The Time Scales of a Stochastic Network with Failures

Philippe Robert

INRIA Paris—Rocquencourt

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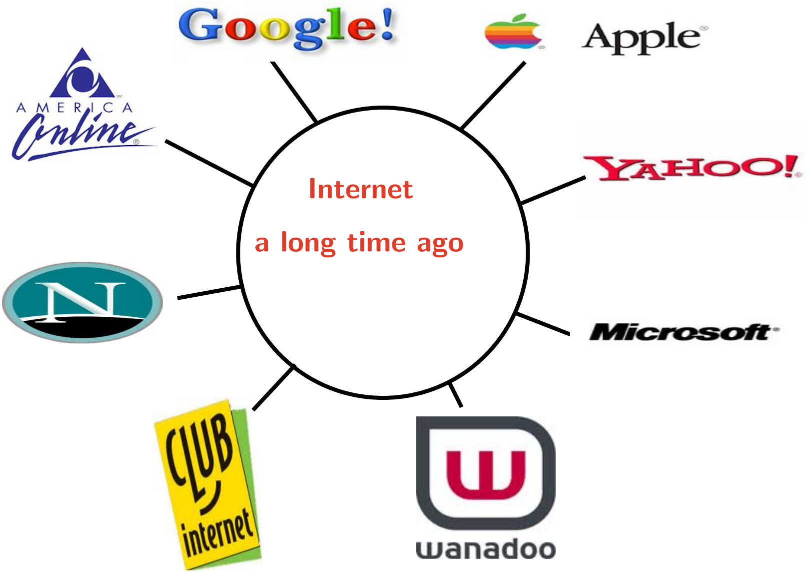
Joint work with Mathieu Feuillet

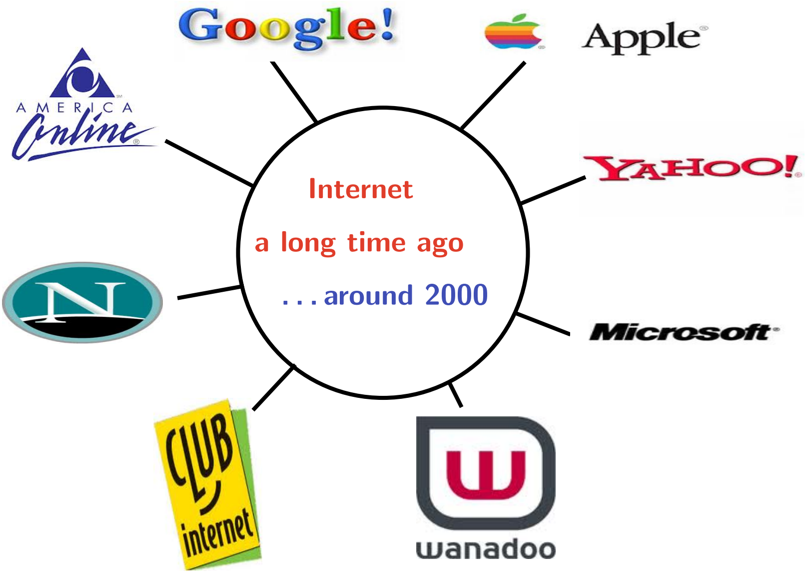
Introduction


Internet
a long time ago
Google!
Apple
AMERICA Online
YAHOO!
Microsoft
CLUB internet
wanadoo

Internet

a long time ago

. . . around 2000



Access to the some information


Internet Now
The Cloud
Google
YouTube
Microsoft
facebook
Instagram
amazon
Apple
iTunes

The Cloud : Computing Facilities and Storage for Data

The Numbers of the Cloud

Number of servers (Estimation) :

- Google : **900 000**
- Microsoft : **518 000**
- Amazon : **445 000**
- HP/EDS : **380 000**
- OVH : **120 000**
- Facebook : **60 000**
- ...

Reliability of Servers in the Cloud

Some Orders of Magnitudes

- **In a large data center**
  Failures occur on a daily basis.

Reliability of Servers in the Cloud

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  200000 servers : 10 servers fail every day
  It takes time to detect failures !

- **In operational context**
  Time to retrieve a 500 GB disk : a day

Duplication

- **To prevent losses of files $\Rightarrow$ duplication policy**
- **Each file may be duplicated on d different servers.**

Duplication

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- **A file with 0 copies is lost for good**

Final Outcome : Empty State

A set of unreliable servers and a set of files

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A set of unreliable servers and a set of files

If failures occur independently

all files will be lost eventually

A Generic Problem

Given a set of unreliable servers

Find an architecture so that

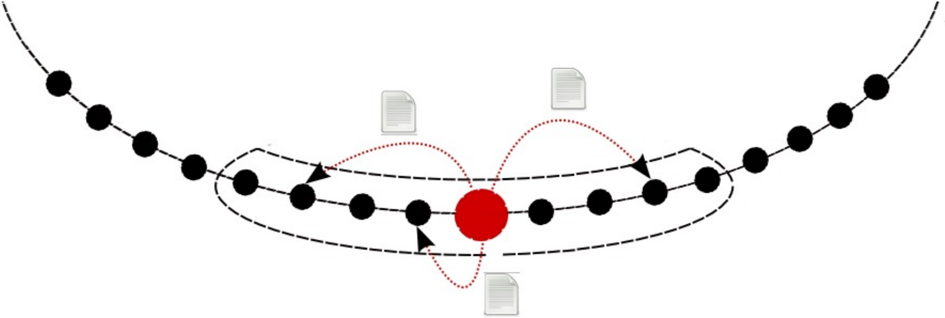
- **the set of files stored is maximal**
- **the duration of time for which "most" of the files can be retrieved is maximal**

Literature : Computer Science

DHT : Distributed Hash Tables

Many Algorithms :

Chord, Pastry, Tapestry, RelaxDHT, . . .



