SIXTH ORDER BLIND IDENTIFICATION OF UNDERDETERMINED MIXTURES (BIRTH) OF SOURCES

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ABSTRACT

Static linear mixtures with more sources than sensors are considered. The Blind Source Identification (BSI) of underdetermined mixtures problem is addressed by taking advantage of Sixth Order (SixO) statistics and the Virtual Array (VA) concept. It is shown how SixO cumulants can be used to increase the effective aperture of an arbitrary antenna array, and so to identify the mixture of more sources than sensors. A computationally simple but efficient algorithm, named BIRTH, is proposed and enables to identify the steering vectors of up to $P = N^2 - N + 1$ sources for arrays of N sensors with space diversity only, and up to $P = N^2$ for those with angular and polarization diversity.

1. INTRODUCTION

If high performance joint Second Order (SO) and Fourth Order (FO) Blind Source Identification and Extraction (BSIE) methods can be found in signal processing literature, most of them [3] [5] [7] can only identify overdetermined mixtures (that means mixtures of fewer sources than sensors) because of the SO prewhitening step. However, in practical situations, such as in HF (High Frequency) radiocommunications contexts, the reception of more sources than sensors is possible and its probability increases with the reception bandwidth.

To face such situations, several BSIE methods, able to identify underdetermined mixtures (i.e. $P > N$) of sources, have been developed. Some papers focus on blind source extraction [11] [8], which is a difficult problem since the underdetermined mixture is not invertible, while others, as herein, favour BSI of the mixture matrix [2] [6] [9] [11] [13]. The methods proposed in [2] [6] [9] only exploit the information contained in the data FO statistics whereas the one proposed in [13] exploits the information contained in the second characteristic function of the observations. As for Lee et al. [11], they maximize the probability of the data conditionally to the mixture matrix. However, all these methods have drawbacks in operational contexts. Indeed, the method [2] is still very difficult to implement and does not ensure the BSI of the source steering vectors when the sources have the same kurtosis. The BSI methods [6] [9] assume non FO circularity and thus fail in separating FO circular sources. Besides, the theory developed in [6] only confines itself to three sources and two sensors. Although the method [11] succeeds in identifying the steering vectors of up to four speech signals with only two sensors, the authors need to assume temporal independence of the samples and that all sources have a sparse distribution. Finally, the method [13] has been developed only for real mixtures of real-valued sources and robustness issue to an over estimation of the number of sources is still to analyse.

To overcome the previous drawbacks a new BSI method, named BIRTH (B*lind* I*dentification of mixtures of sources using* R*edundancies in the da*T*a* H*exacovariance matrix*), is proposed, able to blindly identify the steering vectors of up to $P = N^2 – N+1$ sources for arrays of N sensors with space diversity only, and up to $P = N^2$ for those with angular and polarization diversity. The sources are assumed to have non zero SixO marginal cumulants with the same sign (the latter assumption is generally verified in radiocommunications contexts). Besides, BIRTH exploits the VA concept described in [10] [4] and redundancies in the SixO statistical matrix of the data, called *hexacovariance*, without SO or FO prewhitening.

2. ASSUMPTIONS AND NOTATION

Assume that for any fixed time index k, N complex outputs $x_n(k)$ of a noisy mixture of P statistically sources $s_p(k)$ are available. The vector $\boldsymbol{x}(k)$ of the measured array outputs is given by

$$\boldsymbol{x}(k) = \boldsymbol{A}\,\boldsymbol{s}(k) + \boldsymbol{\nu}(k) \tag{1}$$

where $\boldsymbol{A}$, $\boldsymbol{s}(k)$, $\boldsymbol{\nu}(k)$ are the $(N\times P)$ constant mixing matrix, the source and noise random vectors, respectively. In addition, for any fixed time index k, $\boldsymbol{s}(k)$ and $\boldsymbol{\nu}(k)$ are statistically independent.

For the sake of convenience we need to define, for any k, the entries of the SixO cumulant tensor $\boldsymbol{C_z}$ of a random vector, $\boldsymbol{z}(k)$, stationary and ergodic up to order 6:

$$C^{ghi}_{def,\boldsymbol{z}} = \mathsf{Cum}\{z_d(k), z_e(k), z_f(k), z_g(k)^*, z_h(k)^*, z_i(k)^*\} \quad (2)$$

Such components may be ordered in the $\boldsymbol{H_z}$ hexacovariance matrix as follows:

$$\forall\, 1 \leq d, e, f, g, h, i \leq N,\; H^{N(N(g-1)+h-1)+f}_{N(N(d-1)+e-1)+i,\boldsymbol{z}} = C^{ghi}_{def,\boldsymbol{z}} \quad (3)$$

where $H^{q}_{r,\boldsymbol{z}}$ denotes the $(r,q)^{\text{th}}$ component of $\boldsymbol{H_z}$. Note that SixO cumulants are given as a function of moments in statistics text books, but only in the real case [12]. Nevertheless, their expressions in the complex case can be found in [1] for zero-mean variables which are distributed symmetrically with respect to the origin. Moreover, we further assume the following hypothesis:

A1. For any fixed time index k, sources $s_p(k)$ are stationary, ergodic and mutually uncorrelated at order 6, with values a priori in the complex field;

A2. For any fixed time index k, noise values $\nu_n(k)$ are stationary, ergodic and gaussian with values a priori in the complex field too;

A3. SixO marginal source cumulants, $C^{ppp}_{ppp,\boldsymbol{s}}$, are not null and have all the same sign;

A4. The number of sources is such that $P \leq N^2$;

The goal of BSI consists of determining an estimate of the mixture matrix $\boldsymbol{A}$ of the sources.

3. THE BIRTH METHOD

3.1. Hexacovariance property

The BIRTH method exploits several matrix redundancies in the hexacovariance of the data especially thanks to the multilinearity property under changes of coordinate systems, which enjoy cumulants. This property can be expressed by the following equation:

$$\boldsymbol{H_x} = [\boldsymbol{A} \otimes \boldsymbol{A} \otimes \boldsymbol{A}^*]\, \boldsymbol{H_s}\, [\boldsymbol{A} \otimes \boldsymbol{A} \otimes \boldsymbol{A}^*]^{\mathsf{H}} \quad (4)$$

where the $(N^3 \times N^3)$ $\boldsymbol{H_x}$ and the $(P^3 \times P^3)$ $\boldsymbol{H_s}$ matrices are the hexacovariance matrices of $\boldsymbol{x}(k)$ and $\boldsymbol{s}(k)$ respectively. Furthermore, the $\boldsymbol{H_s}$ matrix is diagonal since the sources are independent but is not full rank. So it is possible to write the $\boldsymbol{H_x}$ matrix as a function of the full rank diagonal $(P \times P)$ $\boldsymbol{\mathcal{H}_s} = \mathrm{diag}\big([C^{111}_{111,\boldsymbol{s}}\, C^{222}_{222,\boldsymbol{s}} \cdots C^{PPP}_{PPP,\boldsymbol{s}}]\big)$ matrix:

$$\boldsymbol{H_x} = \boldsymbol{\mathcal{A}}_1\, \boldsymbol{\mathcal{H}_s}\, \boldsymbol{\mathcal{A}}_1{}^{\mathsf{H}} \quad (5)$$

where the $(N^3 \times P)$ $\boldsymbol{\mathcal{A}}_1$ matrix, assumed full rank, is given by:

$$\begin{aligned} \boldsymbol{\mathcal{A}}_1 &= [\boldsymbol{a}_1 \otimes \boldsymbol{a}_1 \otimes \boldsymbol{a}_1^* \ \cdots \ \boldsymbol{a}_P \otimes \boldsymbol{a}_P \otimes \boldsymbol{a}_P^*] \\ &= [[\boldsymbol{\mathcal{A}}_2 \boldsymbol{\Phi}_1]^{\mathsf{T}}\ [\boldsymbol{\mathcal{A}}_2 \boldsymbol{\Phi}_2]^{\mathsf{T}} \ \cdots \ [\boldsymbol{\mathcal{A}}_2 \boldsymbol{\Phi}_N]^{\mathsf{T}}]^{\mathsf{T}} \end{aligned} \quad (6)$$

with the $(N^2 \times P)$ $\boldsymbol{\mathcal{A}}_2$ matrix, assumed full rank, defined by:

$$\begin{aligned} \boldsymbol{\mathcal{A}}_2 &= [\boldsymbol{a}_1 \otimes \boldsymbol{a}_1^* \ \cdots \ \boldsymbol{a}_P \otimes \boldsymbol{a}_P^*] \\ &= [[\boldsymbol{A}^* \boldsymbol{\Phi}_1]^{\mathsf{T}}\ [\boldsymbol{A}^* \boldsymbol{\Phi}_2]^{\mathsf{T}} \ \cdots \ [\boldsymbol{A}^* \boldsymbol{\Phi}_N]^{\mathsf{T}}]^{\mathsf{T}} \end{aligned} \quad (7)$$

and:

$$\boldsymbol{\Phi}_n = \mathrm{diag}\big([\ \boldsymbol{a}_1(n) \quad \boldsymbol{a}_2(n) \quad \cdots \quad \boldsymbol{a}_P(n)\ ]\big) \quad (8)$$

In other words, the non zero elements of the diagonal $(P \times P)$ $\boldsymbol{\Phi}_n$ matrix are the n^{th} components of the P $\boldsymbol{a_p}$ steering vectors.

