Champ $x = \{x_s, s \in \mathcal{S}\}$; $\forall s$ x_s à valeurs ds E $\downarrow$ Ici $\{-1, 0, 1\}$

Loi conditionelle pour champ de loi :

$$P(x) = K e^{-H(x)}$$

$$= K e^{-\sum_{c \in \mathcal{C}} V_c(x)}$$

On considère $P(x_s / x_{s'}, s' \neq s)$

$$\rightarrow \frac{P(x_s)}{\sum_{x_s \in E} K e^{-\sum_{c \in \mathcal{C}} V_c(x)}}$$

On décompose $\sum_{c \in \mathcal{C}} V_c(x) = \sum_{c : s \in c} V_c(x) + \sum_{c : s \notin c} V_c(x)$

$$\rightarrow \frac{K e^{-\left[\sum_{c : s \in c} V_c(x) + \cancel{\sum_{c : s \notin c} V_c(x)}\right]}}{\left[\sum_{x_s \in E} K e^{-\sum_{c : s \in c} V_c(x)}\right] \cancel{e^{-\sum_{c : s \notin c} V_c(x)}}}$$

$$= K_1 e^{-\sum_{c : s \in c} V_c(x)}$$

$$\boxed{P(x_s / x_{s'}, s' \neq s) \propto e^{-\sum_{c : s \in c} V_c(x)} , \; s \in E}$$

le calcul de la distribution de probabilité conditionnelle s'effectue en n'utilisant que les énergies des cliques locales, celles qui contiennent le point examiné dans l'image si $E = \{-1, 0, 1\}$ cela fait 3 valeurs

à calculer

$$\sum_{C : s \in C} V_C(x_s = \cdot\,, x_{s'}, s' \in C \setminus s)$$

si $\alpha_1, \alpha_2, \alpha_3$ sont ces 3 valeurs

on lit dans $\frac{\alpha_1}{\alpha}, \frac{\alpha_2}{\alpha}, \frac{\alpha_3}{\alpha}$, $\alpha = \alpha_1 + \alpha_2 + \alpha_3$

Applied Optics **Letters to the Editor**

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Noise-immune phase unwrapping algorithm

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A new phase unwrapping algorithm is proposed which combines noise immunity with computational efficiency. It is based on the requirement that the unwrapped map should be independent of the route by which unwrapping takes place.

Automatic fringe analysis by digital computer has been the subject of considerable research activity in recent years. One of the most versatile techniques involves calculating the fringe phase $\Phi(m,n)$ at each pixel, either by shifting the fringes through known phase increments,[1] or by Fourier transformation of a single pattern containing carrier fringes.[2] In either case, the calculated phase is the principal value, lying in the range from $-\pi$ to π. Phase unwrapping must therefore be carried out to restore the unknown multiple of 2π to each pixel. This is normally achieved[2] by working along each row in turn: when the phase difference between a pixel and its predecessor is greater than π, 2π is either added to or subtracted from the remaining pixels in the row. The process is then repeated along the columns. This approach is computationally efficient but has poor noise immunity. More robust methods have recently been presented.[3,4] The first of these, however, requires typically several hundred iterations, with each iteration involving the whole image. In the second method, the image is subdivided into regions containing no phase ambiguities; these regions are then phase shifted with respect to one another to minimize the number of inconsistent boundaries. This Letter proposes a new phase unwrapping algorithm, combining both noise immunity and computational efficiency, based on the simple requirement that the unwrapped map should be independent of the route by which unwrapping takes place. The ideas presented have relevance to the related problem of reconstructing phase maps from measured phase differences (see, for example, Ref. 5).

The first two figures illustrate the problems that can arise with the conventional technique. Figure 1 is a crossed moire fringe pattern from a high speed sequence, recorded at 1 μs frame^{-1} with a Hadland Imacon 790 image converter camera, showing the impact of a steel ball on the edge of a plate of polymethyl methacrylate.[6] The fringe visibility is quite low in places, particularly at the center of the image where ion damage to the photocathode has reduced the sensitivity of the tube. The fringe pattern was digitized to a resolution of 256 × 256 pixels and analyzed by a 2-D Fourier transform method similar to that proposed by Bone *et al.*[7] The horizontal and vertical fringes occupy different regions of the 2-D

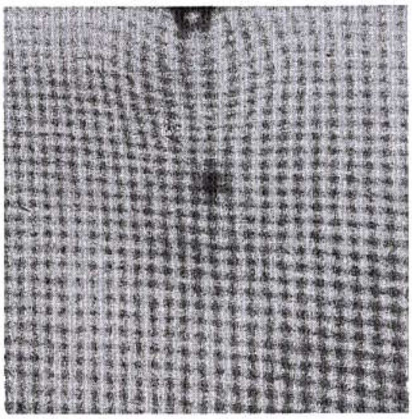

Fig. 1. Crossed moire fringe pattern from high speed photographic sequence showing the impact of a steel ball on a plate of PMMA (spatial frequency of specimen grating = 150 lines mm^{-1}; interframe time = 1 μs).

