FOURTH ORDER BLIND IDENTIFICATION OF UNDERDETERMINED MIXTURES OF SOURCES (FOBIUM)

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ABSTRACT

For about two decades, numerous methods have been developed to blindly identify overdetermined ($P \leq N$) mixtures of P statistically independent narrow band sources received by an array of N sensors. These methods exploit the information contained in the Second Order (SO), the Fourth Order (FO) or both the SO and FO statistics of the data. However in practical situations, the probability of receiving more sources than sensors increases with the reception bandwidth and the use of Blind Identification (BI) methods able to process underdetermined mixtures of sources, for which $P > N$, may be required. Although such methods have been developed these last years, they all present serious limitations in practical situations related to the radiocommunications context. For this reason, the purpose of this paper is to propose a new attractive BI method, exploiting the information contained in the FO data statistics only, able to process underdetermined mixtures of sources without the main limitations of the existing methods, provided that the sources have different trispectrum and non zero kurtosis with the same sign. A new performance criterion, able to quantify the identification quality of a given source and allowing the quantitative comparison of two BI methods for each source, is also proposed in the paper. Finally, an application of the proposed method is presented through the introduction of a powerful direction finding method built from the blindly identified mixture matrix.

Keywords : *Blind source identification, Underdetermined mixtures, Fourth Order statistics, SOBI, Trispectrum, Performance criterion, FO direction finding*

I. INTRODUCTION

For more than two decades and the pioneer work of Godard [30] about blind equalization in SISO (Single Input Single Output) contexts, there has been an increasing interest for BI of both SIMO (Single Input Multiple Outputs) and MIMO (Multiple Inputs Multiple Outputs) systems. While in the SISO case, blind equalization or channel identification require the exploitation of higher order (HO) statistics in the general case of non-minimum phase systems [30], it has been shown recently that for SIMO systems, multichannel identification may be performed from SO statistics only, under quite general assumptions [39] [43] [49]. Extensions of these pioneer works and development of alternative methods for both blind multi-channel identification and equalization in MIMO FIR systems from SO or HO statstics are presented in [1] [23] [31-32] and [17] [29] [35] [38] [50-53] respectively. Other extensions to MIMO IIR systems or taking into account the finite-alphabet property of the sources are presented in [34] [44] and [46] [54] respectively. However, the BI or deconvolution problems in MIMO contexts are not recent but have been considered since the pioneer work of Herault and Jutten [33] [36] about Blind Source Separation (BSS) in 1985. Since these pioneer works, numerous methods have been developed to blindly identify either instantaneous or convolutive mixtures of P statistically independent narrow band sources received by an array of N sensors. Some of these methods [5] [48] exploit the SO data statistics only whereas other methods [6] [9] [14] [22] exploit both the SO and the FO statistics of the data or even the FO data statistics only [2].

Nevertheless, all the previous methods of either blind multichannel identification of MIMO systems or BI of instantaneous or convolutive mixtures of sources, either SO or HO, can only process overdetermined systems, i.e. systems for which the number of sources (or inputs), P, is lower than or equal to the number of sensors (or outputs), N, i.e. such that $P \leq N$.

However in practical situations such as, for example, airborne electronic warfare over dense urban areas, the probability of receiving more sources than sensors increases with the reception bandwidth and the use of BI methods, able to process underdetermined mixtures of sources, for which $P > N$, may be required. To this aim, several methods have been developed this last decade mainly to blindly identify instantaneous mixtures of sources, among which we find the methods [3-4] [8] [15-16] [19-21] [37] [45]. Concerning convolutive mixtures of sources or MIMO FIR systems, only very scarce results exist about BI of underdetermined systems, among which we find [18] [47]. Some of these methods focus on blind source extraction [16] [37], which is a difficult problem since underdetermined mixtures are not linearly invertible, while others, as herein, favour BI of the mixture matrix [3-4] [8] [15-16] [18-21] [37] [45] [47]. The methods proposed in [8] [15] [18-21] [47] only exploit the information contained in the FO statistics of the data whereas the one recently proposed in [3] exploit the Sixth order data statistics only and its extension to an arbitrary even order $2q$ ($q > 2$) is presented in [4]. Finally, the method proposed in [45] exploits the information contained in the second characteristic function of the observations whereas in [37], the probability density of the observations conditionally to the mixture matrix is maximized. Nevertheless, all these methods suffer from serious limitations in operational contexts related to radiocommunications. Indeed, the method [8] and its improvements for both instantaneous [21] and convolutive [18] mixtures of sources remain currently mainly conceptual and has not yet been evaluated by any simulations. The methods [15] [19-20] assume FO non circular sources and thus fail in identifying circular sources, omnipresent in practice. Besides, the theories developed in [15] and [19] confine themselves to the three sources and two sensors case. Although the method [37] succeeds in identifying the steering vectors of up to four speech signals with only two sensors, the authors need sparsity conditions and do not address the general case when all sources are always present. Moreover, the method [45] has been developed only for real mixtures of real-valued sources and the issue of robustness with respect to an over estimation of the source number remains open. Although very promizing, powerful and easy to implement, the methods [3-4] suffer, a priori, from both a higher variance and a higher numerical complexity due to the use of data statistics with an even order strictly greater than four. Finally, for instantaneous mixtures of sources, the method developed in [47] can only process overdetermined systems.

In order to overcome these limitations for underdetermined systems, the purpose of this paper is to propose a new BI method, exploiting the information contained in the FO data statistics only, able to process both over and underdetermined instantaneous mixtures of sources without the drawbacks of the existing methods of this family, but assuming the sources have different trispectrum and have non zero kurtosis with the same sign (the latter assumption is generally verified in radiocommunications contexts). This new BI method, called FOBIUM (Fourth Order Blind Identification of Underdetermined Mixtures of sources), corresponds to the FO extension of the SOBI method [5] and is able to blindly identify the steering vectors of up to $N^2 - N + 1$ sources, from an array of N sensors with space diversity only, and of up to N^2 sources, from an array of N different sensors. Moreover, this method is asymptotically robust to an unknown Gaussian spatially colored noise since it does not exploit the information contained in the SO data statistics. To evaluate the performance of the FOBIUM method and, more generally, of all the BI methods, a new performance criterion, able to quantify the identification quality of the steering vector of each source and allowing the quantitative comparison of two methods for the blind identification of a given source, is also proposed. Finally, an application of the FOBIUM method is presented through the introduction of a FO direction finding method, built from the blindly identified mixing matrix and called MAXCOR (MAXimum of spatial CORrelation), which is shown to be very powerful with respect to SO [42] and FO subspace-based direction finding methods [7] [13] [40]. Note that an extension of the FOBIUM method to HO statistics remains possible.

After the problem formulation and an introduction of some notations, hypotheses and data statistics in section II, the FOBIUM method is presented in Section III. The associated conditions about the identifiability of the mixture matrix are then analysed in Section IV. The new performance criterion is presented in Section V. The application of the FOBIUM method to the direction finding problem through the introduction of the MAXCOR method is described in section VI. All the results of the paper are illustrated in section VII through computer simulations. The numerical complexity of the FOBIUM method compared with the one of some existing methods is briefly presented in section VIII. Finally section IX concludes this paper. Note that the results of the paper have been partially presented in [25] and [11].

II. PROBLEM FORMULATION, HYPOTHESES AND DATA STATISTICS

A. Problem formulation

We consider an array of N narrow-band (NB) sensors and we call $\boldsymbol{x}(t)$ the vector of complex amplitudes of the signals at the output of these sensors. Each sensor is assumed to receive the contribution of P zero-mean stationary and stastistically independent NB sources corrupted by a noise. Under these assumptions, the observation vector, $\boldsymbol{x}(t)$, can be written as follows :

$$\boldsymbol{x}(t) \;=\; \sum_{p=1}^{P} m_p(t)\, \boldsymbol{a}_p \;+\; \boldsymbol{b}(t) \;\overset{\Delta}{=}\; A\, \boldsymbol{m}(t) \;+\; \boldsymbol{b}(t) \qquad (1)\ .$$

where $\boldsymbol{b}(t)$ is the noise vector, assumed zero-mean, stationary and Gaussian, the complex envelope of the source p, $m_p(t)$, is the p-th component of the vector $\boldsymbol{m}(t)$, assumed zero-mean and stationary, $\boldsymbol{a}_p$ corresponds to the steering vector of the source p and A is the (N x P) mixture matrix whose columns are the vectors $\boldsymbol{a}_p$. The instantaneous mixture model defined by (1) have already been considered in numerous papers [2-12] [14-16] [19-22] [24-28] [33] [36-37] [45] [48] and is perfectly suitable for applications such as, for example, airborne or satellite electronic warfare.

Under these assumptions, the problem addressed in this paper is that of FO blind identification of the mixture matrix A. It consists to estimate, from the FO data statistics, the mixing matrix A to within a (P x P) invertible diagonal matrix Λ and a (P x P) permutation matrix Π.

B. Statistics of the data

Under the previous assumptions, the SO statistics of the data used in the paper are characterized by the correlation or covariance matrix, R_x, defined by

$$R_x \stackrel{\Delta}{=} \mathrm{E}[\boldsymbol{x}(t)\, \boldsymbol{x}(t)^{\mathrm{H}}] = \sum_{p=1}^{P} \pi_p\, \boldsymbol{a}_p\, \boldsymbol{a}_p{}^{\mathrm{H}} + \eta_2\, B \stackrel{\Delta}{=} A\, R_m\, A^{\mathrm{H}} + \eta_2\, B \qquad (2)\ .$$

where $\pi_p \stackrel{\Delta}{=} \mathrm{E}[\, |\, m_p(t)|^2\,]$ is the power of source p received by an omnidirectional sensor, η_2 is the mean of the noise power per sensor, B is the spatial coherence of the noise such that $\mathrm{Tr}[B] = N$, where Tr[.] means Trace, $R_m \stackrel{\Delta}{=} \mathrm{E}[\boldsymbol{m}(t)\, \boldsymbol{m}(t)^{\mathrm{H}}]$ is the correlation matrix of the source vector $\boldsymbol{m}(t)$ and the symbol $^{\mathrm{H}}$ means transpose and complex conjugate.

