$$I_9 = \int \frac{dx}{(1+x)\sqrt{x}}$$

$$u = \sqrt{x}$$
$$u^2 = x$$
$$2u\,du = dx$$

$$I = \int \frac{2u\,du}{(u^2+1)u} = 2\int \frac{du}{u^2+1} = 2\left[\text{Arctg}\, u\right] + C$$
$$= 2\,\text{Arctg}\,\sqrt{x} + C$$

$$I_{10} = \int_a^b \frac{x\,dx}{\sqrt{1+x}}$$

$u = 1 + x \qquad dx = du \qquad$ et $\qquad x = u - 1$

$$I = \int \frac{x\,dx}{\sqrt{1+x}} = \int \frac{(u-1)\,du}{\sqrt{u}} = \int \left(\sqrt{u} - \frac{1}{\sqrt{u}}\right)$$

$$I = \int u^{1/2}\,du - \int u^{-1/2}\,du =$$

$$I = \frac{2}{3}u^{3/2} - 2u^{1/2} + K = 2u^{1/2}\left(\frac{u}{3} - 1\right) + K$$

$$I = \frac{2}{3}u^{1/2}(u-3) + K$$

$$\boxed{I = \frac{2}{3}\sqrt{1+x}\,(x-2) + K}$$

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$$I_7 = \int_e^{e^2} \frac{dx}{x(\operatorname{Log} x)^3}$$

Posons $y = \operatorname{Log} x$ $\quad dy = \frac{dx}{x}$

pour $x = e$ $\quad y = 1$

pour $x = e^2$ $\quad y = 2$

$$I_7 = \int_1^2 \frac{dy}{y^3} = \left[-\frac{1}{2y^2}\right]_1^2 = -\frac{1}{8} + \frac{1}{2}$$

$$\boxed{I_7 = \frac{3}{8}}$$

21)

$$I_8 = \int \frac{2x+1}{2x-1}\,dx = \int \frac{2x+1-1+1}{2x-1}\,dx$$

$$= \int \frac{2x-1}{2x-1}\,dx + \int \frac{2\,dx}{2x-1}$$

$$= \int dx + \int \frac{du}{u} = x + \operatorname{Log}(2x-1) + C$$

4] $I_4 = \int_a^b \frac{1}{x^2+2x+2}\,dx = \int_a^b \frac{1}{(x+1)^2+1}\,dx$ [cherché un carré.

On pose : $u = x+1 \Rightarrow du = dx$

$x = a \Rightarrow u = a+1$, $x = b \Rightarrow u = b+1$ || $I_4 = \int_{a+1}^{b+1} \frac{1}{1+u^2}\,du = \left[\text{Arctg}\, u\right]_{a+1}^{b+1}$

$$I_4 = \text{Arctg}(b+1) - \text{Arctg}(a+1)$$

5] $I_5 = \int_a^b \frac{x^2-7x+10}{1-x}\,dx$

$I_5 = \int_a^b \frac{P(x)}{Q(x)}\,dx$

Méthode 1 :

degré de $P = 2$, et degré de $Q = 1$

$\frac{P(x)}{Q(x)} = ax + b + \frac{c}{1-x}$

$\frac{P(x)}{Q(x)} = \frac{-ax^2+(a-b)x+b+c}{1-x} \Rightarrow a = -1,\ b = 6,\ c = 4$

Méthode 2 :

$$\begin{array}{r|l} x^2-7x+10 & -x+1 \\ \hline -(x^2-x) & -x+6 \\ -6x+10 & \\ -(-6x+6) & \\ 4 & \end{array}$$

Donc $I_5 = \int_a^b -x+6+\frac{4}{-x+1}\,dx =$

$= \left[-\frac{x^2}{2}+6x\right]_a^b - 4\int_a^b \frac{dx}{x-1}$,

Changement de variable $u = x-1$, $du = dx$

Bornes : $x = a \Rightarrow u = a-1$, $x = b \Rightarrow u = b-1$ || $I_5 = \left[-\frac{x^2}{2}+6x\right]_a^b - 4\int_{a-1}^{b-1} \frac{du}{u}$

