CANONICAL DECOMPOSITION OF EVEN HIGHER ORDER CUMULANT ARRAYS FOR BLIND UNDERDETERMINED MIXTURE IDENTIFICATION

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ABSTRACT

A new family of methods, named $2q$-ORBIT ($q > 1$), is proposed in this paper in order to blindly identify potentially underdetermined mixtures of statistically independent sources. These methods are based on the canonical decomposition of q-th order ($q \geq 2$) cumulants. The latter decomposition is brought back to the decomposition of a third order array whose one loading matrix is unitary. Such a decomposition is then computed by alterning and repeating two schemes until convergence: the first one consists in solving a Procrustes problem while the second one needs to compute the best rank-1 approximation of several q-th order arrays. Computer results show a good efficiency of the proposed methods with respect to classical cumulant-based algorithms especially in the underdetermined case.

1. INTRODUCTION

CANonical Decomposition (CAND) of $2q$-way ($q > 1$) supersymmetric arrays for Blind Mixture Identification (BMI) is addressed in this paper. A link between CAND and both the well-known *orthogonal Procrustes problem* [1] and the best rank-1 approximation of Higher Order (HO) arrays [2] is established.

On the one hand, while HO arrays are the HO equivalent of vectors (first order) and matrices (second order), CAND extends to HO both the Singular Value Decomposition (SVD) and the rank concept of matrices. CAND was first introduced (around 1970) in psychometrics [3], later it was applied in chemometrics where the term PARAFAC is used instead. Recently, CAND has found widespread applications in signal processing such as biomedical engineering and array processing [4, 5]. On the other hand, the BMI problem, which may require to process more sources than sensors, is often encountered in practice. For instance in radiocommunication contexts, the probability of receiving more sources than sensors increases with the reception bandwidth.

Taleb and Jutten were the first who introduced identifiability results in underdetermined context [6]. Since then, and thanks to many attractive properties of HO cumulants such as their capacity to process more sources than sensors, many cumulant-based methods, which use explicitly or implicitly CAND, were proposed [7–13]. In fact, some of the latter methods [8, 9] achieve CAND using the well-known Alternating Least Square (ALS) algorithm or one of its enhanced versions [9, 10]. Other methods look for a semi-algebraic CAND of cumulants [7, 11–13].

In this paper, we propose iterative methods in order to compute CAND of $2q$-th ($q \geq 2$) order cumulants. These algorithms, named $2q$-ORBIT ($q \geq 2$) (Orthogonal Rotation estimation for Blind Identification of potentially underdeTermined mixTures), allow to solve the BMI problem. In addition the $2q$-ORBIT algorithms outperform the classical cumulant-based approaches [11–13] as shown in the computer results.

2. PROBLEM FORMULATION

Vectors, matrices and arrays with more than two indexes will be denoted in bold lowercase, in bold uppercase and in bold calligraphic uppercase, respectively. Plain uppercases will be mainly used to denote dimensions. The BMI problem can be expressed as following:

Problem 1 *Given a random vector $\boldsymbol{x}$, find a $(N \times P)$ mixing matrix $\boldsymbol{A}$ (P may be greater than N) such that $\boldsymbol{x}$ factorizes into $\boldsymbol{A}\boldsymbol{s} + \boldsymbol{\nu}$ where $\boldsymbol{s} = [s_1, \cdots, s_P]^{\mathsf{T}}$ and $\boldsymbol{\nu}$ are a $(P \times 1)$ source vector with statistically independent components and a $(N \times 1)$ Gaussian noise vector, independent from the source vector, respectively.*

Moreover, the BMI problem can be expressed using the HO array terminology. For this purpose, a few definitions related to q-way ($q \geq 2$) arrays [2] are necessary.

