# ICAR: INDEPENDENT COMPONENT ANALYSIS USING REDUNDANCIES

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## ABSTRACT

A new Blind Source Separation (BSS) algorithm, called ICAR and using only Fourth Order (FO) statistics of the data, is proposed. The latter method is compared by computer experiments with the well-known methods COM1, COM2, JADE, FastICA and SOBI. Since ICAR has given very good convergence results and has performed the source separation in the presence of a Gaussian noise with unknown spatial correlation, it appears as being one of the most attracting BSS algorithms.

## 1. INTRODUCTION

Blind Source separation (BSS) or Independent Component Analysis (ICA) have lately raised great interest. These problems find their place in numerous applications including telecommunications, speech processing, or biomedical engineering. For instance, in antenna array processing, if several sources impinge of an array of discrete sensors, and if the channel time spread associated with every source and every sensor is negligible compared to the symbol period, then the signals received can be modeled as a static mixture of the transmitted sources. BSS aims at extracting the sources from the sole observation of the mixtures received on the array. Several techniques are available, depending on the assumptions made. In this paper, it will be assumed that sources are mutually statistically independent. Contrary to Principal Component Analysis (PCA), which exploits only statistics of order 2, ICA resorts to statistics of higher order, and is thus able to impose some stronger statistical independence than just a mere decorrelation. This is made possible if sources are not Gaussian, and made easier if there are at least as many sensors as sources.

Since the early works of Jutten [11], the concept of ICA has evolved, and most of the material has been presented in the seminal paper of Comon [7], where it is proposed to maximize contrast functionals; an algorithm is proposed there, which shall be referred to as COM2, for Contrast-based Maximization of squared fourth order cumulants. Some years later, he also proposed a simple algorithm [8], which will be called COM1 in the sequel, maximizing a squares-free criterion [13]. On the other hand, Cardoso and Souloumiac have proposed [4] slightly later an efficient algorithm performing Joint Approximate Diagonalization of Eigen cumulant matrices (JADE). Later, Hyvarinen and others propose the so-called FastICA fixed point algorithm, which extracts one source at a time, first for real signals [12], then for complex [3]. All these methods can be sensitive to the presence of additive Gaussian noise, of unknown spatial coherence; such a noise is frequent in applications such as HF radiocommunications. In fact, they perform as a preliminary stage an exact second-order decorrelation by a preliminary “spatial whitening”; this operation is sub-optimal in several respects, because it puts too much emphasis on second order statistics.

The solution (also sub-optimal though) proposed in this paper consists of getting rid of this whitening stage, and of using exclusively higher order statistics, namely fourth order cumulants. More precisely, the redundancy theoretically present in the quadricovariance of the observations is exploited. This concept can be extended to statistics of order strictly higher than 4, allowing for instance to address the case of *underdetermined mixtures*, in which the number of sources present exceeds in permanence the number of sensors. Such extensions to order 6, or more generally to order $m{=}2q$ ($q{\geq}2$), have been recently proposed by the authors under the names of BIRTH [2] and BIOME [1].

## 2. NOTATIONS AND ASSUMPTIONS

Let $N$ sensors be available, and denote $x_n(k)$ the noisy observations received on these sensors, $1 \leq n \leq N$. The vector of observations $\boldsymbol{x}(k) \stackrel{\text{def}}{=} [x_1(k)\ x_2(k)\ \cdots\ x_N(k)]^{\mathsf{T}}$ can be modeled in the form below:

$$\boldsymbol{x}(k) = \boldsymbol{A}\,\boldsymbol{s}(k) + \boldsymbol{\nu}(k) = \sum_{p=1}^{P} \boldsymbol{a}_p\, s_p(k) + \boldsymbol{\nu}(k) \qquad (1)$$

where $s_p(k)$ denotes the unknown sources, and $\boldsymbol{A} \stackrel{\text{def}}{=} [\boldsymbol{a}_1\ \boldsymbol{a}_2\ \cdots\ \boldsymbol{a}_P]$, $\boldsymbol{s}(k) \stackrel{\text{def}}{=} [s_1(k)\ s_2(k)\ \cdots\ s_P(k)]^{\mathsf{T}}$, $\boldsymbol{\nu}(k) \stackrel{\text{def}}{=} [\nu_1(k)\ \nu_2(k)\ \cdots\ \nu_N(k)]^{\mathsf{T}}$ denote respectively the $N \times P$ mixing matrix, and the source and noise random vectors, assumed to be independent. Also define for very index $k$ the entries of the 4-th order cumulant tensor, $\boldsymbol{C_x}$, of a random process $\boldsymbol{x}(k)$, stationary and ergodic up to order 4:

$$C_{de,\boldsymbol{x}}^{fg} \stackrel{\text{def}}{=} \mathsf{Cum}\{x_d(k),\, x_e(k),\, x_f(k)^*,\, x_g(k)^*\} \quad (2)$$

