COMPARATIVE PERFORMANCE ANALYSIS OF EIGHT BLIND SOURCE SEPARATION METHODS ON RADIOCOMMUNICATIONS SIGNALS

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ABSTRACT

For about two decades, many Second Order (SO) and Fourth Order (FO) blind methods have been developed to separate overdetermined mixtures of statistically independent Narrow-band (NB) sources. Besides, mainly to overcome some limitations of these methods, Sixth Order methods have been developed recently. Nevertheless, despite of this great number of methods, the performance of the latter for arbitrary electromagnetic sources are still almost unknown, which limits their use in operational contexts. The purpose of this paper is to fill the gap previously mentioned by presenting a comparative performance analysis of eight Blind Source Separation (BSS) methods for arbitrary overdetermined mixtures of sources borrowed from the radiocommunications context, and to show off both the advantages and the drawbacks of these methods.

1 INTRODUCTION

For about two decades, many blind methods have been developed to separate overdetermined ($P \leq N$) mixtures of P statistically independent NB sources received by an array of N sensors. Among these methods, the SOBI method [4] exploits the SO data statistics only whereas other methods such as COM1 [10], COM2 [9], JADE [6] or Fast ICA [5] exploit both the SO and the FO data statistics. Very recently, new promizing BSS methods exploiting either the FO data statistics only, such as ICAR [1] or FOBIUM [11], or the Sixth Order data statistics only, such as BIRTH [2], have been developed to overcome the limitations associated with the SO whitening step of the older methods. Nevertheless, despite of the great number of BSS methods currently available, the performance of the latter for arbitrary electromagnetic sources are still almost unknown, which prevents from choosing the best method for a given application, and which obviously limits the use of these methods in operational systems. To overcome these limitations, the purpose of this paper is to present a comparative performance analysis of the eight previous methods for arbitrary overdetermined mixtures of sources borrowed from the radiocommunications context and to show off both the advantages and the drawbacks of each method.

2 PROBLEM FORMULATION

A noisy mixture of P statistically independent narrow-band (NB) sources is assumed to be received by an array of N sensors such that $P \leq N$. The vector, $\boldsymbol{x}(t)$, of the complex envelopes of the signals at the output of the sensors is thus given by

$$\boldsymbol{x}(t) = \sum_{p=1}^{P} m_{pc}(t)\, \boldsymbol{a}_p + \boldsymbol{b}(t) \stackrel{\Delta}{=} A\, \boldsymbol{m}_c(t) + \boldsymbol{b}(t) \qquad (1)$$

where $m_{pc}(t) = m_p(t) e^{j(2\pi\Delta f_p t + \phi_p)}$ is the p-th component of the vector $\boldsymbol{m}_c(t)$; $m_p(t)$, Δf_p, ϕ_p and $\boldsymbol{a}_p$ correspond to the complex envelope, assumed cyclostationary, the carrier residu, the phase and the steering vector of the source p respectively, A is the (NxP) matrix whose columns are the vectors $\boldsymbol{a}_p$. The noise vector, $\boldsymbol{b}(t)$, is assumed to be zero-mean, stationary, Gaussian and independent of the sources.

The BSS problem addressed in this paper consists of finding, from the data statistics, the (NxP) *Linear* and *Time Invariant* source separator W, whose (Px1) output vector $\boldsymbol{y}(t) \stackrel{\Delta}{=} W^{H}\, \boldsymbol{x}(t)$ corresponds, to within a diagonal matrix Λ and a permutation matrix Π, to an estimate, $\hat{\boldsymbol{m}}_c(t)$, of the vector $\boldsymbol{m}_c(t)$, where superscript H means conjugate transposition. In this paper, eight separators W are compared to each other for the restitution of the sources.