Literature : Mathematical Models

Reliability Models

King (1990), Falin, Templeton (1997), Artalejo, Gomez-Curral (2008), . . .

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- **A limited Number of Servers (one or two)**
- **Stationary State**

Literature : Mathematical Models

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King (1990), Falin, Templeton (1997),
Artalejo, Gomez-Curral (2008), ...

- A limited Number of Servers (one or two)
- Stationary State
- Does not capture
  the transient characteristics of the model

A Simple Stochastic Model

Framework

- A set of N servers
- Failure rate μ
- Duplication capacity λ per server
- At most d copies of a given file
- F_N files at $t=0$, each one with d copies

Random variables are exponentially distributed

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Total Duplication capacity λN

Some simplification

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- A file with 0 copies is lost

Duplication Policy

Assumption : Optimal Use of Capacity

Recovery capacity λN allocated to the files with the minimum number of copies.

A Transient Markov Process

For $0 \leq i \leq d$:
$X_i(t)$: number of files with i copies

- $X(t) = (X_i(t), 0 \leq i \leq d)$ is Markov
  $X_0(t) + X_1(t) + \cdots + X_d(t) = F_N$
- $X_d(0) = F_N$
- Unique absorbing state $\dagger = (F_N, 0, \ldots, 0)$
  $X(t) = \dagger$ for t sufficiently large

A Transient Markov Process

Transitions
If $X(0) = x = (x_i)$ then

- $X(t) = x + e_{i-1} - e_i$ at rate $i\mu x_i$, $1 \leq i \leq d$

- Duplication Policy :
  $X(t) = x - e_i + e_{i+1}$ at rate λN, $1 \leq i < d$
  if $x_k = 0$, $1 \leq k < i$ and $x_i > 0$

A Scaling Picture

General Problem

Estimate the rate of decay of the network

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For $0 < \delta < 1$,

$$T_N = \inf \left\{ t \geq 0 : X_0^N(t) \geq \delta F_N \right\}$$

A Scaling Picture

General Problem

Estimate the rate of decay of the network

For $0 < \delta < 1$,

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Scaling

$$\lim_{N \to +\infty} \frac{F_N}{N} = \beta > 0,$$

A Scaling Picture

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Estimate the rate of decay of the network

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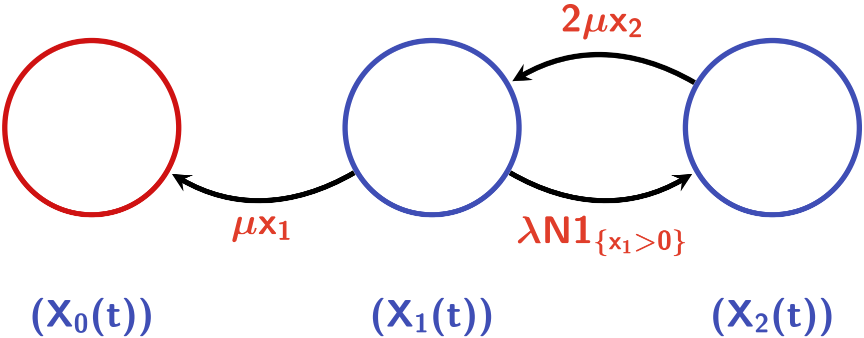
The Simple Case $d = 2$

Network with $d = 2$



Network with $\mathbf{d = 2}$

Three important time scales

$$\begin{cases} \mathbf{t \to t/N} \\ \mathbf{t \to t} \\ \mathbf{t \to Nt} \end{cases}$$

Time scale $t \to t/N$

Stability Condition

Rates of $(X_1(t))$

$$x \mapsto \begin{cases} x+1 & \text{at rate} \quad 2\mu(F_N - X_0(t) - x) \\ x-1 & \qquad\qquad \lambda N \qquad \text{if } x \geq 1 \end{cases}$$

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Stability Condition

Rates of $(X_1(t/N))$ for N large

$$x \mapsto \begin{cases} x+1 & \text{at rate} \quad 2\mu\beta \\ x-1 & \qquad\qquad\quad \lambda \qquad \text{if } x \geq 1 \end{cases}$$

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Reflected random walk on $\mathbb{N}$ ($M/M/1$ queue)

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Reflected random walk on $\mathbb{N}$ ($M/M/1$ queue)

ergodic if $\lambda > 2\beta\mu$

Fluid Limits

Proposition

The sequence $(X_0^N(t)/N, X_1^N(t)/N, t \geq 0)$ converges in distribution to $(L(t))$