$$I_5 = \left[-\frac{x^2}{2}+6x\right]_a^b - 4\left[\ln|u|\right]_{a-1}^{b-1}$$

$$I_5 = \frac{-b^2+a^2}{2} + 6(a-b) - 4\left(\ln|b-1| + \ln|a-1|\right)$$

6) $I_6 = \int_a^b 3^{(2x+1)^{1/2}} dx$

Changement de variable :

on pose : $y = \sqrt{2x+1} \Rightarrow y^2 = 2x+1$

$2y\,dy = 2dx$

. les Borne $a' = \sqrt{2a+1}$

$b' = \sqrt{2b+1}$

$$I_6 = \int_{a'}^{b'} 3^y \, y \, dy$$

faire une IPP $\int u'v = uv - \int v'u$

on pose : $v = y \Rightarrow dv = dy$

$du = 3^y dy \Rightarrow u = \frac{3^y}{\ln 3}$

$$I_6 = \int_{a'}^{b'} 3^y y \, dy = \left[y \frac{3^y}{\ln 3} \right]_{a'}^{b'} - \frac{1}{\ln 3} \int_{a'}^{b'} 3^y dy$$

$$I_6 = \left[\frac{3^y y}{\ln 3} - \frac{3^y}{(\ln 3)^2} \right]_{a'}^{b'} = \frac{3^y}{(\ln(3))^2} \left[y \ln 3 - 1 \right]_{a'}^{b'}$$

Exo 02:

$$I_7 = \int_a^b \frac{1}{(x^2+\alpha^2)^2} dx$$

$$I_7 = \frac{1}{\alpha^2} \int_a^b \frac{\alpha^2 + x^2 - x^2}{(x^2+\alpha^2)^2} dx = \frac{1}{\alpha^2} \left[\underbrace{\int_a^b \frac{\alpha^2+x^2}{(x^2+\alpha^2)^2} dx}_{I_1} - \underbrace{\int_a^b \frac{x^2}{(x^2+\alpha^2)^2} dx}_{I_2} \right]$$

Calcul de I_1 : $I_1 = \frac{1}{\alpha^2} \int_a^b \frac{\alpha^2+x^2}{(x^2+\alpha^2)^2} dx = \frac{1}{\alpha^2} \int_a^b \frac{1}{(x^2+\alpha^2)} dx$

$$I_1 = \frac{1}{\alpha^4} \int_a^b \frac{1}{1 + \frac{x^2}{\alpha^2}} dx = \frac{1}{\alpha^4} \left[\frac{\operatorname{Arctg}\left(\frac{x}{\alpha}\right)}{1/\alpha} \right]_a^b$$

$$I_1 = \frac{1}{\alpha^3} \left[\operatorname{Arctg}\left(\frac{b}{\alpha}\right) - \operatorname{Arctg}\left(\frac{a}{\alpha}\right) \right]$$

Calcul de I_2 : $I_2 = \frac{1}{\alpha^2}\int_a^b \frac{x^2}{(x^2+\alpha^2)^2}\,dx = \frac{1}{\alpha^2}\int_a^b x\ \frac{x}{(x^2+\alpha^2)^2}\,dx$

IPP : $u' = x/(x^2+\alpha^2)^2 \longrightarrow u = -\frac{1}{2}(x^2+\alpha^2)$

$V = x \longrightarrow V' = 1$

$\Rightarrow I_2 = \frac{1}{\alpha^2}\left[\frac{-x}{2(x^2+\alpha^2)}\right]_a^b + \frac{1}{2}\int_a^b \frac{1}{x^2+\alpha^2}\,dx$

$I_2 = \frac{-1}{2\alpha^2}\left[\frac{b}{b^2+\alpha^2} - \frac{a}{a^2+\alpha^2}\right] + \frac{1}{2\alpha^3}\left[\operatorname{Arctg}\left(\frac{b}{\alpha}\right) - \operatorname{Arctg}\left(\frac{a}{\alpha}\right)\right]$

$I_7 = I_1 - I_2 = \frac{1}{\alpha^3}\left[\operatorname{Arctg}\left(\frac{b}{\alpha}\right) - \operatorname{Arctg}\left(\frac{a}{\alpha}\right)\right] + \frac{1}{2\alpha^2}\left[\frac{b}{b^2+\alpha^2} - \frac{a}{a^2+\alpha^2}\right]$