3 STATISTICS OF THE DATA

3.1 Second order statistics

The SO data statistics used by some of the considered BSS methods correspond to the temporal mean, $R_x(\tau) = \langle R_x(t, \tau) \rangle$, over an infinite observation interval, of the data correlation matrix, $R_x(t, \tau) = \mathrm{E}[\boldsymbol{x}(t)\, \boldsymbol{x}(t-\tau)^{H}]$, defined by

$$R_x(\tau) \stackrel{\Delta}{=} \langle \mathrm{E}[\boldsymbol{x}(t)\, \boldsymbol{x}(t-\tau)^{H}] \rangle = A\, R_{mc}(\tau)\, A^{H} + \eta_2 \delta(\tau) V \qquad (2)$$

where V denotes the (NxN) noise spatial coherence matrix such that $\mathrm{Tr}[V] = N$, η_2 is the mean power of the noise per sensor, $\delta(\tau)$ is the Kronecker function and $R_{mc}(\tau) = \langle \mathrm{E}[\boldsymbol{m}_c(t)\, \boldsymbol{m}_c(t-\tau)^{H}] \rangle$ is the temporal mean of the correlation matrix of $\boldsymbol{m}_c(t)$, diagonal for zero-mean statistically independent sources. In the following, $R_x(\tau = 0)$ is noted R_x.

3.2 Fourth order statistics

In a same way, the FO data statistics used by some of the considered BSS methods correspond to the temporal mean, $Q_x(\tau_1,\tau_2,\tau_3) = < Q_x(t,\tau_1,\tau_2,\tau_3) >$, over an infinite observation interval, of the (N^2xN^2) data quadricovariance matrix $Q_x(t,\tau_1,\tau_2,\tau_3)$ whose elements, $Q_x(t,\tau_1,\tau_2,\tau_3)[i, j, k, l]$, are defined by

$$Q_x(t,\tau_1,\tau_2,\tau_3)[i, j, k, l]=\mathrm{Cum}(x_i(t), x_j(t-\tau_1)^*, x_k(t-\tau_2)^*, x_l(t-\tau_3)) \quad (3)$$

where * means complex conjugate and $x_i(t)$ is the i^{th} component of the vector $\boldsymbol{x}(t)$. Using (1) into (3) and assuming that $Q_x(t,\tau_1,\tau_2,\tau_3)[i, j, k, l]$ is the element $[N(i-1) + j, N(k-1) + l]$ of the matrix $Q_x(t,\tau_1,\tau_2,\tau_3)$, we obtain the expression of $Q_x(\tau_1,\tau_2,\tau_3)$ given by

$$Q_x(\tau_1,\tau_2,\tau_3) = (A \otimes A^*)\, Q_{mc}(\tau_1,\tau_2,\tau_3)\, (A \otimes A^*)^{\mathrm{H}} \quad (4)$$

where $Q_{mc}(\tau_1,\tau_2,\tau_3)$ is the temporal mean of the (P^2xP^2) quadricovariance matrix of $\boldsymbol{m}_c(t)$ and $\otimes$ is the Kronecker product. In the following, $Q_x(0,0,0)$ is noted Q_x.

3.3 Sixth order statistics

Finally the Sixth Order data statistics used by the BIRTH method correspond to the temporal mean, $H_x = < H_x(t) >$, over an infinite observation interval, of the (N^3xN^3) data hexacovariance matrix $H_x(t)$ whose elements, $H_x(t)[i, j, k, l, m, n]$, are defined by

$$H_x(t)[i, j, k, l, m, n]=\mathrm{Cum}(x_i(t), x_j(t), x_k(t)^*, x_l(t)^*, x_m(t)^*, x_n(t)) \quad (5)$$

Plugging (1) into (5) and assuming that $H_x(t)[i, j, k, l, m, n]$ is the element $[N^2(i-1) + N(j-1) + k, N^2(l-1) + N(m-1) + n]$ of the matrix $H_x(t)$, we obtain the expression of H_x given by

$$H_x = (A \otimes A \otimes A^*)\, H_{mc}\, (A \otimes A \otimes A^*)^{\mathrm{H}} \quad (6)$$

where H_{mc} is the temporal mean of the (P^3xP^3) hexacovariance matrix of $\boldsymbol{m}_c(t)$.

