Fourth-Order Blind Identification of Underdetermined Mixtures of Sources (FOBIUM)

Anne Ferréol, Laurent Albera, and Pascal Chevalier

***Abstract*—For about two decades, numerous methods have been developed to blindly identify overdetermined ($P \leq N$) mixtures of P statistically independent narrowband (NB) sources received by an array of N sensors. These methods exploit the information contained in the second-order (SO), the fourth-order (FO) or both the SO and FO statistics of the data. However, in practical situations, the probability of receiving more sources than sensors increases with the reception bandwidth and the use of blind identification (BI) methods able to process underdetermined mixtures of sources, for which $P > N$ may be required. Although such methods have been developed over the past few years, they all present serious limitations in practical situations related to the radiocommunications context. For this reason, the purpose of this paper is to propose a new attractive BI method, exploiting the information contained in the FO data statistics only, that is able to process underdetermined mixtures of sources without the main limitations of the existing methods, provided that the sources have different trispectrum and nonzero kurtosis with the same sign. A new performance criterion that is able to quantify the identification quality of a given source and allowing the quantitative comparison of two BI methods for each source, is also proposed in the paper. Finally, an application of the proposed method is presented through the introduction of a powerful direction-finding method built from the blindly identified mixture matrix.**

***Index Terms*—Blind source identification, FO direction finding, fourth-order statistics, performance criterion, SOBI, trispectrum, underdetermined mixtures.**

I. Introduction

FOR more than two decades and the pioneer work of Godard [30] about blind equalization in single-input single-output (SISO) contexts, there has been an increasing interest for blind identification (BI) of both single-input multiple-output (SIMO) and multiple-input multiple-output (MIMO) systems. While, in the SISO case, blind equalization or channel identification require the exploitation of higher order (HO) statistics in the general case of nonminimum phase systems [30], it has been shown recently that for SIMO systems, multichannel identification may be performed from SO statistics only under quite general assumptions [39], [43], [49]. Extensions of these pioneer works and the development of alternative methods for both blind multichannel identification and equalization in MIMO finite impulse response (FIR) systems from SO or HO statstics are presented in [1], [23], [31], [32], and [17], [29], [35], [38], [50]–[53], respectively. Other extensions to MIMO infinite impulse response (IIR) systems or taking into account the finite-alphabet property of the sources are presented in [34], [44] and [46], [54], respectively. However, the BI or deconvolution problems in MIMO contexts are not recent but have been considered since the pioneer work of Herault and Jutten [33], [36] about blind source separation (BSS) in 1985. Since these pioneer works, numerous methods have been developed to blindly identify either instantaneous or convolutive mixtures of P statistically independent NB sources received by an array of N sensors. Some of these methods [5], [48] exploit the SO data statistics only, whereas other methods [6], [9], [14], [22] exploit both the SO and the FO statistics of the data or even the FO data statistics only [2].

Nevertheless, all the previous methods of either blind multichannel identification of MIMO systems or BI of instantaneous or convolutive mixtures of sources, either SO or HO, can only process overdetermined systems, i.e., systems for which the number of sources (or inputs) P is lower than or equal to the number of sensors (or outputs) N, i.e., such that $P \leq N$.

However, in practical situations such as, for example, airborne electronic warfare over dense urban areas, the probability of receiving more sources than sensors increases with the reception bandwidth and the use of BI methods that are able to process underdetermined mixtures of sources, for which $P > N$, may be required. To this aim, several methods have been developed this last decade mainly to blindly identify instantaneous mixtures of sources, among which we find the methods [3], [4], [8], [15], [16], [19]–[21], [37], [45]. Concerning convolutive mixtures of sources or MIMO FIR systems, only very scarce results exist about BI of underdetermined systems, among which we find [18] and [47]. Some of these methods focus on blind source extraction [16], [37], which is a difficult problem since underdetermined mixtures are not linearly invertible, while others, as herein, favor BI of the mixture matrix [3], [4], [8], [15], [16], [18]–[21], [37], [45], [47]. The methods proposed in [8], [15], [18]–[21], and [47] only exploit the information contained in the FO statistics of the data, whereas the one recently proposed in [3] exploits the sixth-order data statistics only, and its extension to an arbitrary even order $2q$ $(q > 2)$ is presented in [4]. Finally, the method proposed in [45] exploits the information contained in the second characteristic function of the observations, whereas in [37], the probability density of the observations conditionally to the mixture matrix is maximized. Nevertheless, all

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A. Ferréol and P. Chevalier are with Thalès-Communications, 92704 Colombes, France (e-mail: pascal.chevalier@fr.thalesgroup.com, Anne.ferreol@fr.thalesgroup.com).

L. Albera is with the Laboratoire Traitement du Signal et de L'Image (LTSI), Rennes University (INSERM U642), 35042 Rennes Cedex, France (e-mail: laurent.albera@univ-rennes1.fr).

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these methods suffer from serious limitations in operational contexts related to radiocommunications. Indeed, the method [8] and its improvements for both instantaneous [21] and convolutive [18] mixtures of sources remain currently mainly conceptual and have not yet been evaluated by any simulations. The methods [15], [19], and [20] assume FO noncircular sources and, thus, fail in identifying circular sources, which are omnipresent in practice. Besides, the theories developed in [15] and [19] confine themselves to the three source and two sensor cases. Although the method [37] succeeds in identifying the steering vectors of up to four speech signals with only two sensors, the authors need sparsity conditions and do not address the general case when all sources are always present. Moreover, the method [45] has been developed only for real mixtures of real-valued sources, and the issue of robustness with respect to an overestimation of the source number remains open. Although very promizing, powerful, and easy to implement, the methods [3] and [4] suffer *a priori* from both a higher variance and a higher numerical complexity due to the use of data statistics with an even order strictly greater than four. Finally, for instantaneous mixtures of sources, the method developed in [47] can only process overdetermined systems.

In order to overcome these limitations for underdetermined systems, the purpose of this paper is to propose a new BI method, exploiting the information contained in the FO data statistics only that is able to process both over and underdetermined instantaneous mixtures of sources without the drawbacks of the existing methods of this family but assuming the sources have a different trispectrum and have nonzero kurtosis with the same sign (the latter assumption is generally verified in radiocommunications contexts). This new BI method, which is called the Fourth-Order Blind Identification of Underdetermined Mixtures of sources (FOBIUM), corresponds to the FO extension of the second-order blind idenification (SOBI) method [5] and is able to blindly identify the steering vectors of up to $N^2 - N + 1$ sources from an array of N sensors with space diversity only and of up to N^2 sources from an array of N different sensors. Moreover, this method is asymptotically robust to an unknown Gaussian spatially colored noise since it does not exploit the information contained in the SO data statistics. To evaluate the performance of the FOBIUM method and, more generally, of all the BI methods, a new performance criterion that is able to quantify the identification quality of the steering vector of each source and allowing the quantitative comparison of two methods for the blind identification of a given source is also proposed. Finally, an application of the FOBIUM method is presented through the introduction of a FO direction-finding method, built from the blindly identified mixing matrix and called MAXimum of spatial CORrelation (MAXCOR), which is shown to be very powerful with respect to SO [42] and FO subspace-based direction-finding methods [7], [13], [40]. Note that an extension of the FOBIUM method to HO statistics remains possible.

After the problem formulation and an introduction of some notations, hypotheses and data statistics in Section II, the FOBIUM method is presented in Section III. The associated conditions about the identifiability of the mixture matrix are then analyzed in Section IV. The new performance criterion is presented in Section V. The application of the FOBIUM method to the direction-finding problem through the introduction of the MAXCOR method is described in Section VI. All the results of the paper are illustrated in Section VII through computer simulations. The numerical complexity of the FOBIUM method compared with the one of some existing methods is briefly presented in Section VIII. Finally, Section IX concludes this paper. Note that the results of the paper have been partially presented in [11] and [25].

II. Problem Formulation, Hypotheses, and Data Statistics

A. Problem Formulation

We consider an array of N NB sensors, and we call $\boldsymbol{x}(t)$ the vector of complex amplitudes of the signals at the output of these sensors. Each sensor is assumed to receive the contribution of P zero-mean stationary and stastistically independent NB sources corrupted by a noise. Under these assumptions, the observation vector $\boldsymbol{x}(t)$ can be written as follows:

$$\boldsymbol{x}(t) = \sum_{p=1}^{P} m_p(t)\boldsymbol{a}_p + \boldsymbol{b}(t) \triangleq A\boldsymbol{m}(t) + \boldsymbol{b}(t) \tag{1}$$

where $\boldsymbol{b}(t)$ is the noise vector that is assumed to be zero-mean, stationary, and Gaussian, the complex envelope of the source p, $m_p(t)$, is the pth component of the vector $\boldsymbol{m}(t)$ that is assumed zero-mean and stationary, $\boldsymbol{a}_p$ corresponds to the steering vector of the source p, and A is the $(N \times P)$ mixture matrix whose columns are the vectors $\boldsymbol{a}_p$. The instantaneous mixture model defined by (1) have already been considered in numerous papers [2]–[12], [14]–[16], [19]–[22], [24]–[28], [33], [36], [37], [45], [48] and is perfectly suitable for applications such as, for example, airborne or satellite electronic warfare.

Under these assumptions, the problem addressed in this paper is that of FO blind identification of the mixture matrix A. It consists of estimating, from the FO data statistics, the mixing matrix A to within a $(P \times P)$ invertible diagonal matrix Λ and a $(P \times P)$ permutation matrix Π.

B. Statistics of the Data

Under the previous assumptions, the SO statistics of the data used in the paper are characterized by the correlation or covariance matrix R_x, which is defined by

$$\begin{aligned} R_x &\triangleq \mathrm{E}[\boldsymbol{x}(t)\boldsymbol{x}(t)^{\mathrm{H}}] = \sum_{p=1}^{P} \pi_p \boldsymbol{a}_p \boldsymbol{a}_p^{\mathrm{H}} + \eta_2 B \\ &\triangleq A R_m A^{\mathrm{H}} + \eta_2 B \end{aligned} \tag{2}$$

where $\pi_p \triangleq \mathrm{E}[|m_p(t)|^2]$ is the power of source p received by an omnidirectional sensor, η_2 is the mean of the noise power per sensor, B is the spatial coherence of the noise such that $\mathrm{Tr}[B] = N$, where $\mathrm{Tr}[\cdot]$ means Trace, $R_m \triangleq \mathrm{E}[\boldsymbol{m}(t)\boldsymbol{m}(t)^{\mathrm{H}}]$ is the correlation matrix of the source vector $\boldsymbol{m}(t)$, and the symbol $^{\mathrm{H}}$ means transpose and complex conjugate.

