG and mECG estimates by a Kalman filtering
  - Multichannel extension of dynamic ECG model for N ECGs
- Results on different data

Tensor Construction of Sources Having Different Symbol Rate



A.L.F.D. Almeida, P. Comon, and X. Luciani, "Deterministic Blind Separation of Sources Having Different Symbol Rates Using Tensor-Based Parallel Deflation", *in Proc. LVA/ICA,* 2010, pp.362-369.

Third-Order Tensor Arrangement of an ECG Signal

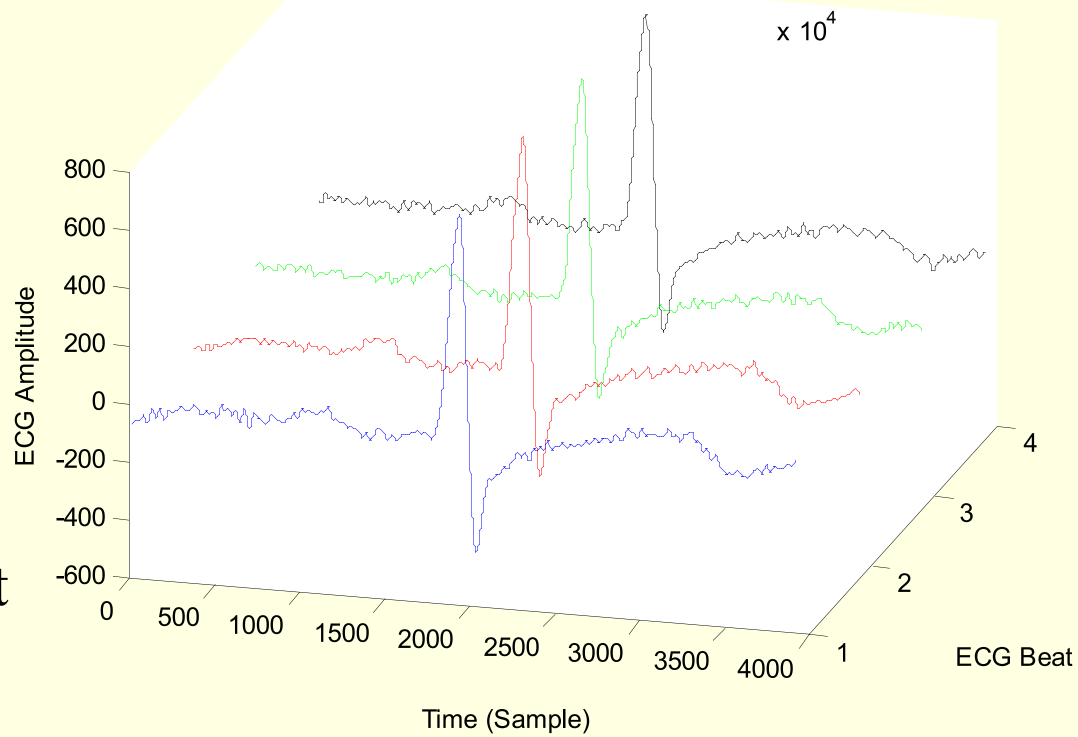


$$\overline{Y}^{(n)} \in \mathbb{C}^{M \times T_n \times L_n}$$

M: Number of sensors

T_n: Number of ECG beats

L_n: Time samples per beat



Third-Order Tensor Decomposition for n-th ECG Using the Canonical Polyadic (CP)

$$\overline{Y}^{(n)} = \sum_{r=1}^{R_n} A_r^{(n)} \otimes \overline{S}_r^{(n)} \otimes \overline{H}_r^{(n)} + N$$