Definition 1 *A rank-1 q-way array $\boldsymbol{\mathcal{T}} \in \mathbb{C}^{N_1 \times \cdots \times N_q}$ is equal to the outer product $\boldsymbol{u}^{(1)} \circ \cdots \circ \boldsymbol{u}^{(q)}$ of q vectors $\boldsymbol{u}^{(i)} \in \mathbb{C}^{N_i}$ $(1 \leq i \leq q)$ where each element of $\boldsymbol{\mathcal{T}}$ is defined by $\mathcal{T}_{i_1, \cdots, i_q} = u^{(1)}_{i_1} \cdots u^{(q)}_{i_q}$.*

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There is a major difference between matrices and multiway arrays when rank properties are concerned.

Definition 2 *The rank of a q-way ($q \geq 2$) array $\mathcal{T}$, denoted by $\mathrm{rk}(\mathcal{T})$, is the minimal number of rank-1 q-way arrays that yield $\mathcal{T} \in \mathbb{C}^{N_1 \times \cdots \times N_q}$ in a linear combination.*

For instance, the rank of a multiway array can exceed its dimensions. From definitions 1 and 2, CAND [9, 10, 14] can be defined as following:

Definition 3 *CAND of a q-way ($q \geq 2$) array $\mathcal{T}$ is the linear combination of $P = \mathrm{rk}(\mathcal{T})$ rank-1 q-way arrays that yields $\mathcal{T}$ exactly.*

Definition 3 shows that the different rank-1 terms can be permuted and scaled without modifying the sum. A CAND is then considered unique when it is only subject to these trivial indeterminacies. Sufficient conditions [15] guarantee the uniqueness of the CAND and can be used to adress the identifiability issue of the ORBIT family. Let's now introduce the i-mode product of a multiway array and a matrix.

Definition 4 *The i-mode product of a q-way array $\mathcal{T} \in \mathbb{C}^{N_1 \times \cdots \times N_q}$ and a matrix $\boldsymbol{U} \in \mathbb{C}^{J_i \times N_i}$ is a q-way array of $\mathbb{C}^{N_1 \times \cdots \times N_{i-1} \times J_i \times N_{i+1} \times \cdots \times N_q}$ given by:*

$$(\mathcal{T} \times_i \boldsymbol{U})_{n_1, \cdots n_{i-1}, j_i, n_{i+1}, \cdots, n_q} = \sum_{n_i=1}^{N_i} \mathcal{T}_{n_1, \cdots, n_i, \cdots, n_q} U_{j_i, n_i}$$

This special product will be as useful in section 3 in order to describe the $2q$-ORBIT methods as the following multiway array-to-matrix transformations:

Definition 5 *Let $\mathcal{T}$ be a square $2q$-way ($q \geq 2$) array of dimension N. Let $\lfloor q/2 \rfloor$ and $\lceil q/2 \rceil$ be the lower and the upper integer part of $q/2$, repectively. Then the (i, j)-th component of the unfolding matrix $\mathrm{mat}_1(\mathcal{T})$ of size ($N^q \times N^q$) is given by:*

$$(\mathrm{mat}_1(\mathcal{T}))_{i,j} = \mathcal{T}_{n_1, \cdots, n_{\lceil q/2 \rceil}, n_{\lceil q/2 \rceil+1}, \cdots, n_q, n_{q+1}, \cdots, n_{q+\lceil q/2 \rceil} n_{2q+\lceil q/2 \rceil+1}, \cdots, n_{2q}}$$

where $i = (n_1 - 1)N^{q-1} + \cdots + (n_{\lceil q/2 \rceil} - 1)N^{\lfloor q/2 \rfloor} + (n_{q+\lceil q/2 \rceil+1} - 1)N^{\lfloor q/2 \rfloor - 1} + \cdots + (n_{2q} - 1)N + n_{2q}$ *and* $j = (n_{q+1} - 1)N^{q-1} + \cdots + (n_{q+\lceil q/2 \rceil} - 1)N^{\lfloor q/2 \rfloor} + (n_{\lceil q/2 \rceil+1} - 1)N^{\lfloor q/2 \rfloor - 1} + \cdots + (n_q - 1)N + n_q$.