Fluid Limits

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The sequence $(X_0^N(t)/N, X_1^N(t)/N, t \geq 0)$ converges in distribution to $(L(t))$ if $\lambda < 2\beta\mu$

$$L(t) = \left[\left(\beta - \frac{\lambda}{2\mu}\right)\left(1 - 2e^{-\mu t} + e^{-2\mu t}\right), \left(2\beta - \frac{\lambda}{\mu}\right)\left(e^{-\mu t} - e^{-2\mu t}\right)\right]$$

Fluid Limits

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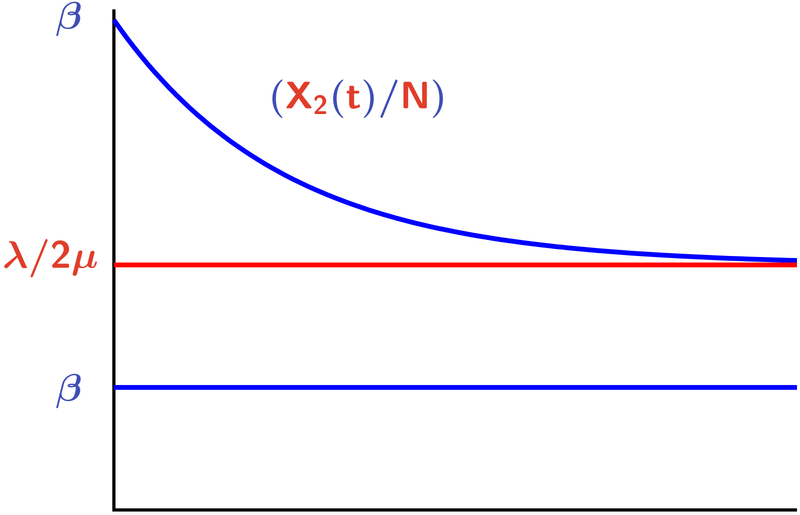
$$\left. \left(2\beta - \frac{\lambda}{\mu}\right)\left(e^{-\mu t} - e^{-2\mu t}\right)\right]$$

and

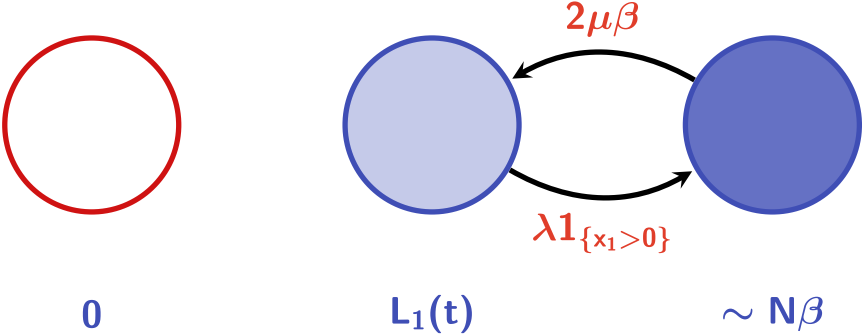
$$L(t) = [0, 0] \qquad \text{if } \lambda \geq 2\beta\mu$$

Fluid Limits



Time scale $t \to t/N$: Asymptotics

$(L_1(t))$: a stable $M/M/1$ queue if $2\beta < \lambda/\mu$



“Normal” Time Scale $\mathbf{t} \rightarrow \mathbf{t}$

“Normal” Time Scale $\mathbf{t} \to \mathbf{t}$

$$\boldsymbol{\lambda} \geq \mathbf{2}\beta\boldsymbol{\mu}$$

Critical Case $\beta = \lambda/2\mu$

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Proposition If $F_N \sim \dfrac{\lambda}{2\mu}N + \gamma\sqrt{N}$ then

$$\lim_{N\to+\infty}\left(\frac{X_0^N(t)}{\sqrt{N}}, \frac{X_1^N(t)}{\sqrt{N}}\right) = \left(\mu\int_0^t Y(u)\,\mathrm{d}u, Y(t)\right)$$

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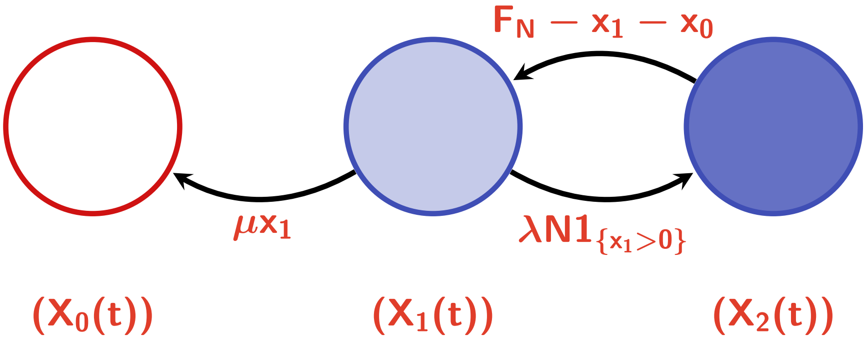
where $(Y(t))$ is the solution of the SDE

$$\mathrm{d}Y(t)=\sqrt{2\lambda}\,\mathrm{d}B(t) + \left[2\gamma-3\mu Y(t)-2\mu^2\int_0^t Y(u)\mathrm{d}u\right]\mathrm{d}t+\mathrm{d}R(t)$$

where $(R(t))$ is a local time of $(Y(t))$ at 0.

