ON THE BEHAVIOR OF CURRENT SECOND ORDER BLIND SOURCE SEPARATION METHODS FOR FIRST AND SECOND ORDER CYCLOSTATIONARY SOURCES – APPLICATION TO CPFSK SOURCES

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ABSTRACT

Most of the current Second Order (SO) and Higher Order (HO) blind source separation (BSS) methods aim at blindly separating statistically independent sources, assumed zero-mean, stationary and ergodic. However in practical situations, such as in radiocommunications contexts, the sources are non stationary and very often cyclostationary. In a previous paper [5] the cumulant-based BSS problem for cyclostationary sources has been analysed assuming zero-mean sources (linear modulations). The purpose of this paper is to analyse the behavior and to propose adaptations of the current SO BSS methods for cyclostationary sources, assumed cyclo-ergodic, which are not zero-mean but first order and SO cyclostationary, such as some CPFSK sources (non linear modulations).

1. INTRODUCTION

For more than a decade, SO [1] and HO [2] [4] blind methods have been developed to separate several statistically independent sources, assumed *zero-mean*, *stationary* and *ergodic*. However, in many applications such as in radiocommunications contexts, the sources are non stationary and very often *cyclostationary* (digital modulations). It then becomes important to analyse the behavior of the current SO and HO BSS methods in cyclostationary contexts.

In a previous paper [5], the behavior of the current SO and Fourth-order (FO) cumulant-based BSS methods has been analysed for cyclostationary sources assumed zero-mean sources. It has been shown in particular that the current SO blind methods are not affected by the cyclostationarity of the sources whereas the current FO blind methods may be strongly affected by this property.

Nevertheless, some cyclostationary sources used in practice are not zero-mean but are first order cyclostationary, which is in particular the case for some non linearly modulated digital sources such as some Continuous Phase Frequency Shift Keying (CPFSK) sources, belonging to the family of Continuous Phase Modulations (CPM) sources [7].

For this reason, our goal in this paper is to analyse the behavior and to propose adaptations of the current SO BSS methods in the presence of statistically independent sources which are both first order and SO cyclostationary.

2. PROBLEM FORMULATION

A noisy mixture of P narrow-band (NB) statistically independent sources is assumed to be received by an array of N sensors. The vector, $\boldsymbol{x}(t)$, of the complex envelopes of the signals at the output of the sensors is thus given by

$$\boldsymbol{x}(t) = \sum_{p=1}^{P} m_{pc}(t)\, \boldsymbol{a}_p + \boldsymbol{b}(t) \overset{\Delta}{=} A\, \boldsymbol{m}_c(t) + \boldsymbol{b}(t) \qquad (1)$$

where $m_{pc}(t) = m_p(t) e^{j(2\pi\, \Delta fp\ t\ +\ \Phi p)}$ is the p-th component of the vector $\boldsymbol{m}_c(t)$, $\boldsymbol{b}(t)$ is the noise vector, assumed zero-mean, stationary, spatially white and temporally white in the reception band, $m_p(t)$, Δf_p, ϕ_p and $\boldsymbol{a}_p$ correspond to the complex envelope, the carrier residu, the phase and the steering vector of the source p respectively, A is the (NxP) matrix whose columns are the vectors $\boldsymbol{a}_p$.

The classical SO blind source separation problem consists to find, from the SO statistics of the observations, the (NxP) *Linear* and *Time Invariant* source separator W, whose (Px1) output vector $\boldsymbol{y}(t) \overset{\Delta}{=} W^{H}\, \boldsymbol{x}(t)$ corresponds, to within a diagonal matrix Λ and a permutation matrix Π, to the best estimate, $\hat{\boldsymbol{m}}_c(t)$, of the vector $\boldsymbol{m}_c(t)$.