Another way to unfold HO arrays is presented hereafter:

Definition 6 *Let $\mathcal{T} \in \mathbb{C}^{N_1 \times \cdots \times N_q}$ be a q-way ($q \geq 3$) array. Then the (n_i, m)-th component of the unfolding matrix $\mathrm{mat}_2^{(i)}(\mathcal{T}) \in \mathbb{C}^{N_i \times N_{i+1} \cdots N_q N_1 \cdots N_{i-1}}$ associated to the i-th mode ($1 \leq i \leq q$) of $\mathcal{T}$ is given by:*

$$(\mathrm{mat}_2^{(i)}(\mathcal{T}))_{n_i, m} = \mathcal{T}_{n_1, \cdots, n_{i-1}, n_i, n_{i+1}, \cdots, n_q}$$

where $m = (n_{i+1} - 1)N_{i+2} \cdots N_q N_1 \cdots N_{i-1} + (n_{i+2} - 1)N_{i+3} \cdots N_q N_1 \cdots N_{i-1} + \cdots + (n_q - 1)N_1 N_2 \cdots N_{i-1} + (n_1 - 1)N_2 N_3 \cdots N_{i-1} + (n_2 - 1)N_3 N_4 \cdots N_{i-1} + \cdots + n_{i-1}$.

Now, let's consider the $2q$-th order ($q > 1$) cumulant array, $\mathcal{C}_{2q, \boldsymbol{x}}$ [11] of the random vector $\boldsymbol{x}$ (see problem (1)) whose entries are denoted by:

$$\mathcal{C}_{n_1, \cdots, n_q, \boldsymbol{x}}^{n_{q+1}, \cdots, n_{2q}} = \mathrm{Cum}\{x_{n_1}, \cdots, x_{n_q}, x_{n_{q+1}}^*, \cdots, x_{n_{2q}}^*\} \quad (1)$$

where $*$ is the complex conjugate operator. Moreover, due to the multilinearity property enjoyed by cumulants, $\mathcal{C}_{2q, \boldsymbol{x}}$ has the following canonical form:

$$\mathcal{C}_{2q, \boldsymbol{x}} = \sum_{p=1}^{P} \mathcal{C}_{p, \cdots, p, \boldsymbol{s}}^{p, \cdots, p} \, \boldsymbol{a}_p^{\circ q} \circ \boldsymbol{a}_p^{* \circ q} \quad (2)$$

where $\boldsymbol{a}_p^{\circ q} = \boldsymbol{a}_p \circ \cdots \circ \boldsymbol{a}_p$, is the q-time outer product of the p-th column vector of the mixture $\boldsymbol{A}$, $\mathcal{C}_{p, \cdots, p, \boldsymbol{s}}^{p, \cdots, p}$ is the $2q$-th order marginal cumulant of the p-th source. Consequently, problem 1 can be reformulated as following:

Problem 2 *Given the $2q$-th order ($q > 1$) cumulant array $\mathcal{C}_{2q, \boldsymbol{x}}$ of $\boldsymbol{x}$, find its CAND.*

3. ALGORITHM

The $2q$-ORBIT ($q > 1$) method is presented here in order to solve problem 2. This method consists, firstly, in finding the P q-way rank-1 arrays $\mathcal{A}^{(p)}$ given by $\mathcal{A}^{(p)} = \boldsymbol{a}_p^{\circ \lceil q/2 \rceil} \circ \boldsymbol{a}_p^{* \circ \lfloor q/2 \rfloor}$ and secondly in identifying matrix $\boldsymbol{A}$.