The FO statistics of the data used in the paper are characterized by the $(N^2 \times N^2)$ quadricovariance matrices

$Q_x(\tau_1,\tau_2,\tau_3)$, whose elements $Q_x(\tau_1,\tau_2,\tau_3)[i,j,k,l](1 \le i,j,k,l \le N)$ are defined by

$$Q_x(\tau_1,\tau_2,\tau_3)[i,j,k,l] \triangleq \text{Cum}\,(x_i(t), xj(t-\tau_1)^*, x_k(t-\tau_2)^*, x_l(t-\tau_3)) \quad (3)$$

where * means complex conjugate, and $x_i(t)$ is the component i of $\boldsymbol{x}(t)$. Using (1) into (3) and assuming that $Q_x(\tau_1,\tau_2,\tau_3)[i,j,k,l]$ is the element $[N(i-1)+j, N(k-1)+l]$ of the matrix $Q_x(\tau_1,\tau_2,\tau_3)$, we obtain the expression of the latter, which is given, under a Gaussian noise assumption, by

$$Q_x(\tau_1,\tau_2,\tau_3) = [A \otimes A^*] Q_m(\tau_1,\tau_2,\tau_3) [A \otimes A^*]^{\text{H}} \quad (4)$$

where $Q_m(\tau_1,\tau_2,\tau_3)$ is the $(P^2 \times P^2)$ quadricovariance matrix of $\boldsymbol{m}(t)$, and $\otimes$ is the Kronecker product.

Under the assumption of statistically independent sources, the matrix $Q_m(\tau_1,\tau_2,\tau_3)$ contains at least $P^4 - P$ zeros, and expression (4) degenerates in a simpler one given by

$$Q_x(\tau_1,\tau_2,\tau_3) = \sum_{p=1}^{P} c_p(\tau_1,\tau_2,\tau_3) \left[\boldsymbol{a}_p \otimes \boldsymbol{a}_p^*\right] \left[\boldsymbol{a}_p \otimes \boldsymbol{a}_p^*\right]^{\text{H}} = A_Q C_m(\tau_1,\tau_2,\tau_3) A_Q^{\text{H}} \quad (5)$$

where A_Q is the $(N^2 \times P)$ matrix defined by $A_Q \triangleq [\boldsymbol{a}_1 \otimes \boldsymbol{a}_1^*, \ldots, \boldsymbol{a}_P \otimes \boldsymbol{a}_P^*]$, $C_m(\tau_1,\tau_2,\tau_3)$ is the $(P \times P)$ diagonal matrix defined by $C_m(\tau_1,\tau_2,\tau_3) \triangleq \text{Diag}[c_1(\tau_1,\tau_2,\tau_3), \ldots, c_p(\tau_1,\tau_2,\tau_3)]$, and $c_p(\tau_1,\tau_2,\tau_3)$ is defined by

$$c_p(\tau_1,\tau_2,\tau_3) \triangleq \text{Cum}(m_p(t), m_p(t-\tau_1)^*, m_p(t-\tau_2)^*, m_p(t-\tau_3)). \quad (6)$$

Expression (5), which has an algebraic structure similar to that of data correlation matrices [5], is the starting point of the FOBIUM method, as it will be shown in Section II-C. To simplify the notations, we note in the following $Q_x \triangleq Q_x(0,0,0)$, $C_m \triangleq C_m(0,0,0)$, and $c_p \triangleq c_p(0,0,0)$, and we obtain from (5)

$$Q_x = A_Q C_m A_Q^{\text{H}}. \quad (7)$$

C. Statistics Estimation

In situations of practical interests, the SO and FO statistics of the data, which are given by (2) and (3), respectively, are not known *a priori* and have to be estimated from L samples of data $\boldsymbol{x}(l) \triangleq \boldsymbol{x}(lT_e)$, $1 \le l \le L$, where T_e is the sample period. For zero-mean stationary observations, using the ergodicity property, empirical estimators [26] may be used since they generate asymptotically unbiased and consistent estimates of the data statistics. However, in radiocommunications contexts, most of the sources are no longer stationary but become cyclostationary (digital modulations). For zero-mean cyclostationary observations, the statistics defined by (2) and (3) become time dependent, and the theory developed in the paper can be extended without any difficulties by considering that R_x and $Q_x(\tau_1,\tau_2,\tau_3)$ are, in this case, the temporal means $\langle R_x(t)\rangle$ and $\langle Q_x(\tau_1,\tau_2,\tau_3)(t)\rangle$ over an infinite interval duration of the instantaneous statistics $R_x(t)$ and $Q_x(\tau_1,\tau_2,\tau_3)(t)$ defined by (2) and (4), respectively. In these conditions, using a cyclo-ergodicity property, the matrix R_x can still be estimated from the sampled data by the SO empirical estimator [26], but the matrix $Q_x(\tau_1,\tau_2,\tau_3)$ has to be estimated by a nonempirical estimator presented in [26], taking into account the SO cyclic frequencies of the data. Note, finally, that this extension can also be applied to nonzero mean cyclostationary sources, such as some nonlinearly digitally modulated sources [41], provided that nonempirical statistics estimators, which are presented in [27] and [28] for SO and FO statistics, respectively, are used. Such SO estimators take into account the first-order cyclic frequencies of the data, whereas such FO estimators take into account both the first and SO cyclic frequencies of the data.

D. Hypotheses

In Sections III–VIII, we further assume the following hypotheses:

- H1) $P \le N^2$.
- H2) A_Q is full rank.
- H3) $c_p \neq 0$ $(1 \le p \le P)$ (i.e., no source is Gaussian).
- H4) $\forall\, 1 \le p, q \le P$, $c_p c_q > 0$ (i.e., sources have FO autocumulant with the same sign).
- H5) $\forall\, 1 \le p, q \le P$, $\exists(\tau_1,\tau_2,\tau_3) \neq (0,0,0)$ such that

$$\frac{c_p(\tau_1,\tau_2,\tau_3)}{|c_p|} \neq \frac{c_q(\tau_1,\tau_2,\tau_3)}{|c_q|}. \quad (8)$$

Note that hypothesis H4 is not restrictive in radiocommunication contexts since most of the digitally modulated sources have negative FO autocumulant. For example, M-PSK constellations [41] have a kurtosis equal to -2 for $M = 2$ and to -1 for $M > 2$. Continuous phase modulation (CPM) [41], among which we find, in particular, that the Continuous Phase Frequency Shift Keying (CPFSK), the Minimum Shift Keying (MSK), and the Gaussian Minimum Shift Keying (GMSK) modulation (GSM standard) have a kurtosis lower than or equal to -1. Moreover, note that (8) requires, in particular, that the sources have a different normalized tri-spectrum, which prevents us, in particular, from considering sources with both the same modulation, the same baud rate, and the same carrier residue.

III. FOBIUM Method

The purpose of the FOBIUM method is to extend the SOBI method [5] to the FO. It first implements an FO prewhitening step aimed at orthonormalizing the so-called *virtual steering vector* [12] of the sources, corresponding to the columns of A_Q. Second, it jointly diagonalizes several well-chosen prewhitened quadricovariance matrices in order to identify the A_Q matrix. Then, in a third step, it identifies the mixing matrix A from the A_Q matrix. The number of sources able to be processed by this method is considered in Section IV.

A. FO Prewhitening Step

The first step of the FOBIUM method is to orthonormalize, in the $Q_x(\tau_1,\tau_2,\tau_3)$ matrices (5), the columns of A_Q, which can be considered to be *virtual steering vectors* of the sources for the considered array of sensors [12]. For this purpose, let us consider the eigendecomposition of the Hermitian matrix Q_x, whose rank is P under the assumptions H1 to H3, which is given by

$$Q_x = E_x \Lambda_x E_x^{\mathrm{H}} \tag{9}$$

where Λ_x is the $(P \times P)$ real-valued diagonal matrix of the P nonzero eigenvalues of Q_x, and E_x is the $(N^2 \times P)$ matrix of the associated orthonormalized eigenvectors.

Proposition 1: Assuming P sources with nonzero kurtosis having the same sign $\varepsilon(\varepsilon = \pm 1)$ (i.e., $H3 + H4$), it is straigthforward to show that the diagonal elements of Λ_x are not zero and also have the same sign corresponding to ε.

We deduce from Proposition 1 that $\varepsilon\Lambda_x$, which contains the nonzero eigenvalues of εQ_x, has square root decompositions such that $\varepsilon\Lambda_x = (\varepsilon\Lambda_x)^{1/2}(\varepsilon\Lambda_x)^{\mathrm{H}/2}$, where $(\varepsilon\Lambda_x)^{1/2}$ is a square root of $\varepsilon\Lambda_x$, and $(\varepsilon\Lambda_x)^{\mathrm{H}/2} \triangleq [(\varepsilon\Lambda_x)^{1/2}]^{\mathrm{H}}$. Thus, the existence of this square root decomposition requires assumption H4. Considering the $(P \times N^2)$ prewhitening matrix T defined by

$$T \triangleq (\varepsilon\Lambda_x)^{-1/2} E_x^{\mathrm{H}} \tag{10}$$

where $(\varepsilon\Lambda_x)^{-1/2}$ is the inverse of $(\varepsilon\Lambda_x)^{1/2}$, we obtain, from (7) and (9)

$$T(\varepsilon Q_x)T^{\mathrm{H}} = T A_Q (\varepsilon C_m) A_Q^{\mathrm{H}} T^{\mathrm{H}} = \mathbf{I}_p \tag{11}$$

where $\mathbf{I}_p$ is the $(P \times P)$ identity matrix and where $\varepsilon C_m = \mathrm{Diag}[|c_1|, \ldots, |c_p|]$. Expression (11) shows that the $(P \times P)$ matrix $T A_Q (\varepsilon C_m)^{1/2}$ is a unitary matrix $U (U U^{\mathrm{H}} = \mathbf{I}_p)$, and we obtain

$$T A_Q = U (\varepsilon C_m)^{-1/2} \tag{12}$$

which means that the columns of A_Q have been orthonormalized to within a diagonal matrix.

B. FO Blind Identification of A_Q

The second step of the FOBIUM method is to blindly identify the A_Q matrix from some FO statistics of the data. For this purpose, we deduce from (5) and (12) that

$$T Q_x(\tau_1, \tau_2, \tau_3) T^{\mathrm{H}} = U (\varepsilon C_m)^{-1/2} \times C_m(\tau_1, \tau_2, \tau_3)(\varepsilon C_m)^{-1/2} U^{\mathrm{H}} \tag{13}$$

which shows that the unitary matrix U diagonalizes the matrices $T Q_x(\tau_1, \tau_2, \tau_3) T^{\mathrm{H}}$ whatever the set of delays (τ_1, τ_2, τ_3), and the associated eigenvalues correspond to the diagonal terms of the diagonal matrix $(\varepsilon C_m)^{-1/2} C_m(\tau_1, \tau_2, \tau_3)(\varepsilon C_m)^{-1/2}$.