The FO statistics of the data used in the paper are characterized by the (N^2 x N^2) quadricovariance matrices $Q_x(\tau_1, \tau_2, \tau_3)$, whose elements, $Q_x(\tau_1, \tau_2, \tau_3)[i, j, k, l]$ ($1 \leq i, j, k, l \leq N$), are defined by

$$Q_x(\tau_1, \tau_2, \tau_3)[i, j, k, l] \stackrel{\Delta}{=} \mathrm{Cum}(x_i(t), x_j(t-\tau_1)^*, x_k(t-\tau_2)^*, x_l(t-\tau_3)) \qquad (3)\ .$$

where * means complex conjugate and $x_i(t)$ is the component i of $\boldsymbol{x}(t)$. Using (1) into (3) and assuming that $Q_x(\tau_1, \tau_2, \tau_3)[i, j, k, l]$ is the element $[N(i-1)+j,\ N(k-1)+l]$ of the matrix $Q_x(\tau_1, \tau_2, \tau_3)$, we obtain the expression of the latter, given, under a Gaussian noise assumption, by

$$Q_x(\tau_1, \tau_2, \tau_3) = [A \otimes A^*]\, Q_m(\tau_1, \tau_2, \tau_3)\, [A \otimes A^*]^{\mathrm{H}} \qquad (4)\ .$$

where $Q_m(\tau_1, \tau_2, \tau_3)$ is the (P^2 x P^2) quadricovariance matrix of $\boldsymbol{m}(t)$ and $\otimes$ is the Kronecker product.

Under the assumption of statistically independent sources, the matrix $Q_m(\tau_1, \tau_2, \tau_3)$ contains at least $P^4 - P$ zeros and expression (4) degenerates in a simpler one given by

$$Q_x(\tau_1, \tau_2, \tau_3) = \sum_{p=1}^{P} c_p(\tau_1, \tau_2, \tau_3) \; [\, \boldsymbol{a}_p \otimes \boldsymbol{a}_p^*] \, [\, \boldsymbol{a}_p \otimes \boldsymbol{a}_p^*]^{\mathrm{H}}$$
$$= A_Q \; C_m(\tau_1, \tau_2, \tau_3) \, A_Q^{\;\mathrm{H}} \quad (5) .$$

where A_Q is the $(N^2 \times P)$ matrix defined by $A_Q \overset{\Delta}{=} [\, \boldsymbol{a}_1 \otimes \boldsymbol{a}_1^*, \ldots\ldots, \boldsymbol{a}_P \otimes \boldsymbol{a}_P^*]$, $C_m(\tau_1, \tau_2, \tau_3)$ is the $(P \times P)$ diagonal matrix defined by $C_m(\tau_1, \tau_2, \tau_3) \overset{\Delta}{=} \mathrm{Diag}[c_1(\tau_1, \tau_2, \tau_3), \ldots.., c_P(\tau_1, \tau_2, \tau_3)]$ and $c_p(\tau_1, \tau_2, \tau_3)$ is defined by

$$c_p(\tau_1, \tau_2, \tau_3) \overset{\Delta}{=} \mathrm{Cum}(m_p(t), m_p(t-\tau_1)^*, m_p(t-\tau_2)^*, m_p(t-\tau_3)) \quad (6) .$$

The expression (5), which has an algebraic structure similar to that of data correlation matrices [5], is the starting point of the FOBIUM method as it will be shown in the next section. To simplify the notations, we note in the following $Q_x \overset{\Delta}{=} Q_x(0, 0, 0)$, $C_m \overset{\Delta}{=} C_m(0, 0, 0)$, $c_p \overset{\Delta}{=} c_p(0, 0, 0)$ and we obtain from (5)

$$Q_x = A_Q \, C_m \, A_Q^{\;\mathrm{H}} \quad (7) .$$

C. Statistics estimation

In situations of practical interests, the SO and FO statistics of the data, given by (2) and (3) respectively, are not known a priori and have to be estimated from L samples of data, $\boldsymbol{x}(l) \overset{\Delta}{=} \boldsymbol{x}(lT_e)$, $1 \leq l \leq L$, where T_e is the sample period. For zero-mean stationary observations, using the ergodicity property, empirical estimators [26] may be used since they generate asymptotically unbiased and consistent estimates of the data statistics. However, in radiocommunications contexts, most of the sources are no longer stationary but become cyclostationary (digital modulations). For zero-mean cyclostationary observations, the statistics defined by (2) and (3) become time dependent and the theory developed in the paper can be extended without any difficulties by considering that R_x and $Q_x(\tau_1, \tau_2, \tau_3)$ are, in this case, the temporal means, $< R_x(t) >$ and $< Q_x(\tau_1, \tau_2, \tau_3)(t) >$, over an infinite interval duration, of the instantaneous statistics, $R_x(t)$ and $Q_x(\tau_1, \tau_2, \tau_3)(t)$ defined by (2) and (4) respectively. In these conditions, using a cyclo-ergodicity property, the matrix R_x can still be estimated from the sampled data by the SO empirical estimator [26] but the matrix $Q_x(\tau_1, \tau_2, \tau_3)$ has to be estimated by a non empirical estimator presented in [26], taking into account the SO cyclic frequencies of the data. Note finally that this extension can also be applied to non zero mean cyclostationary sources, such as some non linearly digitally modulated sources [41], provided that non empirical statistics estimators, presented in [28] and [27] for SO and FO statistics respectively, are used. Such SO estimators take into account the first order cyclic frequencies of the data whereas such FO estimators take into account both the first and SO cyclic frequencies of the data.

D. Hypotheses

In the next sections, we further assume the following hypotheses :

- H1 : $P \leq N^2$
- H2 : A_Q is full rank
- H3 : $c_p \neq 0$ $(1 \leq p \leq P)$ (i.e. no source is Gaussian)
- H4 : $\forall\ 1 \leq p, q \leq P$, $c_p\, c_q > 0$ (i.e. sources have FO autocumulant with the same sign)
- H5 : $\forall\ 1 \leq p, q \leq P$, $\exists\ (\tau_1, \tau_2, \tau_3) \neq (0, 0, 0)$ such that

$$c_p(\tau_1, \tau_2, \tau_3) / |c_p| \neq c_q(\tau_1, \tau_2, \tau_3) / |c_q| \quad (8)\ .$$

Note that hypothesis H4 is not restrictive in radiocommunication contexts since most of the digitally modulated sources have negative FO autocumulant. For example, M-PSK constellations [41] have a kurtosis equal to –2 for M = 2 and to –1 for M > 2. Continuous phase modulation (CPM) [41], among which we find in particular the CPFSK, the MSK and the GMSK modulation (GSM standard) have a kurtosis lower than or equal to –1. Moreover, note that the condition (8) requires in particular that the sources have different normalized tri-spectrum, which prevents in particular from considering sources with both the same modulation, the same baud rate and the same carrier residu.

III. THE FOBIUM METHOD

The purpose of the FOBIUM method is to extend the SOBI method [5] to the FO. It firstly implements a FO pre-whitening step aiming at orthonormalizing the so-called *virtual steering vector* [12] of the sources, corresponding to the columns of A_Q. Secondly it jointly diagonalizes several well chosen pre-whitened quadricovariance matrices in order to identify the A_Q matrix. Then, in a third step, it identifies the mixing matrix A from the A_Q matrix. The number of sources able to be processed by this method is considered in section IV.

A. FO pre-whitening step

The first step of the FOBIUM method is to orthonormalize, in the $Q_x(\tau_1, \tau_2, \tau_3)$ matrices (5), the columns of A_Q, which can be considered as *virtual steering vectors* of the sources for the considered array of sensors [12]. For this purpose, let us consider the eigen decomposition of the Hermitian matrix Q_x, whose rank is P under the assumptions H1 to H3, given by

$$Q_x = E_x \Lambda_x E_x^{\mathrm{H}} \quad (9) .$$

where Λ_x is the $(P \times P)$ real-valued diagonal matrix of the P non zero eigen-values of Q_x and E_x is the $(N^2 \times P)$ matrix of the associated orthonormalized eigen-vectors.

Proposition 1 : *Assuming P sources with non zero kurtosis having the same sign ε (ε = ±1) (i.e. H3 + H4), it is straigthforward to show that the diagonal elements of Λ_x are not zero and have also the same sign corresponding to* ε

We deduce from proposition 1 that $\varepsilon\Lambda_x$, which contains the non zero eigenvalues of εQ_x, has square root decompositions such that $\varepsilon\Lambda_x = (\varepsilon\Lambda_x)^{1/2} (\varepsilon\Lambda_x)^{\mathrm{H}/2}$ where $(\varepsilon\Lambda_x)^{1/2}$ is a square root of $\varepsilon\Lambda_x$ and $(\varepsilon\Lambda_x)^{\mathrm{H}/2} \overset{\Delta}{=} [(\varepsilon\Lambda_x)^{1/2}]^{\mathrm{H}}$. Thus the existence of this square root decomposition requires assumption H4. Considering the $(P \times N^2)$ pre-whitening matrix T defined by

$$T \overset{\Delta}{=} (\varepsilon\Lambda_x)^{-1/2} E_x^{\mathrm{H}} \quad (10)$$

where $(\varepsilon\Lambda_x)^{-1/2}$ is the inverse of $(\varepsilon\Lambda_x)^{1/2}$, we obtain, from (7) and (9)

$$T (\varepsilon Q_x) T^{\mathrm{H}} = T A_Q (\varepsilon C_m) A_Q^{\mathrm{H}} T^{\mathrm{H}} = \mathrm{I}_p \quad (11)$$

where I_p is the $(P \times P)$ identity matrix and where $\varepsilon C_m = \mathrm{Diag}[|c_1|, \ldots, |c_p|]$. The expression (11) shows that the $(P \times P)$ matrix $T A_Q (\varepsilon C_m)^{1/2}$ is a unitary matrix U ($UU^{\mathrm{H}} = \mathrm{I}_p$) and we obtain

$$T A_Q = U (\varepsilon C_m)^{-1/2} \quad (12)$$

which means that the columns of A_Q have been orthonormalized to within a diagonal matrix.

B. FO blind identification of A_Q

The second step of the FOBIUM method is to blindly identify the A_Q matrix from some FO statistics of the data. For this purpose, we deduce from (5) and (12) that

$$T\,Q_x(\tau_1, \tau_2, \tau_3)\,T^{\mathrm{H}} \;=\; U\,(\varepsilon C_m)^{-1/2}\,C_m(\tau_1, \tau_2, \tau_3)\,(\varepsilon C_m)^{-1/2}\,U^{\mathrm{H}} \tag{13}$$

which shows that the unitary matrix U diagonalizes the matrices $T\,Q_x(\tau_1, \tau_2, \tau_3)\,T^{\mathrm{H}}$ whatever the set of delays (τ_1,τ_2,τ_3) and the associated eigen-values correspond to the diagonal terms of the diagonal matrix $(\varepsilon C_m)^{-1/2}\,C_m(\tau_1, \tau_2, \tau_3)\,(\varepsilon C_m)^{-1/2}$.