3.4 Statistics estimation

In situations of practical interests, the data statistics temporal mean are not known a priori and have to be estimated from L samples of data $\boldsymbol{x}(l) = \boldsymbol{x}(lT_e)$, $1 \le l \le L$, where T_e is the sample period.

Empirical estimators of the data statistics temporal means may be built from the well-known Leonov-Shiryaev [14] formula, giving the expression of the n-th order cumulant of $\boldsymbol{x}(t)$ as a function of its p-th order moments ($1 \le p \le n$). The idea is to replace in this formula all the moments by their sample estimate. More precisely, the Leonov-Shiryaev formula is given by

$$\mathrm{Cum}(x_{i1}, x_{i2}, ., x_{in}) = \sum_{p=1}^{n} (-1)^{(p-1)}\, \mathrm{E}[\prod_{j\in S1} x_{ij}] \ldots \mathrm{E}[\prod_{j\in Sp} x_{ij}] \quad (7)$$

where (S1, S2, …, Sp) describes all the partitions in p sets of (1, 2, …, n) and an empirical estimate of (7) is obtained by replacing in (7) all the moments $\mathrm{E}[x_{i1}\, x_{i2} \ldots x_{ip}]$ ($1 \le p \le n$) by their sample estimate given by

$$\hat{\mathrm{E}}[x_{i1}\, x_{i2} .. x_{ip}](L) \triangleq \frac{1}{L}\sum_{l=1}^{L} x_{i1}(l)\, x_{i2}(l) \ldots x_{ip}(l) \quad (8)$$

For zero-mean stationary and ergodic observations, these sample estimators generate asymptotically unbiased and consistent estimates of the data statistics temporal means. However, for zero-mean cyclostationary and cycloergodic observations, the previous empirical estimators are generally biased for statistics higher than or equal to 4 [12] but, in most cases, this bias does not prevent from separating the sources [12] but simply degrades the performance of the latter. For these reasons, the sample statistics estimators are used in this paper. Note that asymptotically unbiased second and FO statistics estimators, using the cyclic frequencies of the observations, can be found in [12] and [13] for zero-mean and non-zero mean cyclostationary signals respectively.

4 OPTIMAL AND BLIND SOURCE SEPARATORS

4.1 Source separator performances

The concept of performance at the output of a (N x P) source separator W has been introduced in [7]. It corresponds to the P-uplet $\mathsf{P}(W)$ defined by

$$\mathsf{P}(W) = (\mathrm{SINRM1}[W], \ldots\ldots, \mathrm{SINRM}P[W]) \quad (9)$$

where SINRM$k[W]$ corresponds to the maximum Signal to Interference plus Noise Ratio (SINR) of the source k at the output of W, defined by the maximum value of the SINR of the source k, over all the outputs of the separator. If we note $\boldsymbol{w}_i$ the i-th column of W, SINRM$k[W]$ corresponds to the maximum value of SINR$k[\boldsymbol{w}_i]$ when i varies from 1 to P, where SINR$k[\boldsymbol{w}_i]$ is defined by

$$\mathrm{SINR}k[\boldsymbol{w}_i] = \pi_k \frac{|\boldsymbol{w}_i^{\mathrm{H}}\, \boldsymbol{a}_k|^2}{\boldsymbol{w}_i^{\mathrm{H}}\, R_{bk}\, \boldsymbol{w}_i} \quad (10)$$

where π_k is the temporal mean of the source k input power received by an omnidirectional sensor and $R_{bk} = R_x - \pi_k\, \boldsymbol{a}_k\, \boldsymbol{a}_k^{\mathrm{H}}$ is the temporal mean of the noise plus interference correlation matrix for the source k.