For a given set (τ_1, τ_2, τ_3) and a given order of the sources, U is unique to within a unitary diagonal matrix if and only if the diagonal elements of the matrix $(\varepsilon C_m)^{-1/2} C_m(\tau_1, \tau_2, \tau_3)(\varepsilon C_m)^{-1/2}$ are all different. If it is not the case, following the results of [5], we have to consider several sets $(\tau_1^k, \tau_2^k, \tau_3^k)$, $1 \le k \le K$, such that for each couple of sources (p, q), there exists at least a set $(\tau_1^k, \tau_2^k, \tau_3^k)$, such that (8) is verified for this set, which corresponds to hypothesis H5. Under this assumption, the unitary matrix U becomes, to within a permutation and a unitary diagonal matrix, the only one that jointly diagonalizes the K matrices $T Q_x(\tau_1^k, \tau_2^k, \tau_3^k) T^{\mathrm{H}}$. In other words, the unitary matrix U_{sol}, which is solution to the previous problem of joint diagonalization, can be written as

$$U_{\mathrm{sol}} = U \Lambda \Pi \tag{14}$$

where Λ and Π are unitary diagonal and permutation matrices, respectively.

Noting $T^{\#} \triangleq E_x(\varepsilon\Lambda_x)^{1/2}$, the pseudo-inverse of T, such that $T T^{\#} = \mathbf{I}_p$, we deduce from (14) that

$$T^{\#} U_{\mathrm{sol}} \triangleq E_x(\varepsilon\Lambda_x)^{1/2} U_{\mathrm{sol}} = E_x(\varepsilon\Lambda_x)^{1/2} U \Lambda \Pi \tag{15}$$

and using (10) and (12) into (15), we obtain

$$T^{\#} U_{\mathrm{sol}} = E_x(\varepsilon\Lambda_x)^{1/2} U \Lambda \Pi = E_x E_x^{\mathrm{H}} A_Q (\varepsilon C_m)^{1/2} \Lambda \Pi. \tag{16}$$

From (7) and (9), we deduce that $\mathrm{Span}(A_Q) = \mathrm{Span}(E_x)$, which implies that the orthogonal projection of A_Q on the space spanned by the columns of E_x, $E_x E_x^{\mathrm{H}} A_Q$ corresponds to A_Q. Using this result in (16), we finally obtain

$$T^{\#} U_{\mathrm{sol}} = A_Q (\varepsilon C_m)^{1/2} \Lambda \Pi \tag{17}$$

which shows that the matrix A_Q can be identified to within a diagonal and a permutation matrix from the matrix $T^{\#} U_{\mathrm{sol}}$.

C. Blind Identification of A

The third step of the FOBIUM method is to identify the mixing matrix A from A_Q. For this purpose, we note from (17) and the definition of A_Q that each column $\boldsymbol{b}_p$ $(1 \le p \le P)$ of $T^{\#} U_{\mathrm{sol}}$ corresponds to a vector $\mu_q |c_q|^{1/2} (\boldsymbol{a}_q \otimes \boldsymbol{a}_q^*)$, $1 \le q \le P$, where μ_q, such that $|\mu_q| = 1$, is an element of the diagonal matrix Λ. Thus, mapping the components of each column $\boldsymbol{b}_p$ of $T^{\#} U_{\mathrm{sol}}$ into an $(N \times N)$ matrix B_p such that $B_p[i, j] = \boldsymbol{b}_p((i-1)N + j)$, $1 \le i, j \le N$ consists of building the matrices $\mu_q |c_q|^{1/2} \boldsymbol{a}_q \boldsymbol{a}_q^{\mathrm{H}}$, $1 \le q \le P$. We then deduce that the steering vector $\boldsymbol{a}_q$ of the source q corresponds, to within a scalar, to the eigenvector of B_p associated with the eigenvalue having the strongest modulus. Thus, the eigendecomposition of all the B_p matrices $1 \le p \le P$ allows the identification of A to within a diagonal and a permutation matrix.

D. Implementation of the FOBIUM Method

The different steps of the FOBIUM method are summarized hereafter when L snapshots of the observations $\boldsymbol{x}(l)$ $(1 \le l \le L)$ are available.

Step 1 Estimation $\hat{Q}_x$ of the Q_x matrix from the L snapshots $\boldsymbol{x}(l)$ using a suitable estimator of the FO cumulants [26], [27];

Step 2 Eigen Value Decomposition (EVD) of the matrix $\hat{Q}_x$.

- From this EVD, estimation $\hat{P}$ of the number of sources P by a classical source number detection test;
- Evaluation of the sign ε of the eigenvalues;
- Restriction of this EVD to the $\hat{P}$ principal components: $\hat{Q}_x \approx \hat{E}_x \hat{\Lambda}_x \hat{E}_x^{\mathrm{H}}$, where $\hat{\Lambda}_x$ is the diagonal matrix of the $\hat{P}$ eigenvalues with the strongest modulus, and $\hat{E}_x$ is the matrix of the associated eigenvectors.

Step 3 Estimation $\hat{T}$ of the prewhitening matrix T by $\hat{T} = (\varepsilon_x \hat{\Lambda}_x)^{-1/2} \hat{E}_x^{\mathrm{H}}$;

Step 4 Selection of K appropriate set of delays $(\tau_1^k, \tau_2^k, \tau_3^k) \neq (0,0,0)$, $1 \leq k \leq K$. For example, one may choose these sets such that $\tau_1^k \neq 0$ and $\tau_2^k = \tau_3^k = 0$ or such that $\tau_1^k = \tau_2^k = \tau_3^k \neq 0$, where τ_1^k may be lower than or equal to $1/\hat{B}_x$, where $\hat{B}_x$ is an estimate of the observation bandwidth B_x;

Step 5 Estimation $\hat{Q}_x(\tau_1^k, \tau_2^k, \tau_3^k)$ of the K matrices $Q_x(\tau_1^k, \tau_2^k, \tau_3^k)$ for the K delays sets using a suitable estimator;

Step 6

- Computation of the matrices $\hat{T}_x \hat{Q}_x (\tau_1^k, \tau_2^k, \tau_3^k) \hat{T}^{\mathrm{H}}$, $1 \leq k \leq K$.
- Estimation $\hat{U}_{\mathrm{sol}}$ of the unitary matrix U_{sol} from the joint diagonalization of the K matrices $\hat{T}\hat{Q}_x(\tau_1^k, \tau_2^k, \tau_3^k)\hat{T}^{\mathrm{H}}$ (the joint diagonalization process is decribed in [5] and [9]);

Step 7 Computation of $\hat{T}^{\#}\hat{U}_{\mathrm{sol}} = \hat{E}_x(\varepsilon\hat{\Lambda}_x)^{1/2}\hat{U}_{\mathrm{sol}}$;

Step 8

- Mapping each column $\hat{\boldsymbol{b}}_p$ $(1 \leq p \leq \hat{P})$ of $\hat{T}^{\#}\hat{U}_{\mathrm{sol}}$ into an $(N \times N)$ matrix $\hat{B}_p$.
- EVD of the $\hat{P}$ matrices $\hat{B}_p$ $(1 \leq p \leq \hat{P})$
- An estimate $\hat{A}$ of the mixing matrix A to within a diagonal and a permutation matrix is obtained by considering that each of the $\hat{P}$ columns of $\hat{A}$ corresponds to the eigenvector of a matrix $\hat{B}_p$ $(1 \leq p \leq \hat{P})$ associated with the eigenvalue having the strongest modulus.

IV. Identifiability Conditions

Following the developments of the previous section, we deduce that the FOBIUM method is able to identify the steering vectors of P sources from an array of N sensors, provided hypotheses H1 to H5 are verified. In other words, the FOBIUM method is able to identify $P(P \leq N^2)$ non-Gaussian sources having different trispectrum and kurtosis with the same sign, provided that the A_Q matrix has full rank P, i.e., that the *virtual steering vectors* $\boldsymbol{a}_q \otimes \boldsymbol{a}_q^*$ $(1 \leq q \leq P)$ for the considered array of N sensors remain linearly independent. However, it has been shown in [24] and [12] that the vector $\boldsymbol{a}_q \otimes \boldsymbol{a}_q^*$ can also be considered as a *true steering vector* but for an FO *virtual array* of $N_e(N_e \leq N^2)$ different sensors, where N_e is directly related to both the pattern of the true sensors and the geometry of the true array of N sensors. This means, in particular, that $N^2 - N_e$ components of each vector $\boldsymbol{a}_q \otimes \boldsymbol{a}_q^*$ are redundant components that bring no information. As a consequence, $N^2 - N_e$ rows of the A_Q matrix bring no information and are linear combinations of the others, which means that the rank of A_Q cannot be greater than N_e. In these conditions, the A_Q matrix may have a rank equal to P only if $P \leq N_e$. Conversely, for an FO virtual array without any ambiguities up to order N_e, P sources coming from P different directions generate an A_Q matrix with a full rank P as long as $P \leq N_e$. Thus, the FOBIUM method is able to process up to N_e sources, where N_e is the number of different sensors of the FO virtual array associated with the considered array of N sensors. For example, for a uniform linear array (ULA) of N identical sensors, $N_e = 2N - 1$, whereas for most of other arrays with space diversity only, $N_e = N^2 - N + 1$ [12]. Finally, for an array with N sensors having all a different angular and polarization pattern, $N_e = N^2$ [12].

V. New Performance Criterion

Most of the existing performance criterions used to evaluate the quality of a blind identification method [14], [15], [45] are *global criterions*, which evaluate a distance between the true mixing matrix A and its blind estimate $\hat{A}$. Although useful, a global performance criterion necessarily contains implicitly a part of arbitrary considerations in the manner of combining the distances between the vectors $\boldsymbol{a}_q$ and $\hat{\boldsymbol{a}}_q$, for $1 \leq q \leq P$, to generate a unique scalar criterion. Moreover, it is possible to find that an estimate $\hat{A}_1$ of A is better than an estimate $\hat{A}_2$, with respect to the global criterion, while some columns of $\hat{A}_2$ estimate the associated true steering vectors in a better way than those of $\hat{A}_1$, which may generate some confusion in the interpretations.

To overcome these drawbacks, we propose in this section a new performance criterion for the evaluation of a blind identification method. This new criterion is no longer global and allows both the quantitative evaluation of the identification quality of each source by a given method and the quantitative comparison of two methods for the blind identification of a given source. It corresponds, for the blind identification problem, to a performance criterion similar, with respect to the spirit, to the one proposed in [10] for the extraction problem. It is defined by the following P-uplet

$$D(A, \hat{A}) \triangleq (\alpha_1, \alpha_2, \ldots, \alpha_P) \tag{18}$$

where α_p, $1 \leq p \leq P$, such that $0 \leq \alpha_p \leq 1$, is defined by

$$\alpha_p \triangleq \underset{1 \leq i \leq \hat{P}}{\mathrm{Min}} [\mathrm{d}(\boldsymbol{a}_p, \hat{\boldsymbol{a}}_i)] \tag{19}$$

where $\mathrm{d}(\boldsymbol{u}, \boldsymbol{v})$ is the pseudo-distance between the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, which is defined by

$$\mathrm{d}(\boldsymbol{u}, \boldsymbol{v}) \triangleq 1 - \frac{|\boldsymbol{u}^{\mathrm{H}}\boldsymbol{v}|^2}{(\boldsymbol{v}^{\mathrm{H}}\boldsymbol{v})(\boldsymbol{u}^{\mathrm{H}}\boldsymbol{u})}. \tag{20}$$

Thus, the identification quality of the source p is evaluated by the parameter α_p, which decreases toward zero as the identification quality of the source p increases. In particular, the source p is perfectly identified when $\alpha_p = 0$. Although arbitrary, we consider in the following that a source p is blindly identified with a very high quality if $\alpha_p \leq 0.01$, with a high quality if $\alpha_p \leq 0.03$, with a good quality if $\alpha_p \leq 0.05$, and with a poor quality otherwise. Besides, we will say that a method M1 is better than a method M2 for the identification of the source p if α_p(M1) $< \alpha_p$(M2), where α_p(Mi) corresponds to the parameter α_p generated by the method Mi. Moreover, we will say that a method M1 is better than a method M2 if it is better for each source, i.e., if α_p(M1) $< \alpha_p$(M2) for $1 \leq p \leq P$. Finally, we verify that, whatever the $(\hat{P} \times \hat{P})$ diagonal matrix $\hat{\Lambda}$ and permutation matrix $\hat{\Pi}$, we obtain

$$D(A, \hat{A}) = D(A, \hat{A}\hat{\Lambda}\hat{\Pi}) \tag{21}$$

which means that two mixing matrix estimates that are equal to within a diagonal and a permutation matrix generate the same performance for all the sources, which is satisfactory.