For a given set (τ_1,τ_2,τ_3) and a given order of the sources, U is unique to within a unitary diagonal matrix if and only if the diagonal elements of the matrix $(\varepsilon C_m)^{-1/2}\,C_m(\tau_1, \tau_2, \tau_3)\,(\varepsilon C_m)^{-1/2}$ are all different. If it is not the case, following the results of [5], we have to consider several sets $(\tau_1{}^k,\tau_2{}^k,\tau_3{}^k)$, $1 \leq k \leq K$, such that for each couple of sources (p, q), there exists at least a set $(\tau_1{}^k,\tau_2{}^k,\tau_3{}^k)$, such that the condition (8) is verified for this set, which corresponds to hypothesis H5. Under this assumption, the unitary matrix U becomes, to within a permutation and a unitary diagonal matrix, the only one which jointly diagonalizes the K matrices $T\,Q_x(\tau_1{}^k,\tau_2{}^k,\tau_3{}^k)\,T^{\mathrm{H}}$. In other words, the unitary matrix, U_{sol}, solution to the previous problem of joint diagonalization can be written as

$$U_{sol} \;=\; U\,\Lambda\,\Pi \tag{14}$$

where Λ and Π are unitary diagonal and permutation matrices respectively.

Noting $T^{\#} \stackrel{\Delta}{=} E_x\,(\varepsilon\Lambda_x)^{1/2}$ the pseudo-inverse of T, such that $T\,T^{\#} = \mathrm{I}_p$, we deduce from (14) that

$$T^{\#}\,U_{sol} \stackrel{\Delta}{=} E_x\,(\varepsilon\Lambda_x)^{1/2}\,U_{sol} \;=\; E_x\,(\varepsilon\Lambda_x)^{1/2}\,U\,\Lambda\,\Pi \tag{15}$$

and using (12) and (10) into (15) we obtain

$$T^{\#}\,U_{sol} \;=\; E_x\,(\varepsilon\Lambda_x)^{1/2}\,U\,\Lambda\,\Pi \;=\; E_x\,E_x{}^{\mathrm{H}}\,A_Q\,(\varepsilon C_m)^{1/2}\,\Lambda\,\Pi \tag{16}$$

From (7) and (9), we deduce that Span(A_Q) = Span(E_x), which implies that the orthogonal projection of A_Q on the space spanned by the columns of E_x, $E_x\,E_x{}^{\mathrm{H}}A_Q$, corresponds to A_Q. Using this result in (16), we finally obtain

$$T^{\#}\,U_{sol} \;=\; A_Q\,(\varepsilon C_m)^{1/2}\,\Lambda\,\Pi \tag{17}$$

which shows that the matrix A_Q can be identified, to within a diagonal and a permutation matrix, from the matrix $T^{\#}\,U_{sol}$.

C. Blind identification of *A*

The third step of the FOBIUM method is to identify the mixing matrix A from A_Q. For this purpose, we note from (17) and the definition of A_Q that each column, $\boldsymbol{b}_p$ ($1 \leq p \leq P$), of $T^{\#}U_{sol}$ corresponds to a vector $\mu_q |c_q|^{1/2} (\boldsymbol{a}_q \otimes \boldsymbol{a}_q^*)$, $1 \leq q \leq P$, where μ_q, such that $|\mu_q| = 1$, is an element of the diagonal matrix Λ. Thus mapping the components of each column, $\boldsymbol{b}_p$, of $T^{\#}U_{sol}$ into a (N x N) matrix, B_p, such that $B_p[i, j] = \boldsymbol{b}_p((i-1)N + j)$, $1 \leq i, j \leq N$, consists to built the matrices $\mu_q |c_q|^{1/2} \boldsymbol{a}_q \boldsymbol{a}_q^{\mathrm{H}}$, $1 \leq q \leq P$. We then deduce that the steering vector, $\boldsymbol{a}_q$, of the source q corresponds, to within a scalar, to the eigenvector of B_p associated with the eigenvalue having the strongest modulus. Thus the eigendecomposition of all the B_p matrices, $1 \leq p \leq P$, allows the identification of A to within a diagonal and a permutation matrix.

D. Implementation of the FOBIUM method

The different steps of the FOBIUM method are summarized hereafter when L snapshots of the observations, $\boldsymbol{x}(l)$ ($1 \leq l \leq L$), are available.

Step 1 : Estimation, $\hat{Q}_x$, of the Q_x matrix from the L snapshots $x(l)$ using a suitable estimator of the FO cumulants [26-27].

Step 2 : Eigen Value Decomposition (EVD) of the matrix $\hat{Q}_x$.

. From this EVD, estimation, $\hat{P}$, of the number of sources P by a classical source number detection test.

. Evaluation of the sign ε of the eigenvalues.

. Restriction of this EVD to the $\hat{P}$ principal components : $\hat{Q}_x \approx \hat{E}_x \hat{\Lambda}_x \hat{E}_x^{\mathrm{H}}$, where $\hat{\Lambda}_x$ is the diagonal matrix of the $\hat{P}$ eigenvalues with the strongest modulus and $\hat{E}_x$ is the matrix of the associated eigenvectors.

Step 3 : Estimation, $\hat{T}$, of the pre-whitening matrix T by $\hat{T} = (\varepsilon\hat{\Lambda}_x)^{-1/2} \hat{E}_x^{\mathrm{H}}$.

Step 4 : Selection of K appropriate set of delays $(\tau_1^k, \tau_2^k, \tau_3^k) \neq (0, 0, 0)$, $1 \leq k \leq K$. For example, one may choose these sets such that $\tau_1^k \neq 0$ and $\tau_2^k = \tau_3^k = 0$ or such that $\tau_1^k = \tau_2^k = \tau_3^k \neq 0$, where τ_1^k may be lower than or equal to $1/\hat{B}_x$, where $\hat{B}_x$ is an estimate of the observation bandwidth, B_x.

Step 5 : Estimation, $\hat{Q}_x(\tau_1^k, \tau_2^k, \tau_3^k)$, of the K matrices $Q_x(\tau_1^k, \tau_2^k, \tau_3^k)$ for the K delays sets using a suitable estimator.

Step 6 : . Computation of the matrices $\hat{T}\,\hat{Q}_x(\tau_1{}^k,\tau_2{}^k,\tau_3{}^k)\,\hat{T}^{\mathrm{H}}$, $1\leq k\leq K$.

. Estimation, $\hat{U}_{sol}$, of the unitary matrix U_{sol} from the joint diagonalization of the K matrices $\hat{T}\,\hat{Q}_x(\tau_1{}^k,\tau_2{}^k,\tau_3{}^k)\,\hat{T}^{\mathrm{H}}$(the joint diagonalization process is decribed in [5] [9]).

Step 7 : Computation of $\hat{T}^{\#}\hat{U}_{sol} = \hat{E}_x\,(\varepsilon\hat{\Lambda}_x)^{1/2}\,\hat{U}_{sol}$

Step 8 : . Mapping each column, $\hat{\boldsymbol{b}}_p$ $(1\leq p\leq \hat{P})$, of $\hat{T}^{\#}\hat{U}_{sol}$ into a (N x N) matrix $\hat{B}_p$.

. EVD of the $\hat{P}$ matrices $\hat{B}_p$ $(1\leq p\leq \hat{P})$

. An estimate, $\hat{A}$, of the mixing matrix A, to within a diagonal and a permutation matrix, is obtained by considering that each of the $\hat{P}$ columns of $\hat{A}$ corresponds to the eigenvector of a matrix $\hat{B}_p$ $(1\leq p\leq \hat{P})$ associated with the eigenvalue having the strongest modulus.

IV. IDENTIFIABILITY CONDITIONS

Following the developments of the previous section, we deduce that the FOBIUM method is able to identify the steering vectors of P sources from an array of N sensors provided hypotheses H1 to H5 are verified. In other words, the FOBIUM method is able to identify P $(P\leq N^2)$ non Gaussian sources having different tri-spectrum and kurtosis with the same sign provided that the A_Q matrix has full rank P, i.e. that the *virtual steering vectors* $\boldsymbol{a}_q\otimes\boldsymbol{a}_q{}^*$ $(1\leq q\leq P)$ for the considered array of N sensors remain linearly independent. However, it has been shown in [24] and [12] that the vector $\boldsymbol{a}_q\otimes\boldsymbol{a}_q{}^*$ can also be considered as a *true steering vector* but for a FO *virtual array* of N_e $(N_e\leq N^2)$ different sensors, where N_e is directly related to both the pattern of the true sensors and the geometry of the true array of N sensors. This means in particular that N^2-N_e components of each vector $\boldsymbol{a}_q\otimes\boldsymbol{a}_q{}^*$ are redundant components that bring no information. As a consequence, N^2-N_e rows of the A_Q matrix bring no information and are linear combinations of the others, which means that the rank of A_Q cannot be greater than N_e. In these conditions, the A_Q matrix may have a rank equal to P only if $P\leq N_{\mathrm{e}}$. Conversely, for a FO virtual array without any ambiguities up to order N_{e}, P sources coming from P different directions generate an A_Q matrix with a full rank P as long as $P\leq N_{\mathrm{e}}$. Thus the FOBIUM method is able to process up to N_{e} sources, where N_{e} is the number of different sensors of the FO virtual array associated with the considered array of N sensors. For example, for a uniform linear array of N identical sensors, $N_e = 2N - 1$ whereas for most of other arrays with space diversity only, $N_e = N^2 - N + 1$ [12]. Finally for an array with N sensors having all a different angular and polarization pattern, $N_e = N^2$ [12].

V. NEW PERFORMANCE CRITERION

Most of the existing performance criterions used to evaluate the quality of a blind identification method [14] [15] [45] are *global criterions* which evaluate a distance between the true mixing matrix A and its blind estimate $\hat{A}$. Although useful, a global performance criterion necessarily contains implicitly a part of arbitrary considerations in the manner of combining the distances between the vectors $\boldsymbol{a}_q$ and $\hat{\boldsymbol{a}}_q$, for $1 \leq q \leq P$, to generate a unique scalar criterion. Moreover, it is possible to find that an estimate $\hat{A}_1$ of A is better than an estimate $\hat{A}_2$, with respect to the global criterion, while some columns of $\hat{A}_2$ estimate the associated true steering vectors in a better way than those of $\hat{A}_1$, which may generate some confusion in the interpretations.