3.2. Data structure

If SixO marginal source cumulants are strictly positive (**A3**), then a square root of $\boldsymbol{H_x}$, called $\boldsymbol{H_x^{1/2}}$, has to be computed (if these cumulants are strictly negative, the $-\boldsymbol{H_x}$ matrix has to be considered for computing the square root) for example as following :

$$\boldsymbol{H_x^{1/2}} = \boldsymbol{E_s}\, \boldsymbol{L_s^{1/2}} = \boldsymbol{\mathcal{A}}_1\, \boldsymbol{\mathcal{H}_s^{1/2}}\, \boldsymbol{V}^{\mathsf{H}} \quad (9)$$

where $\boldsymbol{L_s}$ ($\boldsymbol{L_s^{1/2}}$ denotes a square root of $\boldsymbol{L_s}$) is the $(P \times P)$ real-valued diagonal matrix of the P non zero eigen-values of $\boldsymbol{H_x}$ and $\boldsymbol{E_s}$ is the $(N^3 \times P)$ matrix of the associated orthonormalized eigen-vectors. For a full rank $\boldsymbol{\mathcal{A}}_1$ matrix, it is possible to verify that (**A3**) is equivalent to assume that the diagonal elements of $\boldsymbol{L_s}$ are not null and have also the same sign. In addition, (9) shows the link between $\boldsymbol{H_x^{1/2}}$ and $\boldsymbol{\mathcal{A}}_1$ where $\boldsymbol{V}$ is an unitary matrix. Finally, (9) and (6) allow to prove the link between $\boldsymbol{H_x^{1/2}}$ and $\boldsymbol{\mathcal{A}}_2$, as follows:

$$\begin{aligned} \boldsymbol{H_x^{1/2}} &= \Big[[\boldsymbol{\mathcal{A}}_2 \boldsymbol{\Phi}_1 \boldsymbol{\mathcal{H}_s^{1/2}} \boldsymbol{V}^{\mathsf{H}}]^{\mathsf{T}} \cdots\ [\boldsymbol{\mathcal{A}}_2 \boldsymbol{\Phi}_N \boldsymbol{\mathcal{H}_s^{1/2}} \boldsymbol{V}^{\mathsf{H}}]^{\mathsf{T}}\Big]^{\mathsf{T}} \\ &= [\ \boldsymbol{\Gamma}_1{}^{\mathsf{T}}\ \ \boldsymbol{\Gamma}_2{}^{\mathsf{T}}\ \ \cdots\ \ \boldsymbol{\Gamma}_N{}^{\mathsf{T}}\ ]^{\mathsf{T}} \end{aligned} \quad (10)$$

where $\boldsymbol{\Gamma}_n = \boldsymbol{\mathcal{A}}_2 \boldsymbol{\Phi}_n\, \boldsymbol{\mathcal{H}_s^{1/2}}\, \boldsymbol{V}^{\mathsf{H}}$ is the n^{th} $(N^2 \times P)$ matrix block of $\boldsymbol{H_x^{1/2}}$.

3.3. SixO blind identification step

In this section, the purpose is to exploit the information contained in the $\boldsymbol{H_x^{1/2}}$ matrix to blindly identify $\boldsymbol{A}$. Indeed, the $\boldsymbol{V}$ matrix diagonalizes the $N(N-1)$ $\boldsymbol{\Theta}_{n_1,n_2}$ matrices described, for all $1 \leq n_1 \neq n_2 \leq N$, by:

$$\boldsymbol{\Theta}_{n_1,n_2} = \boldsymbol{\Gamma}_{n_1}^{\sharp}\, \boldsymbol{\Gamma}_{n_2} = \boldsymbol{V}\, \boldsymbol{\mathcal{H}_s^{-1/2}}\, \boldsymbol{\Phi}_{n_1}^{-1}\, \boldsymbol{\Phi}_{n_2}\, \boldsymbol{\mathcal{H}_s^{1/2}}\, \boldsymbol{V}^{\mathsf{H}} \quad (11)$$