VI. Application of the FOBIUM Method: Direction Finding with the MAXCOR Method

Before presenting some computer simulations in Section VII, we propose, in this section, an application of the FOBIUM method that is usable when the array manifold is known or estimated by calibration. This application consists to find the direction of arrival (DOA) of the detected sources directly from the blindly identified mixing matrix, allowing better DOA estimations than the existing ones in many contexts. Besides, for a given array of sensors, this application allows the interpretation of the α_p coefficient, introduced in the previous section to evaluate the identification quality of the source p, in terms of angular precision.

A. Existing Direction-Finding Methods

When the array manifold is known or estimated by calibration, each component a_{pn} $(1 \leq n \leq N)$ of the steering vector $\boldsymbol{a}_p$ may be written as a function $a_n(\theta_p, \varphi_p)$ of the DOA (θ_p, φ_p) of the source p, where θ_p and φ_p are the azimuth and the elevation angles of source p, respectively (see Fig. 1). The function $a_n(\theta, \varphi)$ is the nth component of the steering vector $\boldsymbol{a}(\theta, \varphi)$ for the direction (θ, φ). In particular, in the absence of modeling errors such as mutual coupling, the component $a_n(\theta_p, \varphi_p)$ can be written, under the far field assumption and in the general case of an array with space, angular, and polarization diversity, as [12] (22), shown at the bottom of the page, where λ is the wavelength, (x_n, y_n, z_n) are the coordinates of sensor n of the array, and $f_n(\theta_p, \varphi_p)$ is a complex number corresponding to the response of sensor n to a unit electric field coming from the direction (θ_p, φ_p). Using the knowledge of the array manifold $\boldsymbol{a}(\theta, \varphi)$, it is possible to estimate the DOA of the sources from some statistics of the data such as the SO or the FO statistics given by (2) and (7), respectively.

Among the existing SO direction-finding methods, the so-called High-Resolution (HR) methods, which have been developed from the beginning of the 1980s, are the most powerful in multisource contexts since they are characterized by an asymptotic resolution that becomes infinite, whatever the source signal-to-noise ratio (SNR). Among these HR methods, the subspace-based methods such as the MUSIC method [42] are the most popular. Recall that after a source number estimation $\hat{P}$, the MUSIC method consists of finding the $\hat{P}$ couples (θ_i, φ_i), minimizing the pseudo-spectrum defined by

$$\hat{C}_{\mathrm{Music2}}(\theta, \varphi) \triangleq \frac{\boldsymbol{a}(\theta, \varphi)^{\mathrm{H}} \hat{\Pi}_{\mathrm{MUSIC2}} \boldsymbol{a}(\theta, \varphi)}{\boldsymbol{a}(\theta, \varphi)^{\mathrm{H}} \boldsymbol{a}(\theta, \varphi)} \tag{23}$$

where $\boldsymbol{a}(\theta, \varphi)$ is the steering vector for the direction (θ, φ) and $\hat{\Pi}_{\mathrm{MUSIC2}} \triangleq \left(\mathrm{I}_N - \hat{L}_x \hat{L}_x^{\mathrm{H}}\right)$, where I_N is the $(N \times N)$ identity matrix, and $\hat{L}_x$ is the $(N \times \hat{P})$ matrix of the $\hat{P}$ orthonormalized eigenvectors of the estimated data correlation matrix $\hat{R}_x$ associated with the $\hat{P}$ strongest eigenvalues.

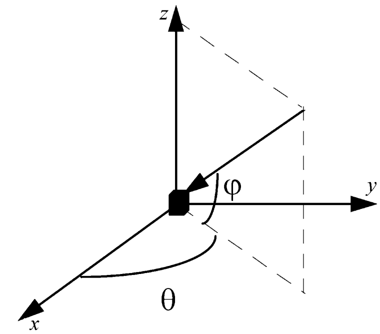

Fig. 1. Incoming signal in three dimensions.

One of the main drawbacks of the SO subspace-based methods such as the MUSIC method is that they are not able to process more than $N-1$ sources from an array of N sensors. Mainly to overcome this limitation, but also to still increase the resolution with respect to that of SO methods for a finite duration observation, higher order HR direction-finding methods [7], [13], [40] have been developed during these two last decades, among which the extension of the MUSIC method to the FO [40], which is called MUSIC4, is the most popular. Recall that after a source number estimation $\hat{P}$, the MUSIC4 method consists of finding the $\hat{P}$ couples (θ_i, φ_i) minimizing the pseudo-spectrum defined by

$$\hat{C}_{\mathrm{Music4}}(\theta, \varphi) \triangleq \frac{[\boldsymbol{a}(\theta, \varphi)^{\otimes 2}]^{\mathrm{H}} \hat{\Pi}_{\mathrm{MUSIC4}} [\boldsymbol{a}(\theta, \varphi)^{\otimes 2}]}{[\boldsymbol{a}(\theta, \varphi)^{\otimes 2}]^{\mathrm{H}} [\boldsymbol{a}(\theta, \varphi)^{\otimes 2}]} \tag{24}$$

where $\boldsymbol{a}(\theta, \varphi)^{\otimes 2} = \boldsymbol{a}(\theta, \varphi) \otimes \boldsymbol{a}(\theta, \varphi)^*$, and $\hat{\Pi}_{\mathrm{MUSIC4}} \triangleq \left(\mathrm{I}_{N^2} - \hat{E}_x \hat{E}_x^{\mathrm{H}}\right)$, where I_{N^2} is the $(N^2 \times N^2)$ identity matrix, and $\hat{E}_x$ is the $(N^2 \times \hat{P})$ matrix of the $\hat{P}$ orthonormalized eigen vectors of the estimated data quadricovariance matrix $\hat{Q}_x$ associated with the $\hat{P}$ strongest eigenvalues. Moreover, it has been shown in [12] that the MUSIC4 method is able to process up to $N_e - 1$ sources where N_e corresponds to the number of different sensors of the FO virtual array associated with the considered array of sensors.

B. Application of the FOBIUM Method: The MAXCOR Method

Despite the interests of both the SO and FO HR subspace-based direction-finding methods described in Section VI-A, the latter keep a source of performance limitation in multisource contexts for a finite duration of observation since they may be qualified as *multidimensionnal* methods insofar as they implement a procedure of searching multiple minima of a pseudo-spectrum function. This multidimensionality character

$$a_n(\theta_p, \varphi_p) = f_n(\theta_p, \varphi_p) \exp\left\{\frac{\mathrm{j}2\pi[x_n \cos(\theta_p)\cos(\varphi_p) + y_n \sin(\theta_p)\cos(\varphi_p) + z_n \sin(\varphi_p)]}{\lambda}\right\} \tag{22}$$

of these methods generates interaction between the sources in the pseudo-spectrum, which is a source of performance limitation, for a finite duration observation, in the presence of modeling errors or for poorly angularly separated sources, for example.

To overcome the previous limitation, it is necessary to transform the multidimensional search of minima into $\hat{P}$ monodimensional searches of optima, which can be easily done from the source steering vector estimates and is precisely the philosophy of the new proposed method. More precisely, from the estimated mixture matrix $\hat{A}$, the new proposed method called MAXCOR (search for a MAXimum of spatial CORrelation), consists of solving, for each estimated source $p(1 \leq p \leq \hat{P})$, a mono-dimensional problem aiming at finding the DOA (θ, φ) that maximizes the square modulus of a certain spatial correlation coefficient, which is defined by

$$\hat{C}_{Cor,p}(\theta, \varphi) \triangleq \frac{\boldsymbol{a}(\theta, \varphi)^{\mathrm{H}} \hat{\Pi}_{\mathrm{Maxcor},p} \boldsymbol{a}(\theta, \varphi)}{\boldsymbol{a}(\theta, \varphi)^{\mathrm{H}} \boldsymbol{a}(\theta, \varphi)} \quad (25)$$

where

$$\hat{\Pi}_{\mathrm{Maxcor},p} = \frac{\hat{\boldsymbol{a}}_p \hat{\boldsymbol{a}}_p^{\mathrm{H}}}{\hat{\boldsymbol{a}}_p^{\mathrm{H}} \hat{\boldsymbol{a}}_p} \quad (26)$$

which is equivalent to minimizing the pseudo-spectrum defined by

$$\hat{C}_{\mathrm{Maxcor},p}(\theta, \varphi) \triangleq 1 - \frac{\boldsymbol{a}(\theta, \varphi)^{\mathrm{H}} \hat{\Pi}_{\mathrm{Maxcor},p} \boldsymbol{a}(\theta, \varphi)}{\boldsymbol{a}(\theta, \varphi)^{\mathrm{H}} \boldsymbol{a}(\theta, \varphi)}. \quad (27)$$

It is obvious that the performance of the MAXCOR method is directly related to those of the BI method, which generates the estimated matrix $\hat{A}$. Performance of the MAXCOR method from a $\hat{A}$ generated by the FOBIUM method are presented in Section VII and compared with those of MUSIC2 and MUSIC4, both with and without modeling errors. Note that following the FOBIUM method, the MAXCOR method is able to process up to N_e statistically independent non-Gaussian sources, whereas MUSIC4 can only process $N_e - 1$ sources [12].

VII. Computer Simulations

Performances of the FOBIUM method are illustrated in Section VII-A, whereas those of the MAXCOR method are presented in Section VII-B. Note that the sources considered for the simulations are zero-mean cyclostationary sources corresponding to quadrature phase shift keying (QPSK) sources, which is not a problem for the FOBIUM method, according to Section II-C, provided the sources do not share the same trispectrum. Nevertheless, for complexity reasons, the empirical estimator of the FO data statistics is still used, despite the cyclostationarity of the sources. This is not a problem since it is shown in [26] that for SO circular sources such as QPSK sources, although biased, the empirical estimator behaves approximately like an unbiased estimator. Finally, the elevation angle of the sources is assumed to be zero.