To overcome these drawbacks, we propose in this section a new performance criterion for the evaluation of a blind identification method. This new criterion is no longer global and allows both the quantitative evaluation of the identification quality of each source by a given method and the quantitative comparison of two methods for the blind identification of a given source. It corresponds, for the blind identification problem, to a performance criterion similar, with respect to the spirit, to the one proposed in [10] for the extraction problem. It is defined by the following P-uplet

$$D(A, \hat{A}) \stackrel{\Delta}{=} (\alpha_1, \alpha_2, \ldots, \alpha_P) \tag{18}$$

where α_p, $1 \leq p \leq P$, such that $0 \leq \alpha_p \leq 1$, is defined by

$$\alpha_p \stackrel{\Delta}{=} \operatorname*{Min}_{1 \leq i \leq \hat{P}} [\mathrm{d}(\boldsymbol{a}_p, \hat{\boldsymbol{a}}_i)] \tag{19}$$

where $\mathrm{d}(\boldsymbol{u}, \boldsymbol{v})$ is the pseudo-distance between the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, defined by

$$\mathrm{d}(\boldsymbol{u}, \boldsymbol{v}) \stackrel{\Delta}{=} 1 - \frac{\left|\boldsymbol{u}^H \boldsymbol{v}\right|^2}{(\boldsymbol{v}^H \boldsymbol{v})(\boldsymbol{u}^H \boldsymbol{u})} \tag{20}$$

Thus the identification quality of the source p is evaluated by the parameter α_p, which decreases toward zero as the identification quality of the source p increases. In particular, the source p is perfectly identified when $\alpha_p = 0$. Although arbitrary, we consider in the following that a source p is blindly identified with a very high quality if $\alpha_p \leq 0.01$, with a high quality if $\alpha_p \leq 0.03$, with a good quality if $\alpha_p \leq 0.05$ and with a poor quality otherwise. Besides, we will say that a method M1 is better than a method M2 for the identification of the source p if $\alpha_p(\mathrm{M1}) < \alpha_p(\mathrm{M2})$, where $\alpha_p(\mathrm{Mi})$ corresponds to the parameter α_p generated by the method Mi. Moreover, we will say that a method M1 is better than a method M2 if it is better for each source, i.e. if $\alpha_p(\mathrm{M1}) < \alpha_p(\mathrm{M2})$ for $1 \leq p \leq P$. Finally we verify that, whatever the $(\hat{P} \times \hat{P})$ diagonal matrix $\hat{\Lambda}$ and permutation matrix $\hat{\Pi}$, we obtain

$$D(A, \hat{A}) = D(A, \hat{A}\,\hat{\Lambda}\hat{\Pi}) \tag{21}$$

which means that two mixing matrix estimates which are equal to within a diagonal and a permutation matrix generate the same performance for all the sources, which is satisfactory.

VI. APPLICATION OF THE FOBIUM METHOD : DIRECTION FINDING WITH THE MAXCOR METHOD

Before presenting some computer simulations in section VII, we propose in this section, an application of the FOBIUM method, usable when the array manifold is known or estimated by calibration. This application consists to find the direction of arrival (DOA) of the detected sources directly from the blindly identified mixing matrix, allowing better DOA estimations than the existing ones in many contexts. Besides, for a given array of sensors, this application allows to interpret the α_p coefficient, introduced in the previous section to evaluate the identification quality of the source p, in terms of angular precision.

A. Existing direction finding methods

When the array manifold is known or estimated by calibration, each component, a_{pn} $(1 \leq n \leq N)$, of the steering vector $\boldsymbol{a}_p$ may be written as a function, $a_n(\theta_p, \varphi_p)$, of the direction of arrival (θ_p, φ_p) of the source p, where θ_p and φ_p are the azimuth and the elevation angles of source p respectively (Figure 1). The function $a_n(\theta, \varphi)$ is the n^{th} component of the steering vector $\boldsymbol{a}(\theta, \varphi)$ for the direction (θ, φ). In particular, in the absence of modelling errors such as mutual coupling, the component $a_n(\theta_p, \varphi_p)$ can be written, under the far field assumption and in the general case of an array with space, angular and polarization patterns diversity, as [12]

$$a_n(\theta_p, \varphi_p) = f_n(\theta_p, \varphi_p) \exp\{\mathrm{j}2\pi[x_n \cos(\theta_p) \cos(\varphi_p) + y_n \sin(\theta_p) \cos(\varphi_p) + z_n \sin(\varphi_p)\,]/\,\lambda\} \tag{22}$$

where λ is the wavelength, (x_n, y_n, z_n) are the coordinates of sensor n of the array, $f_n(\theta_p, \varphi_p)$ is a complex number corresponding to the response of sensor n to a unit electric field coming from the direction (θ_p, φ_p). Using the knowledge of the array manifold $\boldsymbol{a}(\theta, \varphi)$, it is possible to estimate the direction of arrival of the sources from some statistics of the data such as the SO or the FO statistics given by (2) and (7) respectively.

Among the existing SO direction finding methods, the so-called High Resolution (HR) methods, developed from the beginning of the eighties, are the most powerful in multi-sources contexts since they are characterized by an asymptotic resolution which becomes infinite whatever the source Signal to Noise Ratio (SNR). Among these HR methods, the subspace-based methods such as the MUSIC method [42] are the most popular. Recall that, after a source number estimation $\hat{P}$, the MUSIC method consists to find the $\hat{P}$ couples (θ_i, φ_i) minimizing the pseudo-spectrum defined by

$$\hat{C}_{Music2}(\theta, \varphi) \overset{\Delta}{=} \frac{\boldsymbol{a}(\theta,\varphi)^{\mathrm{H}}\, \hat{\Pi}_{MUSIC2}\, \boldsymbol{a}(\theta,\varphi)}{\boldsymbol{a}(\theta,\varphi)^{\mathrm{H}}\boldsymbol{a}(\theta,\varphi)} \tag{23}$$

where $\boldsymbol{a}(\theta, \varphi)$ is the steering vector for the direction (θ, φ) and $\hat{\Pi}_{MUSIC2} \overset{\Delta}{=} (\mathrm{I}_N - \hat{L}_x \hat{L}_x^{\mathrm{H}})$ where I_N is the $(N \times N)$ identity matrix and $\hat{L}_x$ is the $(N \times \hat{P})$ matrix of the $\hat{P}$ orthonormalized eigenvectors of the estimated data correlation matrix, $\hat{R}_x$, associated with the $\hat{P}$ strongest eigenvalues.

One of the main drawbacks of the SO subspace-based methods such as the MUSIC method is that they are not able to process more than $N - 1$ sources from an array of N sensors. Mainly to overcome this limitation, but also to still increase the resolution with respect to that of SO methods for a finite duration observation, higher order HR direction finding methods [7] [13] [40] have been developed these two last decades, among which the extension of the MUSIC method to the FO [40], called MUSIC4, is the most popular. Recall that, after a source number estimation $\hat{P}$, the MUSIC4 method consists to find the $\hat{P}$ couples (θ_i, φ_i) minimizing the pseudo-spectrum defined by

$$\hat{C}_{Music4}(\theta, \varphi) \overset{\Delta}{=} \frac{\left[\boldsymbol{a}(\theta,\varphi)^{\otimes 2}\right]^{\mathrm{H}} \hat{\Pi}_{MUSIC4} \left[\boldsymbol{a}(\theta,\varphi)^{\otimes 2}\right]}{\left[\boldsymbol{a}(\theta,\varphi)^{\otimes 2}\right]^{\mathrm{H}}\left[\boldsymbol{a}(\theta,\varphi)^{\otimes 2}\right]} \tag{24}$$

where $\boldsymbol{a}(\theta, \varphi)^{\otimes 2} = \boldsymbol{a}(\theta, \varphi) \otimes \boldsymbol{a}(\theta, \varphi)^*$ and $\hat{\Pi}_{MUSIC4} \overset{\Delta}{=} (\mathrm{I}_{N^2} - \hat{E}_x \hat{E}_x^{\mathrm{H}})$, with I_{N^2} the $(N^2 \times N^2)$ identity matrix and $\hat{E}_x$ the $(N^2 \times \hat{P})$ matrix of the $\hat{P}$ orthonormalized eigen vectors of the estimated data quadricovariance matrix, $\hat{Q}_x$, associated with the $\hat{P}$ strongest eigenvalues. Moreover it has been shown in [12] that the MUSIC4 method is able to process up to $N_e - 1$ sources where N_e corresponds to the number of different sensors of the FO virtual array associated with the considered array of sensors.

Figure 1

C. Application of the FOBIUM method : the MAXCOR method

Despite of the interests of both the SO and FO HR subspace-based direction finding methods described in section A, the latter keep a source of performance limitation in multi-sources contexts for a finite duration of observation, since they may be qualified as *multidimensionnal* methods insofar as they implement a procedure of searching multiple minima of a pseudo-spectrum function. This multidimensionality character of these methods generates interaction between the sources in the pseudo-spectrum, which is a source of performance limitation, for a finite duration observation, in the presence of modelling errors or for poorly angularly separated sources for example.

To overcome the previous limitation, it is necessary to transform the multidimensional search of minima into $\hat{P}$ monodimensional searches of optima, which can be easily done from the source steering vectors estimates and which is precisely the philosophy of the new proposed method. More precisely, from the estimated mixture matrix $\hat{A}$, the new proposed method, called MAXCOR (search for a MAXimum of spatial CORrelation), consists to solve, for each estimated source p ($1 \leq p \leq \hat{P}$), a mono-dimensional problem aiming at finding the direction of arrival (θ, φ) which maximizes the square modulus of a certain spatial correlation coefficient, defined by

$$\hat{C}_{Cor,p}(\theta, \varphi) \overset{\Delta}{=} \frac{\boldsymbol{a}(\theta,\varphi)^{\mathrm{H}} \, \hat{\Pi}_{Maxcor,p} \, \boldsymbol{a}(\theta,\varphi)}{\boldsymbol{a}(\theta,\varphi)^{\mathrm{H}} \boldsymbol{a}(\theta,\varphi)} \tag{25}$$

where

$$\hat{\Pi}_{Maxcor,p} = \frac{\hat{\boldsymbol{a}}_p \, \hat{\boldsymbol{a}}_p^{\mathrm{H}}}{\hat{\boldsymbol{a}}_p^{\mathrm{H}} \hat{\boldsymbol{a}}_p} \tag{26}$$

which is equivalent to minimize the pseudo-spectrum defined by

$$\hat{C}_{Maxcor,p}(\theta, \varphi) \overset{\Delta}{=} 1 - \frac{\boldsymbol{a}(\theta,\varphi)^{\mathrm{H}} \, \hat{\Pi}_{Maxcor,p} \, \boldsymbol{a}(\theta,\varphi)}{\boldsymbol{a}(\theta,\varphi)^{\mathrm{H}} \boldsymbol{a}(\theta,\varphi)} \tag{27}$$

It is obvious that the performance of the MAXCOR method are directly related to those of the blind identification method which generates the estimated matrix $\hat{A}$. Performance of the MAXCOR method from a $\hat{A}$ generated by the FOBIUM method are presented in the next section and compared with those of MUSIC2 and MUSIC4 both with and without modelling errors. Note that following the FOBIUM method, the MAXCOR method is able to process up to N_e statistically independent non Gaussian sources, while MUSIC4 can only process $N_e - 1$ sources [12].