where $\sharp$ denotes the pseudo-inverse operator and where the $\boldsymbol{D}_{n_1,n_2} = \boldsymbol{\mathcal{H}}_{\boldsymbol{s}}^{-1/2}\boldsymbol{\Phi}_{n_1}^{-1}\boldsymbol{\Phi}_{n_2}\boldsymbol{\mathcal{H}}_{\boldsymbol{s}}^{1/2}$ matrices are diagonal. Thus, by construction, the rank of $\boldsymbol{\Theta}_{n_1,n_2}$, denoted by $\mathrm{rk}(\boldsymbol{\Theta}_{n_1,n_2})$, cannot exceed the $\min(\mathrm{rk}(\boldsymbol{\Gamma}_{n_1}), \mathrm{rk}(\boldsymbol{\Gamma}_{n_2})) = \min(P, \mathrm{rk}(\boldsymbol{\mathcal{A}}_2))$ value, hence another bound of the maxi number of sources, P. The unitary $\boldsymbol{V}_{sol} = \boldsymbol{V}\boldsymbol{\mathcal{T}}$ matrix, solution to the previous problem of joint diagonalization to within an unitary trivial matrix $\boldsymbol{\mathcal{T}}$ (a trivial matrix is of the form $\boldsymbol{\Lambda}\boldsymbol{\Pi}$ where $\boldsymbol{\Lambda}$ is an invertible diagonal matrix and $\boldsymbol{\Pi}$ a permutation), allows, in accordance with (9), to recover $\boldsymbol{\mathcal{A}}_1$ to within a trivial matrix as follows:

$$\boldsymbol{H}_{\boldsymbol{x}}^{1/2}\,\boldsymbol{V}_{sol} = \boldsymbol{\mathcal{A}}_1\,\boldsymbol{\mathcal{H}}_{\boldsymbol{s}}^{1/2}\,\boldsymbol{\mathcal{T}} \tag{12}$$

Since, consistent with (6) and (7), the (12) equation can also be written as follows:

$$\begin{aligned}\boldsymbol{H}_{\boldsymbol{x}}^{1/2}\,\boldsymbol{V} &= \Big[[\boldsymbol{A}^*\boldsymbol{\Phi}_1\boldsymbol{\Phi}_1\boldsymbol{\mathcal{H}}_{\boldsymbol{s}}^{1/2}]^{\mathsf{T}}\ \cdots[\boldsymbol{A}^*\boldsymbol{\Phi}_N\boldsymbol{\Phi}_N\boldsymbol{\mathcal{H}}_{\boldsymbol{s}}^{1/2}]^{\mathsf{T}}\Big]^{\mathsf{T}} \\ &= [\boldsymbol{\Sigma}_1{}^{\mathsf{T}}\ \boldsymbol{\Sigma}_2{}^{\mathsf{T}}\cdots\boldsymbol{\Sigma}_{N^2}{}^{\mathsf{T}}]^{\mathsf{T}}\end{aligned} \tag{13}$$

So, the $\boldsymbol{\Sigma}_1$ matrix block made up of the first N^{th} rows of the $\boldsymbol{H}_{\boldsymbol{x}}^{1/2}\boldsymbol{V}_{sol}$ matrix corresponds to within a trivial matrix to $\boldsymbol{A}^*$ such as:

$$\boldsymbol{\Sigma}_1 = \boldsymbol{A}^*\ [\boldsymbol{\Phi}_1]^2\ \boldsymbol{\mathcal{H}}_{\boldsymbol{s}}^{1/2}\,\boldsymbol{\mathcal{T}} \tag{14}$$

where $\boldsymbol{\mathcal{H}}_{\boldsymbol{s}}^{1/2}$ and $\boldsymbol{\Phi}_n$, for all $1 \leq n \leq N$, are diagonal matrices.

3.4. Implementation of the BIRTH method

The different steps of the BIRTH method are summarized hereafter when K samples of the observations, $\boldsymbol{x}(k)$ ($1 \leq k \leq K$), are available.

***Step*1:** Compute the estimate $\widehat{\boldsymbol{H}}_{\boldsymbol{x}}$ of $\boldsymbol{H}_{\boldsymbol{x}}$ from the K samples $\boldsymbol{x}(k)$ using for instance [1] and the empirical estimate of moments, unbiased and consistent for ergodic stationary sources.
***Step*2:** Eigen Value Decomposition (EVD) of the matrix $\widehat{\boldsymbol{H}}_{\boldsymbol{x}}$, estimation of the number of sources P and restriction of this EVD to the P principal components: $\widehat{\boldsymbol{H}}_{\boldsymbol{x}} = \widehat{\boldsymbol{E}}_{\boldsymbol{s}}\,\widehat{\boldsymbol{L}}_{\boldsymbol{s}}\,\widehat{\boldsymbol{E}}_{\boldsymbol{s}}^{\mathsf{H}}$, where $\widehat{\boldsymbol{L}}_{\boldsymbol{s}}$ is the diagonal matrix of the P eigenvalues with the strongest modulus and $\widehat{\boldsymbol{E}}_{\boldsymbol{s}}$ is the matrix of the associated eigen-vectors.
***Step*3:** Computation of a square root matrix $\widehat{\boldsymbol{H}}_{\boldsymbol{x}}^{1/2}$ of $\widehat{\boldsymbol{H}}_{\boldsymbol{x}}$: $\widehat{\boldsymbol{H}}_{\boldsymbol{x}}^{1/2} = \widehat{\boldsymbol{E}}_{\boldsymbol{s}}\,|\widehat{\boldsymbol{L}}_{\boldsymbol{s}}|^{1/2}$, where $|\cdot|$ denotes the complex modulus operator.
***Step*4:** Computation from $\widehat{\boldsymbol{H}}_{\boldsymbol{x}}^{1/2}$ of the $\widehat{\boldsymbol{\Theta}}_{n_1,n_2} = [\widehat{\boldsymbol{\Gamma}}_{n_1}^{\sharp}\,\widehat{\boldsymbol{\Gamma}}_{n_2}]$ matrices for all $1 \leq n_1 \neq n_2 \leq N$, and estimation, $\widehat{\boldsymbol{V}}_{sol}$, of the unitary matrix $\boldsymbol{V}_{sol}$ from the joint diagonalization of the $N(N-1)$ matrices $\widehat{\boldsymbol{\Theta}}_{n_1,n_2}$.
***Step*5:** Estimation $\widehat{\boldsymbol{A}}$ of the $\boldsymbol{A}$ mixture matrix taking the matrix block made up of the first N^{th} rows of $[\widehat{\boldsymbol{H}}_{\boldsymbol{x}}^{1/2}\,\widehat{\boldsymbol{V}}_{sol}]^*$.