It is then possible to store every entry of tensor $\boldsymbol{C_x}$ in a matrix $\boldsymbol{Q_x}$, sometimes called the *quadricovariance*:

$$\forall\, 1 \leq d, e, f, g \leq N,\ Q_{\boldsymbol{x}}(N(d-1)+g, N(f-1)+e) = C_{de,\boldsymbol{x}}^{fg} \quad (3)$$

where $Q_{\boldsymbol{x}}(r, q)$ corresponds to the $(r, q)$-th component of $\boldsymbol{Q_x}$. Note that this arrangement is of course not unique, but this is the one that yields the best performances in terms of identifiability [6]. Matrix $\boldsymbol{Q_x}$ can be denoted more generally as $\boldsymbol{\mathcal{C}}_{4,\boldsymbol{x}}^{1}$ [1].

Let us add that the expression of 4-th order cumulants of zero-mean complex variables as a function of moments appears in many papers; see for instance [10] for non zero mean variables. We also assume the following:

**H1**. For every index $k$, the source vector $\boldsymbol{s}(k)$ is stationary and ergodic (extension to cyclostationary and cycloergodic is straightforward), with complex values, mutually independent at order 4 (i.e. all cross cumulants of order 4 are null);

**H2**. For every $k$, the noise vector $\boldsymbol{\nu}(k)$ is stationary and ergodic;

**H3**. For every $k$, $\boldsymbol{s}(k)$ and $\boldsymbol{\nu}(k)$ are statistically independent;

**H4**. Source *kurtoses* (standardized autocumulants of order 4), $\kappa_{ppp,\boldsymbol{s}}^{ppp} = C_{ppp,\boldsymbol{s}}^{ppp}/\pi_p^2$, are non zero and have all the same sign ($\pi_p \stackrel{\text{def}}{=} \mathsf{E}[|s_p(k)|^2]$ denotes the power of the $p$-th source);

**H5**. The number of sources is smaller than the number of sensors: $P \leq N$. The mixture is then referred to as *overdetermined*;

**H6**. The mixing matrix $\boldsymbol{A}$ is full rank and does not contain any null entry.

The goal is to determine a separating matrix, $\boldsymbol{W}$, such that

$$\boldsymbol{y}(n) \stackrel{\text{def}}{=} \boldsymbol{W}^{\mathsf{H}}\, \boldsymbol{x}(n) \quad (4)$$

is an estimate of vector $\boldsymbol{s}(k)$ up to a multiplicative *trivial* matrix (i.e. of the form $\boldsymbol{\Lambda\Pi}$ where $\boldsymbol{\Lambda}$ is invertible diagonal and $\boldsymbol{\Pi}$ is a permutation). In ICAR, as in numerous other BSS algorithms, the construction of $\boldsymbol{W}$ requires the identification of mixture $\boldsymbol{A}$.

## 3. THE CORE OF THE ICAR METHOD

### 3.1. Properties of the quadricovariance

The multilinearity property enjoyed by cumulants allows to write

$$\boldsymbol{Q_x} = [\boldsymbol{A} \otimes \boldsymbol{A}^*]\, \boldsymbol{Q_s}\, [\boldsymbol{A} \otimes \boldsymbol{A}^*]^{\mathsf{H}} \quad (5)$$

where $\boldsymbol{Q_x}$ and $\boldsymbol{Q_s}$ denote quadricovariance matrices of $\boldsymbol{x}(k)$ and $\boldsymbol{s}(k)$, of size $N^2 \times N^2$ and $P^2 \times P^2$, respectively. Because sources are independent, $\boldsymbol{Q_s}$ is diagonal. However, it is not full rank. Another decomposition,