VII. COMPUTER SIMULATIONS

Performance of the FOBIUM method are illustrated in section A whereas those of the MAXCOR method are presented in section B. Note that the sources considered for the simulations are zero-mean cyclostationary sources corresponding to QPSK sources, which is not a problem for the FOBIUM method, according to section II.C, provided the sources do not share the same trispectrum. Nevertheless, for complexity reasons, the empirical estimator of the FO data statistics is still use despite of the cyclostationarity of the sources. This is not a problem since it is shown in [26] that for SO circular sources such as QPSK sources, although biased, the empirical estimator behaves approximatly like an unbiased estimator. Finally the elevation angle of the sources is assumed to be zero.

A. FOBIUM method performance

The performance of the FOBIUM method are presented in this section both for overdetermined and underdetermined mixtures of sources.

A1. *Overdetermined mixtures of sources*

To illustrate the performance of the FOBIUM method for overdetermined mixtures of sources, we assume that 2 statistically independent QPSK sources with a raise cosine pulse shape are received by a Uniform Linear Array (ULA) of $N = 3$ omnidirectional sensors spaced half a wavelength apart. The 2 QPSK sources have the same symbol duration $T = 4T_e$, where T_e is the sample period, the same roll-off $\mu = 0.3$, the same input SNR, have a carrier residu such that $\Delta f_1 \times T_e = 0$, $\Delta f_2 \times T_e = 0.5$, and a direction of arrival equal to θ_1 and θ_2 respectively. The performance for the source q, α_q, is computed and averaged over 300 realizations.

Under these assumptions, the figures 2 to 5, show, for several configurations of SNR and spatial correlation between the sources, the variations of α_1 (α_2 behaves in a same way) at the output of both JADE [9], SOBI [5] and FOBIUM methods, as a function of the number of snapshots L. For figures 2 and 3, the sources are well angularly separated (θ_1 = 90°, θ_2 = 131.76°) and such that their SNR is equal to 10 and 0 dB respectively. For figures 4 and 5, the sources are poorly angularly separated (θ_1 = 90°, θ_2 = 82.7°) and such that their SNR is equal to 10 and 0 dB respectively. For the SOBI method $K = 8$ delays, τ^k ($1 \leq k \leq 8$), are considered such that $\tau^k = k\ T_e$, whereas for the FOBIUM method $K = 8$ delays set, $(\tau_1{}^k, \tau_2{}^k, \tau_3{}^k)$, are taken into account such that $\tau_1{}^k = \tau^k$ and $\tau_2{}^k = \tau_3{}^k = 0$.

Figures 2 and 3 show that for well angularly separated non Gaussian sources having different spectrum and trispectrum, JADE, SOBI and FOBIUM methods succeed in blindly identifying the sources steering vectors with a very high quality ($\alpha_i \leq 0.01$, $1 \leq i \leq 2$) from a relatively weak number of snapshots and even for weak sources ($L \approx 100$ for SNR = 10 dB and $L \approx 600$ for SNR = 0 dB). Nevertheless, in such situations, we note the best behavior of the SOBI method with respect to FO methods and the best behavior of JADE with respect to FOBIUM, whatever the source SNR and the number of snapshots, due to a higher variance of the FO statistics estimators.

Figure 4 confirms the very good behavior of the three methods from a very weak number of snapshots ($L \approx 100$) even when the sources are poorly angularly separated provided the SNR is not too low (SNR = 10 dB for figure 4). However, figures 4 and 5 show that for poorly angularly separated sources, there exists a number of snapshots, L_0, increasing with the source SNR ($L_0 \approx 265$ for SNR = 10 dB and $L_0 \approx 150$ for SNR = 0 dB), over which the FOBIUM method becomes much more efficient than the JADE and SOBI methods. In such situations, the resolution gain obtained with FOBIUM is higher than the loss due to a higher variance in the statistics estimates. In particular, for sources with a SNR equal to 0 dB, figure 5 shows a very high source identification quality ($\alpha_i \leq 0.01$, $1 \leq i \leq 2$) with the FOBIUM method for $L > 2600$ while the JADE and SOBI methods generate coefficients α_i only around 0.05 for $L = 4000$.

Figure 2

Figure 3

Figure 4

Figure 5

A2. *Underdetermined mixtures of sources*

To illustrate the performance of the FOBIUM method for underdetermined mixtures of sources, we assume first that 5 statistically independent QPSK sources with a raise cosine pulse shape are received by an array of $N = 3$ omnidirectional sensors. The 5 QPSK sources have the same symbol duration $T = 4T_e$, the same roll-off $\mu = 0.3$, the same input SNR of 20 dB, a carrier residu such that $\Delta f_1 \mathrm{x} T_e = 0$, $\Delta f_2 \mathrm{x} T_e = 1/2$, $\Delta f_3 \mathrm{x} T_e = 1/3$, $\Delta f_4 \mathrm{x} T_e = 1/5$, $\Delta f_5 \mathrm{x} T_e = 1/7$ and a direction of arrival given by $\theta_1 = 90°$, $\theta_2 = 120.22°$, $\theta_3 = 150.65°$, $\theta_4 = -52.05°$, $\theta_5 = -76.32°$ respectively. The performance for the source q, α_q, is still computed and averaged over 300 realizations. For the FOBIUM method $K = 8$ delays set, $(\tau_1^k, \tau_2^k, \tau_3^k)$, are taken into account such that $\tau_1^k = k\ T_e$ and $\tau_2^k = \tau_3^k = 0$ $(1 \leq k \leq 8)$. Under these assumptions, the figures 6 and 7 show the variations of all the coefficients α_q $(1 \leq q \leq 5)$, at the output of the FOBIUM method, as a function of the number of snapshots L. For figure 6, a ULA of 3 sensors spaced half a wavelength apart is considered whereas for figure 7, the array of sensors corresponds to an Uniformly Circular Array (UCA) such that $r/\lambda = 0.5$ (r is the radius and λ is the wavelength). Note that the two considered array of sensors have the same aperture on the x-axis if the sensors of the ULA lie on this axis.

Figures 6 and 7 show that for both the ULA and UCA arrays, as long as $P \leq N_e$ ($N_e = 5$ for the ULA and $N_e = 7$ for the UCA), the FOBIUM method succeeds in blindly identifying the sources steering vectors with a high quality ($\alpha_i \leq 0.03$, $1 \leq i \leq 5$) in underdetermined contexts as soon as the number of snapshots is enougth ($L > 6460$ for the ULA and $L > 4930$ for the UCA). Nevertheless, the comparison of figures 2 and 6 shows that, for a given array of sensors, the number of snapshots L required to obtain a high blind identification quality of all the source steering vectors increases with the number of sources ($L < 100$ for $P = 2$ and $L = 6460$ for $P = 5$ for a ULA of 3 sensors). On the other hand, the comparison of figures 6 and 7 shows that for a given number P and scenario of sources, the required number of snapshots K ensuring a high quality of source steering vectors identification increases as the quantity $N_e - P$ decreases. Note that the quantity $N_e - P$ (0 for the ULA and 2 for the UCA) corresponds to the number of degrees of freedom in excess for the FO virtual array associated with the considered array of sensors.

Figure 6

Figure 7

We now decide to add one QPSK source with a raise cosine pulse shape to the 5 previous ones. The source 6 has the symbol duration $T = 3T_e$, the same roll-off $\mu = 0.3$, the same input SNR of 20 dB, a carrier residu such that $\Delta f_6 \times T_e = 0.5$ and a direction of arrival given by $\theta_6 = 66.24°$. For the FOBIUM method $K = 8$ delays set, $(\tau_1^k, \tau_2^k, \tau_3^k)$, are still taken into account such that $\tau_1^k = kT_e$ and $\tau_2^k = \tau_3^k = 0$. Under these new assumptions, the figures 8 and 9 show again the variations of all the coefficients α_q $(1 \leq q \leq 6)$, at the output of the FOBIUM method, as a function of the number of snapshots L. For figure 8, a ULA of 3 sensors is considered whereas for figure 9, the UCA of 3 sensors is considered.

The comparison of figures 7 and 9 confirms, for a given array of sensors and as long as $P \leq N_e$, the increasing value of L required to obtain a good blind identification of all the source steering vectors ($\alpha_i \leq 0.05$, $1 \leq i \leq 6$) as the number of sources increases ($L > 2400$ for $P = 5$ and $L > 8400$ for $P = 6$, for a UCA of 3 sensors). However, the comparison of figures 6 and 8, for the ULA with 3 sensors, shows off the limitations of the FOBIUM method and the poor identification quality of some sources ($\exists\, i$ such that $\alpha_i > 0.09$), even for large values of L ($L = 10000$) as soon as $P > N_e$.

Figure 8

Figure 9

B. MAXCOR method performance

The performance of the MAXCOR method, which extracts the direction of arrival of the sources from the source steering vectors blindly identified by the FOBIUM method, are presented in this section both in the absence and in the presence of modelling errors.

B1. *Performance criterion*

For each of the P considered sources and for each of the three considered direction finding methods, two criterions are used in the following to quantify the quality of the associated direction of arrival estimation. For a given source, the first criterion is a probability of aberrant results generated by a given method for this source and the second one is an averaged Root Mean Square Error (RMSE), computed from the non aberrant results, generated by a given method for this source.

More precisely, for a given method, a given number of snapshots, L, and a particular realization of the L observation vectors $\boldsymbol{x}(l)$ $(1 \leq l \leq L)$, the estimation, $\hat{\theta}_p$, of the direction of arrival of the source p $(1 \leq p \leq P)$ is defined by

$$\hat{\theta}_p \overset{\Delta}{=} \underset{\zeta_i}{\mathrm{Arg}} \, (\underset{i}{\mathrm{Min}} \, |\zeta_i - \theta_p|) \tag{28}$$

where, for MUSIC2 and MUSIC4 methods, the quantities ζ_i $(1 \leq i \leq \hat{P})$ correspond to the $\hat{P}$ minima of the pseudo-spectrum $\hat{C}_{Music2}(\theta)$ and $\hat{C}_{Music4}(\theta)$ respectively, defined by (23) and (24), and where, for MAXCOR method, ζ_i corresponds to the minimum of $\hat{C}_{Maxcor,i}(\theta)$, $(1 \leq i \leq \hat{P})$. To each estimate $\hat{\theta}_p$ $(1 \leq p \leq P)$, we associate the corresponding value of the pseudo-spectrum, defined by $\eta_p = \hat{C}_{Music2}(\hat{\theta}_p)$ for MUSIC2, $\eta_p = \hat{C}_{Music4}(\hat{\theta}_p)$ for MUSIC4 and $\eta_p = \hat{C}_{Maxcor,ip}(\hat{\theta}_p)$ for MAXCOR, where ip is the integer i which minimizes $|\zeta_i - \theta_p|$. In this context, the estimate $\hat{\theta}_p$ is considered to be aberrant if $\eta_p > \eta$, where η is a treshold to be defined. In the following $\eta = 0.1$.