Normal time scale $t \to t$: Asymptotics



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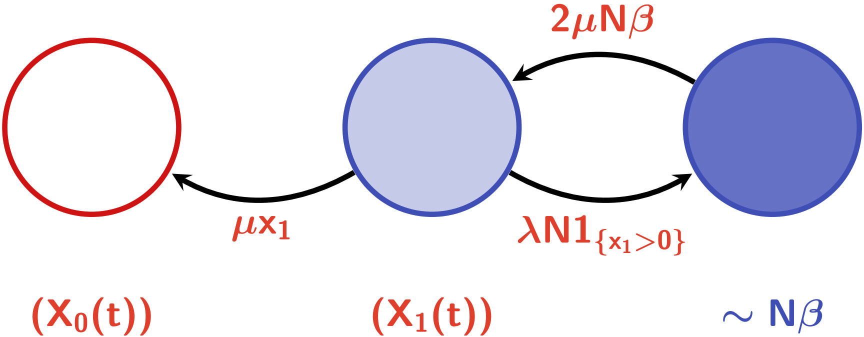
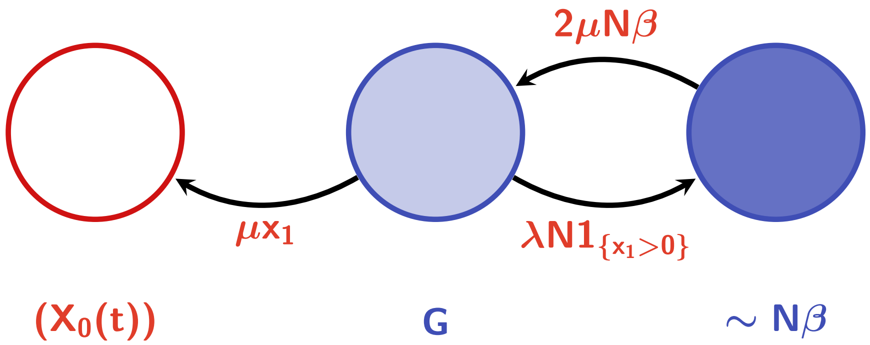


Normal time scale $t \to t$: Asymptotics

G Geometric r.v. with par. $2\beta\mu/\lambda$

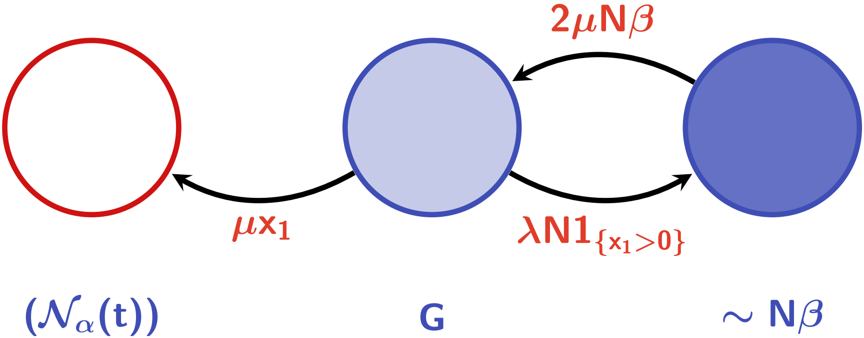


Normal time scale $t \to t$: Asymptotics

$\mathbf{G}$ Geometric r.v. with par. $2\beta\mu/\lambda$

$(\mathcal{N}_\alpha(t))$ Poisson process,

rate $\alpha = 2\mu^2\beta/(\lambda - 2\beta\mu)$



Time Scale $t \to Nt$

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$$\lambda > 2\beta\mu$$

Time scale $t \to Nt$: Asymptotics

$$\lim_{N\to+\infty}\left(\frac{X_0(Nt)}{N}\right) = \Psi(t),$$

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$$(1 - y/\beta)^{\lambda/2\mu}\, e^{y+t} = 1.$$

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As $t \to \infty$ then $\Psi(t) \sim \beta - e^{-2\mu(\beta+t)/\lambda}$

Time scale $t \to Nt$: Asymptotics

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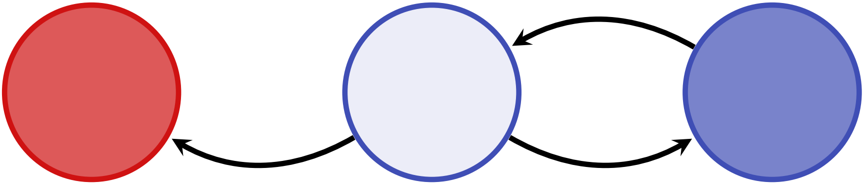
$t \to Nt$ "correct" time scale to describe decay.

Time scale $t \to Nt$: Asymptotics

G_t Geometric r.v. with par. $2(\beta - \Psi(t))/\rho$



$\sim N\Psi(t)$ G_t $\sim N(\beta - \Psi(t))$

Time scale $t \to Nt$: Stochastic Averaging

At "time" Nt, $(X_1(\cdot))$ behaves as an $M/M/1$ process at equilibrium

$+1$ **at rate** $2\mu(\beta - \Psi(t))$

-1 **at rate** λ.

Time scale $t \to Nt$: Stochastic Averaging

$(X_1(Nt + u/N), u \geq 0)$ is an $M/M/1$ process at equilibrium

$+1$ at rate $2\mu(\beta - \Psi(t))$
-1 at rate λ.