3.1. First step: estimation of the P rank-1 arrays $\mathcal{A}^{(p)}$

According to equation (2) and definition 5, the ($N^q \times N^q$) unfolding matrix of $\mathcal{C}_{2q, \boldsymbol{x}}$, $\boldsymbol{C}_{2q, \boldsymbol{x}} = \mathrm{mat}_1(\mathcal{C}_{2q, \boldsymbol{x}})$, can be written as following:

$$\boldsymbol{C}_{2q, \boldsymbol{x}} = \boldsymbol{A}_q \, \boldsymbol{\zeta}_{2q, \boldsymbol{s}} \, \boldsymbol{A}_q^{\mathsf{H}} \quad (3)$$

where $\boldsymbol{A}_q = \boldsymbol{A}^{\oslash \lceil q/2 \rceil} \oslash \boldsymbol{A}^{* \lfloor q/2 \rfloor}$, $\oslash$ is the *Khatri-Rao product* (columns-wise kronecker product) operator [11], $^{\oslash}$ is the *Khatri-Rao power* operator [11] and $\boldsymbol{\zeta}_{2q, \boldsymbol{s}}$ the diagonal matrix $\mathrm{diag}\{[\mathcal{C}_{1, \cdots, 1, \boldsymbol{s}}^{1, \cdots, 1}, \cdots, \mathcal{C}_{P, \cdots, P, \boldsymbol{s}}^{P, \cdots, P}]\}$. Now, assume that the $2q$-th order marginal source cumulants are non-zero and have the same sign $\epsilon = \pm 1$ and if $\boldsymbol{A}_q$ is full column rank P, and construct the ($N^q \times P$) matrix $\boldsymbol{C}_{2q, \boldsymbol{x}}^{1/2} = \boldsymbol{E}_{\boldsymbol{s}} \boldsymbol{L}_{\boldsymbol{s}}^{1/2}$ where $\boldsymbol{L}_{\boldsymbol{s}}^{1/2}$, $\boldsymbol{E}_{\boldsymbol{s}}$ are the square root of the real-valued diagonal matrix of the P non-zero eigenvalues of $\epsilon \boldsymbol{C}_{2q, \boldsymbol{x}}$ and the corresponding unitary ($N^q \times P$) eigenmatrix, repectively. Then we have the following proposition:

Proposition 1 *Under the previous assumptions, $\boldsymbol{C}_{2q, \boldsymbol{x}}^{1/2}$ is a square root of $\epsilon \boldsymbol{C}_{2q, \boldsymbol{x}}$ and $\boldsymbol{B} = (\boldsymbol{L}_{\boldsymbol{s}}^{1/2})^{-1} \boldsymbol{E}_{\boldsymbol{s}}^{\mathsf{H}} \boldsymbol{A}_q \boldsymbol{\zeta}_{2q, \boldsymbol{s}}^{1/2}$ is a ($P \times P$) unitary matrix, which is real when q is even and where the diagonal matrix $\boldsymbol{\zeta}_{2q, \boldsymbol{s}}^{1/2}$ denotes a square root of $\boldsymbol{\zeta}_{2q, \boldsymbol{s}}$.*

Proof is derived from [13, Theorem 2]. The first result of proposition 1 derives from the eigenvalue decomposition of $\epsilon \boldsymbol{C}_{2q,\boldsymbol{x}}^{\ell}$. The second result comes from equation (3) and the fact that two square root matrices are equal up to an orthogonal matrix. Now, the $(P\times N^q)$ matrix $(\boldsymbol{C}_{2q,\boldsymbol{x}}^{\ell\ 1/2})^{\mathsf{T}}$ can be seen as the unfolding matrix associated to the second mode of a 3-way array $\boldsymbol{\mathcal{T}}$ of size $(N^{q-1}\times P\times N)$ (see definition 6) whose CAND is then given by:

$$\boldsymbol{\mathcal{T}} = \sum_{p=1}^{P} \boldsymbol{a}_{q-1,p} \circ \boldsymbol{b}_p^{*} \circ \boldsymbol{d}_p \tag{4}$$