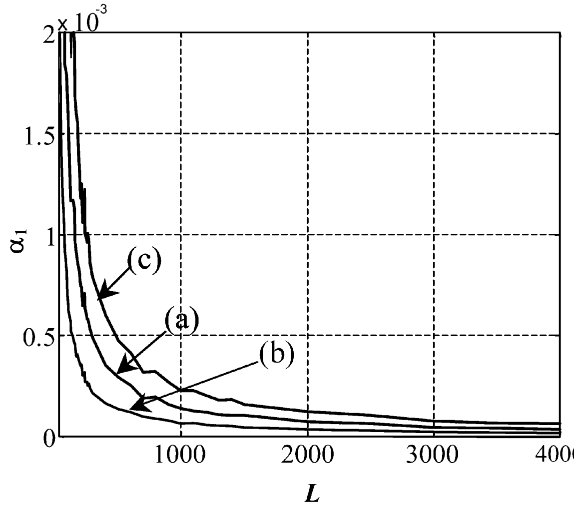

Fig. 2. α_1 as a function of L. (a) JADE. (b) SOBI. (c) FOBIUM. $P = 2$, $N = 3$, ULA, $\theta_1 = 90°$, $\theta_2 = 131.76°$, SNR = 10 dB.

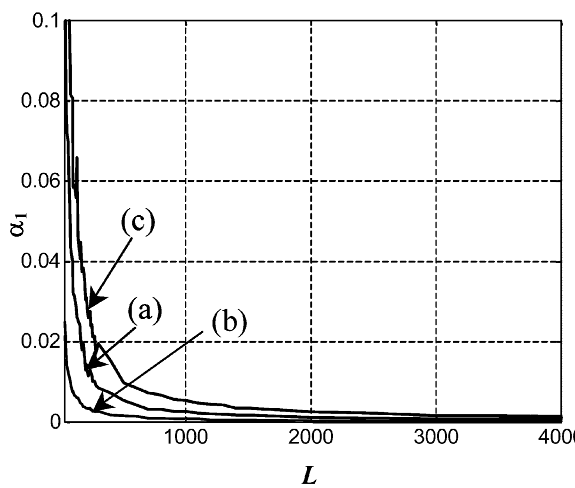

Fig. 3. α_1 as a function of L, (a) JADE. (b) SOBI. (c) FOBIUM. $P = 2$, $N = 3$, ULA, $\theta_1 = 90°$, $\theta_2 = 131.76°$, SNR = 0 dB.

A. FOBIUM Method Performance

The performances of the FOBIUM method are presented in this section both for the overdetermined and underdetermined mixtures of sources.

1) Overdetermined Mixtures of Sources: To illustrate the performance of the FOBIUM method for overdetermined mixtures of sources, we assume that two statistically independent QPSK sources with a raise cosine pulse shape filter are received by a ULA of $N = 3$ omnidirectional sensors spaced half a wavelength apart. The two QPSK sources have the same symbol duration $T = 4T_e$ (where T_e is the sample period), the same roll-off $\mu = 0.3$, the same input SNR and have a carrier residue such that $\Delta f_1 \times T_e = 0$, $\Delta f_2 \times T_e = 0.5$, and a DOA equal to θ_1 and θ_2, respectively. The performance for the source q, α_q is computed and averaged over 300 realizations.

Under these assumptions, Figs. 2–5 show, for several configurations of SNR and spatial correlation between the sources, the variations of α_1 (α_2 behaves in a same way) at the output of both Joint Approximated Diagonalization of Eigenmatrices (JADE) [9], SOBI [5], and FOBIUM methods, as a function of

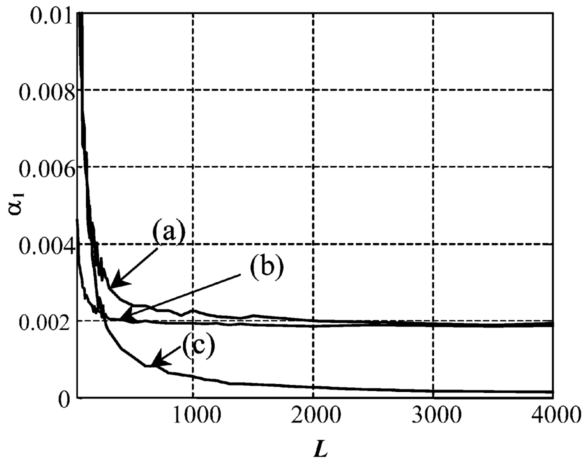

Fig. 4. α_1 as a function of L. (a) JADE. (b) SOBI. (c) FOBIUM. $P = 2$, $N = 3$, ULA, $\theta_1 = 90°$, $\theta_2 = 82.7°$, SNR $= 10$ dB.

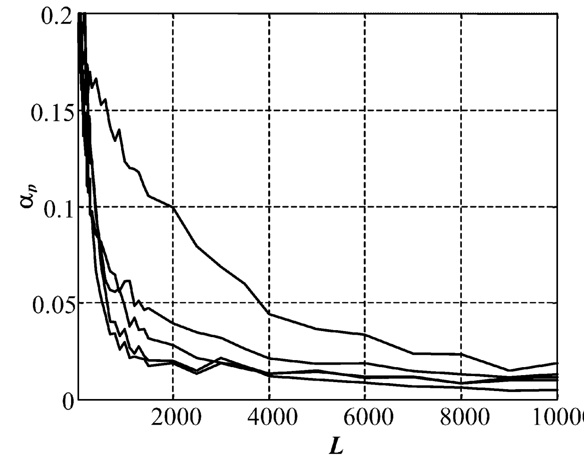

Fig. 6. α_p $(1 \leq p \leq 5)$ as a function of L at the output of FOBIUM, $P = 5$, $N = 3$, ULA, $\theta_p = 90°$, 120.22 °, 150.65 °, $-52.05°$, and $-76.32°$.

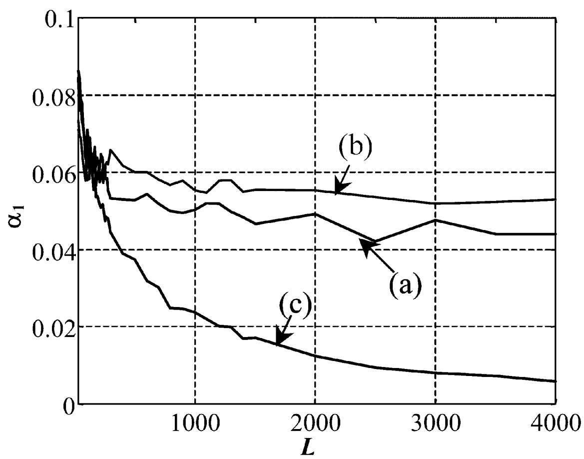

Fig. 5. α_1 as a function of L. (a) JADE. (b) SOBI. (c) FOBIUM. $P = 2$, $N = 3$, ULA, $\theta_1 = 90°$, $\theta_2 = 82.7°$, SNR $= 0$ dB.

the number of snapshots L. For Figs. 2 and 3, the sources are well angularly separated ($\theta_1 = 90°$, $\theta_2 = 131.76°$) and such that their SNR is equal to 10 and 0 dB, respectively. For Figs. 4 and 5, the sources are poorly angularly separated ($\theta_1 = 90°$, $\theta_2 = 82.7°$) and such that their SNR is equal to 10 and 0 dB, respectively. For the SOBI method, $K = 8$ delays, τ^k $(1 \leq k \leq 8)$ are considered such that $\tau^k = kT_e$, whereas for the FOBIUM method, $K = 8$ delays set, $\left(\tau_1^k, \tau_2^k, \tau_3^k\right)$ are taken into account such that $\tau_1^k = \tau^k$ and $\tau_2^k = \tau_3^k = 0$.

Figs. 2 and 3 show that for well angularly separated non-Gaussian sources having different spectrum and trispectrum, the JADE, SOBI, and FOBIUM methods succeed in blindly identifying the sources steering vectors with a very high quality ($\alpha_i \leq 0.01, 1 \leq i \leq 2$) from a relatively weak number of snapshots and even for weak sources ($L \approx 100$ for SNR $= 10$ dB and $L \approx 600$ for SNR $= 0$ dB). Nevertheless, in such situations, we note the best behavior of the SOBI method with respect to FO methods and the best behavior of JADE with respect to FOBIUM, whatever the source SNR and the number of snapshots, due to a higher variance of the FO statistics estimators.

Fig. 4 confirms the very good behavior of the three methods from a very weak number of snapshots $(L \approx 100)$ even when the sources are poorly angularly separated, provided the SNR is not too low (SNR $= 10$ dB for Fig. 4). However, Figs. 4 and 5 show that for poorly angularly separated sources, there exists a number of snapshots L_0 increasing with the source SNR ($L_0 \approx 265$ for SNR $= 10$ dB and $L_0 \approx 150$ for SNR $= 0$ dB), over which the FOBIUM method becomes much more efficient than the JADE and SOBI methods. In such situations, the resolution gain obtained with FOBIUM is higher than the loss due to a higher variance in the statistics estimates. In particular, for sources with an SNR equal to 0 dB, Fig. 5 shows a very high source identification quality ($\alpha_i \leq 0.01$, $1 \leq i \leq 2$) with the FOBIUM method for $L > 2600$, whereas the JADE and SOBI methods generate coefficients α_i only around 0.05 for $L = 4000$.

2) Underdetermined Mixtures of Sources: To illustrate the performance of the FOBIUM method for underdetermined mixtures of sources, we assume first that five statistically independent QPSK sources with a raised-cosine pulse shape filter are received by an array of $N = 3$ omnidirectional sensors. The five QPSK sources have the same symbol duration $T = 4T_e$, the same roll-off $\mu = 0.3$, the same input SNR of 20 dB, a carrier residue such that $\Delta f_1 \times T_e = 0$, $\Delta f_2 \times T_e = 1/2$, $\Delta f_3 \times T_e = 1/3$, $\Delta f_4 \times T_e = 1/5$, and $\Delta f_5 \times T_e = 1/7$, and a DOA given by $\theta_1 = 90°$, $\theta_2 = 120.22°$, $\theta_3 = 150.65°$, $\theta_4 = -52.05°$, and $\theta_5 = -76.32°$, respectively. The performance for the source q, α_q is still computed and averaged over 300 realizations. For the FOBIUM method, $K = 8$ delays set, $\left(\tau_1^k, \tau_2^k, \tau_3^k\right)$ are taken into account such that $\tau_1^k = kT_e$, and $\tau_2^k = \tau_3^k = 0$ $(1 \leq k \leq 8)$. Under these assumptions, Figs. 6 and 7 show the variations of all the coefficients α_q $(1 \leq q \leq 5)$ at the output of the FOBIUM method, as a function of the number of snapshots L. For Fig. 6, a ULA of three sensors spaced half a wavelength apart is considered, whereas for Fig. 7, the array of sensors corresponds to a uniformly circular array (UCA) such that $r/\lambda = 0.5$ (r is the radius, and λ is the wavelength). Note that the two considered arrays of sensors have the same aperture on the x-axis if the sensors of the ULA lie on this axis.