Estimation of Decay

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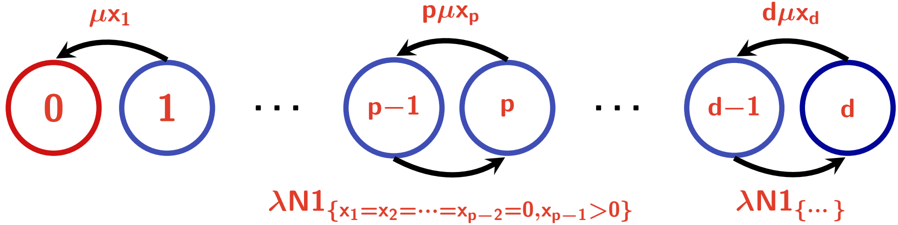
$$T_N(\delta) = \inf\left\{t \geq 0 : \frac{X_0(t)}{N} \geq \delta\beta\right\}$$

For the convergence in distribution

$$\lim_{N\to+\infty} \frac{T_N(\delta)}{N} = \frac{1}{\mu}\left(-\frac{\lambda}{2\mu}\log(1-\delta) - \delta\beta\right)$$

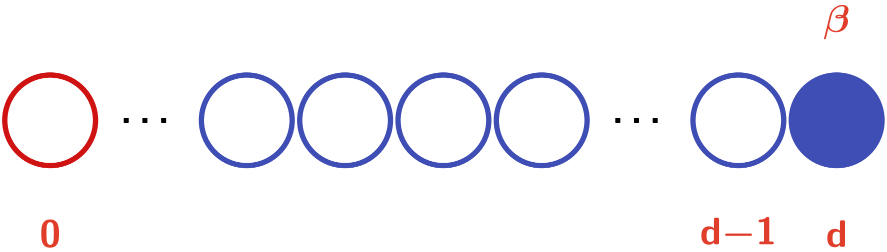
The General Case $d \geq 2$

Network with d copies



Normal time scale : Fluid Limits $(X_i^N(t)/N)$

$\lambda > d\beta\mu$



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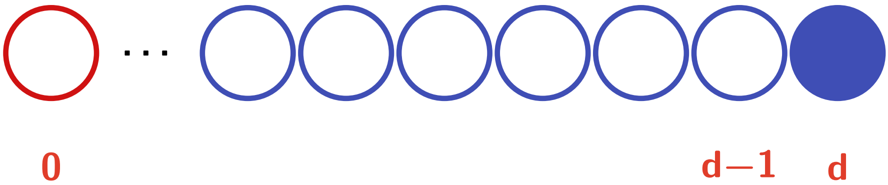

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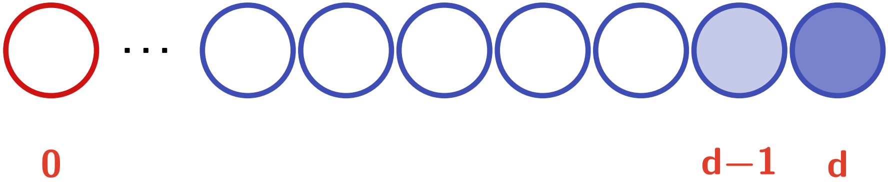

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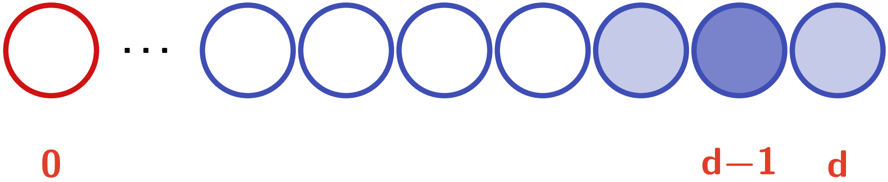

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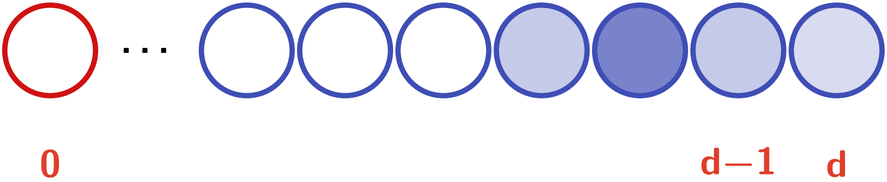

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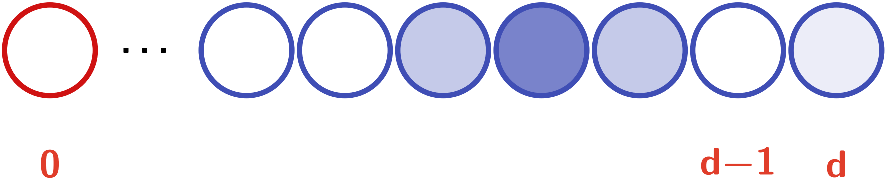