$$\boldsymbol{Q_x} = \boldsymbol{\mathcal{A}}_Q\, \boldsymbol{\mathcal{Q}_s}\, \boldsymbol{\mathcal{A}}_Q^{\mathsf{H}} \quad (6)$$

exhibits two new matrices. The first one, $\boldsymbol{\mathcal{Q}_s} \stackrel{\text{def}}{=} \operatorname{diag}(\left[C_{11,\boldsymbol{s}}^{11}\ C_{22,\boldsymbol{s}}^{22}\ \cdots\ C_{PP,\boldsymbol{s}}^{PP}\right])$, is diagonal of size $P \times P$, and invertible from (**H4**). The second one, $\boldsymbol{\mathcal{A}}_Q$, of size $N^2 \times P$, is also of full rank from (**H6**) and [1, prop. 1], can be written as:

$$\begin{aligned} \boldsymbol{\mathcal{A}}_Q &\stackrel{\text{def}}{=} [\boldsymbol{a}_1 \otimes \boldsymbol{a}_1^*\ \boldsymbol{a}_2 \otimes \boldsymbol{a}_2^*\ \cdots\ \boldsymbol{a}_P \otimes \boldsymbol{a}_P^*] \\ &= [[\boldsymbol{A}^* \boldsymbol{\Phi}_1]^{\mathsf{T}}\ [\boldsymbol{A}^* \boldsymbol{\Phi}_2]^{\mathsf{T}}\ \cdots\ [\boldsymbol{A}^* \boldsymbol{\Phi}_N]^{\mathsf{T}}]^{\mathsf{T}} \end{aligned} \quad (7)$$

where

$$\boldsymbol{\Phi}_n \stackrel{\text{def}}{=} \operatorname{diag}([A(n,1)\ A(n,2)\ \cdots\ A(n,P)]) \quad (8)$$

Entries $A(n, \cdot)$ of the diagonal matrix $\boldsymbol{\Phi}_n$, of size $P \times P$, form the $n$-th line of $\boldsymbol{A}$. (Matrices $\boldsymbol{\mathcal{Q}_s}$ and $\boldsymbol{\mathcal{A}}_Q$ are referred to as $\boldsymbol{\zeta}_{4,\boldsymbol{s}}$ et $\boldsymbol{\mathcal{A}}_2^1$ in [1]).

### 3.2. Principle of ICAR

the principle is similar to that described in [2] with the difference that statistics of order 4 are utilized instead of 6. This difference is important because 4-th order statistics do not allow any more to address *underdetermined mixtures*, at least in the present framework. Since sources have the same kurtosis sign, it is possible to determine a unique real square root matrix of $\boldsymbol{Q_x}$ or $-\boldsymbol{Q_x}$. Without restricting the generality, assume source kurtoses are all positive. The square root of $\boldsymbol{Q_x}$ can be computed by an Eigen Value Decomposition (EVD), from (6):

$$\boldsymbol{Q}_{\boldsymbol{x}}^{1/2} = \boldsymbol{E_s}\, \boldsymbol{L}_{\boldsymbol{s}}^{1/2} = \boldsymbol{\mathcal{A}}_Q\, \boldsymbol{\mathcal{Q}}_{\boldsymbol{s}}^{1/2}\, \boldsymbol{V}^{\mathsf{H}} \quad (9)$$

where $\boldsymbol{L_s}$ ($\boldsymbol{L}_{\boldsymbol{s}}^{1/2}$ denotes a square root of $\boldsymbol{L_s}$) is the $P \times P$ diagonal matrix containing the $P$ non-zero eigenvalues of $\boldsymbol{Q_x}$, and $\boldsymbol{E_s}$ is the $N^2 \times P$ matrix of associated normalized eigenvectors. Because $\boldsymbol{\mathcal{A}}_Q$ is full rank, it can be shown that (**H4**) amounts to assuming that diagonal elements of $\boldsymbol{L_s}$ are non zero and have the same sign [1, prop. 2], here positive. Moreover, (9) establishes a link between $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}$ and $\boldsymbol{\mathcal{A}}_Q$, where $\boldsymbol{V}$ is a unitary matrix, uniquely defined once $\boldsymbol{L_s}$ and $\boldsymbol{E_s}$ are fixed. Next, (9) and (7) relate $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}$ and $\boldsymbol{A}$ such that:

$$\begin{aligned} \boldsymbol{Q}_{\boldsymbol{x}}^{1/2} &= \left[[\boldsymbol{A}^* \boldsymbol{\Phi}_1 \boldsymbol{\mathcal{Q}}_{\boldsymbol{s}}^{1/2} \boldsymbol{V}^{\mathsf{H}}]^{\mathsf{T}} \cdots [\boldsymbol{A}^* \boldsymbol{\Phi}_N \boldsymbol{\mathcal{Q}}_{\boldsymbol{s}}^{1/2} \boldsymbol{V}^{\mathsf{H}}]^{\mathsf{T}}\right]^{\mathsf{T}} \\ &\stackrel{\text{def}}{=} [\ \boldsymbol{\Gamma}_1{}^{\mathsf{T}}\ \ \boldsymbol{\Gamma}_2{}^{\mathsf{T}}\ \cdots\ \boldsymbol{\Gamma}_N{}^{\mathsf{T}}\ ]^{\mathsf{T}} \end{aligned} \quad (10)$$

where $\mathbf{\Gamma}_n = \boldsymbol{A}^* \mathbf{\Phi}_n \, \boldsymbol{Q}_{\boldsymbol{s}}^{1/2} \, \boldsymbol{V}^{\mathsf{H}}$ is the $n$-th matrix block of $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}$, of size $N \times P$. Consequently, matrix $\boldsymbol{V}$ diagonalizes the $N(N-1)$ matrices $\mathbf{\Theta}_{n_1,n_2}$ given by

$$\forall \, 1 \leq n_1 \neq n_2 \leq N, \qquad \mathbf{\Theta}_{n_1,n_2} = \mathbf{\Gamma}_{n_1}^{\sharp} \, \mathbf{\Gamma}_{n_2} \tag{11}$$

where $\sharp$ denotes pseudo-inversion. In fact, from the expression of $\mathbf{\Gamma}_n$ yielded by (10), and under assumptions **(H5)**-**(H6)**, the pseudo-inverse of $\mathbf{\Gamma}_n$ can be written as $\mathbf{\Gamma}_n^{\sharp} \stackrel{\text{def}}{=} (\mathbf{\Gamma}_n{}^{\mathsf{H}} \mathbf{\Gamma}_n)^{-1} \mathbf{\Gamma}_n{}^{\mathsf{H}}$. As a consequence, matrices $\mathbf{\Theta}_{n_1,n_2}$ can be rewritten in the form below:

$$\forall \, 1 \leq n_1 \neq n_2 \leq N, \qquad \mathbf{\Theta}_{n_1,n_2} = \boldsymbol{V} \, D_{n_1,n_2} \, \boldsymbol{V}^{\mathsf{H}} \tag{12}$$

where matrices $D_{n_1,n_2} \stackrel{\text{def}}{=} \mathbf{\Phi}_{n_1}^{-1} \, \mathbf{\Phi}_{n_2}$ are diagonal. Denote $\boldsymbol{V}_{sol}$ the unitary matrix that jointly diagonalizes matrices $\mathbf{\Theta}_{n_1,n_2}$. Then $\boldsymbol{V}_{sol} = \boldsymbol{V} \boldsymbol{\mathcal{T}}$ with $\boldsymbol{\mathcal{T}}$ unitary. Thus, from (9), $\boldsymbol{V}_{sol}$ allows to identify $\boldsymbol{\mathcal{A}}_Q$ up to a unitary matrix:

$$\boldsymbol{Q}_{\boldsymbol{x}}^{1/2} \, \boldsymbol{V}_{sol} = \boldsymbol{\mathcal{A}}_Q \, \boldsymbol{Q}_{\boldsymbol{s}}^{1/2} \, \boldsymbol{\mathcal{T}} \stackrel{\text{def}}{=} \widehat{\boldsymbol{\mathcal{A}}_Q} \tag{13}$$

On the other hand, from assumptions **(H5)**-**(H6)**, for every pair $(p_1,p_2)|_{p_1 \neq p_2}$ belonging to $\{1,2,\ldots,P\}^2$, there exists at least a pair $(n_1,n_2)|_{n_1 \neq n_2}$ belonging to $\{1,2,\ldots,N\}^2$ such that $D_{n_1,n_2}(p_1,p_1) \neq D_{n_1,n_2}(p_2,p_2)$. This implies that $\boldsymbol{\mathcal{T}}$ is trivial. Then from (13), matrix $\widehat{\boldsymbol{\mathcal{A}}_Q}$ is consequently an estimate of $\boldsymbol{\mathcal{A}}_Q$, up to a trivial matrix.