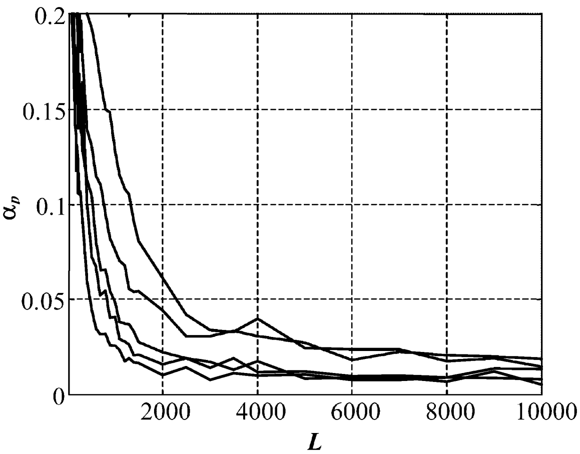

Fig. 7. α_p $(1 \leq p \leq 5)$ as a function of L at the output of FOBIUM, $P = 5$, $N = 3$, UCA, $\theta_p = 90°$, 120.22 °, 150.65 °, $-52.05°$, and $-76.32°$.

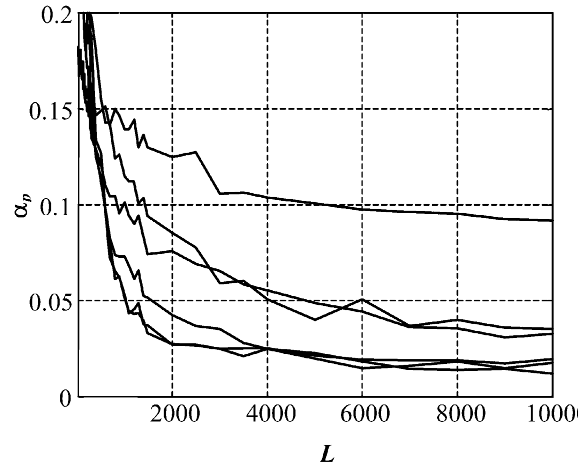

Fig. 8. α_p $(1 \leq p \leq 6)$ as a function of L at the output of FOBIUM, $P = 6$, $N = 3$, ULA, $\theta_p = 90°$, 120.22 °, 150.65 °, $-52.05°$, $-76.32°$, and 66.24 °.

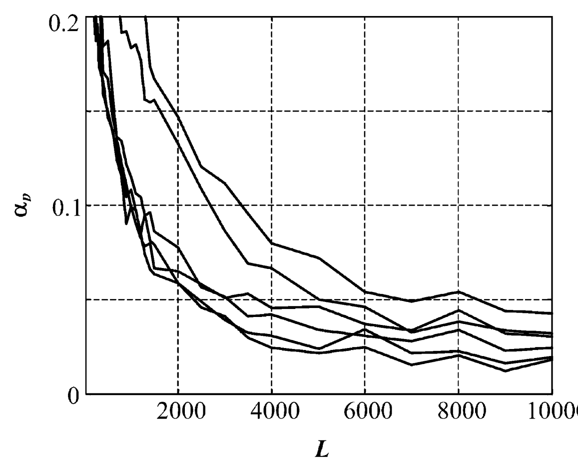

Fig. 9. α_p $(1 \leq p \leq 6)$ as a function of L at the output of FOBIUM, $P = 6$, $N = 3$, UCA, $\theta_p = 90°$, 120.22 °, 15.65 °, $-52.05°$, $-76.32°$, and 66.24 °.

Figs. 6 and 7 show that for both the ULA and UCA arrays, as long as $P \leq N_e$ ($N_e = 5$ for the ULA and $N_e = 7$ for the UCA), the FOBIUM method succeeds in blindly identifying the source steering vectors with a high quality ($\alpha_i \leq 0.03$, $1 \leq i \leq 5$) in underdetermined contexts as soon as there are enough snapshots ($L > 6460$ for the ULA and $L > 4930$ for the UCA). Nevertheless, the comparison of Figs. 2 and 6 shows that for a given array of sensors, the number of snapshots L required to obtain a high BI quality of all the source steering vectors increases with the number of sources ($L < 100$ for $P = 2$ and $L = 6460$ for $P = 5$ for a ULA of three sensors). On the other hand, the comparison of Figs. 6 and 7 show that for a given number P and scenario of sources, the required number of snapshots L ensuring a high quality of source steering vector identification increases as the quantity $N_e - P$ decreases. Note that the quantity $N_e - P$ (0 for the ULA and two for the UCA) corresponds to the number of degrees of freedom in excess for the FO virtual array associated with the considered array of sensors.

We now decide to add one QPSK source with a raised-cosine pulse shape filter to the five previous ones. Source 6 has the symbol duration $T = 3T_e$, the same roll-off $\mu = 0.3$, the same input SNR of 20 dB, a carrier residue such that $\Delta f_6 \times T_e = 0.5$, and a DOA given by $\theta_6 = 66.24°$. For the FOBIUM method, a $K = 8$ delays set, $\left(\tau_1^k, \tau_2^k, \tau_3^k\right)$, are still taken into account such that $\tau_1^k = kT_e$, and $\tau_2^k = \tau_3^k = 0$. Under these new assumptions, Figs. 8 and 9 again show the variations of all the coefficients α_q $(1 \leq q \leq 6)$ at the output of the FOBIUM method, as a function of the number of snapshots L. For Fig. 8, a ULA of three sensors is considered, whereas for Fig. 9, the UCA of three sensors is considered.

The comparison of Figs. 7 and 9 confirms, for a given array of sensors and as long as $P \leq N_e$, the increasing value of L required to obtain a good blind identification of all the source steering vectors ($\alpha_i \leq 0.05$, $1 \leq i \leq 6$) as the number of sources increases ($L > 2400$ for $P = 5$ and $L > 8400$ for $P = 6$ for a UCA of three sensors). However, the comparison of Figs. 6 and 8, for the ULA with three sensors, shows off the limitations of the FOBIUM method and the poor identification quality of some sources ($\exists i$ such that $\alpha_i > 0.09$), even for large values of $L (L = 10\,000)$ as soon as $P > N_e$.

B. MAXCOR Method Performance

The performance of the MAXCOR method, which extracts the DOA of the sources from the source steering vectors blindly identified by the FOBIUM method, are presented in this section both in the absence and in the presence of modeling errors.

1) Performance Criterion: For each of the P considered sources and for each of the three considered direction finding methods, two criterions are used in the following to quantify the quality of the associated DOA estimation. For a given source, the first criterion is a probability of aberrant results generated by a given method for this source, and the second one is an averaged Root Mean Square Error (RMSE), computed from the nonaberrant results, generated by a given method for this source.

More precisely, for a given method, a given number of snapshots L, and a particular realization of the L observation vectors $\boldsymbol{x}(l)$ $(1 \leq l \leq L)$, the estimation $\hat{\theta}_p$ of the DOA of the source $p(1 \leq p \leq P)$ is defined by

$$\hat{\theta}_p \triangleq \underset{\zeta_i}{\text{Arg}}(\underset{i}{\text{Min}}|\zeta_i - \theta_p|) \tag{28}$$

where, for the MUSIC2 and MUSIC4 methods, the quantities $\zeta_i (1 \leq i \leq \hat{P})$ correspond to the $\hat{P}$ minima of the pseudo-spec-

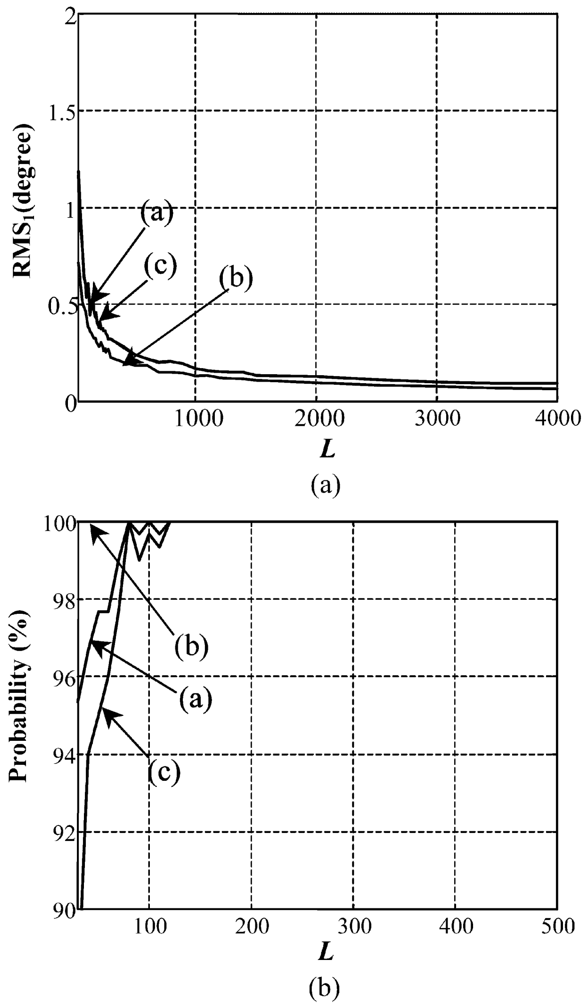

Fig. 10. RMS error of the source 1 and $p(\eta_1 \leq \eta)$ as a function of L. (a) MAXCOR. (b) MUSIC2. (c) MUSIC4. $P = 2$, $N = 3$, ULA, SNR $= 10$ dB, $\theta_1 = 90°$, and $\theta_2 = 131.76°$; no modeling errors.

