CENTRE NATIONAL DE LA RECHERCHE SCIENTIFIQUE

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Non-Newtonian Flows in Porous Media: upscaling problems

https://www.dropbox.com/s/mcgg0ifpogsznv2/non_newtonian_V00.pdf?dl=0

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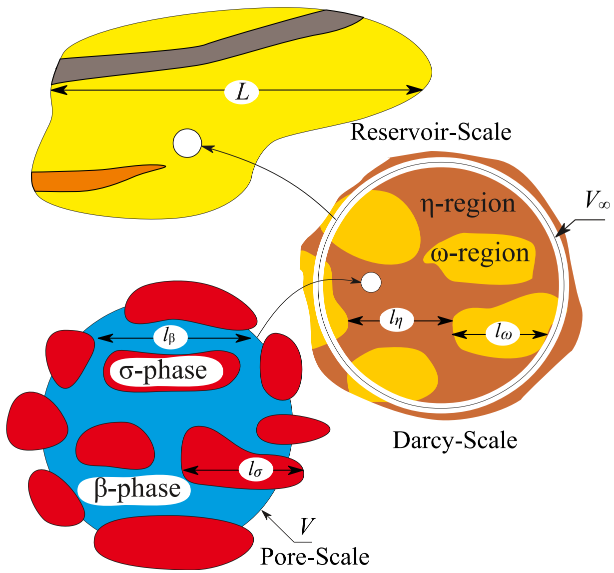
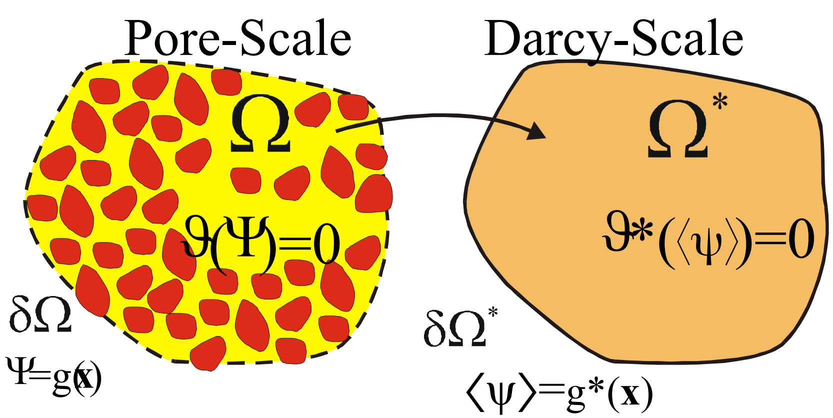
Objective/Outline

- *Motivation: flow of polymer solutions, question about heuristic models in Res. Engng*
- Upscaling
  - Introduction (generalized Stokes)
  - Transition
  - Induced anisotropy, effect of disorder, effect of size of the UC, ...
- Further problems: exclusion zone, viscoelastic
- Conclusions

Multi-Scale Analysis





- **Sequential multi-scale pattern**
- **Used in DRP, Res. Engng, Hydro., etc...**
- **Objectives of macro-scale theories:**
  - **Smoothing operator ⟨.⟩ → Macro variables, Eqs & BCs**
  - **Micro-macro link → Determination of Effective Properties**
- **Needs Scale Separation:**

$$l_\beta , l_\sigma \ll \mathrm{REV?} \ll L$$

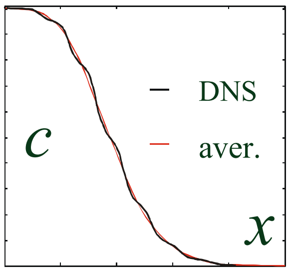
(process dependent)

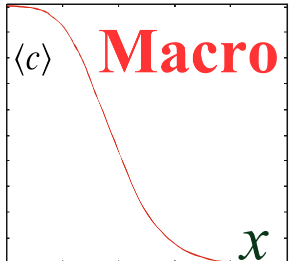
Multi-Scale Analysis: Upscaling Techniques