4.2 Optimal source separator

The optimal source separator is the one that gives the best performance for the restitution of each source. It maximizes the quantity SINRM$k[W]$ for each source k and corresponds to the separator W whose columns are the Spatial Matched Filters (SMF) associated with the different sources. It is defined to within a diagonal matrix Λ and a permutation matrix Π and can be written as

$$W_{smf} = R_x^{-1} A \ \Lambda \Pi \quad (11)$$

4.3 Considered blind source separators

The blind source separators considered in the paper aim at implementing, to within a diagonal and a permutation matrix, an estimate, $\hat{W}_{smf} = \hat{R}_x^{-1} \hat{A}$, of the optimal separator (11) from both an estimate, $\hat{R}_x$, of R_x, common to all the separators, and the blindly estimated matrix A, noted $\hat{A}$, specific of each separator. In the SOBI method [4], after a SO data prewhitening step, the $\hat{A}$ matrix is generated from the joint diagonalization of an estimate of several $R_z(\tau)$ matrices, where $z(t)$ is the whitened observation vector and $R_z(\tau)$ is defined by (2) with z instead of x. In the JADE method [6], $\hat{A}$ is obtained from the joint diagonalization of P eigenmatrices of an estimate of Q_z, the temporal mean of the quadricovariance of $z(t)$. In COM1 [10] and COM2 [9] methods, $\hat{A}$ is obtained from the maximization of a FO contrast function built from the FO autocumulant of the outputs and the square modulus of the latter respectively. In Fast ICA $\hat{A}$ is generated from a deflation process, optimizing a FO contrast function under a decorrelation constraint of the outputs. The FOBIUM method [11], which is the extension of the SOBI method at the FO, generates $\hat{A}$ from a FO prewhitening step followed by the joint diagonalization of the estimate of several $Q_z(\tau_1,\tau_2,\tau_3)$ matrices. Finally ICAR [1] and BIRTH [2] correspond to the FO and Sixth order version of a family of $2q$-order methods, named BIOME [3], generating $\hat{A}$ from the exploitation of the redundancies of a $2q$-th order statistical matrix of the data.

5 COMPARATIVE PERFORMANCE ANALYSIS

A parametric comparative performance analysis of the eight previous methods is presented in this section. For this purpose, we consider a Uniform Linear Array (ULA) of $N = 4$ sensors, equispaced half a wavelength apart. The P sources are linearly modulated with a pulse shape filter corresponding to a ½ Nyquist filters with a roll off μ. The background noise is temporally and spatially white except for section 5.6. For the SOBI method, two matrices $R_z(\tau)$ are jointly diagonalized, with $\tau = T_e$ and $\tau = 2T_e$. For the FOBIUM method, two matrices $Q_z(\tau_1,\tau_2,\tau_3)$ are jointly diagonalized with $(\tau_1,\tau_2,\tau_3) = (T_e, 0, 0)$ and $(\tau_1,\tau_2,\tau_3) = (2T_e, 0, 0)$. Finally, the SINRM of the sources is averaged over 200 realizations.

5.1 SNR influence

Figures 1 and 2 show the variations of SINRM1 at the output of the eight separators (similar results are obtained for the source 2) as a function of the number of snapshots L, when $P = 2$ QPSK sources with the same roll-off $\mu = 0.3$, same symbol period T and same Signal to Noise ratio (SNR) equal to 15 and 0 dB respectively, are received by the array. The sources are assumed to be well angularly separated ($\theta 1 = 0°$ and $\theta 2 = 20°$), which generates a spatial correlation coefficient of 0.16, with carrier residues such that $\Delta f_1 \times T = 0$, $\Delta f_2 \times T = (1 + \mu)/2$. The sample period T_e corresponds to the symbol period T.

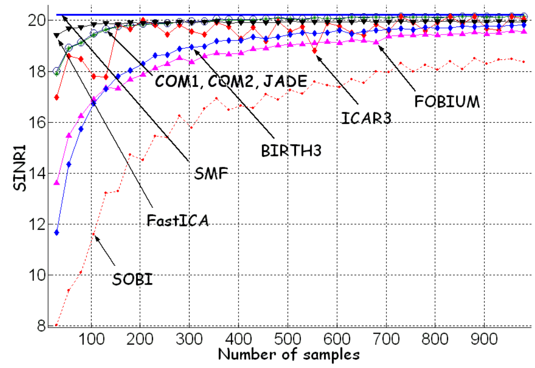

Fig.1 - *SINRM1 as a function of L, N = 4, P = 2 QPSK, $T_e = T$, SNR = 15 dB, $\theta 1 = 0°$, $\theta 2 = 20°$*