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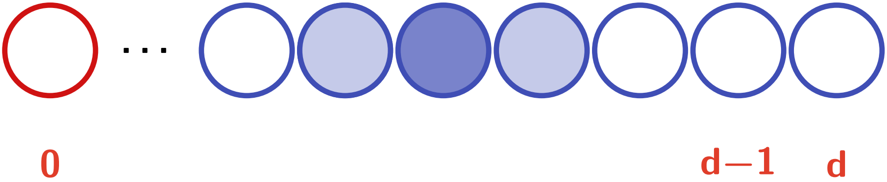
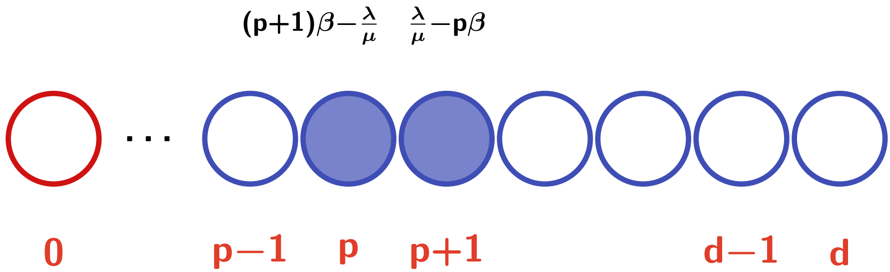


Normal time scale : Fluid Limits $(X_i^N(t)/N)$

$p\beta\mu < \lambda < (p+1)\beta\mu$ **for** $1 \leq p < d$

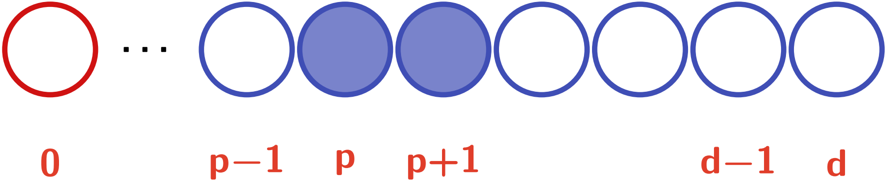


A Piecewise deterministic function

Decay : A Cascade of Times Scales

Unstable Case : $\lambda < d\beta\mu$

$$p\beta\mu < \lambda < (p+1)\beta\mu,$$



Unstable Case : $\lambda < d\beta\mu$

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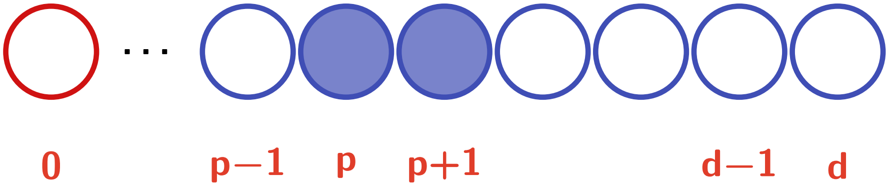
Time Scale $t \mapsto N^{p-1}t$



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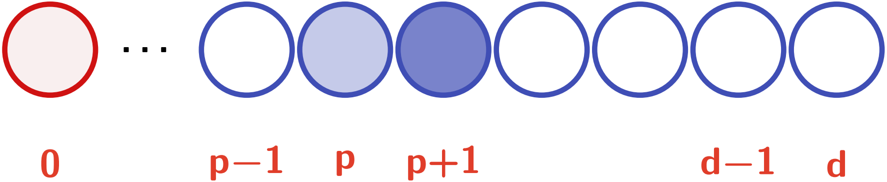

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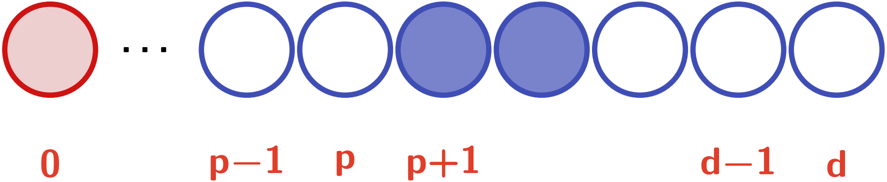
Time Scale $t \mapsto N^{p-1}t$ **: Final State**



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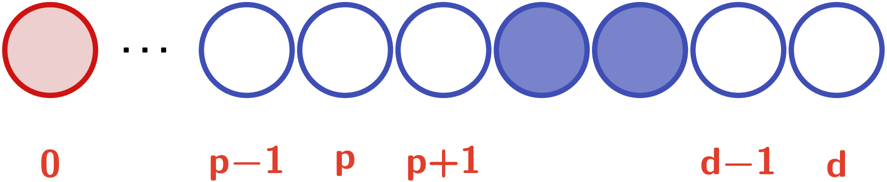
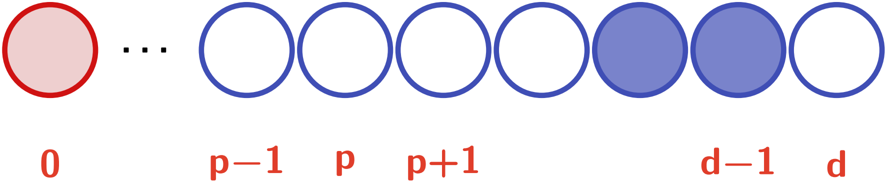
Time Scale $t \mapsto N^p t$ **: Final State**



Unstable Case : $\lambda < d\beta\mu$

$$p\beta\mu < \lambda < (p+1)\beta\mu,$$

Time Scale $t \mapsto N^{d-2}t$:



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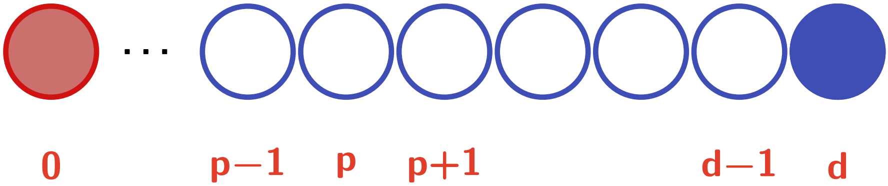

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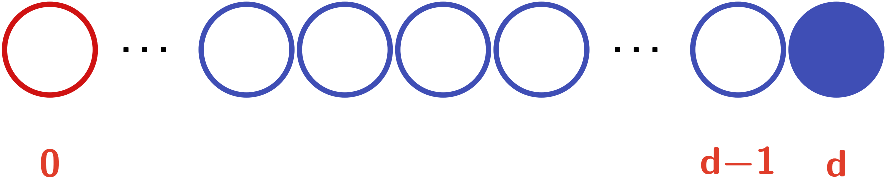

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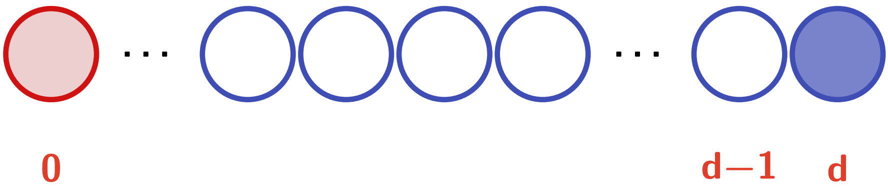
Time Scale $t \mapsto N^{d-1}t$



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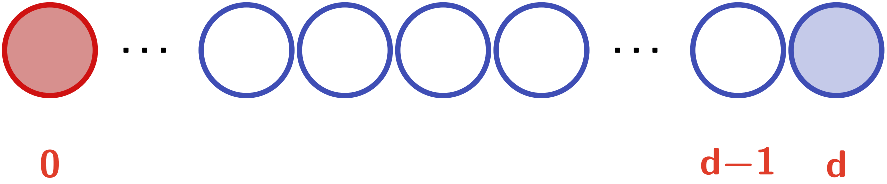
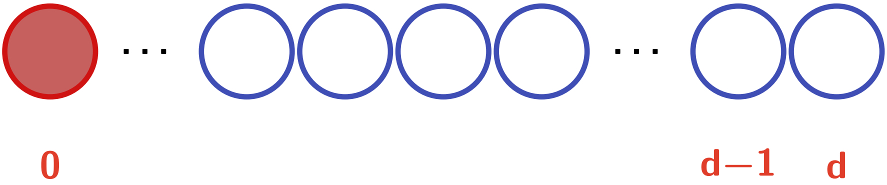


Stable Case : $\lambda > d\beta\mu$

Time Scale $t \mapsto N^{d-1}t$: Final State



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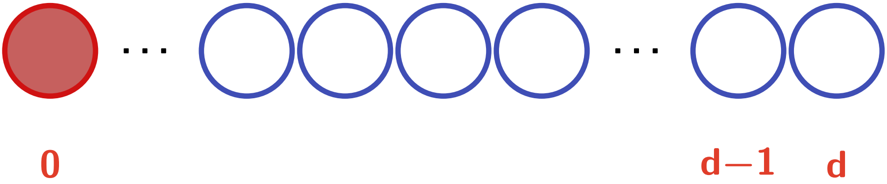

Stochastic Averaging for each step

Some Technical Points

Generalized Skorokhod Problems

Skorokhod Problems

- $(Y(t)) : \mathbb{R}_+ \to \mathbb{R}^K$ cadlag function
- A a non-negative $K \times K$ matrix

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$((X(t)), (R(t)))$ solution of

Skorokhod Problem

- $X(t) = Y(t) + (I - A) \cdot R(t)$
- $X_k(t) \geq 0,\ 1 \leq k \leq d$
- $(R_k(t)) \nearrow, \qquad R_k(0) = 0$

$$\int_{\mathbb{R}_+} X_k(t) dR_k(t) = 0$$

Generalized Skorokhod Problems

- $G : \mathcal{D}(\mathbb{R}_+, \mathbb{R}^K) \to \mathcal{D}(\mathbb{R}_+, \mathbb{R}^K)$ Borelian
- A a non-negative $K \times K$ matrix

$((X(t)), (R(t)))$ solution of

Generalized Skorokhod Problem

- $X(t) = G(X)(t) + (I - A) \cdot R(t)$
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Skorokhod Problems

Literature

- **Skorokhod (1962), Anderson/Orey (1972),**
- **El Karoui/Chaleyat-Maurel (1978), Harrison/Reiman (1981), . . .**

Skorokhod Problems

Literature

- Skorokhod (1962), Anderson/Orey (1972),
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Main Property

If problem is "regular" and $(G_n) \to G$, then

$$\lim_{n \to +\infty} ((X_{G_n,A}(t)), (R_{G_n,A}(t))) = ((X_{G,A}(t)), (R_{G,A}(t)))$$