3. SO BLIND SOURCE SEPARATION FOR ZERO MEAN STATIONARY SOURCES

3.1. SO Statistics of the data

Under the assumption of stationary sources, the SO statistics of the observations are characterized by the correlation matrices $R_x(\tau)$, given by

$$R_x(\tau) \overset{\Delta}{=} \mathrm{E}[\boldsymbol{x}(t)\, \boldsymbol{x}(t-\tau)^{H}] = A\, R_{mc}(\tau)\, A^{H} + \sigma^2\, \delta(\tau)\, \mathrm{I} \qquad (2)$$

where $\delta(\tau)$ is the Kronecker function, I denotes the (NxN) identity matrix, σ^2 is the input noise power per sensor, $R_{mc}(\tau) \overset{\Delta}{=} \mathrm{E}[\boldsymbol{m}_c(t)\, \boldsymbol{m}_c(t-\tau)^{H}]$ is the correlation matrix of the vector $\boldsymbol{m}_c(t)$**,** diagonal for zero-mean statistically independent sources.

3.2. Philosophy of the SOBI method

Under the previous assumptions, assuming the sources have different spectral contents, the current SOBI method [1] aims at separating the sources from the SO blind identification of the A matrix. This requires the prewhitening of the data which orthonormalizes the sources steering vectors so as to search for the latter through a unitary (PxP) matrix U simpler to handle. If we note $\boldsymbol{z}(t)$ the prewhitened observation vector, the matrix U is chosen so as to jointly diagonalize several correlation matrices, $R_z(\tau)$, of $\boldsymbol{z}(t)$, for several non zero values τ_κ, of τ where $R_z(\tau)$ is given by

$$R_z(\tau) \triangleq \mathrm{E}[\boldsymbol{z}(t)\, \boldsymbol{z}(t-\tau)^{\mathrm{H}}] = A'\, R_{mc'}(\tau)\, A'^{\mathrm{H}} \quad (\tau \neq 0) \quad (3)$$

where A' is the unitary matrix of the prewhitened source steering vectors and $R_{mc'}(\tau)$ is the correlation matrix of $\boldsymbol{m}_{c'}$(t), the normalized vector $\boldsymbol{m}_c(t)$ such that each component has a unit power. Under some conditions [1], it is easy to verify that the unitary matrix A' is, to within a permutation and an unitary diagonal matrix, the only one which jointly diagonalizes the set of K matrices $R_z(\tau_\kappa)$.

3.3. Implementation of the SOBI method

In practical situations, the SO statistics of the data have to be estimated, by temporal averaging operations, using the SO ergodicity property of the data. Under these assumptions, noting T_e the sample period and $\boldsymbol{x}(l)$ the l-th sample of the observation vector $\boldsymbol{x}(t)$, the empirical estimator $\hat{R}_x(qT_e)(L)$ of the matrix $R_x(\tau)$ for $\tau = qT_e$, from L independent data snapshots, is defined by

$$\hat{R}_x(qT_e)(L) \triangleq \frac{1}{L}\sum_{l=1}^{L} x(l)\, x(l-q)^{\mathrm{H}} \quad (4)$$

It is well known that for stationary and ergodic observations, this empirical estimator generates, as L becomes infinite, an unbiased and consistent estimate of the true statistic $R_x(qT_\mathrm{e})$.

4. SO BSS FOR FIRST AND SECOND ORDER CYCLOSTATIONARY SOURCES

4.1. First and SO Statistics of the data

We now assume that the sources are both first and second order cyclostationary, which means that their first and second order statistics are (quasi)-periodic functions of the time.

4.1.1. First order statistics

The first order statistic of $\boldsymbol{x}(t)$, given by (1), is defined by

$$\boldsymbol{e}_x(t) \triangleq \mathrm{E}[\boldsymbol{x}(t)] = A\, \mathrm{E}[\boldsymbol{m}_c(t)] \triangleq A\, \boldsymbol{e}_{mc}(t) \quad (5)$$

and the first order cyclostationarity property of the sources implies that $\boldsymbol{e}_x(t)$ has a Fourier serial expansion given by