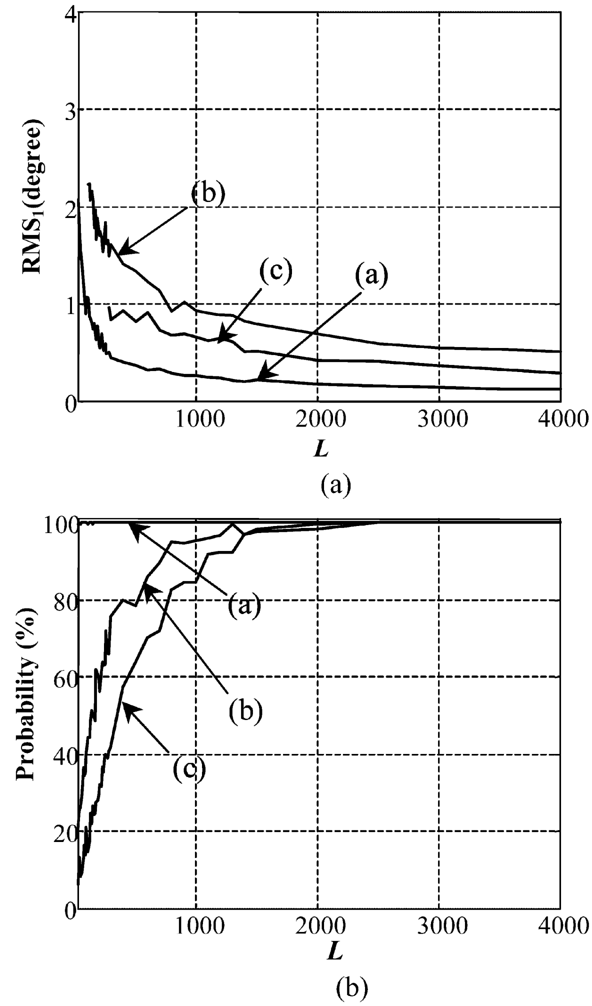

Fig. 11. RMS error of the source 1 and $p(\eta_1 \leq \eta)$ as a function of L. (a) MAXCOR. (b) MUSIC2. (c) MUSIC4. $P = 2$, $N = 3$, ULA, SNR $= 10$ dB, $\theta_1 = 90°$, $\theta_2 = 82, 7\ °$; no modeling errors.

trum $\hat{C}_{\mathrm{Music2}}(\theta)$, and $\hat{C}_{\mathrm{Music4}}(\theta)$, respectively, which are defined by (23) and (24), and where, for the MAXCOR method, ζ_i corresponds to the minimum of $\hat{C}_{\mathrm{Maxcor,i}}(\theta)$, $(1 \leq i \leq \hat{P})$. To each estimate $\hat{\theta}_p$ $(1 \leq p \leq P)$, we associate the corresponding value of the pseudo-spectrum, which is defined by $\eta_p = \hat{C}_{\mathrm{Music2}}(\hat{\theta}_p)$ for MUSIC2, $\eta_p = \hat{C}_{\mathrm{Music4}}(\hat{\theta}_p)$ for MUSIC4, and $\eta_p = \hat{C}_{\mathrm{Maxcor,ip}}(\hat{\theta}_p)$ for MAXCOR, where ip is the integer i that minimizes $|\zeta_i - \theta_p|$. In this context, the estimate $\hat{\theta}_p$ is considered to be aberrant if $\eta_p > \eta$, where η is a threshold to be defined. In the following, $\eta = 0.1$.

Let us now consider M realizations of the L observation vectors $\boldsymbol{x}(l)$ $(1 \leq l \leq L)$. For a given method, the probability of abberant results for a given source p, $p(\eta_p > \eta)$ is defined by the ratio between the number of realizations for which $\hat{\theta}_p$ is aberrant, and the number M of realizations. From the nonaberrant realizations for the source p, we then define the averaged RMS error for the source p (RMSE_p) by the quantity

$$\mathrm{RMSE}_p \triangleq \sqrt{\frac{1}{M_p}\sum_{m=1}^{M_p}|\hat{\theta}_{pm} - \theta_p|^2} \quad (29)$$

where M_p is the number of nonaberrant realizations for the source p, and $\hat{\theta}_{pm}$ is the estimate of θ_p for the nonaberrant realization m.

2) Absence of Modeling Errors: To illustrate the performance of the MAXCOR method in the absence of modeling errors, we consider the scenarios of Figs. 2 and 4, respectively, for which two QPSK sources that are well and poorly angularly separated, respectively, and such that SNR $=$ 10 dB, are received by an ULA of three sensors.

Under the assumptions of Fig. 2 (sources with a large angular separation), Fig. 10 shows the variations, as a function of the number of snapshots L, of the RMS error for the source 1 (RMSE_1) and the associated probability of nonabberant results $p(\eta_1 \leq \eta)$ (we obtain similar results for the source 2), estimated from $M = 300$ realizations at the output of the MAXCOR, MUSIC2, and MUSIC4 methods. Fig. 11 shows the same variations as those of Fig. 10 but under the assumptions of Fig. 4 (sources with a weak angular separation).

Fig. 10(b) shows that the probability of aberrant realizations for source 1 is zero for all the methods as soon as L becomes greater than 120. In this context, Fig. 10(a) shows that for well angularly separated non-Gaussian sources having different spectrum and trispectrum and a SNR equal to 10 dB, the three methods succeed in estimating the DOA of the two sources with a high precision ($\text{RMSE}_i \leq 0.5°$, $1 \leq i \leq 2$) from a relatively weak number of snapshots ($L \approx 90$ for MUSIC2 and $L \approx 180$ for MUSIC4 and MAXCOR). Nevertheless, in such situations, we note the best behavior of the MUSIC2 method with respect to HO methods MUSIC4 and MAXCOR, which give the same results, due to a higher variance of the FO statistics estimators.

Fig. 11(b) shows that the probability of aberrant realizations for the source 1 is equal to 0 for MAXCOR, whatever the value of L, but remains greater than 20% for $L < 480$ for MUSIC2 and MUSIC4. Both in terms of probability of nonaberrant results and estimation precision, Fig. 11(a) and (b) shows, for poorly angularly separated sources, the best behavior of the MAXCOR method, which becomes much more efficient than the MUSIC4 and MUSIC2 methods. Indeed, MAXCOR succeeds in estimating the DOA of the two sources with a high precision ($\text{RMSE}_i \leq 0.5°$, $1 \leq i \leq 2$) from a relatively weak number of snapshots ($L \approx 230$), whereas MUSIC4 and MUSIC2 require $L \approx 1500$ and $L \approx 3200$ snapshots, respectively, to obtain the same precision. In such situations, the resolution gain obtained with MAXCOR and MUSIC4 with respect to MUSIC2 is higher than the loss due to a higher variance in the statistics estimates. Besides, the monodimensionality character of the MAXCOR method with respect to MUSIC4 jointly with the very high resolution power of the FOBIUM method explain the best behavior of MAXCOR with respect to MUSIC4.

3) Presence of Modeling Errors: We now consider the simulations of Section VII-B2 but with modeling errors due for instance to a nonperfect equalization of the reception chains. In the presence of such errors, the steering vector $\boldsymbol{a}_p$ of the source p is not the known function $\boldsymbol{a}(\theta_p, \varphi_p)$ of the DOA (θ_p, φ_p) but becomes an unknown function $\tilde{\boldsymbol{a}}(\theta_p, \varphi_p) = \boldsymbol{a}(\theta_p, \varphi_p) + \boldsymbol{e}(\theta_p, \varphi_p)$ of (θ_p, φ_p), where $\boldsymbol{e}(\theta_p, \varphi_p)$ is a modeling error vector. In such conditions, the previous HR methods lose their infinite asymptotic resolution, and the question is to search for a method that presents some robustness to the modeling errors. To solve this problem, we assume that the vector $\boldsymbol{e}(\theta_p, \varphi_p)$ is a zero-mean, Gaussian, circular vector with independent components such that $\text{E}[\boldsymbol{e}(\theta_p, \varphi_p)\boldsymbol{e}(\theta_p, \varphi_p)^{\text{H}}] = \sigma_e^2 \mathbf{I}_N$. Note that for omnidirectional sensors and small errors, σ_e^2 is the sum of the phase and amplitude error variances per reception chain. For the simulations, σ_e is chosen to be equal to 0.0174, which corresponds, for example, to a phase error with a standard deviation of 1° with no amplitude error.

In this context, under the assumptions of Fig. 10 (sources with a large angular separation) but with modeling errors, Fig. 12 shows the variations, as a function of the number of snapshots L, of the RMS error for the source 1 (RMSE_1) and the associated probability of nonabberant results $p(\eta_1 \leq \eta)$ (we obtain similar results for the source 2) estimated from $M = 300$ realizations at the output of the MAXCOR, MUSIC2, and MUSIC4 methods. Fig. 13 shows the same variations as those of Fig. 12 but under the assumptions of Fig. 11 (sources with a weak angular separation) with modeling errors.

Fig. 12(b) shows that the probability of aberrant realizations for the source 1 is zero for all the methods as soon as L becomes greater than 135. In this context, comparison of Figs. 10

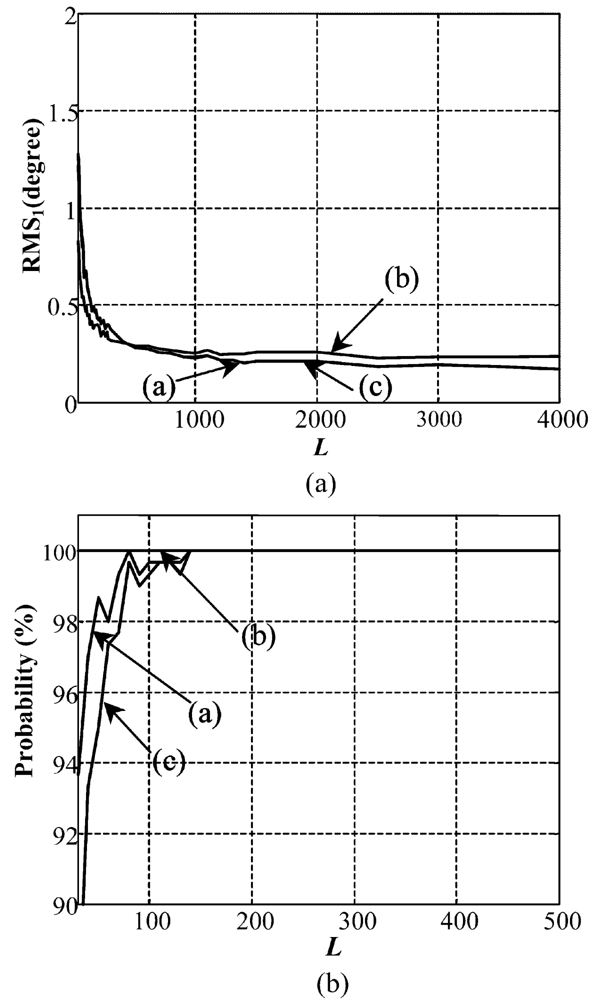

Fig. 12. RMS error of the source 1 and $p(\eta_1 \leq \eta)$ as a function of L. (a) MAXCOR. (b) MUSIC2. (c) MUSIC4. $P = 2$, $N = 3$, ULA, SNR = 10 dB, $\theta_1 = 90°$, and $\theta_2 = 131.76°$; with modeling errors.

and 12 show a degradation of the performance of each method in the presence of modeling errors. However, for well angularly separated sources, MUSIC2 is more affected by the presence of modeling errors than FO methods as soon as the number of snapshots is sufficient. Indeed, while MUSIC2 remains better than FO methods for a relatively weak number of snapshots ($L < 500$), due to a higher variance of HO methods, MUSIC4 and MAXCOR, which are equivalent to each other, become better than MUSIC2 as soon as the number of snapshots is sufficient ($L > 500$). In this latter case, the higher number of sensors of the FO virtual array ($N_e = 5$) with respect to that of the true array ($N = 3$) reduces the effect of modeling errors on the performances of FO methods.