where $\boldsymbol{a}_{q-1,p} = \boldsymbol{a}_p^{\otimes\lceil(q-1)/2\rceil} \otimes \boldsymbol{a}_p^{*\otimes\lfloor(q-1)/2\rfloor}$ is the p-th column of matrix $\boldsymbol{A}_{q-1}$, $\otimes$ is the *Kronecker product* operator, $\boldsymbol{a}^{\otimes\ell}$ is the ℓ-time Kronecker product of $\boldsymbol{a}$ [11]. In addition, $\boldsymbol{b}_p$ and $\boldsymbol{d}_p$ are the p-th columns of $\boldsymbol{B}$ and $\boldsymbol{D}$, respectively, where the $(N\times P)$ matrix $\boldsymbol{D}$ is given by $\boldsymbol{D}=\boldsymbol{A}\boldsymbol{\zeta}_{2q,\boldsymbol{s}}^{1/2}$. Thus, each n-th frontal slice of $\boldsymbol{\mathcal{T}}$ can be written as following:

$$\boldsymbol{\mathcal{T}}(:,:,n) = \boldsymbol{A}_{q-1}\mathrm{diag}\{\boldsymbol{D}(n,:)\}\boldsymbol{B}^{\mathsf{H}} \tag{5}$$

where $\boldsymbol{D}(n,:)$ denotes the n-th row of $\boldsymbol{D}$. Then, equation (5) can be seen as the core equation of an extended version of the following *Procrustes problem*:

Problem 3 *Given two matrices $\boldsymbol{G}$ and $\boldsymbol{F}$ of the same size, find a unitary matrix $\boldsymbol{B}$ such that $\boldsymbol{G}=\boldsymbol{F}\boldsymbol{B}^{\mathsf{H}}$.*

The solution of the latter constrained problem is given by $\boldsymbol{B} = \boldsymbol{U}\boldsymbol{V}^{\mathsf{H}}$, where $\boldsymbol{U}$ and $\boldsymbol{V}$ are the left and right singular matrices of matrix $\boldsymbol{G}^{\mathsf{H}}\boldsymbol{F}$ respectively [1]. Nevertheless, such a solution cannot be directly used since, according to (5), the N matrices $\boldsymbol{F}_n = \boldsymbol{A}_{q-1}\mathrm{diag}\{\boldsymbol{D}(n,:)\}$ are unknown. So, how can we solve this more generalized Procrustes problem? The solution proposed in this paper consists in identifying iteratively the unknown matrices $\boldsymbol{A}_{q-1}$, $\boldsymbol{D}$ and $\boldsymbol{B}$. First, with matrix $\boldsymbol{B}$ fixed to an initial value, the p-th column vector, $\boldsymbol{a}_{q-1,p}$, of matrix $\boldsymbol{A}_{q-1}$ is computed as the left singular vector associated to the largest singular value of the p-th vertical slice, $\boldsymbol{\mathcal{T}}_1(:,p,:)$, of the 3-way array $\boldsymbol{\mathcal{T}}_1 = \boldsymbol{\mathcal{T}}\times_2\boldsymbol{B}^{\mathsf{T}}$. Indeed, each p-th vertical slice of the latter 3-way array $\boldsymbol{\mathcal{T}}_1$ can be written as a rank-1 matrix of the following form:

$$\boldsymbol{\mathcal{T}}_1(:,p,:) = \boldsymbol{A}_{q-1}\mathrm{diag}\{\mathbf{I}_P(p,:)\}\boldsymbol{D}^{\mathsf{T}} = \boldsymbol{a}_{q-1,p}\,\boldsymbol{d}_p^{\mathsf{T}} \tag{6}$$

where $\mathbf{I}_P$ is the $(P\times P)$ identity matrix. Secondly, the n-th row of matrix $\boldsymbol{D}$ is obtained by taking the diagonal of the n-th frontal slice of the 3-way array $\boldsymbol{\mathcal{T}}_2 = \boldsymbol{\mathcal{T}}\times_1\boldsymbol{A}_{q-1}^{\sharp}\times_2\boldsymbol{B}^{\mathsf{T}}$ where $\sharp$ denotes the *pseudo-inverse* operator. Indeed, we have:

$$\boldsymbol{\mathcal{T}}_2(:,:,n) = \boldsymbol{A}_{q-1}^{\sharp}\boldsymbol{\mathcal{T}}(:,:,n)\boldsymbol{B} = \mathrm{diag}\{\boldsymbol{D}(n,:)\} \tag{7}$$