$$\boldsymbol{e}_x(t) = \sum_{\gamma\in\Gamma} \boldsymbol{e}_x^{\gamma} e^{\mathrm{j}2\pi\gamma t} = \sum_{p=1}^{P}\sum_{\gamma_p\in\Gamma_p} e_{pc}^{\gamma_p} e^{\mathrm{j}2\pi\gamma_p t}\boldsymbol{a}_p \quad (6)$$

where $\boldsymbol{e}_x^{\gamma} = \langle \boldsymbol{e}_x(t)\, \mathrm{e}^{-\mathrm{j}2\pi\gamma t} \rangle_\mathrm{c}$ is called the cyclic mean of $\boldsymbol{x}(t)$ for the cyclic frequency γ, $\langle.\rangle_\mathrm{c}$ is the continuous-time temporal mean operation, Γ_p defines the set of cyclic frequencies γ_p of $e_{pc}(t) = \mathrm{E}[m_{pc}(t)]$, $\Gamma = \mathrm{U}_{1\le p\le P}\{\Gamma_p\}$ is the set of the cyclic frequencies γ of $\boldsymbol{e}_x(t)$ and $\boldsymbol{e}_{mc}(t)$. Note that we can link the cyclic mean $e_{pc}^{\gamma_p}$ of $m_{pc}(t)$ and the cyclic mean $e_p^{\gamma_p}$ of $m_p(t)$ by $e_{pc}^{\gamma_p} = e_p^{\gamma_p-\Delta f_p} e^{\mathrm{j}\phi_p}$.

4.1.2. Second order statistics

The first correlation matrix of $\boldsymbol{x}(t)$ becomes now Time Dependent (TD) and is given by

$$R_x(t,\tau) \triangleq \mathrm{E}[\boldsymbol{x}(t)\, \boldsymbol{x}(t-\tau)^{\mathrm{H}}] = A\, R_{mc}(t,\tau)\, A^{\mathrm{H}} + \sigma^2\, \delta(\tau)\, \mathrm{I} \quad (7)$$

Introducing the zero-mean vector $\Delta\boldsymbol{m}_c(t) = \boldsymbol{m}_c(t) - \boldsymbol{e}_{mc}(t)$, the correlation matrix of $\boldsymbol{m}_c(t)$, $R_{mc}(t,\tau)$, can be written as $R_{mc}(t,\tau) = R_{\Delta mc}(t,\tau) + E_{mc}(t,\tau)$ where the matrices $R_{\Delta mc}(t,\tau)$ and $E_{mc}(t,\tau)$ are defined by $R_{\Delta mc}(t,\tau) \triangleq \mathrm{E}[\Delta\boldsymbol{m}_c(t)\Delta\boldsymbol{m}_c(t-\tau)^{\mathrm{H}}]$ and $E_{mc}(t,\tau) = \boldsymbol{e}_{mc}(t)\, \boldsymbol{e}_{mc}(t-\tau)^{\mathrm{H}}$ respectively. Moreover, the SO cyclostationary property of the sources implies that the matrices $R_{mc}(t,\tau)$ and $R_{\Delta mc}(t,\tau)$ and thus, the matrices $R_x(t,\tau)$ have Fourier serial expansions introducing the SO cyclic frequencies of $\boldsymbol{m}_c(t)$ and $\boldsymbol{x}(t)$. In particular, the cyclic correlation matrix of $\boldsymbol{x}(t)$ for the zero cyclic frequency corresponds to the temporal mean, $R_\mathrm{x}(\tau)$, of $R_x(t,\tau)$, which has the same form as (2) and is given by

$$R_x(\tau) \triangleq \langle R_x(t,\tau)\rangle_\mathrm{c} = A\, R_{mc}(\tau)\, A^{\mathrm{H}} + \sigma^2\, \delta(\tau)\, \mathrm{I} \quad (8)$$

where $R_{mc}(\tau) \triangleq \langle R_{mc}(t,\tau)\rangle_\mathrm{c} = R_{\Delta mc}(\tau) + E_{mc}(\tau)$.