$\otimes$: Outer product

R_n : Tensor rank

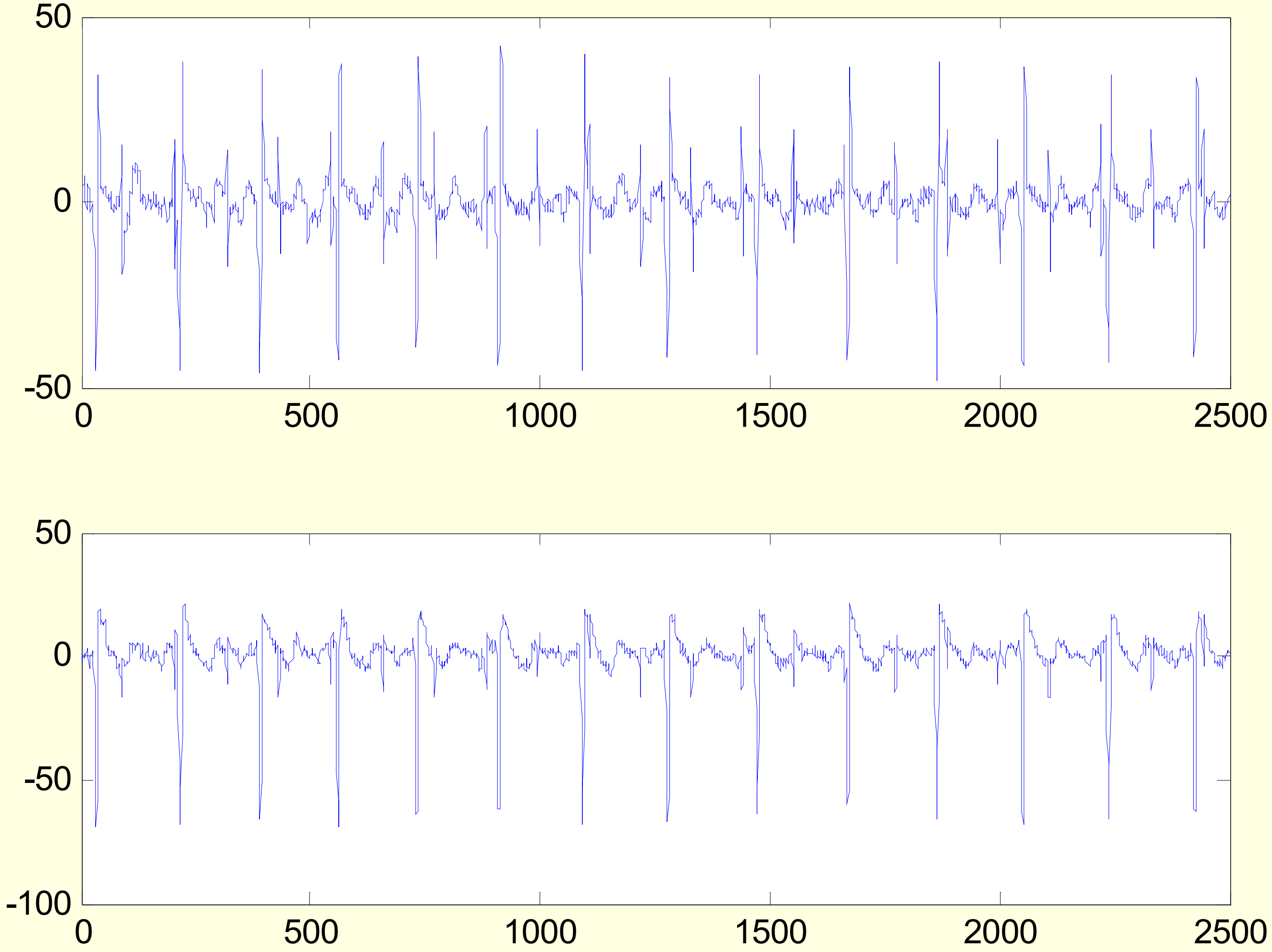
$$\min_{\left\{\hat{\mathbf{A}}^{(n)}, \hat{\overline{\mathbf{S}}}^{(n)}, \hat{\overline{\mathbf{H}}}^{(n)}\right\}} \sum_{i,j,k} \left\| y_{ijk}^{(n)} - \sum_{r=1}^{R_n} a_{ir}^{(n)} \overline{s}_{jr}^{(n)} \overline{h}_{kr}^{(n)} \right\|_F^2$$

A: Mixing matrix

$\overline{\mathrm{S}}$: ECG beat amplitude

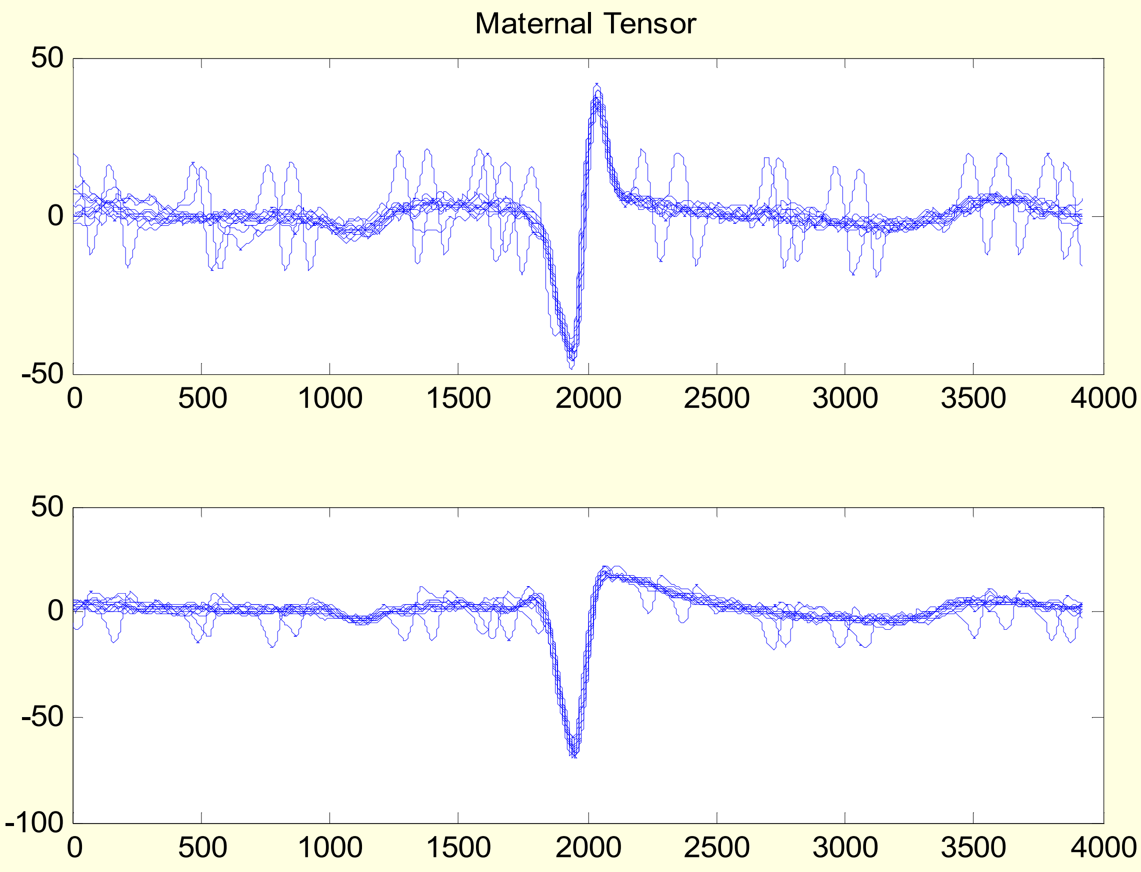
$\overline{\mathrm{H}}$: ECG beat temporal pattern