Thirdly, according to the solution of Procrustes problem mentionned above, the orthogonal matrix $\boldsymbol{B}$ is obtained by computing the Singular Value Decomposition (SVD) of matrix $\sum_{n=1}^{N}(\mathrm{diag}\{\boldsymbol{D}(n,:)\})^{\mathsf{H}}\boldsymbol{\mathcal{T}}_3(:,:,n)$ where $\boldsymbol{\mathcal{T}}_3(:,:,n)$ is the n-th frontal slice of the 3-way array $\boldsymbol{\mathcal{T}}_3 = \boldsymbol{A}_{q-1}^{\mathsf{H}}\times_1\boldsymbol{\mathcal{T}}$. Next, repeat the same previous steps until convergence. This approach describes the first step of the $2q$-ORBIT$_1$ method. Another solution consists in computing each column vector of $\boldsymbol{D}$ by taking the right singular vector associated to the largest singular value of each vertical slice of $\boldsymbol{\mathcal{T}}_1$. The latter approach does not require to compute the 3-way array $\boldsymbol{\mathcal{T}}_2$ and will be referred to as $2q$-ORBIT$_2$ in the sequel. Now, once the unitary matrix $\boldsymbol{B}$ is identified, the $(n_1,\cdots,n_q)$-th component, $\mathcal{A}_{n_1,\cdots,n_q}^{(p)}$, of the p-th $(1\le p\le P)$ rank-1 array $\boldsymbol{\mathcal{A}}^{(p)}$ can be computed by taking the $(n_q+(n_{q-1}-1)N+\cdots+(n_1-1)N^{q-1})$-th component of the p-th column vector of $\boldsymbol{C}_{2q,\boldsymbol{x}}^{\ell\ 1/2}\boldsymbol{B}$.

3.2. Second step: identification of mixture $\boldsymbol{A}$

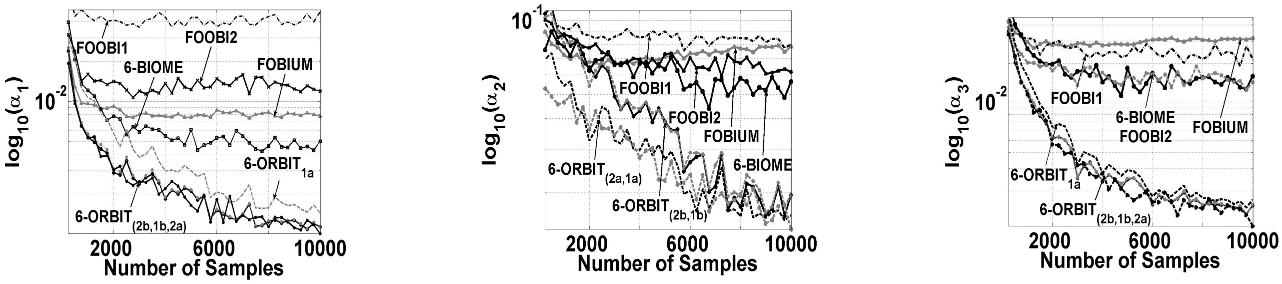
Two ways to identify mixture $\boldsymbol{A}$ are proposed hereafter. The first one consists in taking matrix $\boldsymbol{D}$ since $\boldsymbol{D} = \boldsymbol{A}\,\boldsymbol{\zeta}_{2q,\boldsymbol{s}}^{1/2}$, which leads up to the $2q$-ORBIT$_{1a}$ and $2q$-ORBIT$_{2a}$. The second one consists in canonically decomposing each rank-1 array $\boldsymbol{\mathcal{A}}^{(p)}$ $(1\le p\le P)$ in order to identify each column vector $\boldsymbol{a}_p$ of $\boldsymbol{A}$. The HO power iteration method [2] allows for such a decomposition. Let $\{\boldsymbol{w}_{it}^{(i)}\}$ be the set of q N-dimensionnal vectors computed during the it-th iteration of the HO power method applied to $\boldsymbol{\mathcal{A}}^{(p)}$. Vector $\boldsymbol{w}_{it+1}^{(i)}$ is then given by the following equation:

$$\begin{aligned}\boldsymbol{w}_{it+1}^{(i)} = \boldsymbol{\mathcal{A}}^{(p)}\times_1(\boldsymbol{w}_{it+1}^{(1)})^{\mathsf{T}}\times_2\cdots\times_{i-1}(\boldsymbol{w}_{it+1}^{(i-1)})^{\mathsf{T}}\\ \times_{i+1}(\boldsymbol{w}_{it}^{(i+1)})^{\mathsf{T}}\times_{i+2}\cdots\times_q(\boldsymbol{w}_{it}^{(q)})^{\mathsf{T}}\end{aligned} \tag{8}$$

and then converges to vectors $\boldsymbol{a}_p$ and $\boldsymbol{a}_p^{*}$ for $(1\le i\le q)$ and $(q+1\le i\le 2q)$, respectively. This procedure leads us up to the $2q$-ORBIT$_{1b}$ and $2q$-ORBIT$_{2b}$ methods.

3.3. Identifiability

The number maximal P_{max} of sources which can be processed using the ORBIT approach is briefly presented hereafter. We can show that this number is generically equal to N^{q-1}. Recall that a property is called generic when it holds everywhere except for a set of Lebesgue mesure 0. As far as radiocommunication contexts are concerned, P_{max} is equal to $\mathcal{N}_{2(q-1)}^{\lfloor(q-1)/2\rfloor}$ where $\mathcal{N}_{2q}^{\lfloor q/2\rfloor}$ denotes the number of different virtual sensors of the more ‘efficient’ q-th order virtual array associated with the true array of N sensors [16]. Word ‘efficient’ has to be interpreted in terms of maximal number of sources which can be processed at the concerned statistical q-th order (more details are given in [16]). For lack of place, the computation of P_{max} is not detailed in this section and will be addressed in a longer paper. Even so, we can say that the proof lies in part on the theoretical results



(a) source 1 (b) source 2 (c) source 3

Fig. 1. Criteria α_i ($1 \leq i \leq 3$) at the output of the 6-ORBIT methods and four classical cumulant-based methods, for a ULA of $N = 2$ sensors and $P = 3$ equispaced ($\Delta\theta = 30°$) QPSK sources with the same SNR= 20dB.

proposed in [15]. Besides, in order to illustrate this upper bound in radiocommunications, table 1 gives it for different cumulant-based methods, namely 6-BIOME [11], FOBIUM [12], FOOBI1 [13], FOOBI2 [13] and 6-ORBIT, as a function of the number N of sensors of a Uniform Linear Array (ULA).

N		2	3	4	5	6	7	8
P_{max}	FOOBI1	2	4	7	9	11	13	15
	FOOBI2	3	5	7	9	11	13	15
	FOBIUM	3	5	7	9	11	13	15
	6-BIOME	3	5	7	9	11	13	15
	6-ORBIT	3	5	7	9	11	13	15