Let us now consider M realizations of the L observation vectors $\boldsymbol{x}(l)$ $(1 \leq l \leq L)$. For a given method, the probability of abberant results for a given source p, $p(\eta_p > \eta)$, is defined by the ratio between the number of realizations for which $\hat{\theta}_p$ is aberrant and the number of realizations M. From the non aberrant realizations for the source p, we then define the averaged RMS error for the source p, RMSE_p, by the quantity

$$\mathrm{RMSE}_p \overset{\Delta}{=} \sqrt{\frac{1}{M_p} \sum_{m=1}^{M_p} \left| \hat{\theta}_{pm} - \theta_p \right|^2} \tag{29}$$

where M_p is the number of non aberrant realizations for the source p and $\hat{\theta}_{pm}$ is the estimate of θ_p for the non aberrant realization m.

B2. *Absence of modelling errors*

To illustrate the performance of the MAXCOR method in the absence of modelling errors, we consider the scenarios of figures 2 and 4 respectively for which two QPSK sources, well and poorly angularly separated respectively, and such that SNR = 10 dB, are received by an ULA of 3 sensors.

Under the assumptions of figure 2 (sources with a large angular separation), the figure 10 shows the variations, as a function of the number of snapshots L, of the RMS error for the source 1, RMSE_1, and the associated probability of non abberant results, $p(\eta_1 \leq \eta)$, (we obtain similar results for the source 2), estimated from $M = 300$ realizations, at the output of both MAXCOR, MUSIC2 and MUSIC4 methods. The figure 11 shows the same variations as those of figure 10 but under the assumptions of figure 4 (sources with a weak angular separation).

Figure 10 (b) shows that the probability of aberrant realizations for the source 1 is zero for all the methods as soon as L becomes greater than 120. In this context, figure 10 (a) shows that for well angularly separated non Gaussian sources having different spectrum and trispectrum and a SNR equal to 10 dB, the three methods succeed in estimating the direction of arrival of the two sources with a high precision ($\text{RMSE}_i \leq 0.5°$, $1 \leq i \leq 2$) from a relatively weak number of snapshots ($L \approx 90$ for MUSIC2 and $L \approx 180$ for MUSIC4 and MAXCOR). Nevertheless, in such situations, we note the best behavior of MUSIC2 method with respect to HO methods MUSIC4 and MAXCOR, which give the same results, due to a higher variance of the FO statistics estimators.

Figure 11 (b) shows that the probability of aberrant realizations for the source 1 is equal to 0 for MAXCOR whatever the value of L but remains greater than 20% for $L < 480$ for MUSIC2 and MUSIC4. Both in terms of probability of non aberrant results and estimation precision, figures 11 (a) and 11 (b) show, for poorly angularly separated sources, the best behavior of the MAXCOR method which becomes much more efficient than MUSIC4 and MUSIC2 methods. Indeed, MAXCOR succeeds in estimating the direction of arrival of the two sources with a high precision ($\text{RMSE}_i \leq 0.5°$, $1 \leq i \leq 2$) from a relatively weak number of snapshots ($L \approx 230$) while MUSIC4 and MUSIC2 require $L \approx 1500$ and $L \approx 3200$ snapshots respectively to obtain the same precision. In such situations, the resolution gain obtained with MAXCOR and MUSIC4 with respect to MUSIC2 is higher than the loss due to a higher variance in the statistics estimates. Besides, the monodimensionality character of the MAXCOR method with respect to MUSIC4 jointly with the very high resolution power of the FOBIUM method explain the best behavior of MAXCOR with respect to MUSIC4.

Figure 10

Figure 11

B3. *Presence of modelling errors*

We now consider the simulations of section B2 but with modelling errors due for instance to a non perfect equalization of the reception chains. In the presence of such errors, the steering vector $\boldsymbol{a}_p$ of the source p is not the known function, $\boldsymbol{a}(\theta_p, \varphi_p)$, of the direction of arrival (θ_p, φ_p) but becomes an unknown function, $\tilde{\boldsymbol{a}}(\theta_p, \varphi_p) = \boldsymbol{a}(\theta_p, \varphi_p) + \boldsymbol{e}(\theta_p, \varphi_p)$, of (θ_p, φ_p), where $\boldsymbol{e}(\theta_p, \varphi_p)$ is a modelling error vector. In such conditions, the previous HR methods loss their infinite asymptotic resolution and the question is to search for a method which presents some robustness to the modelling errors. To solve this problem, we assume that the vector $\boldsymbol{e}(\theta_p, \varphi_p)$ is a zero-mean, Gaussian, circular vector with independent components such that $E[\mathbf{e}_p\ \mathbf{e}_p^H] = \sigma_e^2\ \mathbf{I}_N$. Note that for omnidirectional sensors and small errors, σ_e^2 is the sum of the phase and amplitude error variances per reception chain. For the simulations, σ_e is chosen to be equal to 0.0174, which corresponds for example to a phase error with a standard deviation of 1° without any amplitude error.

In this context, under the assumptions of figure 10 (sources with a large angular separation) but with modelling errors, the figure 12 shows the variations, as a function of the number of snapshots L, of the RMS error for the source 1, $RMSE_1$, and the associated probability of non abberant results, $p(\eta_1 \leq \eta)$, (we obtain similar results for the source 2), estimated from $M = 300$ realizations, at the output of both MAXCOR, MUSIC2 and MUSIC4 methods. The figure 13 shows the same variations as those of figure 12 but under the assumptions of figure 11 (sources with a weak angular separation) with modelling errors.

Figure 12 (b) shows that the probability of aberrant realizations for the source 1 is zero for all the methods as soon as L becomes greater than 135. In this context, comparison of figures 10 and 12 shows a degradation of the performance of each method in the presence of modelling errors. However, for well angularly separated sources, MUSIC2 is more affected by the presence of modelling errors than FO methods as soon as the number of snapshots is sufficient. Indeed, while MUSIC2 remains better than FO methods for a relatively weak number of snapshots ($L < 500$), due to a higher variance of HO methods, MUSIC4 and MAXCOR, equivalent to each other, become better than MUSIC2 as soon as the number of snapshots is sufficient ($L > 500$). In this latter case, the higher number of sensors of the FO virtual array ($N_e = 5$) with respect to that of the true array ($N = 3$) reduces the effect of modelling errors on the performances of FO methods.

Figure 13 (b) shows that the probability of aberrant realizations for the source 1 is equal to 0 for MAXCOR whatever the value of L but remains greater than 20% for $L < 1180$ for MUSIC2 and MUSIC4. Both in terms of probability of non aberrant results and estimation precision, comparison of figures 11 and 13 shows again a degradation of the performance of all the methods in the presence of modelling errors. However, for poorly angularly separated sources, whatever the value of the number of snapshots, MUSIC2 is much more affected by the modelling errors than FO methods, as soon as L

> 700, due to a greater aperture and number of sensors of the FO virtual array with respect to the true array. Note again, in the presence of modelling errors, the best performance of MAXCOR with respect to MUSIC4 for poorly angularly separated sources.

Figure 12

Figure 13

VIII. NUMERICAL COMPLEXITY COMPUTATION

This section aims at giving some insights into the relative numerical complexity of SOBI, JADE and FOBIUM methods for given values of N, P, L and the number of sweeps, I, required by the joint diagonalization process [5] [9]. The numerical complexity of the methods is presented in terms of number of flotting complex operations (Flocops) required to identify the mixture matrix A from L snapshots of the data. Note that a flocop corresponds to the sum of a complex multiplication and a complex addition.

The number of flocops required by JADE, SOBI and FOBIUM methods for given values of N, P, L and I are given by :

$$\text{Comp[JADE]} = \text{Min}[\,LN^2/2 + 4N^3/3 + PNL,\ 2LN^2] + \text{Min}[\,4P^6/3,\ 8P^3(P^2+3)] + 3LP^3(1 + P/2)/4 + IP^2(75 + 21P + 4P^2)/2 + LP^2 \quad (30)$$

$$\text{Comp[SOBI]} = MLN^2/2 + 4N^3/3 + (M-1)N^3/2 + IP(P-1)[\,4P(M-1) + 17(M-1) + 4P + 75]/2 \quad (31)$$

$$\text{Comp[FOBIUM]} = 3MLN^6(1 + N^2/2)/4 + 4N^6/3 + (M-1)N^6/2 + IP(P-1)[\,4P(M-1) + 17(M-1) + 4P + 75]/2 \quad (32)$$

where M is the number of correlation and quadricovariance matrices jointly diagonalized by the SOBI and FOBIUM methods respectively.

For a given number of sources P, the minimum complexity of the previous methods is obtained by minimizing the values of I, N, M and L ensuring the good identification of the mixture matrix A. It is said in [14] that the minimum value of I is $I_{\min} = 1 + \text{Int}(P^{1/2})$ where Int means integer part. The minimum value of M depends on the spectral difference between the sources and is chosen to be equal to $M_{\min} = 2$ in the following. The minimum value of N is equal to $N_{\min} = P$ for JADE and SOBI whereas for FOBIUM, assuming an array with space diversity only, it corresponds to the minimum value, $N_{\min}$, such that $N_{\min} \leq P \leq N_{\min}^2 - N_{\min} + 1$. Finally, the minimum value of L depends on several parameters such as P, N, the FO autocumulant and the SNR of the sources…. For this reason L is chosen to be the same for all the methods in the following.

Under these assumptions, figure 14 shows the variations of the minimum numerical complexity of JADE, SOBI and FOBIUM as a function of the number of sources P for $L = 1000$. Note the higher complexity of FOBIUM with respect to JADE and SOBI which requires about 1 Mflocops to process 4 sources from 1000 snapshots.

Figure 14

IX. CONCLUSION

A new BI method, exploiting the FO data statistics only and called FOBIUM, has been presented in this paper to process both overdetermined and underdetermined instantaneous mixtures of statistically independent sources. This method has not the drawbacks of the existing methods capable of processing underdetermined mixtures of sources and is able to put up with any kind of sources, analogical or digital, circular or not, i.i.d or not, with potential different symbol duration… It only requires non Gaussian sources having kurtosis with the same sign (practically always verified in radiocommunications contexts) and sources having different trispectrum, which is the only limitation of the method. The FOBIUM method is capable of processing up to $N^2 - N + 1$ sources, from an array of N sensors with space diversity only, and up to N^2sources, from an array of N different sensors. A consequence of this result is that it allows to drastically reduce or to minimize the number of sensors for a given number of sources, which finally may generate a receiver much less expensive than a receiver developed to process overdetermined mixtures only. The FOBIUM method has been shown to require a relatively weak number of snapshots to generate good output performances for currently used radiocommunications sources such as QPSK sources. Besides, exploiting the FO data statistics only, the FOBIUM method is robust to the presence of a Gaussian noise whose spatial coherence is unknown. Finally, an application of the FOBIUM method has been presented through the introduction of a new FO direction finding method, built from the blindly identified mixing matrix and called MAXCOR. The comparison of this method to both SO and FO HR subspace-based direction finding methods shows the better resolution and the better robustness to modelling errors of the MAXCOR method with respect to MUSIC2 and MUSIC4 and its ability to process underdetermined mixtures of up to N^2 statistically independent non Gaussian sources.