Ex 003

$I_8 = \int_0^{\pi/2} (\sin(x))^2 \cdot (1+\cos(x))^2\,dx$

Méthode : $I_8 = \int_0^{\pi/2} \sin^2(x)\,dx + 2\cos(x)\sin^2(x) + \cos^2(x)\sin^2(x)\,dx$

On pose : $I_1 = \int_0^{\pi/2} \sin^2(x)\,dx$, on a $\cos(2x) = 2\cos^2(x) - 1 = 1 - 2\sin^2(x)$

$\Rightarrow \sin^2(x) = \frac{1-\cos(2x)}{2}$

Donc : $I_1 = \int_0^{\pi/2} \sin^2(x)\,dx = \int_0^{\pi/2} \frac{1-\cos(2x)}{2}\,dx = \frac{1}{2}\left[x - \frac{\sin(2x)}{2}\right]_0^{\pi/2} = \frac{\pi}{4}$

On pose : $I_2 = 2\int_0^{\pi/2} \sin^2(x)\cos(x)\,dx$

. Changement de variable : $u = \sin(x) \Rightarrow du = \cos(x)\,dx$

. Les Bornes : $x = 0 \longrightarrow u = 0$

$x = \pi/2 \longrightarrow u = 1$

Donc : $I_2 = 2\int_0^1 u^2\,du = \frac{2}{3}u^3\Big]_0^1 = 2/3$

Posons : $I_3 = \int_0^{\pi/2} \sin^2(x)\cos^2(x)\,dx = \int_0^{\pi/2} \left(\sin(x)\cos(x)\right)^2 dx$

on a : $\sin 2x = 2\sin(x)\cos(x)$

$\Rightarrow I_3 = \frac{1}{4}\int_0^{\pi/2} \sin^2(2x)\,dx = \frac{1}{4}\int_0^{\pi/2} \frac{1 - \cos(4x)}{2}\,dx$

$I_3 = \frac{1}{8}\left[x\right]_0^{\pi/2} - \left[\frac{1}{32}\sin(4x)\right]_0^{\pi/2} = \frac{1}{16}\pi$

$\Rightarrow I_8 = I_1 + I_2 + I_3 = \pi/4 + 2/3 + \pi/16 = \frac{5\pi}{16} + 2/3 = \frac{15\pi + 32}{48}$

Méthode 2 : poser $\sin(x) = \frac{e^{ix} - e^{-ix}}{2}$ et $\cos(x) = \frac{e^{ix} + e^{-ix}}{2}$

Devellopper et revenir par des termes en $\cos(2x)$, $\sin(2x)$

Exo 09

$I_9 = \int_a^b \frac{dx}{\sin(2x)}$

. Posons $t = f(x) = tg(x) \Rightarrow x = Arctg(y)$

$f'(x) = 1 + tg^2(x)$ et $dx = \frac{dt}{1 + t^2}$

. Les Bornes : $x = a \rightarrow t = tg(a) = a'$

$x = b \rightarrow t = tg(b) = b'$

. Nous avons aussi : . $\cos(2x) = 1 - 2\sin^2(x)$

. $\sin(2x) = 2\sin(x)\cos(x)$

. $1 + tg^2(x) = \frac{1}{\cos^2(x)}$

. D[illegible] . $tg(x) = \frac{\sin(x)}{\cos(x)}$

. Donc : $\sin(2x) = 2\sin(x)\cos(x) = 2\,tg(x)\cos(x)\cos(x) = 2\,tg(x)\cos^2$

$= 2\,tg(x)\,\frac{1}{1 + tg^2(x)} = \frac{2t}{1 + t^2}$

. De même : $\cos 2(x) = 1 - 2\sin^2(x) = 1 - 2\,tg^2(x)\cos^2(x) = 1 - 2\,tg^2(x)\,\frac{1}{1 + tg^2(x)}$

$= \frac{1 + tg^2(x) - 2\,tg^2(x)}{1 + tg^2(x)} = \frac{1 - t^2}{1 + t^2}$

Ce resultats peut aussi se verifier en exploitant les formules d'Euler

Donc $I_9 = \int_a^b \frac{1}{\sin(2x)} dx = \int_{a'}^{b'} \frac{1+t^2}{2t} \frac{dt}{1+t^2} = \int_{a'}^{b'} \frac{dt}{2t} = \frac{1}{2} \ln|t| \Big]_{a'}^{b'}$