4.2. Behavior analysis of the SO statistics empirical estimator

For cyclostationary sources, SO BSS methods such as the SOBI method has to exploit the information contained in several matrices $R_x(\tau)$ empirically estimated from (4). For band-limited, cyclo-ergodic and sufficiently oversampled observations, the empirical estimator, $\hat{R}_x(qT_e)(L)$, of $R_x(qT_e)$ is asymptotically unbiased and consistent.

However, while for zero-mean independent sources, the matrix $R_{mc}(\tau)$ is diagonal, it is not necessary the case for first order cyclostationary independent sources for which only the matrix $R_{\Delta mc}(\tau)$ is diagonal while $E_{mc}(\tau) = \langle\boldsymbol{e}_{mc}(t)\boldsymbol{e}_{mc}(t-\tau)^{\mathrm{H}}\rangle_\mathrm{c}$ may be not diagonal. As a consequence, while the current SO BSS methods are not affected by the cyclostationarity of zero-mean sources, they may be affected for first order cyclostationary ones for which, an *apparent SO correlation* of the sources may appear in the $R_x(\tau)$ matrix.

4.3. Structure analysis of the $E_{mc}(\tau)$ matrix

According to (6), the $[i,j]$ element, $E_{mc}(\tau)[i,j] = \langle e_{ic}(t)e_{jc}(t-\tau)^* \rangle_c$, of the $E_{mc}(\tau)$ matrix is given by

$$E_{mc}(\tau)[i,j] = \sum_{\gamma_{ij} \in \{\Gamma_i \cap \Gamma_j\}} e_{ic}^{\gamma_{ij}} e_{jc}^{\gamma_{ij}*} e^{j2\pi\tau\gamma_{ij}} \quad (9)$$

where the γ_{ij}'s are the common first order cyclic frequencies of the processes $m_{ic}(t)$ and $m_{jc}(t)$. This expression shows that $E_{mc}(\tau)[i,j]$ is generally not zero, i.e. that the two sources i and j become *apparently SO correlated* in the matrice $R_x(\tau)$, if $e_{ic}(t)$ and $e_{jc}(t)$ share at least one cyclic frequency.

4.4. Example of CPFSK sources

4.4.1. CPFSK : a particular case of CPM sources

The good spectral efficiency and the constant amplitude of the complex envelope of the CPM [7] are well appreciated, especially in radiocommunications contexts. The CPFSK source is a particular case of the mono-indice full response CPM source which can be written as

$$m_p(t) = \pi_p^{1/2} e^{j2\pi f_{dp}\left[a_n^p(t-nT_p)+T_p\sum_{k=-\infty}^{n-1} a_k^p\right]} Rect_p(t-nT_p) \quad (10)$$

where T_p, h_p, a_n^p, $Rect_p(t)$, $f_{dp} \triangleq h_p/2T_p$, $\pi_p^{1/2}$ are the symbol duration, the modulation indice, the transmitted M_p-ary symbols assumed i.i.d. and taking their values in the alphabet $\pm 1, \pm 3, \ldots, \pm (M_p-1)$, the rectangular pulse of amplitude 1 and of duration T_p, the peak frequency deviation and the amplitude of the source p respectively. Note that M_p is generally a power of two.

4.4.2. Structure analysis of the $E_{mc}(\tau)$ matrix

It is shown in [6] that for a M_p-CPFSK source p having an integer modulation indice, the cyclic mean appearing in (9) take the form

$$e_{pc}^{\gamma_p} = \pm\left(\sqrt{\pi_p}\,/M_p\right)e^{j\Phi_p} \quad (11)$$

where the set of the first order cyclic frequencies of the M_p-CPFSK source p is defined by $\Gamma_p=\{\gamma_p=\Delta f_p \pm (2q_p+1)f_{dp},\ 0\leq q_p \leq (M_p-2)/2\}$. From this result, it is possible to show [6] that for a M_i-CPFSK source i and a M_j-CPFSK source j, a necessary and sufficient condition to obtain $E_{mc}(\tau)[i,j] \neq 0$ is that the conditions a) and b) are verified :

a) *The modulation indices* h_i and h_j are *integer*

b) *$\exists$ (q_i,q_j) with $0\leq q_i \leq (M_i-2)/2$ and $0\leq q_j \leq (M_j-2)/2$ such that $\Delta f_i \pm (2q_i+1)f_{di} = \Delta f_j \pm (2q_j+1)f_{dj}$*