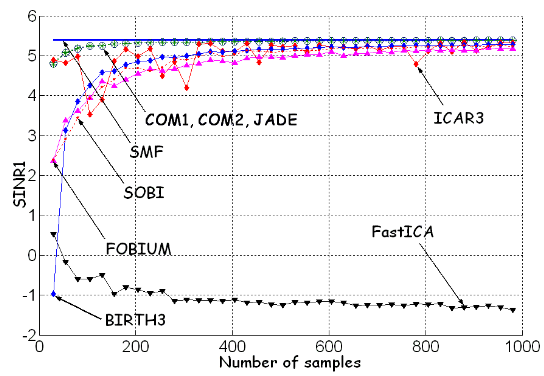

Fig.2 - *SINRM1 as a function of L, N = 4, P = 2 QPSK, $T_e = T$, SNR = 0 dB, $\theta 1 = 0°$, $\theta 2 = 20°$*

Figure 1 shows that for strong and well angularly non Gaussian sources, all the methods succeed in separating the 2 sources from a limited number of snapshots. However, the convergence speed is not the same for all of the methods. Methods exploiting both the SO and FO statistics such as JADE, COM1, COM2 and Fast ICA have the best behavior. Indeed, the SINRM of the sources at the output of these methods is greater than the optimal one minus 3 dB as soon as L is greater than 30. To obtain similar results, ICAR requires about 100 snapshots whereas FOBIUM and BIRTH require about 120 snapshots, due to a higher variance of the statistics estimators. Finally the SOBI method requires about 400 snapshots to obtain such results, due to a mild difference between the spectral densities of the sources. We note that the FOBIUM method seems to be more robust than SOBI with respect to a weak spectrum difference of the sources.

The results of figure 1 remain qualitatively valid for weak sources, as shown by Figure 2, except for Fast ICA which does not succeed in separating the sources. This result is due to the decorrelation constraint of the outputs imposed in the method, which prevents from identifying the A matrix since, as shown in [8], the outputs of both the optimal and the Least Square separators become strongly correlated for weak sources.

To complete the previous results, Figure 3 shows, in the same context, the variations of SINRM2 at the output of the eight separators as a function of the input SNR of the two QPSK sources for $L = 400$.

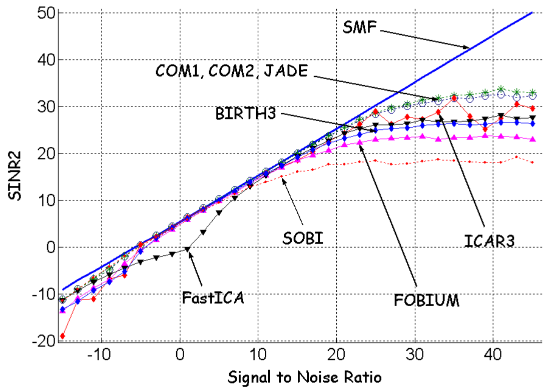

Fig.3 - *SINRM2 as a function of SNR, N = 4, P = 2 QPSK,* $T_e = T$*, L = 400,* θ1 = 0°, θ2 = 20°

Figure 3 shows that for SNR values such that – 5 dB ≤ SNR ≤ 15 dB, the performance at the output of all the separators are approximately optimal for $L = 400$, except for Fast ICA whose performance become suboptimal for weak sources and more precisely for SNR values such that – 5 dB ≤ SNR ≤ 5 dB. However, for SNR values greater than 15 dB, the convergence speed of the eight methods decreases and the output performance, although good, becomes sub-optimal for $L = 400$. Similar results have been reported in [15] for the convergence of the optimal separator when A is known. In this context, we find again that JADE, COM1, COM2, Fast ICA and ICAR seem to be better than FOBIUM and BIRTH, the latter remaining better than SOBI.