4. IDENTIFIABILITY

4.1. The BIRTH approach

Following the development of the previous sections, it appears that the BIRTH method is able to identify, from an array of N sensors, the steering vectors of P ($P \leq N^2$) non Gaussian sources having SixO marginal cumulants with the same sign, provided that the $\boldsymbol{\mathcal{A}}_2$ matrix has full rank P, i.e. that the *virtual steering vectors* $[\boldsymbol{a}_p \otimes \boldsymbol{a}_p^*]$ ($1 \leq p \leq P$) for the considered array of N sensors remain linearly independent. In addition, it has been shown in [4] that the vector $[\boldsymbol{a}_p \otimes \boldsymbol{a}_p^*]$ can also be considered as a *true steering vector* but for a *virtual array* of N_e different sensors. This especially means that $N^2 - N_e$ components of each vector $\left[\boldsymbol{a}_p \otimes \boldsymbol{a}_p^*\right]$ are redundant elements which bring no information. The rank of $\boldsymbol{\mathcal{A}}_2$ cannot therefore be greater than N_e and is equal to $\min(N_e, P)$ when $\boldsymbol{A}$ is full rank. In these conditions, since $\boldsymbol{\mathcal{A}}_2$ has full rank P, $\min(N_e, P)$ is equal to P, which implies $P \leq N_e$. So the BIRTH algorithm is able to process up to N_e sources, where N_e is the number of different Virtual Sensors (VS) of the VA associated with the chosen array of N sensors. So, it is shown in [4] that using an array with space diversity only, as for instance an Uniformly spaced Circular Array (UCA), N_e may be equal to $N^2 - N + 1$, whereas using an array with angular and polarization diversity, the N_e number may attain N^2.

4.2. Impact of the hexacovariance structure

According to [4], the N_e number is directly related to both kind of sensors and geometry of the true array of N sensors. For example, an Uniform Linear Array (ULA) of identical sensors generates a VA of $N_e = 2N - 1$ different VS, whereas for most of other arrays $N_e = N^2 - N + 1$. Nevertheless, both kind of sensors and geometry of the true array are not the only factor which the N_e number depends on. Indeed the way data SixO cumulants are mapped in $\boldsymbol{H}_{\boldsymbol{x}}$ is also a parameter which affects the number of VS. To show this, consider the following way to sort SixO Cumulants in the hexacovariance matrix:

$$\forall\, 1 \leq d, e, f, g, h, i \leq N,\ H_{N(N(d-1)+e-1)+f,\boldsymbol{z}}^{N(N(g-1)+h-1)+i} = C_{def,\boldsymbol{z}}^{ghi} \tag{15}$$

what implies:

$$\boldsymbol{H}_{\boldsymbol{x}} = [\boldsymbol{A} \otimes \boldsymbol{A} \otimes \boldsymbol{A}]\,\boldsymbol{H}_{\boldsymbol{s}}\,[\boldsymbol{A} \otimes \boldsymbol{A} \otimes \boldsymbol{A}]^{\mathsf{H}} \tag{16}$$

The FO virtual array associated with this expression (the corresponding *virtual steering vectors*, for all $1 \leq p \leq P$, are thus of the form $[\boldsymbol{a}_p \otimes \boldsymbol{a}_p]$) is generally different from the one obtained from (4). In particular, the VA associated with (16) and an UCA of odd N identical sensors, is caracterized by $N_e = N(N+1)/2$ different VS, whereas the one associated with (4) and an UCA of odd N identical sensors, is caracterized by $N_e = N^2 - N + 1$ different VS. For any $N \geq 2$, the

$N^2 - N + 1$ value is obviously greater than $N(N+1)/2$.