4.5. Behavior of the SOBI method

While, for apparently SO uncorrelated sources, which is in particular the case for zero-mean sources, the whitened mixed matrix A' is an unitary matrix, it is no longer the case for apparently SO correlated sources, for which the vectors $\boldsymbol{a}_p'$ are neither normalized nor orthogonal.

Proof1 : *To show the previous result let us firstly assume that the matrix A' is orthogonal. Under this assumption, as the matrix $R_{s'}(0) \triangleq A'R_{mc'}(0)A'^H$ corresponds to the identity matrix, the matrix $A'^H R_{s'}(0)A' = A'^H A'$ is diagonal and equal to $A'^H A' R_{mc'}(0)A'^H A'$, implying that the matrix $R_{mc'}(0)$ is diagonal, which is not the case for apparently SO correlated sources.*

Proof2 : *Let us now assume that the columns of A' are normalized. In this case, as $A'^H R_{s'}(0)A' = A'^H A' = A'^H A' R_{mc'}(0)A'^H A'$, we obtain that $R_{mc'}(0)A'^H A' = I$, which is not possible, as shown in* [6], *under the previous assumptions if the matrix $R_{mc'}(0)$ is not diagonal.*

A consequence of this result is that, contrary to the zero-mean sources case, the matrix A' does not jointly diagonalizes the set of K matrices $R_z(\tau_k)$ defined by (8), with the indice z instead of x. In other words, the blindly identified source steering vectors are only a linear combination of the source steering vectors, which shows that the SOBI method as well as the SO BSS methods are affected by the presence of apparently SO correlated sources.

4.6. Adaptation : an exhaustive estimator

Since the correlation matrices $R_{mc}(\tau)$ may be non diagonal in the presence of first order cyclostationary sources, we have to exploit the information contained in the covariance matrices $R_{\Delta mc}(\tau)$ which are always diagonal for statistically independent sources, zero-mean or not. In other words, we have to implement the SO BSS methods from the covariance matrix $R_{\Delta x}(\tau)$ defined by

$$R_{\Delta x}(\tau) \triangleq \langle R_{\Delta x}(t,\tau)\rangle_c = R_x(\tau) - E_x(\tau) \quad (12)$$

where $R_x(\tau)$ is defined by (8) and $E_x(\tau)$ given by

$$E_x(\tau) \triangleq \langle \boldsymbol{e}_x(t)\, \boldsymbol{e}_x(t-\tau)^H \rangle_c = \sum_{\gamma \in \Gamma} \boldsymbol{e}_x^{\gamma} \boldsymbol{e}_x^{\gamma H} e^{j2\pi\tau\gamma} \quad (13)$$

where Γ is the set of the first order cyclic frequencies of $\boldsymbol{x}(t)$. So, for first order and SO cyclostationary and band-limited vectors $\boldsymbol{x}(t)$ having a SO cyclo-ergodicity property and for sufficiently oversampled data, after a preliminary step of first order cyclic frequencies estimation, we define an asymptotic unbiased and consistent estimator $\hat{R}_{\Delta x}(qT_e)(L)$ of the covariance matrix $R_{\Delta mc}(\tau)$ for $\tau = qT_e$ by

$$\hat{R}_{\Delta x}(qT_e)(L) \triangleq \hat{R}_x(qT_e)(L) - \sum_{k=1}^{K} \hat{\boldsymbol{e}}_x^{\gamma(k)} \hat{\boldsymbol{e}}_x^{\gamma(k)\mathrm{H}} e^{\,\mathrm{j}2\pi q T_e \gamma(k)} \quad (14)$$

where $\hat{R}_x(qT_e)(L)$ is defined by (4) and

$$\hat{\boldsymbol{e}}_x^{\gamma} \triangleq \frac{1}{L} \sum_{l=1}^{L} \boldsymbol{x}(l) e^{-\mathrm{j}2\pi l T_e \gamma} \quad (15)$$