5.2 Source spatial correlation influence

To evaluate the influence of the spatial correlation coefficient between the sources, we consider the scenario of Figure 1 but where the angular separation of the sources is now equal to 5° (θ1 = 0° and θ2 = 5°), which generates a spatial correlation coefficient of 0.9. In this context, Figure 4 shows the variations of SINRM1 at the output of the eight separators as a function of L. We note the very good behavior of COM1, COM2 and JADE whose output SINRM is equal to the optimal one minus 3 dB with less than 30 snapshots. Such performances are obtained for a number of snapshots equal to less than 50 for ICAR, 100 for SOBI, 130 for BIRTH and 150 for FOBIUM. Note also the absence of source separation for Fast ICA due again to the decorrelation constraint imposed by the algorithm while the outputs of the ultimate source separators are strongly correlated for strong sources spatial correlation [8].

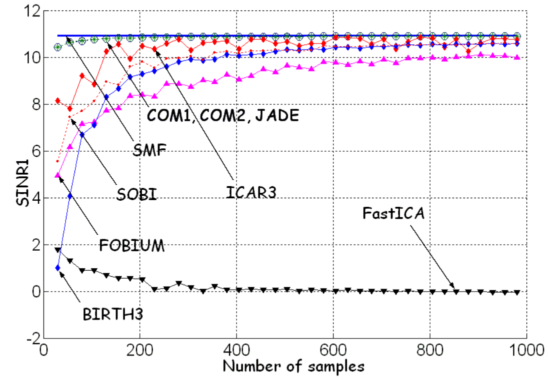

Fig.4 - *SINRM1 as a function of L, N = 4, P = 2 QPSK,* $T_e = T$*, SNR = 15 dB,* θ1 = 0°, θ2 = 5°

5.3 Source non gaussiannity degree influence

To evaluate the influence of the non Gaussiannity degree of the sources, we consider the scenario of Figure 1 but where the QPSK sources are replaced by sources whose complex envelope are linear combination of ½ Nyquist QPSK sources and filtered Gaussian noise. This allows the generation of sources with arbitrary FO and Sixth order normalized autocumulant between [-0.96, 0] and [3.75, 0] respectively. Under these assumptions, Figure 5 shows the variations of SINRM1 at the output of the eight separators as a function of L for sources having a normalized FO autocumulant c equal to – 0.5, instead of –0.96 for Figure 1. We note the decreasing convergence speed of all the methods, except SOBI, as the Gaussianity degree of the sources increases. Note again that the methods that do not use the SO data statistics (FOBIUM, ICAR, BIRTH) are more affected by the non Gaussianity than the others (Fast ICA, JADE, COM1, COM2), SOBI being the most robust since it is not dependent on the Gaussianity degree of the sources.

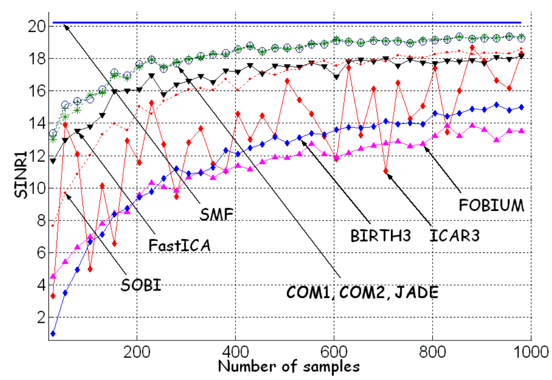

Fig.5 - *SINRM1 as a function of L, N = 4, P = 2,* $T_e = T$*, SNR = 15 dB,* θ1 = 0°, θ2 = 20°, c = – 0.5

To complete the previous results, Figure 6 shows, in the same context, the variations of SINRM1 at the output of the eight separators as a function of the source normalized autocumulant for $L = 400$.