↳ faire l'étude en fct de a' et b'

Ex n° 05

$I_{10} = \int_a^b \frac{dx}{\sin(x) - \cos(x)}$

Même de l'ex n° 04

on pose $t = f(x) = tg(x/2) \Rightarrow x = \operatorname{Arctg}(t) / 2$

$dx = \frac{2dt}{1+t^2}$

et : $\sin(x) = \frac{2t}{1+t^2}$ $\quad$ $a' = tg(a/2)$, $b' = tg(b/2)$ bornes

$\cos(x) = \frac{1-t^2}{1+t^2}$

Donc :

$I_{10} = 2\int_{a'}^{b'} \frac{1}{\frac{2t + 1 - t^2}{1+t^2}} \cdot \frac{1}{1+t^2} dt = 2\int_{a'}^{b'} \frac{-1}{-1 + 2t + t^2} dt$

. Il faut chercher à décomposer en element simple (cours)

Suppo $\frac{+1}{c + b\cancel{t} + a\cancel{t^2}} = \left(\frac{A}{(\cancel{t} - t_1)} + \frac{B}{(\cancel{t} - t_2)} \right) \times \operatorname{sgn}(a)$.

donc : $f(t) = \frac{1}{t^2 + 2t - 1} = \frac{\alpha_1}{(t - t_1)} + \frac{\alpha_2}{(t - t_2)}$

$= \frac{1}{(t - t_1)(t - t_2)}$ (ou t_1 et t_2 sont les racines de $t^2 + 2t - 1$

. **But** : Il s'agit donc de determiner α_1 et α_2.

= (7)

. On a : $\dfrac{1}{(t-t_1)(t-t_2)} = \dfrac{\alpha_1}{(t-t_1)} + \dfrac{\alpha_2}{(t-t_2)}$

$$\frac{(t-t_1)}{(t-t_1)(t-t_2)} = \alpha_1 + \frac{\alpha_2 (t-t_1)}{(t-t_2)}$$

en posant : $t = t_1 \Rightarrow \dfrac{1}{t_1 - t_2} = \alpha_1$

. De même :

$$\frac{(t-t_2)}{(t-t_1)(t-t_2)} = \frac{\alpha_1 (t-t_2)}{(t-t_1)} + \alpha_2$$

en posant : $t = t_2 \Rightarrow \dfrac{1}{(t_2 - t_1)} = \alpha_2 = -\alpha_1$

Donc $I_{AD} = -2 \displaystyle\int_{a'}^{b'} \left(\frac{\alpha_1}{(t-t_1)} + \frac{\alpha_2}{(t-t_2)} \right) dt$

$$= -2 \int_{a'}^{b'} \frac{\alpha_1}{(t-t_1)} - \frac{\alpha_1}{(t-t_2)}\, dt = -2\alpha_1 \int_{a'}^{b'} \frac{1}{(t-t_1)} - \frac{1}{(t-t_2)}\, dt$$

$$= \frac{-2}{(t_1 - t_2)} \int_{a'}^{b'} \frac{1}{(t-t_1)} - \frac{1}{(t-t_2)}\, dt$$

. Calcul de t_1, t_2

$f(t) = t^2 - 2t - 1$

Discriminant : $\Delta = b^2 - 4ac = 4 + 4 = 8$

$$t_1 = \frac{-b + \sqrt{\Delta}}{2a} = \frac{2 + 2\sqrt{2}}{2} = 1 + \sqrt{2}$$

$$t_1 = \frac{-b - \sqrt{\Delta}}{2a} = \frac{2 - 2\sqrt{2}}{2} = 1 - \sqrt{2}$$

$$\Rightarrow I_{AD} = \frac{-2}{(t_1 - t_2)} \Big[\ln(|t - t_1|) - \ln(|t - t_2|) \Big]_{a'}^{b'} = \frac{-2}{t_1 - t_2} \left[\ln \left(\left| \frac{t - t_1}{t - t_2} \right| \right) \right]_{a'}^{b'}$$

$$I_{AD} = \frac{-1}{\sqrt{2}} \ln \left(\left| \frac{t - 1 - \sqrt{2}}{t - 1 + \sqrt{2}} \right| \right)_{a'}^{b'}$$