Mixed ECGs on Two Electrodes



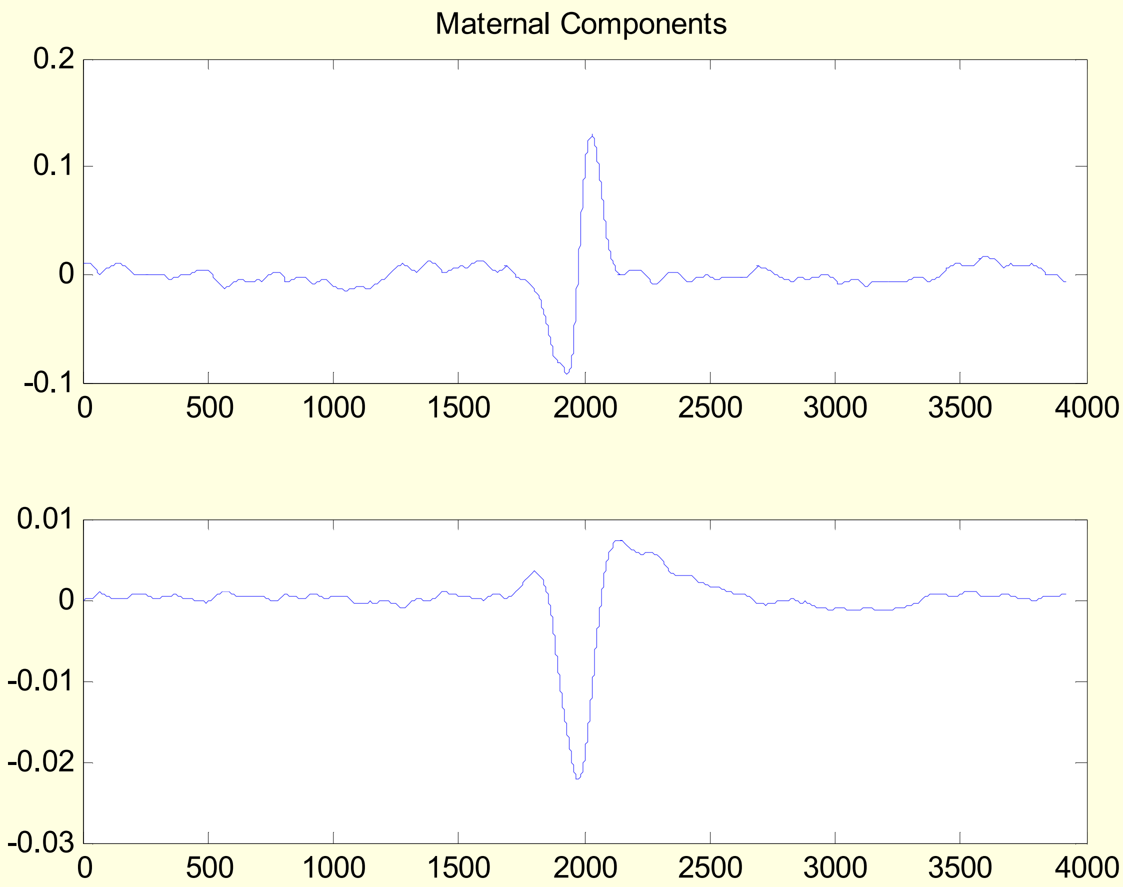
Maternal Tensor

- Two components



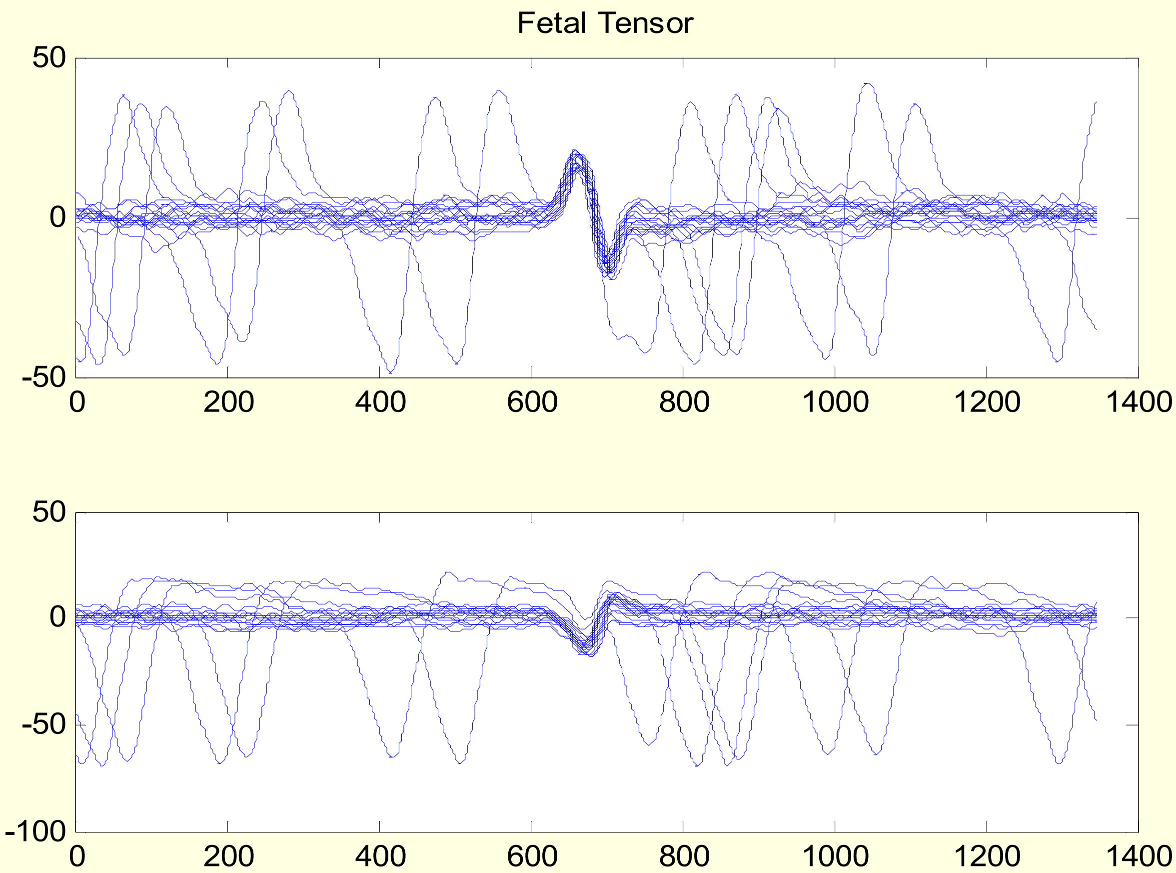
Maternal Components

- Two extracted components



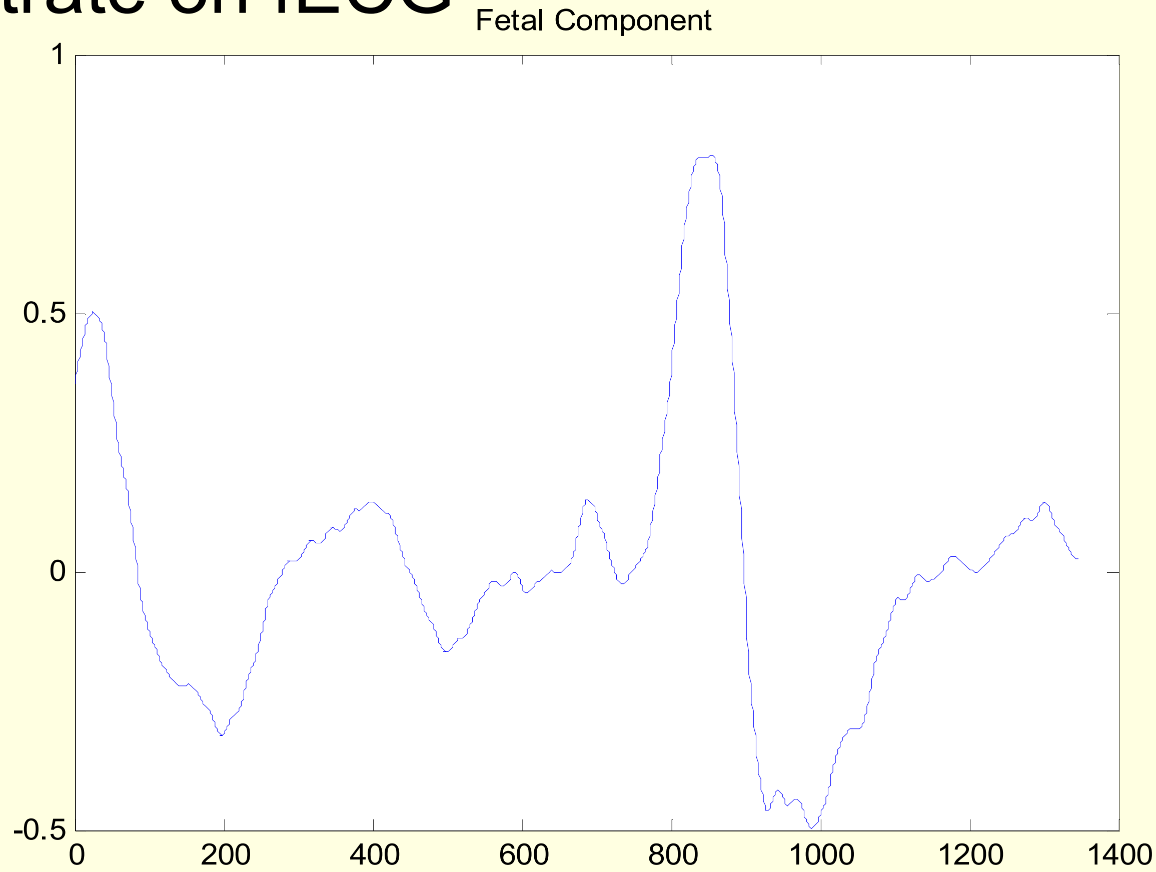
Fetal Tensor

- fECG is weak: One component



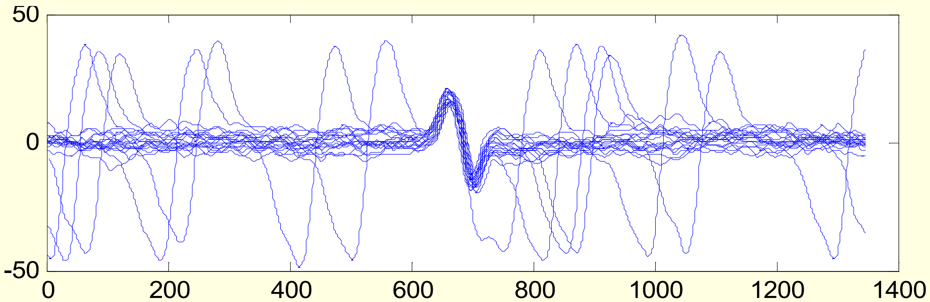
Fetal Component

- mECG prevents the algorithms to concentrate on fECG



Weighted CP Decomposition (WCPD) and Gaussian-shaped Cost Function (GCF)

$$\min_{\{\hat{\mathbf{A}}^{(n)},\hat{\bar{\mathbf{S}}}^{(n)},\hat{\bar{\mathbf{H}}}^{(n)}\}} \sum_{i,j,k} \left\| w_{ijk}^{(n)} \left(y_{ijk}^{(n)} - \sum_{r=1}^{R_n} a_{ir}^{(n)} \bar{s}_{jr}^{(n)} \bar{h}_{kr}^{(n)} \right) \right\|_F^2$$