Next from (10), equation (13) can be rewritten as

$$\begin{aligned} \boldsymbol{Q}_{\boldsymbol{x}}^{1/2} \, \boldsymbol{V}_{sol} \;&=\; \left[ [\boldsymbol{A}^* \mathbf{\Phi}_1 \boldsymbol{Q}_{\boldsymbol{s}}^{1/2} \, \boldsymbol{\mathcal{T}}]^{\mathsf{T}} \cdots [\boldsymbol{A}^* \mathbf{\Phi}_N \boldsymbol{Q}_{\boldsymbol{s}}^{1/2} \, \boldsymbol{\mathcal{T}}]^{\mathsf{T}} \right]^{\mathsf{T}} \\ &\stackrel{\text{def}}{=}\; [\mathbf{\Sigma}_1{}^{\mathsf{T}} \, \mathbf{\Sigma}_2{}^{\mathsf{T}} \cdots \mathbf{\Sigma}_{N^2}{}^{\mathsf{T}}]^{\mathsf{T}} = \widehat{\boldsymbol{\mathcal{A}}_Q} \end{aligned} \tag{14}$$

The matrix block $\mathbf{\Sigma}_1$, formed of the $N$ first rows of $\boldsymbol{Q}_{\boldsymbol{x}}^{1/2}$ $\boldsymbol{V}_{sol}$, corresponds to $\boldsymbol{A}^*$, up to a trivial matrix:

$$\mathbf{\Sigma}_1 = \boldsymbol{A}^* \, \mathbf{\Phi}_1 \, \boldsymbol{Q}_{\boldsymbol{s}}^{1/2} \, \boldsymbol{\mathcal{T}} \tag{15}$$

where $\boldsymbol{Q}_{\boldsymbol{s}}^{1/2}$ and $\mathbf{\Phi}_n$ are diagonal for every $n$, $1 \leq n \leq N$. This method is named ICAR1. One could also think of several improvements, for instance by averaging the $N$ blocks $\mathbf{\Sigma}_n$ for estimating $\boldsymbol{A}^*$, giving rise to ICAR2.

### 3.3. Refinement of the method

In order to fully exploit the information contained in matrix $\widehat{\boldsymbol{\mathcal{A}}_Q}$ (14), namely its redundancies of $\boldsymbol{\mathcal{A}}_Q$ in (7), it is possible to mimic the last step of FOBIUM [9]. Indeed, from (13) and (7):

$$\begin{aligned} \widehat{\boldsymbol{\mathcal{A}}_Q} \;&=\; \left[ \lambda_{\xi(1)} \left[ \boldsymbol{a}_{\xi(1)} \otimes \boldsymbol{a}_{\xi(1)}^* \right] \; \cdots \; \lambda_{\xi(P)} \left[ \boldsymbol{a}_{\xi(P)} \otimes \boldsymbol{a}_{\xi(P)}^* \right] \right] \\ &\stackrel{\text{def}}{=}\; \left[ \boldsymbol{b}_{\xi(1)} \; \cdots \; \boldsymbol{b}_{\xi(P)} \right] \end{aligned} \tag{16}$$

where $|\lambda_p| = \left| C_{ppp,\boldsymbol{s}}^{ppp} \right|^{1/2}$, and $\xi$ is a permutation mapping on $\{1,2,\ldots,P\}$. It is then possible to associate every vector $\boldsymbol{b}_{\xi(p)}$ of size $N^2 \times 1$ with a matrix $\boldsymbol{B}_{\xi(p)}$ of size $N \times N$, whose columns are precisely the $N$ successive vectors $\boldsymbol{b}_{\xi(p)}$ formed of $N$ values:

$$\boldsymbol{B}_{\xi(p)} = \lambda_{\xi(p)} \left[ \boldsymbol{a}_{\xi(p)} \boldsymbol{a}_{\xi(p)}^{\mathsf{H}} \right]^* \tag{17}$$

A mere diagonalization of matrices $\boldsymbol{B}_{\xi(p)}^*$ allows to yield the $P$ directional vectors $\boldsymbol{a}_p$, by retaining each time the eigenvector associated with the dominant eigenvalue, and up to a permutation and a scalar multiplicative factor. The method with the latter improvement is named ICAR3.