Table 1. P_{max} for a ULA of N sensors

4. COMPUTER RESULTS

A comparative study between the $2q$-ORBIT methods for $q = 3$) and classical cumulant-based methods such as 6-BIOME [11], FOBIUM [12], FOOBI1 [13], FOOBI2 [13] is presented hereafter. A ULA of $N = 2$ sensors and $P = 3$ QPSK sources linearly modulated with pulse shape filter corresponding to a $1/2$ Nyquist filter with a roll-off equal to 0.3 are used. In addition, all sources have the same symbol period $T_s = 5T_e$ and the same Signal-to-Noise Ratio (SNR), where T_e denotes the sample period. The source direction angles are $\theta_1 = 10°, \theta_2 = 40°$ and $\theta_3 = 70°$. The source carrier residuals are such that $f_{c_1}T_e = 0, f_{c_2}T_e = 0.3$ and $f_{c_3}T_e = 0.6$. Noise is assumed to be Gaussian, temporally and spatially white. Eventually, the simulation results are averaged over 200 trials wherein the sources and the noise are resampled at each trial. A distance criterion between mixture $\boldsymbol{A}$ and its estimate $\widehat{\boldsymbol{A}}$ is used as a performance criterion, that is, $D(\boldsymbol{A}, \widehat{\boldsymbol{A}}) = (\alpha_1, \alpha_2, \cdots, \alpha_P)$ with $\alpha_p = \min_{1 \leq i \leq P}\{d(\boldsymbol{a}_p, \widehat{\boldsymbol{a}}_i)\}$ where d is the pseudo-distance between vectors [12] and defined by $d(\boldsymbol{u}, \boldsymbol{v}) = 1 - |\boldsymbol{u}^{\mathsf{H}}\boldsymbol{v}|^2/(||\boldsymbol{u}||\,||\boldsymbol{v}||)$. Figure 1 shows the variation of α_i ($1 \leq i \leq 3$) at the output of the considered methods as a function of data samples for a SNR of 20 dB. We note a faster convergence of the 6-ORBIT methods with respect to the other algorithms. Figure 2 shows the variation of α_i ($1 \leq i \leq 3$) as a function of SNR and for 1000 data samples. We note a good robustness of the 6-ORBIT method to a low SNR and a very good behaviour, especially for 6-ORBIT$_{1a}$, when SNR increases.

5. CONCLUSION

We propose a new family of methods, named $2q$-ORBIT ($q > 1$), in order to solve the BUMI problem. These algorithms are based on a CAND of a special semi-definite positive (or negative) $2q$-th order cumulant array. In fact, the latter decomposition is brought back to the decomposition of a third order array whose one loading matrix is unitary. Such a decomposition is thus performed in two steps: the first one consists in solving a generalized Procrustes problem while the second one needs to compute the best rank-1 approximation of several q-th order arrays. Computer results show the efficiency of the 6-ORBIT methods with respect to classical cumulant-based methods. An identifiability study will be more detailed in a longer version of this paper in order to compute the maximum number of sources which can be processed by the 2q-ORBIT method family. Moreover, additional simulations will show the efficiency of 2q-ORBIT algorithms compared to the ALS-based BUMI approaches especially in terms of resolution and convergence speed.

6. ACKNOWLEDGMENT

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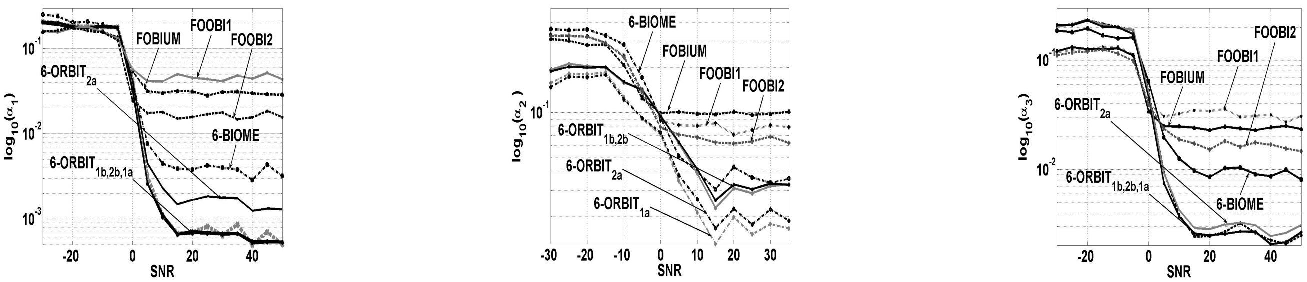

(a) source 1 (b) source 2 (c) source 3

Fig. 2. Criterion α_i ($1 \leq i \leq 3$) at the output of the 6-ORBIT methods and four classical cumulant-based methods as a function of SNR, for a ULA of $N = 2$ sensors and $P = 3$ equispaced ($\delta\theta = 30°$) QPSK sources of 1000 samples.

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