Use of Skorokhod Problems

$$S(t) = \left(\sum_{j=1}^{k} X_j(t), 1 \leq k \leq d-1 \right)$$

is the first coordinate of the solution of a generalized Skorokhod Problem ass. to (G, A)
$A = (\mathbb{1}_{\{(i,j)=(i,i-1)\}}, 1 \leq i, j \leq d-1)$, $G = (G_k)$

$$G_k(y)(t) = \mu \int_0^t ((k+1)y_{k+1} - (k+2)y_k(u))\, du$$
$$+ M_k(t), \qquad 2 \leq k \leq d-2$$

Stochastic Averaging Principle (SAP)

Case $d = 2, \lambda > 2\beta\mu$

Transition rates of

- $X_1(t)$ of the order of N
- $X_0(t)$ 1

X_1 "fast" component, X_0 "slow" component.

Case $d = 2, \lambda > 2\beta\mu$

Transition rates of

- $X_1(Nt)$ of the order of N^2
- $X_0(Nt)$ N

X_1 "fast" component, X_0 "slow" component.

Stochastic Averaging : Literature

PDE

- **Singular Perturbation Theory**

Stochastic Averaging : Literature

PDE

- Singular Perturbation Theory

Probability

- From Khasminskii (1968) to Freidlin, Wentzell (1979)
- Papanicolaou, Stroock, Varadhan (1977) mainly for diffusion processes
- Kurtz (1992) jump processes

SAP for $d = 2$

It amounts to prove

$$\lim_{N\to+\infty}\left(\frac{1}{N}\int_0^{Nt} f(X_1(u))\,du\right) = \left(\int_0^t \pi_u(f)\,du\right)$$

where $(\pi_u, u \geq 0)$ is some random measure.

Stochastic Averaging Principle

(1) $\left(\frac{1}{N}\int_0^{Nt} f(X_1(u))\,du\right)$ is tight

(2) $\int_0^{t} \pi_u(1)\,du = t$, almost surely $\forall t \geq 0$.

SAP : Tightness

Kurtz (1992)

- **Tightness + Predictable Projections**
- **Extensions of bi-measures**
  Morando's Theorem + Kingman.

Some Technicalities. . .

SAP : Tightness

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Some Technicalities...

An alternative Approach

- Tightness of (μ_N)

$$\langle \mu_N, g \rangle = \int_{\mathbb{R}_+} g(X_1^N(Nt), t)\, dt.$$

+classical results of measure theory.

SAP : Conservation of Mass

$$\int_0^t \pi_u(1)\,du = t$$

A problem of non-compact state spaces

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An alternative formulation :

if Ω_u generator of ergodic Markov process
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Delicate problem
Regularity properties of $u \to \Omega_u$

Conservation of Mass : Literature

- **Freidlin and Wentzel (1979)**
  Criterion of SAP in terms of uniform control of ergodic averages.
- **Papanicolaou, Stroock, Varadhan (1977)**
  Uniform control through a Lyapounov function.

Conservation of Mass : Literature

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A problem sometimes underestimated/forgotten in the literature

Conservation of Mass : Our Case

Monotonicity Properties

of the family of Markov processes

Conclusion

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For given parameters λ, μ and N

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For given parameters λ, μ and N the quantities $\beta = F_N/N$ and d must be chosen so that

$$d = \left\lfloor \frac{\lambda}{\beta\mu} \right\rfloor$$

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- **The system is stable if $\mathbf{d} \geq 2$**

Conclusion

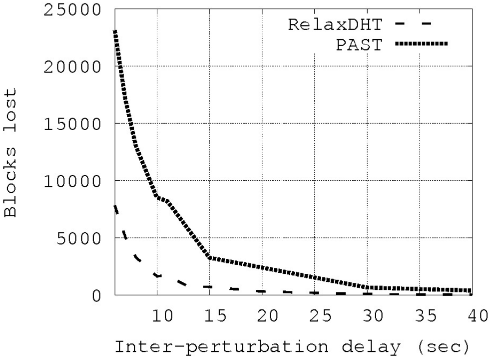
For given parameters λ, μ and N
the quantities $\beta = F_N/N$ and d
must be chosen so that

$$d = \left\lfloor \frac{\lambda}{\beta\mu} \right\rfloor$$

- The system is stable if $d \geq 2$
- The time to observe a decay is of the order of N^{d-1}

Back to Literature in Computer Science

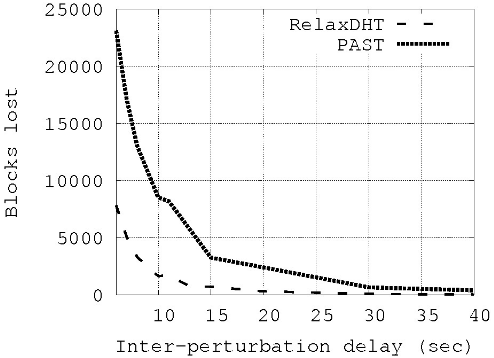
Comparison of Algorithms with 100 000 files



Back to Literature in Computer Science

Comparison of Algorithms with 100 000 files

no stable regime !



Limitations of the Model

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- **Heterogeneity**

$\Rightarrow$ **Mean Field Approach ?**

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  ? ? ?

On-going work

with S. Monnet, P. Sens and V. Simon (LIP6)

The End