## 4. SIMULATIONS

Two computer experiments show the performances of ICAR and some efficient BSS techniques (COM1, JADE, FastICA, SOBI). $P=4$ statistically independent sources, i.e. 2 BPSK and 2 QPSK, all with a raised cosine pulse shape of roll-off equal to $0.25$, are received by a UCA of $N=4$ identical sensors of radius $R$ such that $R/\lambda = 0.55$ ($\lambda$: wavelength). The four sources, assumed synchronized, have the same input SNR (Signal to Noise Ratio) of 20dB and the noise is spatially and temporally white Gaussian. The symbol period $T_1$ associated with the first BPSK is equal to three times the sample period $T_e$. The other sources have a symbol period equal to twice the sample period. The directions of arrival of the sources are such that the source steering vectors are orthogonal and the associated carrier residus are such that $f_{c1} \, T_e = 0$, $f_{c2} \, T_e = 0.3$, $f_{c3} \, T_e = 0.2$ and $f_{c4} \, T_e = 0.1$. The performance measure assumed to evaluate the quality of the extraction of source $p$ is the maximal signal to interference plus noise ratio associated with source $p$, denoted *SINRM*$_p$ [5]. It can be compared to the optimal *SINRM*$_p$ computed using the exact mixing matrix instead of the estimated one and denoted by *Optimum SMF*. These are precisely the comparisons that are drawn now. Figure 1

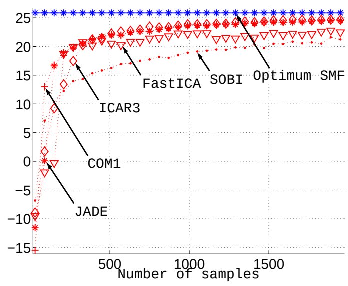


**Fig. 1**. *SINRM*$_3$ for a SNR of 20 dB

displays the SINRM of source 3 averaged over 200 realizations, associated with ICAR3, and BSS techniques JADE, COM1, FastICA and SOBI, as a function of the number of samples. Figure 1 shows the good performance of the ICAR3 algorithm facing the other methods. Note that the SOBI method gives in this simulation good results since sources have been chosen with different spectral densities, especially taking different carrier residus. Similar results are observed for the other sources, and are not reported here.

The influence of the noise spatial coherence coefficient $\rho$ is next studied. Now we have 3 sources, i.e. 2 BPSK and 1 QPSK, all with a raised cosine pulse shape of roll-off equal to $0.25$, are assumed to be received by a UCA of $N = 5$ identical sensors of radius $R$ such that $R/\lambda = 0.55$. Their symbol periods are equal to $T_1 = 2T_e$, $T_2 = 3T_e$ and $T_3 = 4T_e$ respectively. Their carrier residus are chosen equal to zero. The source steering vectors are built orthogonal, the SNR is $0$ dB and $1500$ samples are used for separation in this scenario. We apply the COM1, COM2, JADE, SOBI and ICAR3 methods, and the SINRM associated with each source is computed and averaged still over $200$ realizations. The Gaussian noise is modeled as a sum of two noises, $\boldsymbol{\nu_{in}}(k)$ and $\boldsymbol{\nu_{out}}(k)$, of covariance $\boldsymbol{R_{\nu}^{in}}$ and $\boldsymbol{R_{\nu}^{out}}$ respectively, such that:

$$\boldsymbol{R_{\nu}^{in}}(r,q) \stackrel{\text{def}}{=} \sigma^2\delta(r-q)/2 \quad \boldsymbol{R_{\nu}^{out}}(r,q) \stackrel{\text{def}}{=} \sigma^2\rho^{|r-q|}/2 \quad (18)$$

where $\sigma^2$ is the global noise variance per sensor. Note that $\boldsymbol{R_{\nu}}(r,q) \stackrel{\text{def}}{=} \boldsymbol{R_{\nu}^{in}}(r,q) + \boldsymbol{R_{\nu}^{out}}(r,q)$ is the $(r,q)$-th component of the global noise covariance. Contrary to COM1, COM2, JADE and SOBI, algorithm ICAR3 is totally insensitive to the increase in coefficient $\rho$. In fact, the classical methods such as COM1, COM2, JADE, FastICA and SOBI require a prior spatial whitening based on second order moments. This stage theoretically needs the perfect knowledge of the noise covariance. If this is not the case, a whitening of the observed data is performed instead, which is biased. ICAR3 does not suffer from this drawback, since it uses only 4-th order cumulants, which are (asymptotically) insensitive to Gaussian noise, regardless of its space/time color. Again, similar results have been observed for sources 1 and 2.