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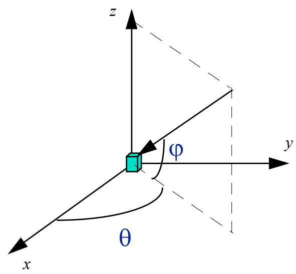


Figure 1 - *An incoming signal in three dimensions*

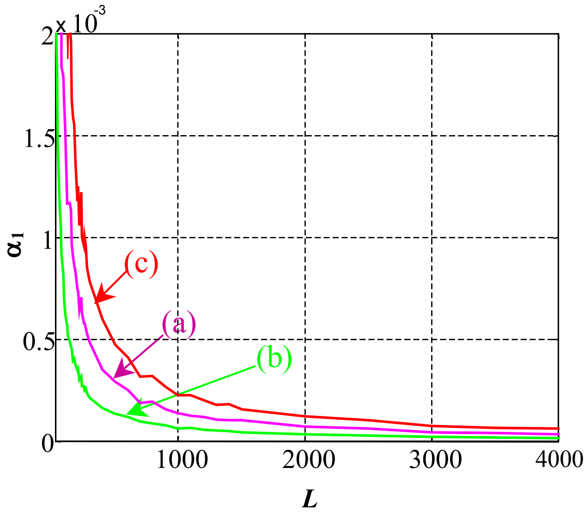


Figure 2 – *α_1 as a function of L, (a) JADE, (b) SOBI, (c) FOBIUM,*
P = 2, N = 3, ULA, θ_1 = 90°, θ_2 = 131.76°, SNR = 10 dB

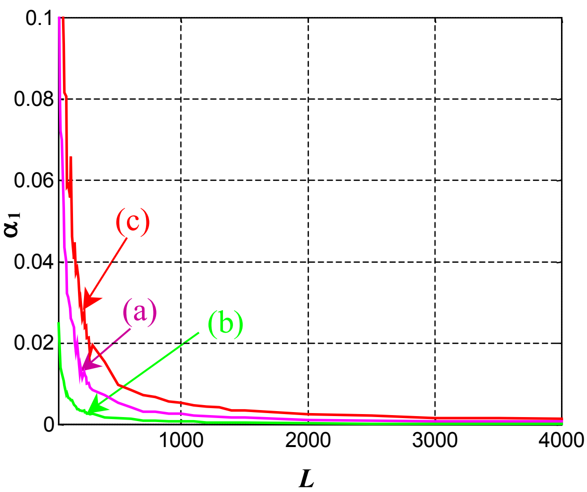


Figure 3 – *α_1 as a function of L, (a) JADE, (b) SOBI, (c) FOBIUM,*
P = 2, N = 3, ULA, θ_1 = 90°, θ_2 = 131.76°, SNR = 0 dB

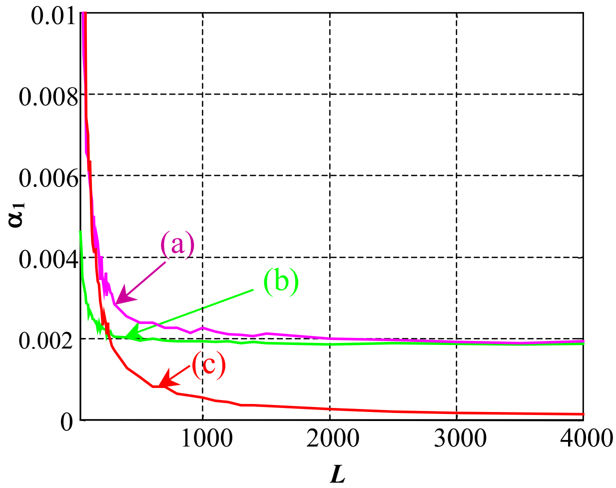


Figure 4 – α_1 *as a function of L, (a) JADE, (b) SOBI, (c) FOBIUM,*
P = 2, N = 3, ULA, $\theta_1 = 90°$, $\theta_2 = 82{,}7°$, *SNR =10dB*

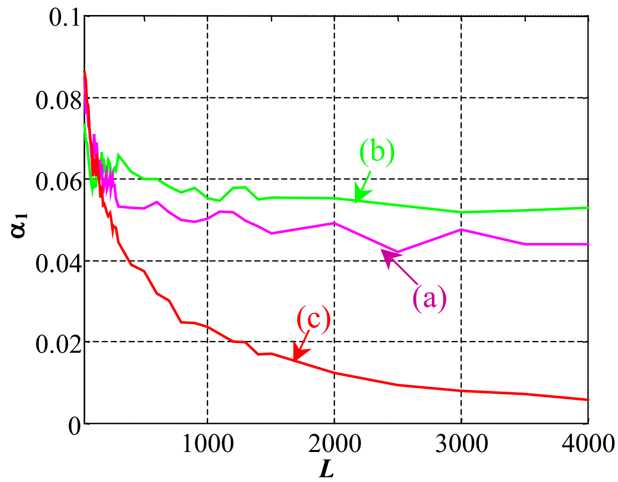


Figure 5 – α_1 *as a function of L, (a) JADE, (b) SOBI, (c) FOBIUM,*
P = 2, N = 3, ULA, $\theta_1 = 90°$, $\theta_2 = 82{,}7°$, *SNR = 0 dB*

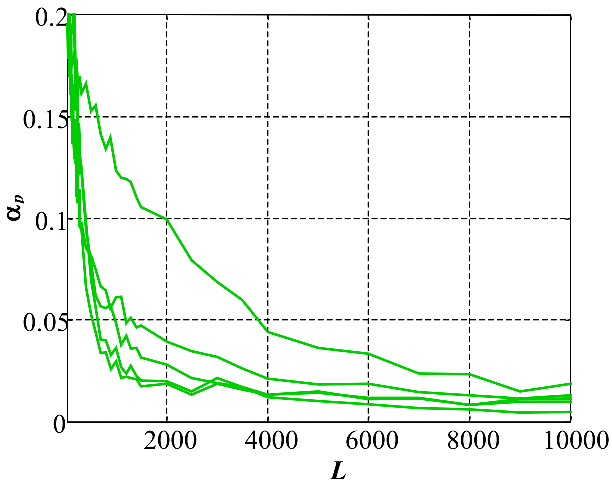


Figure 6 – *α_p (1 ≤ p ≤ 5) as a function of L at the output of FOBIUM, (p) : α_p*
P = 5, N = 3, ULA, θ_p = 90°, 120.22°, 150.65°, - 52.05°, -76.32°

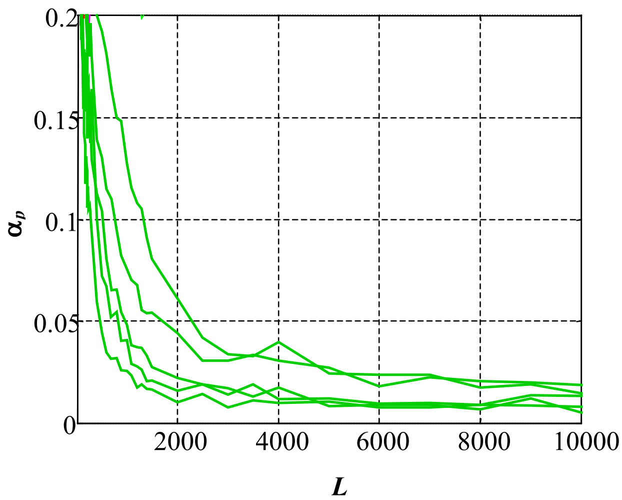


Figure 7 – *α_p (1 ≤ p ≤ 5) as a function of L at the output of FOBIUM, (p) : α_p*
P = 5, N = 3, UCA, θ_p = 90°, 120.22°, 150.65°, - 52.05°, -76.32°

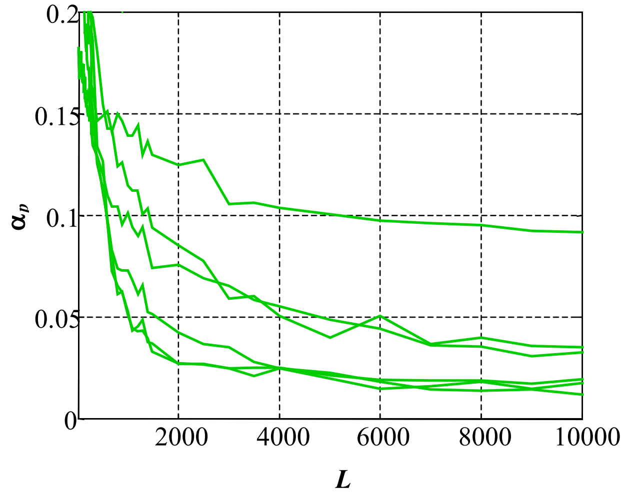


Figure 8 – *α_p ($1 \le p \le 6$) as a function of L at the output of FOBIUM, (p) : α_p*
P = 6, N = 3, ULA, θ_p = 90°, 120.22°, 150.65°, - 52.05°, -76.32°, 66.24°