Fig. 13(b) shows that the probability of aberrant realizations for the source 1 is equal to 0 for MAXCOR, whatever the value of L, but remains greater than 20% for $L < 1180$ for MUSIC2 and MUSIC4. Both in terms of probability of nonaberrant results and estimation precision, comparison of Figs. 11 and 13 again show a degradation of the performance of all the methods in the presence of modeling errors. However, for poorly angularly separated sources, whatever the value of the number of snapshots, MUSIC2 is much more affected by the modeling er-

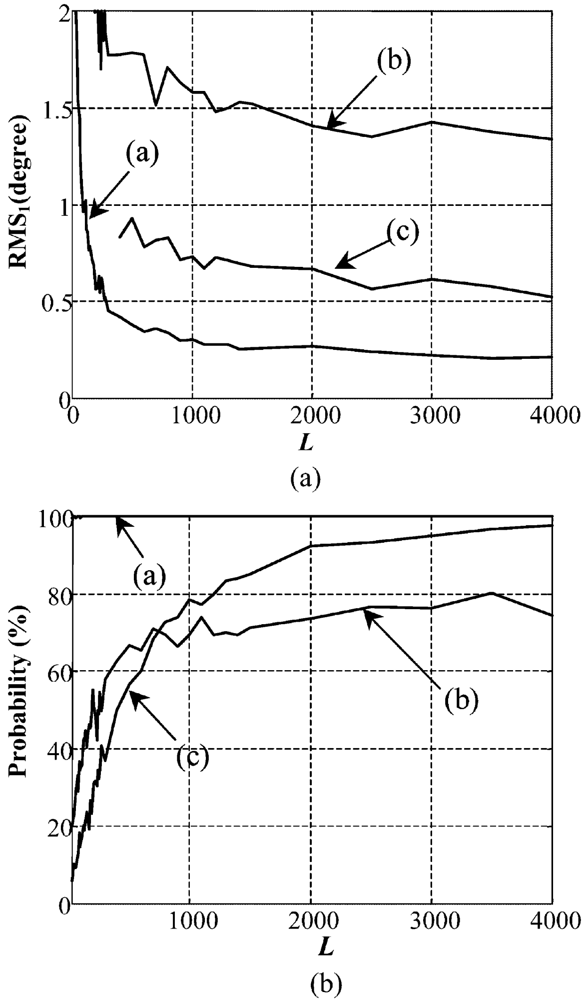

Fig. 13. RMS of the source 1 and $p(\eta_1 \leq \eta)$ as a function of L. (a) MAXCOR. (b) MUSIC2. (c) MUSIC4. $P = 2$, $N = 3$, ULA, SNR $= 10$ dB, $\theta_1 = 90°$, $\theta_2 = 82, 7°$; with modeling errors.

rors than FO methods, as soon as $L > 700$, due to a greater aperture and number of sensors of the FO virtual array with respect to the true array. Note again, in the presence of modeling errors, the best performance of MAXCOR with respect to MUSIC4 for poorly angularly separated sources.

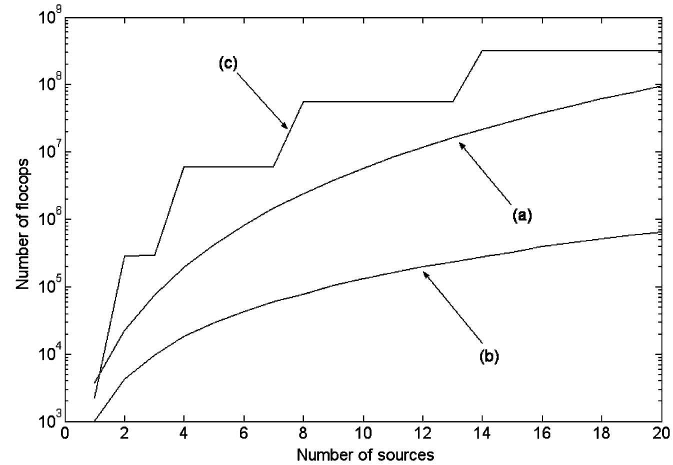

Fig. 14. Minimum numerical complexity as a function of P. (a) JADE. (b) SOBI. (c) FOBIUM.

VIII. Numerical Complexity Computation

This section aims at giving some insight into the relative numerical complexity of the SOBI, JADE, and FOBIUM methods for given values of N, P, L, and the number of sweeps I required by the joint diagonalization process [5], [9]. The numerical complexity of the methods is presented in terms of the number of floating complex operations (Flocops) required to identify the mixture matrix A from L snapshots of the data. Note that a flocop corresponds to the sum of a complex multiplication and a complex addition.

The number of flocops required by the JADE, SOBI, and FOBIUM methods for given values of N, P, L, and I are given by (30)–(32), shown at the bottom of the page, where M is the number of correlation and quadricovariance matrices jointly diagonalized by the SOBI and FOBIUM methods, respectively.

For a given number of sources P, the minimum complexity of the previous methods is obtained by minimizing the values of I, N, M, and L, ensuring the good identification of the mixture matrix A. It is said in [14] that the minimum value of I is $I_{\min} = 1 + \text{Int}(P^{1/2})$, where Int means integer part. The minimum value of M depends on the spectral difference between

$$\begin{aligned}\text{Comp[JADE]} &= \text{Min}\left[\frac{LN^2}{2} + \frac{4N^3}{3} + PNL, 2LN^2\right] + \text{Min}\left[\frac{4P^6}{3}, 8P^3(P^2+3)\right] \\ &\quad + \frac{3LP^3\left(1+\frac{P}{2}\right)}{4} + \frac{IP^2(75+21P+4P^2)}{2} + LP^2 \end{aligned} \tag{30}$$

$$\begin{aligned}\text{Comp[SOBI]} &= \frac{MLN^2}{2} + \frac{4N^3}{3} + \frac{(M-1)N^3}{2} \\ &\quad + \frac{IP(P-1)[4P(M-1)+17(M-1)+4P+75]}{2} \end{aligned} \tag{31}$$

$$\begin{aligned}\text{Comp[FOBIUM]} &= \frac{3MLN^6\left(1+\frac{N^2}{2}\right)}{4} + \frac{4N^6}{3} + \frac{(M-1)N^6}{2} \\ &\quad + \frac{IP(P-1)[4P(M-1)+17(M-1)+4P+75]}{2} \end{aligned} \tag{32}$$

the sources and is chosen to be equal to $M_{\min} = 2$ in the following. The minimum value of N is equal to $N_{\min} = P$ for JADE and SOBI, whereas for FOBIUM, assuming an array with space diversity only, it corresponds to the minimum value $N_{\min}$, such that $N_{\min} \leq P \leq N_{\min}^2 - N_{\min} + 1$. Finally, the minimum value of L depends on several parameters such as P, N, the FO autocumulant, and the SNR of the sources…. For this reason, L is chosen to be the same for all the methods in the following.

Under these assumptions, Fig. 14 shows the variations of the minimum numerical complexity of JADE, SOBI, and FOBIUM as a function of the number of sources P for $L = 1000$. Note the higher complexity of FOBIUM with respect to JADE and SOBI, which requires about 1 Mflocops to process four sources from 1000 snapshots.

IX. Conclusion

A new BI method that exploits the FO data statistics only (called FOBIUM) has been presented in this paper to process both overdetermined and underdetermined instantaneous mixtures of statistically independent sources. This method does not have the drawbacks of the existing methods that are capable of processing underdetermined mixtures of sources and is able to put up with any kind of sources (analogical or digital, circular or not, i.i.d or not) with potential different symbol duration … It only requires non-Gaussian sources having kurtosis with the same sign (practically always verified in radiocommunications contexts) and sources having different trispectrum, which is the only limitation of the method. The FOBIUM method is capable of processing up to $N^2 - N + 1$ sources from an array of N sensors with space diversity only and up to N^2 sources from an array of N different sensors. A consequence of this result is that it allows a drastic reduction or minimization in the number of sensors for a given number of sources, which finally may generate a receiver that is much less expensive than a receiver developed to process overdetermined mixtures only. The FOBIUM method has been shown to require a relatively weak number of snapshots to generate good output performances for currently used radiocommunications sources, such as QPSK sources. Besides exploiting the FO data statistics only, the FOBIUM method is robust to the presence of a Gaussian noise whose spatial coherence is unknown. Finally, an application of the FOBIUM method has been presented through the introduction of a new FO direction-finding method that is built from the blindly identified mixing matrix and called MAXCOR. The comparison of this method to both the SO and FO HR subspace-based direction-finding methods shows better resolution and better robustness to modeling errors of the MAXCOR method with respect to MUSIC2 and MUSIC4 and its ability to process underdetermined mixtures of up to N^2 statistically independent non-Gaussian sources.

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Anne Ferréol was born in 1964 in Lyon, France. She received the M.Sc. degree from ICPI-Lyon, Lyon, France, and the Mastère degree from Ecole Nationale Supérieure des Télécommunications (ENST), Paris, France, in 1988 and 1989, respectively. She is currently pursuing the Ph.D. degree with the Ecole Normale Supérieure de Cachan, France, in collaboration with both SATIE Laboratory and THALES Communications.

Since 1989, she has been with Thomson-CSF-Communications, Gennevilliers, France, in the array processing department.

Ms. Ferréol co-received the 2003 "Science and Defense" Award from the French Ministry of Defence for its work as a whole about array processing for military radiocomunications.

Laurent Albera was born in Massy, France, in 1976. In 2001, he received the DESS degree in mathematics and the DEA degree in authomatic and signal processing from the University of Science (Paris XI), Orsay, France, and, in 2003, the Ph.D. degree in science from the University of Nice, Sophia-Antipolis, France.

He is now Associate Professor with the University of Rennes I, Rennes, France, and is affiliated with the Laboratoire Traitement du Signal et de l'Image (LTSI). His research interests include high order statistics, multidimensional algebra, blind deconvolution and equalization, digital communications, statistical signal and array processing, and numerical analysis. More exactly, since 2000, he has been involved with blind source separation (BSS) and independent component analysis (ICA) by processing both the cyclostationary source case and the underdetermined mixture identification problem.

Pascal Chevalier received the M.Sc. degree from Ecole Nationale Supérieure des Techniques Avancées (ENSTA), Paris, France, and the Ph.D. degree from South-Paris University in 1985 and 1991, respectively.

Since 1991, he has been with Thalés-Communications, Colombes, France, where he has shared industrial activities (studies, experimentations, expertise, management), teaching activities both in French engineering schools (Supelec, ENST, ENSTA), and French Universities (Cergy-Pontoise), and research activities. Since 2000, he has also been acting as Technical Manager and Architect of the array processing subsystem as part of a national program of military satellite telecommunications. He has been a Thalés Expert since 2003. His present research interests are in array processing techniques, either blind or informed, second order or higher order, spatial- or spatio-temporal, time-invariant or time-varying, especially for cyclostationary signals, linear or nonlinear, and particularly widely linear for noncircular signals, for applications such as TDMA and CDMA radiocommunications networks, satellite telecommunications, spectrum monitoring, and HF/VUHF passive listening. He is author or co-author of about 100 papers, including journals, conferences, patents, and chapters of books.

Dr. Chevalier was a member of the THOMSON-CSF Technical and Scientific Council from 1993 to 1998. He co-received the "2003 Science and Defense" Award from the french Ministry of Defence for its work as a whole about array processing for military radiocommunications. He is presently a EURASIP member and an emeritus member of the Societé des Electriciens et des Electroniciens (SEE).