Rem : $A(t) \perp\!\!\!\perp B_1(t') \Rightarrow \left| \begin{array}{l} \alpha_{S1} \perp\!\!\!\perp \alpha_{B1} \\ \beta_{S1} \perp\!\!\!\perp \beta_{B1} \end{array} \right.$

On a :

$$\gamma_\alpha(\nu) = \gamma_{\alpha_{S1}}(\nu) + \gamma_{\alpha_{B1}}(\nu) = \gamma_{\beta_{S1}}(\nu) + \gamma_{\beta_{B1}}(\nu) = \gamma_\beta(\nu).$$

De plus :

$\gamma_{\alpha,\beta}(\nu) = 0.$ Car $\Gamma_{\alpha,\beta}(t,t') = \Gamma_{(\alpha_{S1}+\alpha_{B1}),(\beta_{S1}+\beta_{B1})}(t,t') = (\dots).$

2) $$Y_1(t) = \sqrt{\alpha(t)^2 + \beta(t)^2}$$
$$= \sqrt{(\alpha_{S1}(t) + \alpha_{B1}(t))^2 + (\beta_{S1}(t) + \beta_{B1}(t))^2}$$
$$= \sqrt{\alpha_{S1}(t)^2 + \cancel{\alpha_{B1}(t)^2} + 2\,\alpha_{S1}(t).\alpha_{B1}(t) + \beta_{S1}(t)^2 + \cancel{\beta_{B1}(t)^2} + 2\,\beta_{S1}(t).\beta_{B1}(t)}.$$

On néglige $\alpha_{B1}(t)^2$ et $\beta_{B1}(t)^2$ car le bruit est supposé faible.

D'autre part, on a :

$$s_1(t) = \alpha_{S1}(t).\cos(2\pi f_0 t) - \beta_{S1}(t)\sin(2\pi f_0 t)$$
$$\parallel$$
$$S_2(t).P(t)$$
$$\parallel$$
$$S_2(t).\cos(2\pi f_0 t + \phi) = S_2(t).\cos(2\pi f_0 t).\cos(\phi) - S_2(t).\sin(2\pi f_0 t).\sin(\phi).$$

D'où :

$$\alpha_{S1}(t) = S_2(t).\cos(\phi) \quad \text{et} \quad \beta_{S1}(t) = S_2(t).\sin(\phi).$$

De ce fait, on a :

$$Y_1(t) \simeq \sqrt{S_2(t)^2 . \cos^2(\phi) + S_2(t)^2 . \sin^2(\phi) + 2 S_2(t) . \left(\alpha_{B_1}(t) . \cos(\phi) + \beta_{S_1}(t) . \sin(\phi) \right)}$$

On pose :

$$z(t) = \alpha_{B_1}(t) . \cos(\phi) + \beta_{B_1}(t) . \sin(\phi)$$

On a alors :

$$Y_1(t) \simeq \sqrt{S_2(t)^2 + 2 S_2(t) . z(t)} = \pm S_2(t) \sqrt{1 + 2 . \frac{z(t)}{S_2(t)}}$$

$\frac{z(t)}{S_2(t)}$ étant très petit, on fait un DL autour de 0 à l'ordre 1, d'où :

$$Y_1(t) \simeq \pm S_2(t) . \left(1 + \frac{1}{2} . 2 \frac{z(t)}{S_2(t)} \right)$$

$$\simeq \pm S_2(t) \pm z(t)$$

composante de bruit.

$$E[z(t) . z(t)^*] \underset{z \in \mathbb{R}}{=} E[z(t)^2] = E\left[\alpha_{B_1}(t)^2 \cos^2(\phi) + \beta_{B_1}(t)^2 \sin^2(\phi) + 2 . \alpha_{B_1}(t) . \beta_{B_1}(t) \cos(\phi) . \sin(\phi) \right]$$

$$= \Gamma_{\alpha_{B_1}}(0) . \cos^2(\phi) + \underbrace{\Gamma_{\beta_{B_1}}(0)}_{= \Gamma_{\alpha_{B_1}}(0)} . \sin^2(\phi) + 2 . \underbrace{\Gamma_{\alpha_{B_1}, \beta_{B_1}}(0)}_{= 0} . \cos(\phi) \sin(\phi)$$

$$= \Gamma_{\alpha_{B_1}}(0) = \int_{\mathbb{R}} \chi_{\alpha_{B_1}}(\nu) . d\nu = \int_{\mathbb{R}} N_0 . \mathbb{1}_{[-B/2, B/2]}(\nu) . d\nu$$

$$= \int_{-B/2}^{B/2} N_0 . d\nu = \boxed{N_0 . B = E[z(t)^2]}$$

Puissance stat. moy. du bruit

erreur quadratique moy. de démodul.