Note that the K first order cyclic frequencies $\gamma(k)$ of $\boldsymbol{x}(t)$ may be estimated by searching for the ones which make the criterion $V(\gamma)$, defined by (16), greater than a threshold,

$$V(\gamma) = \sum_{n=1}^{N} \left| \sum_{l=1}^{L} x_n(l) e^{-\mathrm{j}2\pi l T_e \gamma} \right|^2 \left(L \sum_{n=1}^{N} \sum_{l=1}^{L} |x_n(l)|^2 \right)^{-1} \quad (16)$$

5. SIMULATIONS

To illustrate the previous results, we assume that two statistically independent NB and orthogonal ($A^{\mathrm{H}}A = N$ I) 2-CPFSK sources are received by an array of N=5 sensors. These two sources have the same input SNR (Signal Noise Ratio) of 10 dB and are synchronized. Their symbol durations and their modulation indices are such that $h_1/T_1 = h_2/T_2 = (4T_e)^{-1}$ for h_1=2 and h_2=4. Moreover, we apply the SOBI method for only one $R_z(\tau_1)$ correlation matrix where $\tau_1 = 4Te$. Finally, the SINRMk (Maximal Signal to Interference plus Noise Ratio of the source k, defined in [3], at the output of the SOBI separator for k=1,2 are averaged over 200 realizations.

Under the previous assumptions, the figure 1 shows the variations of the SINRM1 of the first source at the output of the SOBI separator, implemented from both the SO statistics estimators (4) and (14-15), as a function of the number of snapshots L. Taking the carrier frequencies of the two sources such that $\Delta f_1 = \Delta f_2 = h_1/2T_1$, the two sources share the same first order cyclic frequencies $\gamma(1)$=0 and $\gamma(2) = h_1/T_1$ and are apparently SO correlated. As planned, the figure 1 shows the poor separation of the sources when the SOBI method uses the empirical SO estimator (4) : the SINR1 converges toward 2.467dB. On the contrary, the exhaustive SO estimator (14-15) using the cyclic frequencies $\gamma(1)$ and $\gamma(2)$ allows the separation of the two 2-CPFSK sources.

6. CONCLUSION

In this paper, we showed that the current SO BSS methods, such as the SOBI method, may be affected by the presence of statistically independent NB sources which are first order cyclostationary. It is in particular the case for CPFSK sources having an integer modulation indice and sharing some first order cyclic frequencies. This problem is directly related to the fact that the current SO BSS method aim at exploiting the information contained in the temporal mean of some correlation matrices instead of some covariance matrices.

To solve this problem, we must exploit the information contained in the temporal mean of some covariance matrices of the observations and we introduced an unbiased and consistent estimator of these matrices for first and SO cyclostationary observations, assuming the first order cyclic frequencies have been estimated previously. The extensions of these results to HO BSS methods will be the subject of an other paper.

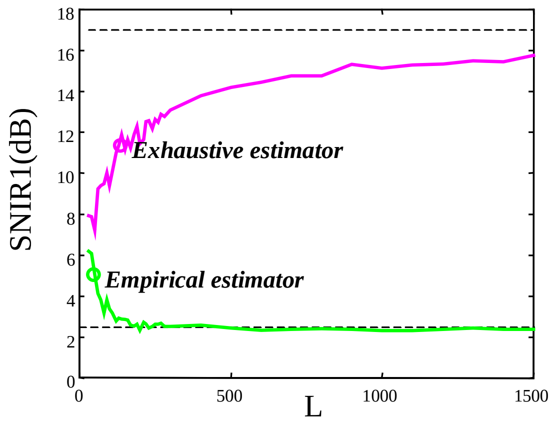

Fig.1 - SINRM1 as a function of L, $\Delta f_1 = \Delta f_2 = h_1/2T_1$

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