***Proof*:** *Note that the $(r,q)^{\text{th}}$ VS associated with the p^{th} source and the UCA of N sensors is such that:*

$$[\boldsymbol{a}_p \otimes \boldsymbol{a}_p]_r^q = \exp\{\mathrm{j}2\pi[x_r^q \cos(\theta_p)\cos(\phi_p) + y_r^q \sin(\theta_p)\cos(\phi_p)]\} \quad (17)$$

$(x_r^q, y_r^q, 0) = ((R_r^q/\lambda)\cos(\varphi_r^q), (R_r^q/\lambda)\sin(\varphi_r^q), 0)$ are the coordinates of the $(r,q)^{\text{th}}$ VS ($1 \le r, q \le N$) where $R_r^q = 2R\cos((\varphi_r - \varphi_q)/2)$ and $\varphi_r^q = (\varphi_r + \varphi_q)/2$ since it is always possible to choose a coordinate system in which the n^{th} sensor of the true array has the coordinates $(x_n, y_n, 0) = (R\cos(\varphi_n), R\sin(\varphi_n), 0)$ where R is the radius and $\varphi_n = 2\pi(n-1)/N$. It is thus easy to deduce from the previous equations that the VS that are not at coordinates $(0,0,0)$ lie on $(N+1)/2$ different circles if N is odd or $N/2$ if N is even and that there are VS at coordinates $(0,0,0)$ only if N is even. Moreover, for odd values of N, N different VS lie on each circle of the VA uniformly spaced. As a consequence, this VA, for odd values of N, has $N_e = N(N+1)/2$ different VS. As to the second result, it is given by [4].

It is important to explain that if both FO VA obtained from (4) and (16) are not equivalent, however, they have the same radiation pattern.

***Proof*:** *The radiation pattern of a $[\boldsymbol{b}(\theta_p, \phi_p)]_{1 \le p \le P}$ VA is defined by:*

$$\forall\ (\theta, \phi), \quad \forall 1 \le p \le P,$$
$$c((\theta,\phi), \boldsymbol{b}(\theta_p,\phi_p)) = \frac{|\langle\, \boldsymbol{b}(\theta,\phi), \boldsymbol{b}(\theta_p,\phi_p)\,\rangle|}{\|\boldsymbol{b}(\theta,\phi)\|^2\, \|\boldsymbol{b}(\theta_p,\phi_p)\|^2} \quad (18)$$

where θ_p, ϕ_p, $|\cdot|$, $\langle\cdot,\cdot\rangle$, $\|\cdot\|$ denote azimuth and elevation angles of the p^{th} source, the complex modulus, the scalar product and the norm operators, respectively. Since for any (θ,ϕ) and for each source p, both $c\big((\theta,\phi), [\boldsymbol{a}_p \otimes \boldsymbol{a}_p^*]\big)$ and $c((\theta,\phi), [\boldsymbol{a}_p \otimes \boldsymbol{a}_p])$ values are equal.

These results are illustrated by figures 1 to 3, which show the identical radiation pattern of both FO VA of an UCA of five identical sensors, and the geometry of each VA, respectively.

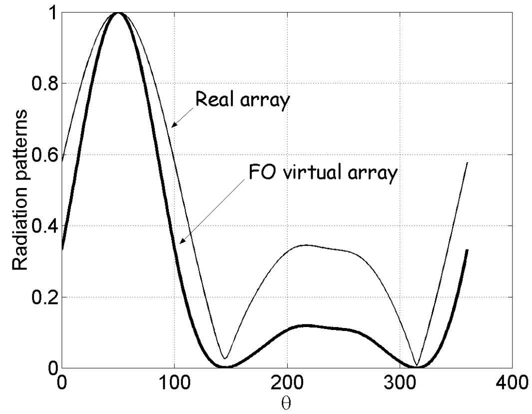

Fig. 1. FO virtual array radiation pattern ($N = 5$)

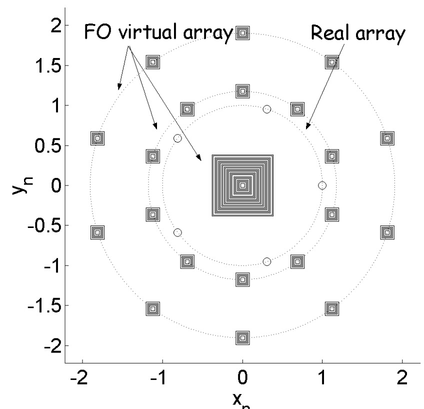

Fig. 2. FO* virtual array defined by $[\boldsymbol{a}_p \otimes \boldsymbol{a}_p^*]$ ($N = 5$)