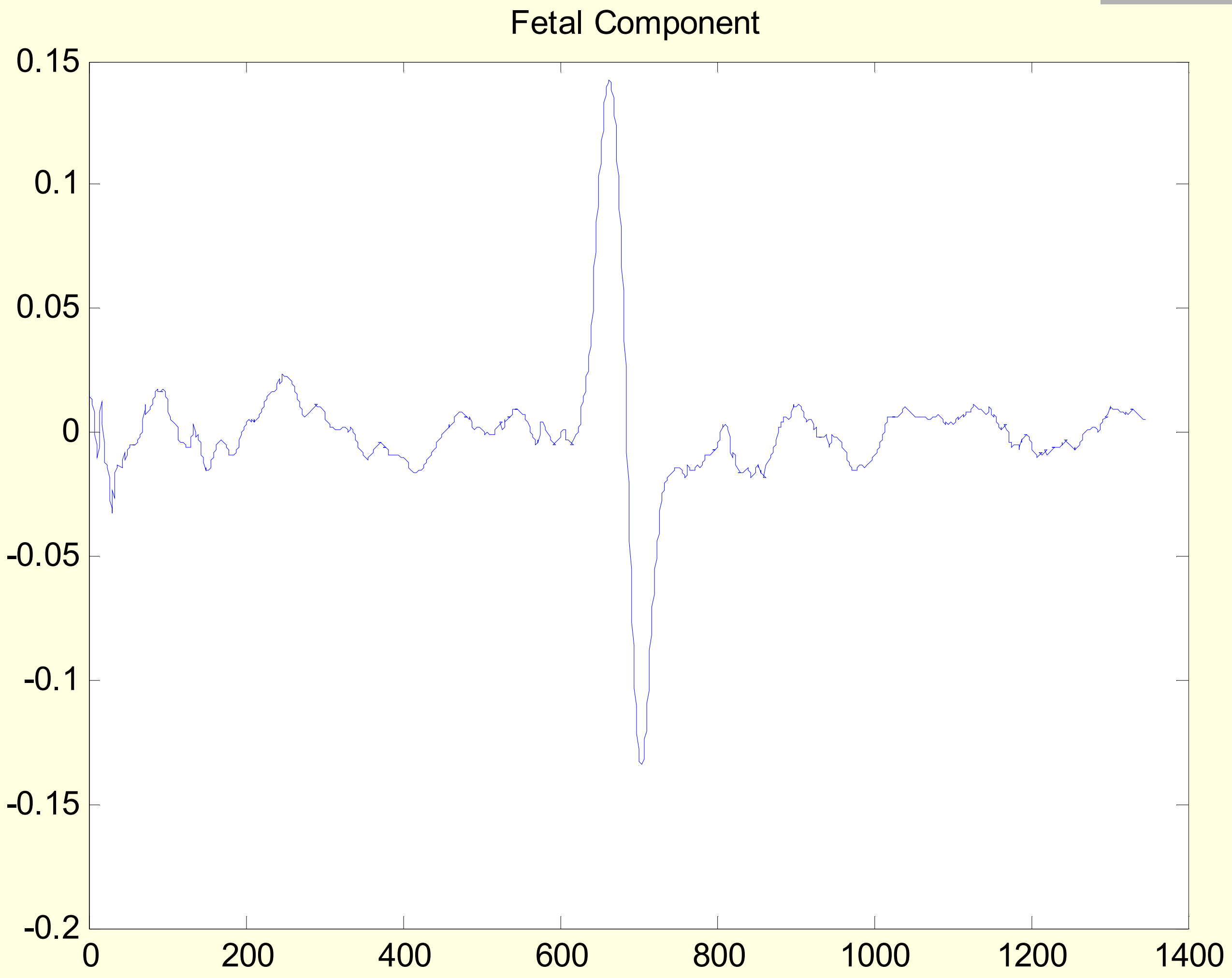


$$w_{ijk}^{(n)} = \exp\left\{ -\frac{(y_{ijk}^{(n)} - \mu_{ik})^2}{\sigma_{ik}^2} \right\}$$

$$\min_{\{\hat{\mathbf{A}}^{(n)},\hat{\bar{\mathbf{S}}}^{(n)},\hat{\bar{\mathbf{H}}}^{(n)}\}} \sum_{i,j,k} \left\| y_{ijk}^{(n)} - \sum_{r=1}^{R_n} a_{ir}^{(n)} \bar{s}_{jr}^{(n)} \bar{h}_{kr}^{(n)} \right\|_F^2$$

$$\min_{\{\hat{\mathbf{A}}^{(n)},\hat{\bar{\mathbf{S}}}^{(n)},\hat{\bar{\mathbf{H}}}^{(n)}\}} \sum_{i,j,k} \psi\left(y_{ijk}^{(n)} - \sum_{r=1}^{R_n} a_{ir}^{(n)} \bar{s}_{jr}^{(n)} \bar{h}_{kr}^{(n)} \right) \quad \psi(u) = 1 - \exp\{-\frac{u^2}{2\sigma^2}\}$$

Fetal Components via Weighted Tensor Decomposition

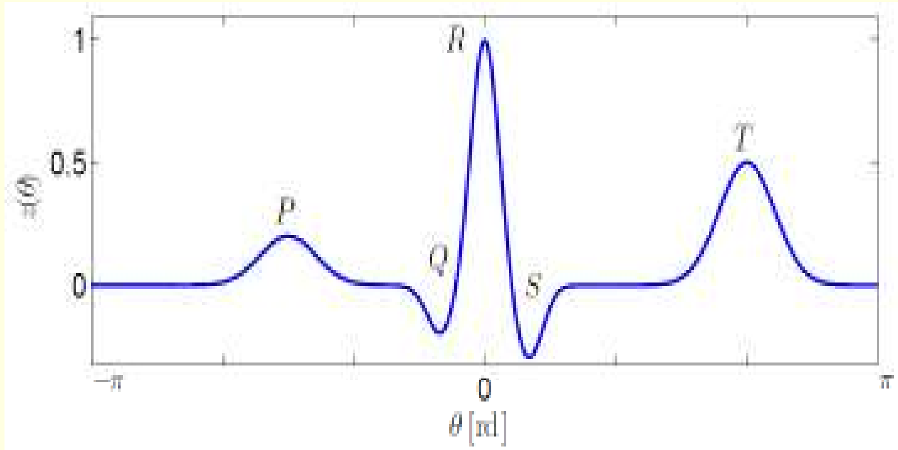


Extended Kalman Filter and Dynamic ECG Model

A: Mixing matrix

$\overline{\mathrm{S}}$: ECG beat amplitude

$\overline{\mathrm{H}}$: ECG beat temporal pattern



$$\begin{cases} \theta_{k+1} = (\theta_k + \omega\delta)\mathrm{mod}(2\pi) \\ z_{k+1} = -\sum_{i\in\{P,Q,R,S,T\}} \delta\frac{\alpha_i\omega}{b_i^2}\Delta\theta_i\exp(-\frac{\Delta\theta_i^2}{2b_i^2}) + z_k + \eta \end{cases}$$

$$\mathbf{x}_k = [\theta_k, z_k]^T \qquad \begin{bmatrix}\varphi_k \\ s_k\end{bmatrix} = \begin{pmatrix}1 & 0 \\ 0 & 1\end{pmatrix}\begin{bmatrix}\theta_k \\ z_k\end{bmatrix} + \begin{bmatrix}u_k \\ v_k\end{bmatrix}$$

R. Sameni, M. B. Shamsollahi, C. Jutten, and G. D. Clifford, “A Nonlinear Bayesian Filtering Framework for ECG Denoising,” *IEEE Trans. Biomed. Eng.*, vol. 54, no. 12, pp. 2172-2185, December 2007.