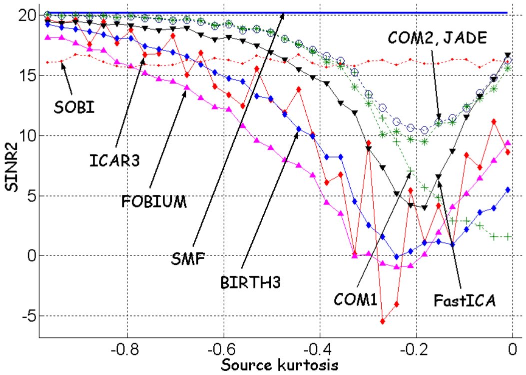

Fig.6 - *SINRM1 as a function of c, N = 4, P = 2,* $T_e = T$, *L = 400,* θ1 = 0°, θ2 = 20°, *SNR = 15 dB*

Figure 6 shows the performance degradation of all the methods using Higher Order data statistics as the non Gaussiannity degree of the sources increases. More precisely, COM1, COM2, JADE and Fast ICA are weakly changed as long as $|c|$ remains greater than or equal to 0.4. For ICAR, FOBIUM and BIRTH, the lower bound on $|c|$ is comprised between 0.6 and 0.75. The apparent increasing performance as $|c|$ decreases under 0.2 is due to the high variance of the estimated statistics for $L = 400$.

5.4 Sample period influence

To evaluate the influence of the sample period on the separators performance, we consider the scenario of Figure 1 but where the sample period T_e is now a fraction of the symbol period T. In this context, Figures 7 and 8 show the variations of SINRM1 at the output of the eight separators as a function of L for $T_e = T/2$ and $T_e = T/4$ respectively. Note that due oversampling, the snapshots are no longer independents.

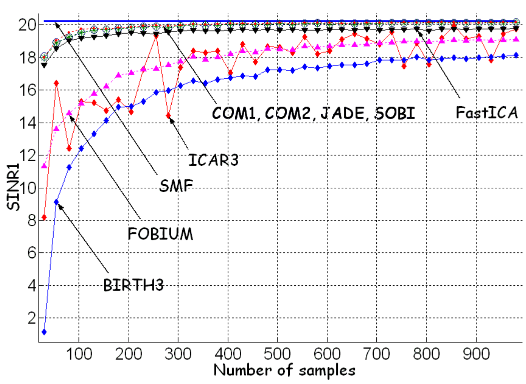

Fig. 7 - *SINRM1 as a function of L, N = 4, P = 2 QPSK,* $T_e = T/2$, *SNR = 15 dB,* θ1 = 0°, θ2 = 20°

When $T_e < T$, the cyclostationary character of the QPSK sources appears explicitly and the empirical statistics estimators become biased at the FO and the Sixth Order [12]. This bias is weak for QPSK sources and Figures 7 and 8 show that the convergence speed, in terms of required independent snapshots, of all the considered methods does not seem to be very affected by the oversampling operation. Note even a better behavior of SOBI in the presence of oversampling due to an increase of the spectral difference of the sources.

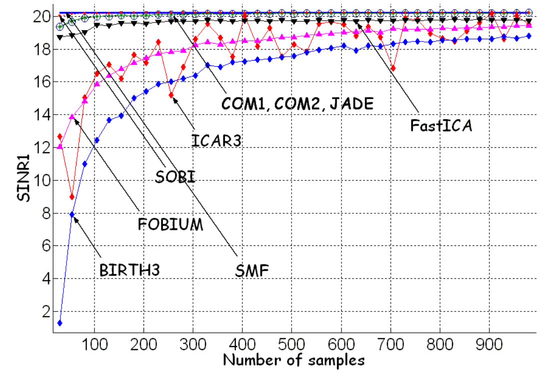

Fig. 8 - *SINRM1 as a function of L, N = 4, P = 2 QPSK,* $T_e = T/4$, *SNR = 15 dB,* θ1 = 0°, θ2 = 20°

5.5 Source number estimation influence

To evaluate the influence of the source number overestimation, we consider the scenario of Figure 1 but where we assume that 3 sources are received on the array. In this context, Figure 9 shows the variations of SINRM1 at the output of the eight separators as a function of L. Note that Fast ICA, BIRTH and ICAR are not perturbed by the source overestimation. On the contrary the convergence speed of all the other methods decreases when the number of sources is overestimated. In particular, the performance of COM2, FOBIUM, SOBI and COM1 seem to be degraded of about 4 dB by the overestimation for $L = 200$. Finally JADE is the most perturbed and looses about 16 dB.