5. SIMULATIONS

5.1. Performance criterion

Most of the existing performance criteria used to evaluate the quality of the BSI process [5], [6] [13] are global criteria, which evaluate a distance between the true mixing matrix $\boldsymbol{A}$ and its blind estimate $\widehat{\boldsymbol{A}}$. Although practical, a global performance criterion necessarily contains a part of arbitrary considerations in the manner of combining all the distances between the vectors $\boldsymbol{a}_p$ and $\widehat{\boldsymbol{a}}_p$. Moreover, it is possible to find that an estimate $\widehat{\boldsymbol{A}}_1$ of $\boldsymbol{A}$ is better than an estimate $\widehat{\boldsymbol{A}}_2$, with respect to the global criterion, while some columns of $\widehat{\boldsymbol{A}}_2$ estimate the associated true steering vectors in a better way than $\widehat{\boldsymbol{A}}_1$. For these reasons, it may be more appropriate to use a non global criterion for the evaluation of the BSI process, which is defined by the P-uplet:

$$\mathrm{D}\Big(\boldsymbol{A}, \widehat{\boldsymbol{A}}\Big) = (\alpha_1, \alpha_2, \ldots, \alpha_P) \quad (19)$$

where

$$\alpha_p = \underbrace{\min}_{1 \le i \le P} [\mathrm{d}(\boldsymbol{a}_p, \widehat{\boldsymbol{a}}_i)] \quad (20)$$

and where $\mathrm{d}(\boldsymbol{u}, \boldsymbol{v})$ is the pseudo-distance between the vectors $\boldsymbol{u}$ and $\boldsymbol{v}$, defined by:

$$\mathrm{d}(\boldsymbol{u}, \boldsymbol{v}) = 1 - \frac{|\langle \boldsymbol{u}, \boldsymbol{v} \rangle|^2}{\|\boldsymbol{u}\|^2 \|\boldsymbol{v}\|^2} \quad (21)$$

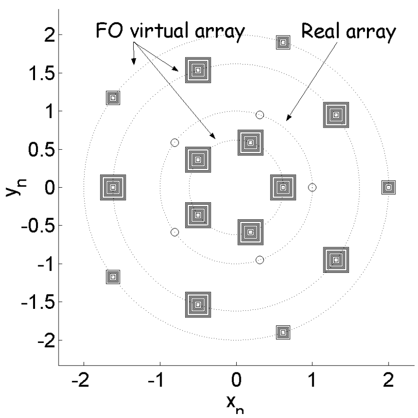

Fig. 3. FO virtual array defined by $[\boldsymbol{a}_p \otimes \boldsymbol{a}_p]$ $(N = 5)$

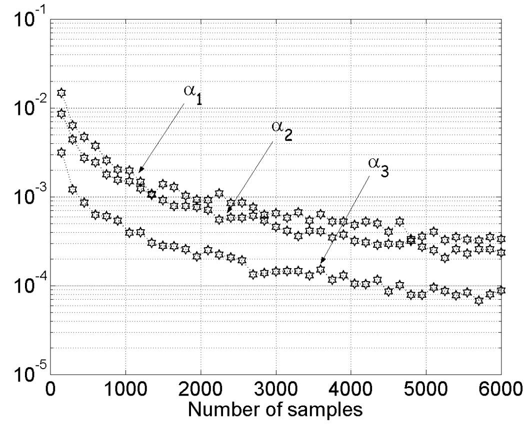

Fig. 5. $\mathrm{D}\left(\boldsymbol{A}, \widehat{\boldsymbol{A}}\right)$ associated with the BIRTH method