Multichannel Extension of Dynamical ECG Model for N ECGs

$$\mathbf{x}_k = [\theta_k^{(1)}, \cdots, \theta_k^{(N)}, z_k^{(1)}, \cdots, z_k^{(N)}]^T$$

$$\mathbf{y}_k = \left[\varphi_k^{(1)}, \cdots, \varphi_k^{(N)}, s_k^{(1)}, \cdots, s_k^{(M)}\right]^T$$

$$\mathbf{y}_k = \begin{pmatrix} I & 0 \\ 0 & A \end{pmatrix} \mathbf{x}_k + \mathbf{u}_k$$

$$A = \begin{bmatrix} a_{11} & \ldots & a_{1N} \\ \vdots & \ddots & \vdots \\ a_{M1} & \ldots & a_{MN} \end{bmatrix} = ?$$

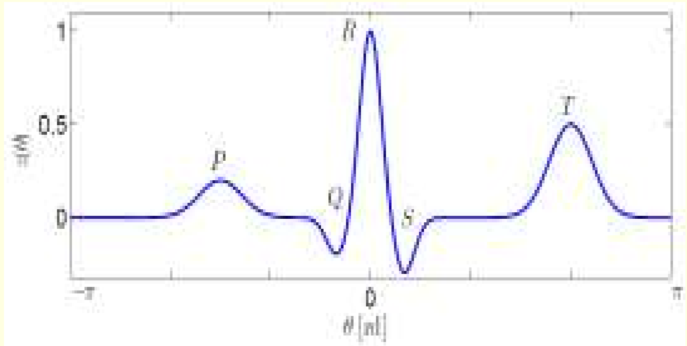
$$\{\alpha_i^{(n)}, b_i^{(n)}, \psi_i^{(n)}, \omega^{(n)}\}_{i \in P,Q,R,S,T} = ?$$