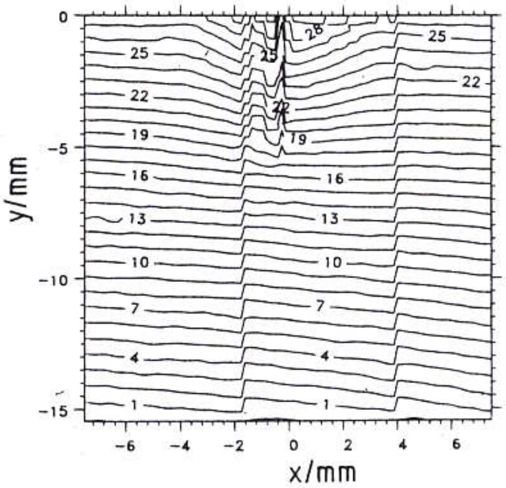

Fig. 2. Phase map of the horizontal fringes from Fig. 1 after unwrapping by the conventional algorithm. Phase values have been divided by 2π; each contour represents one fringe.

Fourier transform plane, so that the phase distribution of the two patterns can be separated.[6] Figure 2 shows the unwrapped phase map from the horizontal fringes. Unwrapping was carried out row by row in the $+x$ direction, and then column by column in the $+y$ direction. This method of unwrapping has two undesirable consequences. First, a low signal-to-noise ratio at a given point results in phase discontinuities, and hence corrupted data in regions well away from this point. Second, the unwrapped map is not unique: if

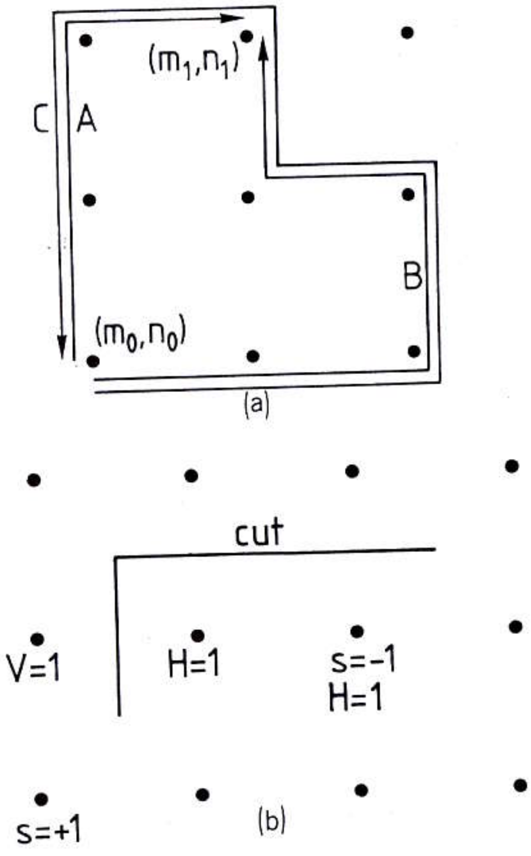

Fig. 3. (a) A and B are two alternative paths for unwrapping the phase at data point (m_1,n_1), given the phase at (m_0,n_0) (data points are represented by the symbol ●). Path C is a closed loop consisting of path B, and path A reversed. (b) Example of a cut made between two discontinuity sources $s = +1$ at data point (m_2,n_2) and $s = -1$ at (m_3,n_3). The cut is represented by two arrays H and V referred to in the text; nonzero values only are shown here.

the columns are unwrapped before the rows, the resulting discontinuities are horizontal rather than vertical.

The basis for the improved algorithm is the requirement that, given the phase at pixel (m_0,n_0), the phase at any other point (m_1,n_1) in the image should be defined uniquely, independent of the path by which the phases are unwrapped. This is achieved by placing cut lines in the phase map, which act as barriers to unwrapping. Consider two paths A and B in the example shown in Fig. 3(a). For simplicity of notation, the sequences of phase values along the two paths will be relabeled $\Phi_A(i)$ $(i = 0,1,\ldots N_A)$ and $\Phi_B(j)$ $(j = 0,1,\ldots N_B)$, respectively [$N_A = 3$ and $N_B = 5$ in Fig. 3(a)]. Unwrapping along A is achieved by calculating the number of 2π discontinuities, $d_A(i)$ $(i = 1,2,\ldots,N_A)$, between adjacent pixels:

$$d_A(i) = [(\Phi_A(i) - \Phi_A(i-1))/2\pi], \tag{1}$$

where $[\ldots]$ denotes rounding to the nearest integer. $2\pi d_A(i)$ is then subtracted from the phase values along the rest of the path (i.e., from $\Phi_A(i')$, $i' = i, i+1,\ldots,N_A$). The sequence $d_B(j)$ required to unwrap Φ_B is defined in a similar way. Uniqueness of the unwrapped phase at (m_1,n_1) requires the total number of discontinuities along the two paths to be equal; i.e., that the parameter S defined by

$$S = \sum_{j=1}^{N_B} d_B(j) - \sum_{i=1}^{N_A} d_A(i) \tag{2}$$

is equal to zero. If path A is reversed, the $d_A(i)$ all change sign, so that S is just the total number of 2π discontinuities around the counter clockwise closed loop C. The problem, therefore, is to construct the cut lines such that any permissible closed loop (i.e., one which does not cross a cut) has $S = 0$.

To proceed systematically, we consider a closed loop around each of the smallest possible units of the phase map: a square of 4 pixels. The distribution of s (the discontinuity source map) is calculated from $\Phi(m,n)$ as follows:

$$\begin{aligned} s(m,n) = {} & [(\Phi(m+1,n) - \Phi(m,n))/2\pi] \\ & + [(\Phi(m+1,n+1) - \Phi(m+1,n))/2\pi] \\ & + [(\Phi(m,n+1) - \Phi(m+1,n+1))/2\pi] \\ & + [(\Phi(m,n) - \Phi(m,n+1))/2\pi]. \end{aligned} \tag{3}$$

The value of S for larger loops can be easily obtained from $s(m,n)$. For example, path C in Fig. 3(a) has $S = s(m_0,n_0) + s(m_0+1,n_0) + s(m_0,n_0+1)$ because the contributions from the internal paths cancel. In general, S can be calculated for any closed loop as

$$S = \sum_{m,n} s(m,n), \tag{4}$$

where the sum is over all pixels enclosed by the loop.

Combining the requirement that $S = 0$ with Eq. (4) shows that any point (m,n) having nonzero s (i.e., a discontinuity source) is only allowed within a closed loop when accompanied by another source of opposite sign. In terms of cut lines, this means that each source must be at one end of a cut, with the other end attached to a source of opposite sign, or to the boundary of the phase map. The discontinuity sources tend to occur naturally in pairs of opposite sign, although isolated sources can occur near the boundary. The criterion used when deciding how to pair the sources is one of minimizing the length of cut. A cut is constructed between the two sources (or source and boundary) separated by the shortest distance; these are then removed from the list of sources, and the process repeated until the list is empty. When constructing the cut, several different routes will generally give the same minimum cut length. The choice of route will affect the unwrapped phase only in the region between the two sources; since this is the region containing the corrupted phase information, the precise route chosen for the cut is not important.

In the computer, cuts are represented by two arrays of flags, $H(m,n)$ and $V(m,n)$. These can be two bits of a single byte array if the available memory is restricted. H and V are initially set to zero. A cut between the two points (m_2,n_2) and (m_3,n_3) is denoted by setting $V(m_2,n) = 1$ $(n = n_2 + 1,\ldots,n_3$ if $n_2 < n_3$; $n = n_3+1,\ldots,n_2$ if $n_3 < n_2)$ and $H(m,n_3) = 1$ $(m = m_2+1,\ldots,m_3$ if $m_2 < m_3$; $m = m_3+1,\ldots,m_2$ if $m_3 < m_2)$. A simple example is shown in Fig. 3(b).

Once the cut arrays have been set up, the phase map is unique and phase unwrapping can be carried out in any order. Suppose the phase at point (m,n) has been unwrapped, but that at $(m+1,n)$ has not. A valid path is first established between the two points. Normally this would have a single link: the number of discontinuities would be calculated as $d = [(\Phi(m+1,n) - \Phi(m,n))/2\pi]$, and $2\pi d$ would be subtracted from $\Phi(m+1,n)$. However, if $V(m,n) = 1$, indicating a vertical cut between the two data points, the search direction is rotated through 90° counter clockwise, to the point $(m,n+1)$. This is the next valid point in the path,

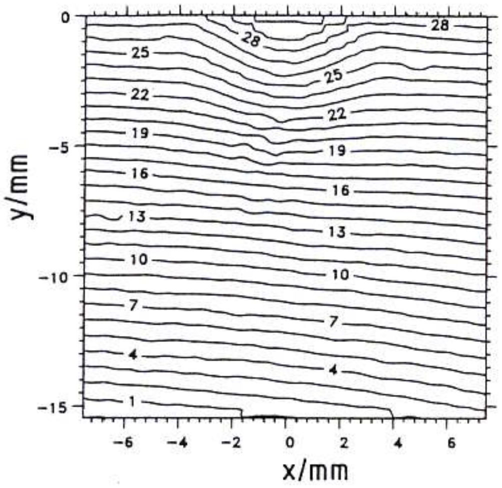

Fig. 4. Phase map of the horizontal fringes from Fig. 1 after unwrapping by the new algorithm. Phase values have been divided by 2π; each contour represents one fringe.