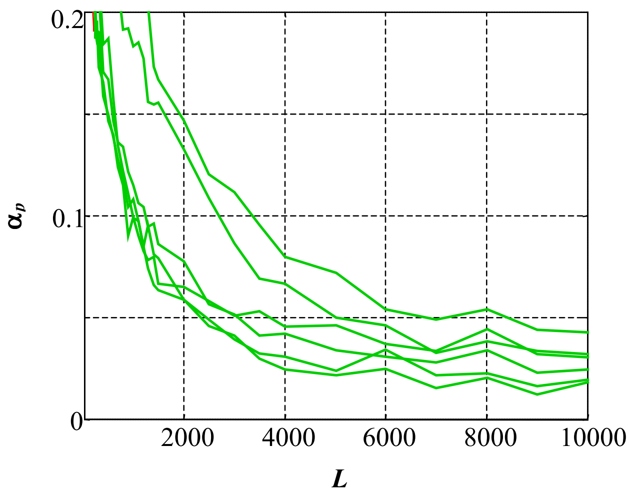
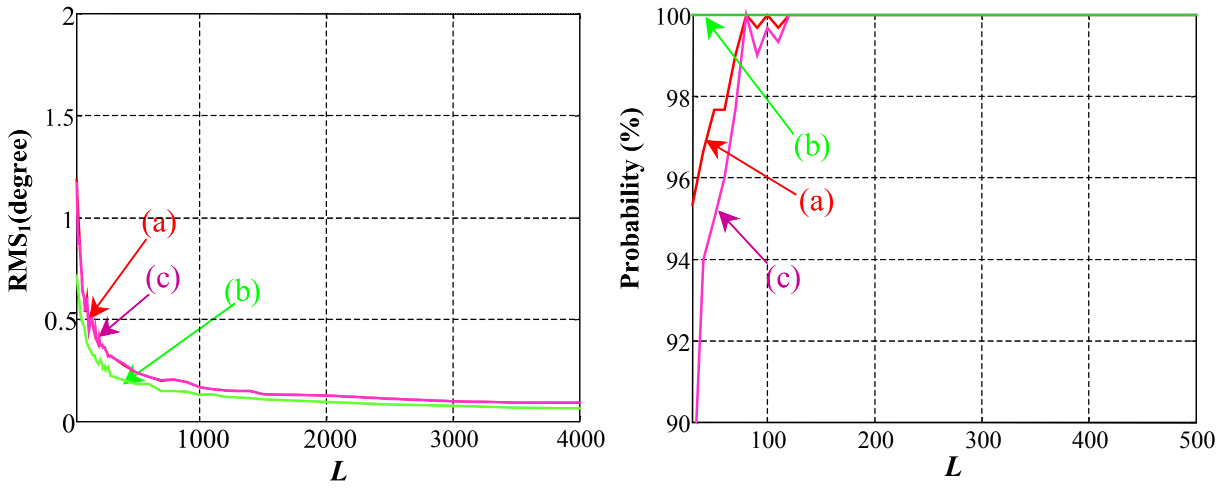
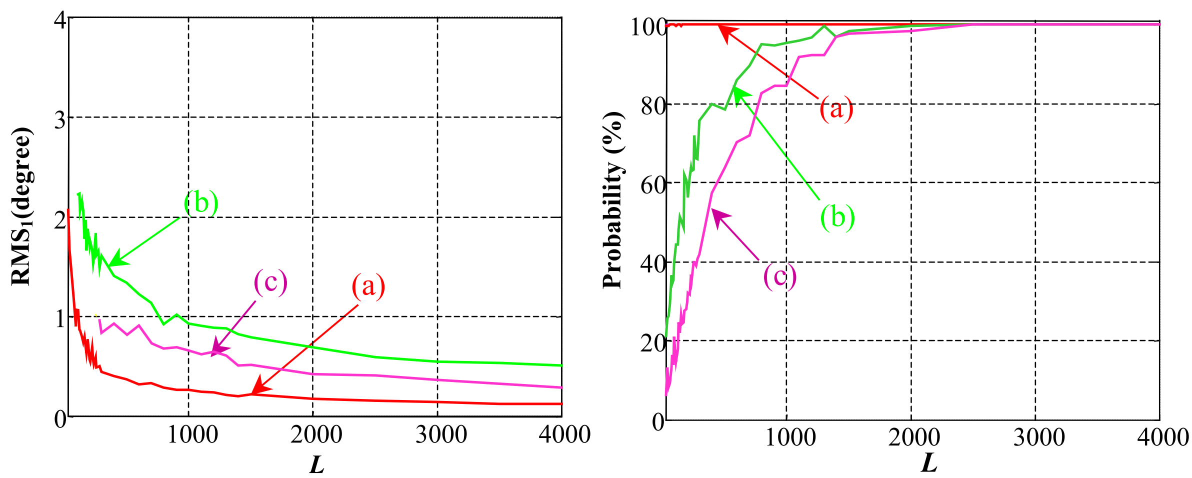


Figure 9 – *α_p ($1 \le p \le 6$) as a function of L at the output of FOBIUM, (p) : α_p*
P = 6, N = 3, UCA, θ_p = 90°, 120.22°, 150.65°, - 52.05°, -76.32°, 66.24°



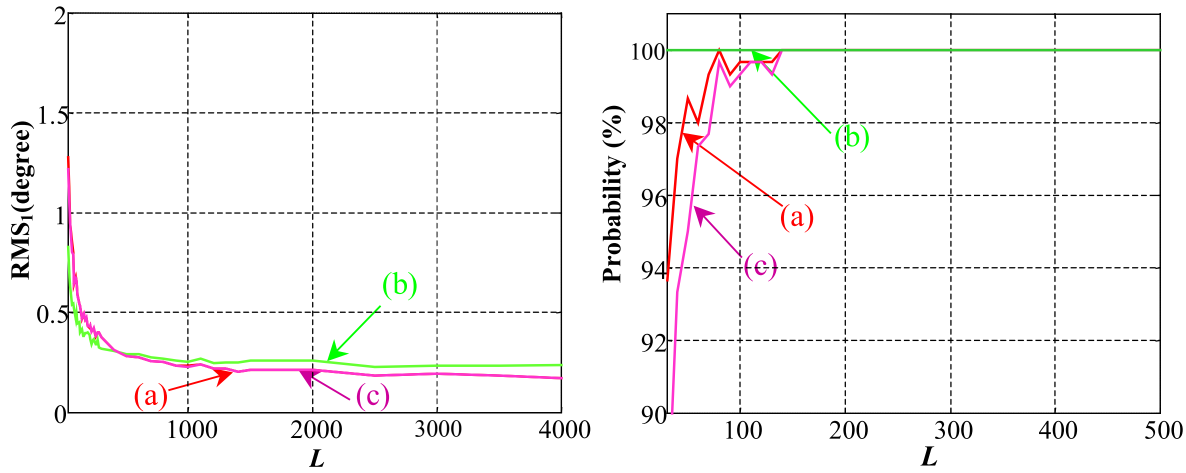
(a) (b)

Figure 10 – *RMS error of the source 1 and $p(\eta_1 \le \eta)$ as a function of L, (a) MAXCOR, (b) MUSIC2, (c) MUSIC4, P = 2, N = 3, ULA, SNR = 10 dB, θ_1 =90°, θ_2 =131.76°, no modelling errors*



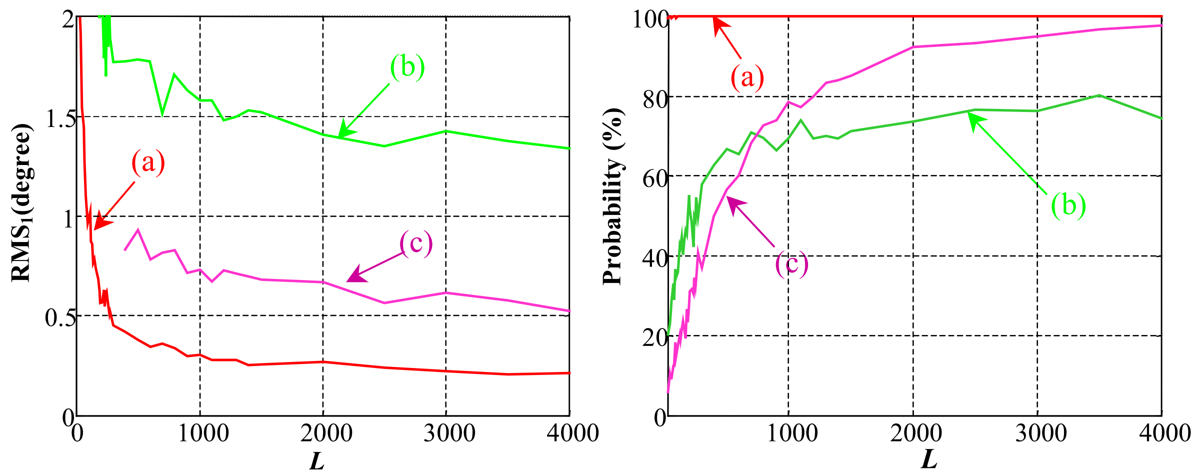
(a) (b)

Figure 11 – *RMS error of the source 1 and $p(\eta_1 \le \eta)$ as a function of L, (a) MAXCOR, (b) MUSIC2, (c) MUSIC4, P = 2, N = 3, ULA, SNR = 10 dB, θ_1 =90°, θ_2 =82,7°, no modelling errors*



(a) (b)

Figure 12 – *RMS error of the source 1 and* $p(\eta_1 \le \eta)$ *as a function of L, (a) MAXCOR, (b) MUSIC2, (c) MUSIC4, P = 2, N = 3, ULA, SNR = 10 dB,* $\theta_1 = 90°$, $\theta_2 = 131.76°$, *with modelling errors*



(a) (b)

Figure 13 – *RMS of the source 1 and* $p(\eta_1 \le \eta)$ *as a function of L, (a) MAXCOR, (b) MUSIC2, (c) MUSIC4, P = 2, N = 3, ULA, SNR = 10 dB,* $\theta_1 = 90°$, $\theta_2 = 82{,}7°$, *with modelling errors*

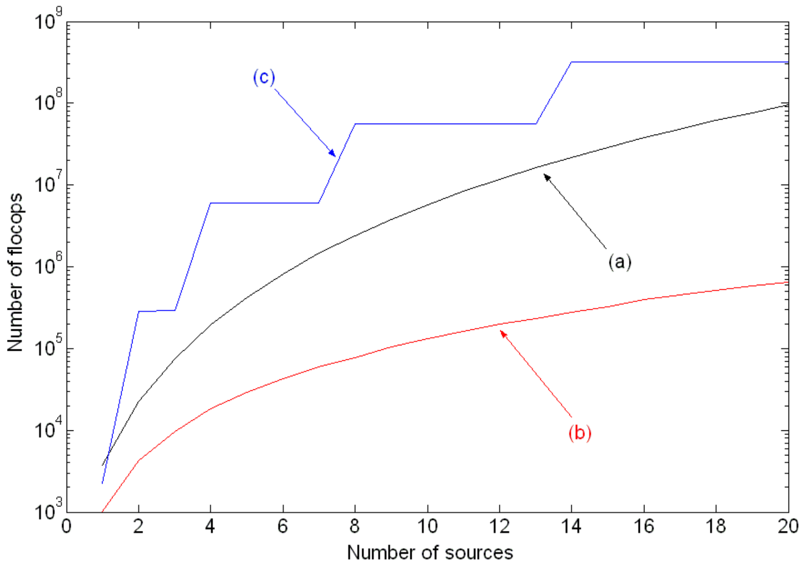


Figure 14 – *Minimum numerical complexity as a function of P*
JADE (a), SOBI (b), FOBIUM (c)

List of figures and tables