Using Loading Matrices for Mixing Matrix and Parameter Estimation

A: Mixing matrix

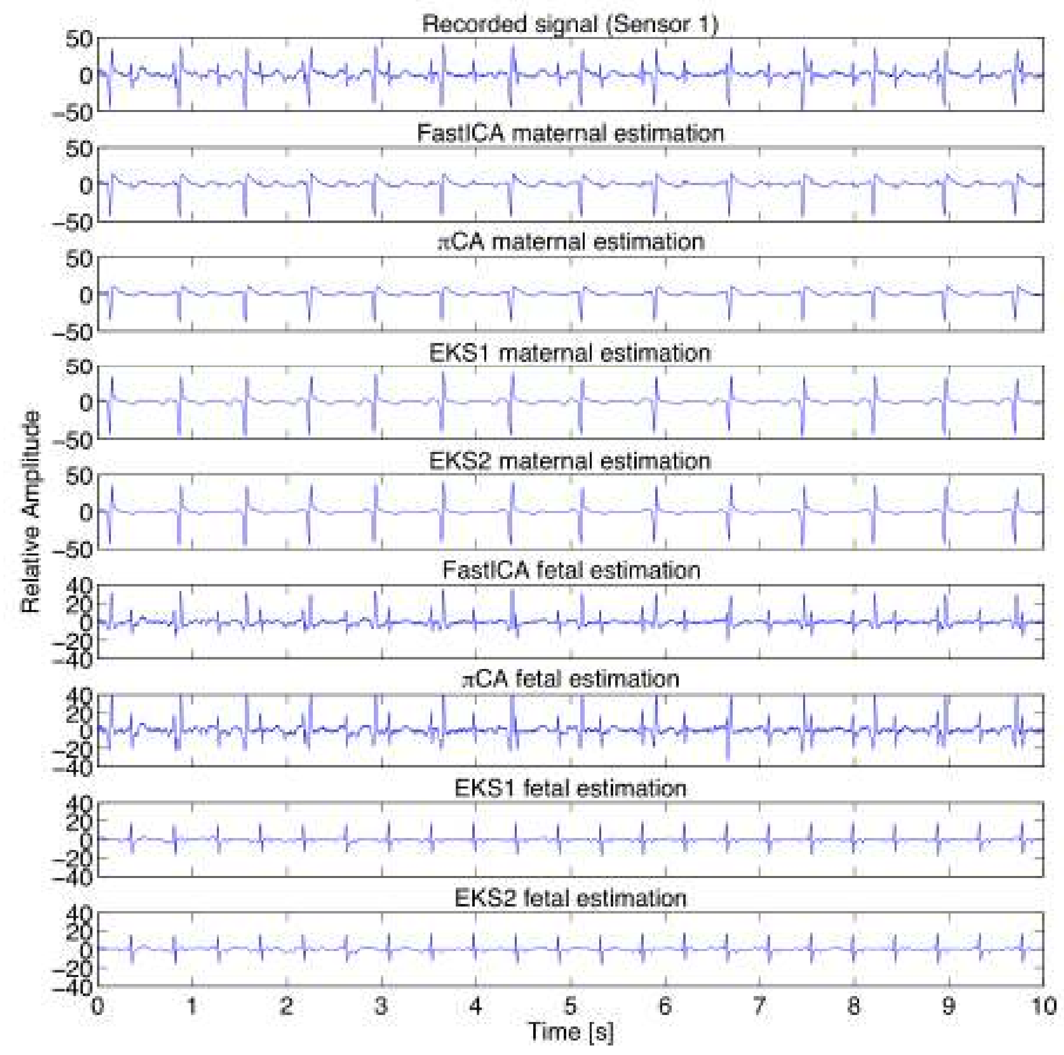
$\overline{S}$: ECG beat amplitude

$\overline{H}$: ECG beat temporal pattern

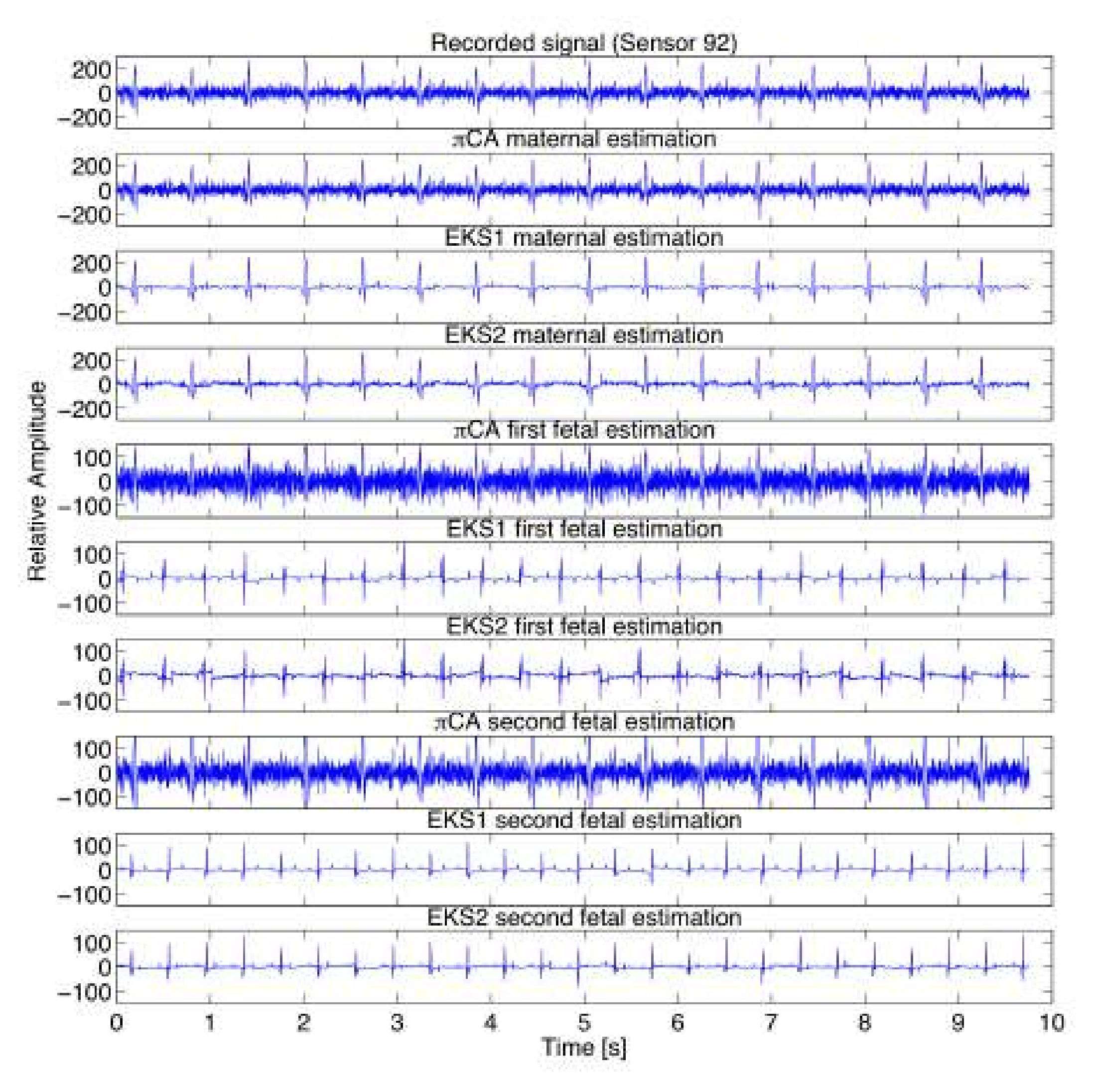


- The mixing matrix is directly defined as the concatenation of the loading matrices **A**$^{(n)}$ related to the fetus and to the mother
- The state parameters $\{\alpha_i^{(n)}, b_i^{(n)}, \psi_i^{(n)}, \omega^{(n)}\}$ are obtained by fitting, for each ECG, the sum of the Gaussian functions with the loading matrix **H**$^{(n)}$
- The ECG variability is estimated using the third loading matrix **S**$^{(n)}$

DaISy ECG Dataset



Twin MCG Dataset



Conclusions and Perspectives

- Robust 3-way tensor decomposition
- Extension of a synthetic dynamical ECG model within a Kalman filtering framework to model several ECGs
- Only two electrodes are utilized
- Deep comparison between tensor decomposition methods
- Application of the proposed method on other datasets

Thank You

Спасибо
Russian

धन्यवाद
Hindi

תודה רבה
Hebrew

Gracias
Spanish

Grazie
Italian

Mulţumesc

شكراً
Arabic

Merci
French

Obrigado
Portuguese

با تشکر
Persian

Danke
German

多謝
Traditional Chinese

ありがとうございました
Japanese

Korean

多谢
Simplified Chinese

ขอบคุณ
Thai

நன்றி
Tamil