provided $H(m,n) = 0$. Each successive link is established by rotating the search direction through 90° clockwise compared with the previous link. If this is unsuccessful (i.e., flags are set in H or V when moving in a vertical or horizontal direction, respectively), the search direction is rotated through successive 90° counter clockwise increments until the link can be made. In this way, cuts are circumnavigated in a clockwise direction. Any other path (e.g., counter clockwise circumnavigation) is of course also valid. The number of 2π phase discontinuities between successive elements along the path $d(i)$ is calculated according to Eq. (1), and $2\pi\Sigma d(i)$ is subtracted from $\Phi(m+1,n)$.

Figure 4 shows the phase map from the fringes in Fig. 1 after unwrapping with the new algorithm. The discontinuities produced by the old method (Fig. 2) no longer occur. The ability to deal with regions of an image that contain no fringe information is one of the main advantages of this technique. The computation time varies according to the signal-to-noise ratio, and on low noise phase maps is comparable to that required by the old algorithm. The time increases in line with the number of cuts, but even in the case of moderately noisy phase maps is usually less than that taken by other stages of the fringe analysis procedure, such as the forward and inverse 2-D Fourier transforms.

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References

1. G. T. Reid, "Automatic Fringe Pattern Analysis: a Review," Opt. Lasers Eng. 7, 37–68 (1986/7).
2. M. Takeda, H. Ina, and S. Kobayashi, "Fourier-Transform Method of Fringe-Pattern Analysis for Computer-based Topography and Interferometry," J. Opt. Soc. Am. 72, 156–160 (1982).
3. D. C. Ghiglia, G. A. Mastin, and L. A. Romero, "Cellular-Automata Method for Phase Unwrapping," J. Opt. Soc. Am. A 4, 267–280 (1987).
4. J. J. Gierloff, "Phase Unwrapping by Regions," Proc. Soc. Photo-Opt. Instrum. Eng. 818, 2–9 (1987).
5. K. Freischlad and C. L. Koliopoulos, "Wavefront Reconstruction from Noisy Slope or Difference Data Using the Discrete Fourier Transform," Proc. Soc. Photo-Opt. Instrum. Eng. 551, 74–80 (1985).
6. J. M. Huntley and J. E. Field, "High Resolution Moire Photography: Application to Dynamic Stress Analysis," Opt. Eng. 28, (1989), accepted for publication.
7. D. J. Bone, H.-A. Bachor, and R. J. Sandeman, "Fringe-Pattern Analysis Using a 2-D Fourier Transform," Appl. Opt. 25, 1653–1660 (1986).

White light reconstruction setup for shearograms

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A reconstruction setup for the shearographic camera is discussed that is simpler than a Fourier processor and gives bright reconstructions with white light.

Shearograms[1] are generally evaluated with an optical correlator. The standard setup is a Fourier processor using coherent light for filtering the spatial frequency spectrum of the speckle patterns stored on the shearograms. In the case of multiaperture speckle interferometers[2,3] the spectrum consists of an array of separate spots and is easy to filter. These modern interferometers, however, have the disadvantage of long exposure times. The multiaperture mask blocks most of the light available at the recording lens.

We have found an alternative reconstruction setup for the original shearographic camera.[1] It is simpler than a Fourier processor. Bright reconstructions with high contrast fringes are visible not only with laser illumination but also with white light.

The setup (Fig. 1) consists of a collimated light source and a camera with a macrolens. The shearogram is obliquely illuminated. The finite aperture D of the reconstruction lens has the effect of a circular spatial filter centered at ($k \sin\gamma$,0) in the FT plane of a Fourier processor (Fig. 2).

The intensity distribution inside the diffraction halo behind the shearogram is proportional to the power spectral density G of the amplitude transmittance field of the film. For a subjective speckle pattern recorded linearly, G can be shown to be proportional to the autocorrelation of the aperture function of the recording lens.[4,5]

The bright and dark fringes on a reconstructed double exposure shearogram correspond to regions where the amplitude fields from both semicircular apertures are stored like coherent and incoherent fields, respectively.[6] The coherent addition leads to the power spectral density shown in Fig. 3(a). It is derived from the autocorrelation of the aperture function for the full circular aperture A_c:

$$A_c(k_x,k_y) = 1 \quad k_x^2 + k_y^2 \le k_0^2/4,$$
$$= 0 \quad \text{elsewhere},$$