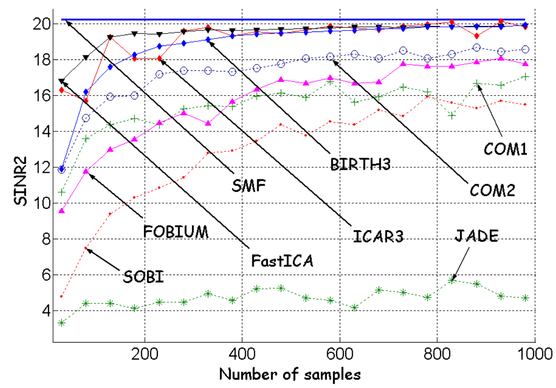

Fig.9 - *SINRM1 as a function of L, N = 4, P = 2 QPSK,* $T_e = T$, *SNR = 15 dB,* θ1 = 0°, θ2 = 20°, $\hat{P} = 3$

5.6 Noise spatial coherence influence

To evaluate the influence of the noise spatial coherence, we consider the scenario of Figure 2 but where we assume that θ1 = 20°, θ2 = 45° and the background noise is no longer spatially white but such that its spatial coherence V is equal to $V = (\mathrm{I} + O)/2$ where I is the identity matrix and O is a matrix such that $O[i, j] = \rho^{|i-j|}$ where $0 \leq \rho \leq 1$. This model corresponds to the presence of both white thermal noise and potentially non white external noise with the same power. Under these assumptions, Figure 10 shows the variations of SINRM2 at the output of the eight separators as a function of ρ for L = 400. Note the poor performance of Fast ICA due to the presence of weak sources. Note the increasing performance degradation of JADE, COM2, COM1 and SOBI as ρ increases beyond 0.4. Note finally the robustness of ICAR, FOBIUM and BIRTH to non identity noise spatial coherence.

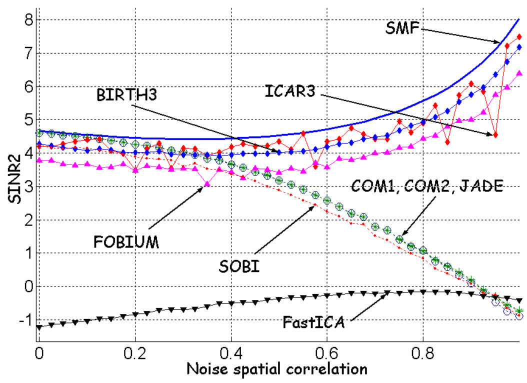

Fig.10 - *SINRM2 as a function of ρ, N = 4, P = 2 QPSK,* $T_e = T$, *SNR = 0 dB,* θ1 = 20°, θ2 = 45°, *K = 400*

6 CONCLUSION

In this paper, a comparative performance analysis of eight methods has been presented for the blind separation of overdetermined instantaneous mixtures of statistically independent sources. This analysis shows that when the sources are far from Gaussianity, strong and not too close to each other, COM2, COM1, Fast ICA, ICAR and JADE show off a very good behavior whatever the spectral difference between the sources, the latter being very sensitive to the source number overestimation while Fast ICA and ICAR being the more robust to this criterion. As the variance of methods that use HO data statistics only is stronger than the variance of the other ones and increases with statistics order, ICAR is better than BIRTH but also than FOBIUM since the latter is sensitive to weak spectral difference between the sources. As SOBI is very sensitive to the latter criterion, it is mainly well suited for Gaussian or quasi-Gaussian contexts when the sources are strongly spectrally separated. For sources approaching Gaussiannity, SOBI is unaltered while ICAR, FOBIUM and BIRTH are the more altered. For weak or close sources, Fast ICA fails in separating the sources while the behavior of the other methods is not altered provided the noise spatial coherence is not a cause of problem. If it is (unknown noise spatial coherence, very correlated noise and weak sources) ICAR and BIRTH seem to be the best methods.

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