5.2. Computer results

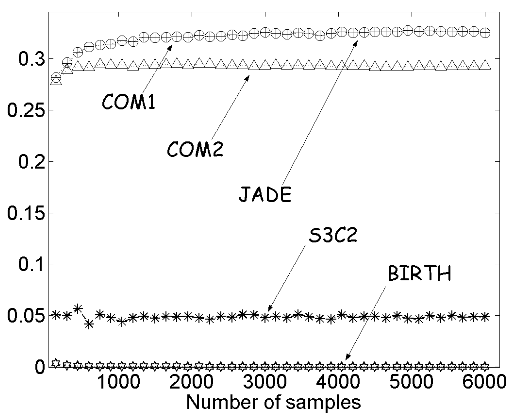

Fig. 4. α_3 for a $\mathrm{SNR} = 20$ dB

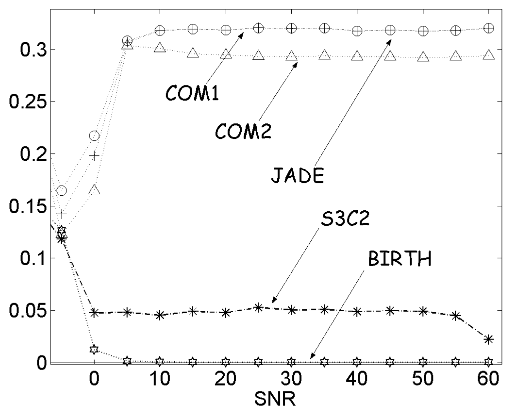

Fig. 6. α_3 for one thousand samples

To illustrate the previous results, we assume that $P = 2$ statistically independent sources, i.e. 2 non filtered QPSK and 1 non filtered BPSK, are received by a linear array of $N = 2$ sensors of radius R such that $R/\lambda = 0.55$ (λ: wavelength). The 3 sources, assumed synchronized, have the same input SNR (Signal to Noise Ratio) of 20 dB with a symbol period $T = 4T_e$, where T_e is the sample period. The normalized marginal source cumulants are $\kappa_{111,\mathrm{QPSK}}^{111} = \kappa_{222,\mathrm{QPSK}}^{222} = 4$ and $\kappa_{333,\mathrm{BPSK}}^{333} = 16$ [1]. The direction of arrival of the sources are such that $\theta_1 = 50°$, $\theta_2 = 136°$, $\theta_3 = 29.5°$, $\phi_1 = \phi_2 = \phi_3 = 0°$ and the associated carrier frequencies verify $\Delta f_1 T_e = 1/3$, $\Delta f_2 T_e = 1/2$ and $\Delta f_3 T_e = 0$. We apply the COM1 [7], COM2 [5], JADE [3], S3C2 [6] and BIRTH methods, and the performance α_p for $p = 1 \ldots 3$ is computed and averaged over 200 realizations.

Under the previous assumptions, figure 4 shows the variations of α_3 (source 3 performance) at the output of the COM1, COM2, JADE, S3C2 and BIRTH algorithms as a function of the number of samples. The COM1, COM2, JADE methods obviously find difficulties in well identifying the steering vector of the source 3 in an underdetermined context. The S3C2 method gives better results. As to the BIRTH process, it completely succeeds in identifying the steering vector. Figure 5 shows, in the same context, all the α_p at the output of the BIRTH method as a function of samples. Note the decreasing values toward zero of all the previous coefficients as the number of samples increases. In addition, figure 6 displays the variations of α_3 (source 3 performance) at the output of the COM1, COM2, JADE, S3C2 and BIRTH methods as a function of SNR. Likewise, the COM1, COM2, JADE algorithms do not identify the steering vector of the source 3 in an underdetermined context even when the SNR increases. The S3C2 results are more pleasing. As to the BIRTH process, it performs the identification of the steering vector even for a small value of SNR.

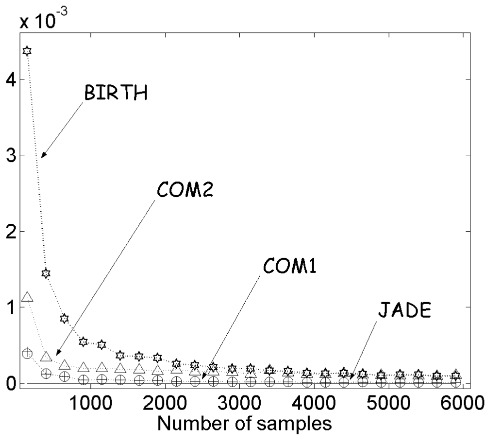

Fig. 7. α_3 for a $\mathrm{SNR} = 20$ dB

Finally, consider the $P = 3$ previous sources are received by a circular array of $N = 3$ sensors such that $R/\lambda = 0.55$. Figure 7 shows the variations of α_3 (source 3 performance) at the output of the COM1, COM2, JADE and BIRTH methods as a function of the number of samples : the BIRTH method obviously works in overdetermined contexts and although SixO cumulants have to be estimated, the BIRTH algorithm converges fast enough compared with the other algorithms.

6. CONCLUSION

This paper presents a new simple BSI method, BIRTH, in an underdetermined context, i.e. allowing to identify the steering vectors of more sources than sensors, using SixO cumulants and the FO VA concept. The BIRTH algorithm succeeds in recovering the mixture matrix even for a small number of samples or a weak SNR. Moreover, new results as for the VA are given : both FO VA, described in this paper, are proved to be not equivalent. As a consequence, the way to store cumulants in the corresponding matrix affects the performance of the method. Finally, the BIRTH algorithm can be improved, in particular the fifth step of (3.4). This will be the subject of a forthcoming paper.

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