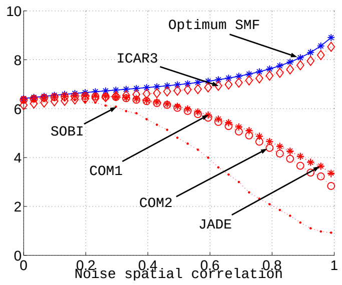


**Fig. 2**. *SINRM*$_3$ for a SNR of $0$ dB

## 5. CONCLUSION

The new algorithm proposed in this paper, referred to as ICAR, utilizes only fourth order statistics of observations, and seems to be much more attractive than previous (classical) BSS techniques needing prior second order decorrelation, according to our computer simulations.We currently work on another solution to the problem presented in section 3.3, in other words, on another technique to extract the mixing matrix $\boldsymbol{A}$ contained in $\boldsymbol{\mathcal{A}}_Q$ (7).

## 6. REFERENCES


[1] L. ALBERA, A. FERREOL, P. COMON, P. CHEVALIER, "Blind Identification of Overdetermined and underdetermined Mixtures of sources (BIOME)", *Linear Algebra Applications*, april 2003, submitted.

[2] L. ALBERA, A. FERREOL, P. COMON, P. CHEVALIER, "Sixth order blind identification of underdetermined mixtures (BIRTH) of sources", in *ICA 03, Fourth International Symposium on Independent Component Analysis and Blind Signal Separation*, Nara, Japan, April 1-4 2003, pp. 909–914.

[3] E. BINGHAM, H. HYVARINEN, "A fast fixed-point algorithm for independent component analysis of complex valued signals", *Int. J. of Neural Systems*, vol. 10, no. 1, pp. 1–8, 2000.

[4] J.-F. CARDOSO, A. SOULOUMIAC, "Blind beamforming for non-gaussian signals", *IEE Proceedings-F*, vol. 140, no. 6, pp. 362–370, December 1993.

[5] P. CHEVALIER, "Optimal separation of independent narrow-band sources : Concept and Performances", *Signal Processing, Elsevier*, vol. 73, pp. 27–47, 1999.

[6] P. CHEVALIER, L. ALBERA, A. FERREOL, P. COMON, "On the virtual array concept for higher order array processing", *IEEE Transactions On Signal Processing*, 2003, submitted.

[7] P. COMON, "Independent Component Analysis, a new concept ?", *Signal Processing, Elsevier*, vol. 36, no. 3, pp. 287–314, April 1994.

[8] P. COMON, "From source separation to blind equalization, contrast-based approaches", in *ICISP 01, Int. Conf. on Image and Signal Processing*, Agadir, Morocco, May 3-5 2001, pp. 20–32.

[9] A. FERREOL, L. ALBERA, P. CHEVALIER, "Fourth order blind identification of underdetermined mixtures of sources (FOBIUM)", in *ICASSP 03, 2003 IEEE International Conference on Acoustics Speech and Signal Processing*, Hong Kong, China, April 6-10 2003, pp. 41–44.

[10] A. FERREOL, P. CHEVALIER, L. ALBERA, "Higher order blind separation of non zero-mean cyclostationary sources", in *EUSIPCO 02, XI European Signal Processing Conference*, Toulouse, France, September 3-6 2002, vol. 5, pp. 103–106.

[11] J. HERAULT, C. JUTTEN, B. ANS, "Détection de grandeurs primitives dans un message composite par une architecture de calcul neuromimétique en apprentissage non supervisé", in *GRETSI 85, Dixième colloque sur le Traitement du Signal et des Images*, Nice, France, Septembre 1985, pp. 1017–1022.

[12] H. HYVARINEN, E. OJA, "A fast fixed-point algorithm for independent component analysis", *Neural Computation*, vol. 9, no. 7, pp. 1483–1492, 1997.

[13] N. THIRION, E. MOREAU, "New criteria for blind signal separation", in *IEEE Workshop on Statistical Signal and Array Processing*, Pennsylvania, US, August 2000, pp. 344–348.