- ***Form of the equations?***
  - **averaging and TIP (**Marle, Gray, Hassanizadeh, …**)**
  - **averaging and closure (**Whitaker, …**)**
  - **homogenization (**Bensoussan et al., Sanchez-Palencia, Tartar, …**), also "closure"**
  - **stochastic approaches (**Dagan, Gelhar, ...**)**
- ***Effective properties* calculations?**
  - **Assuming the form of Eqs: interpret experiments or DNS**
  - **Upscaling with "closure" (averaging, homogenization, stochastic): provides *local* Unit Cell problems**
- ***Many Open Problems*: High non-linearities, Strong couplings, Evolving pore-scale structure, ...**

A simple introduction to upscaling with “closure”

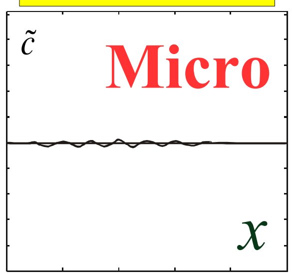
$$\nabla.\left(\mathbf{k}(x).\nabla c\right) = 0$$





$$\nabla.\left\langle \mathbf{k}(\mathbf{x}).\nabla\left(\langle c\rangle + \tilde{c}\right)\right\rangle = 0$$

$$c = \langle c\rangle + \tilde{c}$$

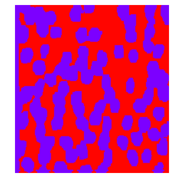


Closure:

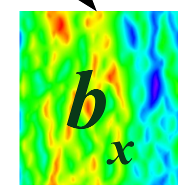
$$\tilde{c} = \mathbf{b}.\nabla\langle c\rangle + O(\ell^2/L^2)$$

$$\nabla\cdot(\mathbf{k}\cdot\nabla\mathbf{b}) = -\nabla\cdot\widetilde{\mathbf{k}}$$

$$\nabla.\left(\mathbf{k}.\nabla\tilde{c}\right) = -\nabla.\left(\tilde{\mathbf{k}}.\nabla\langle c\rangle\right) + \nabla.\left\langle\mathbf{k}.\nabla\tilde{c}\right\rangle$$



- Tomography
- Reconstruction
- Geostatistics
- ...



Macro-scale Equation

$$\nabla.\left(\mathbf{K}_{eff}.\nabla\langle c\rangle\right) = 0$$

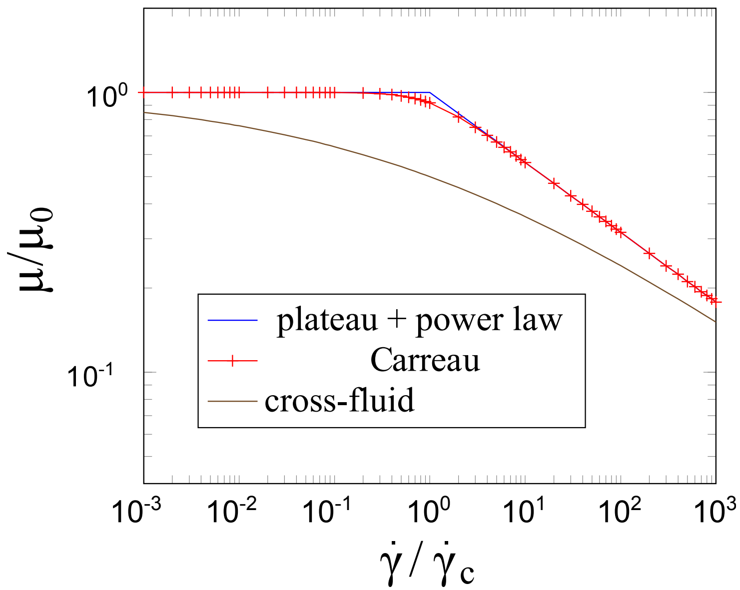
Effective property

$$\mathbf{K}_{eff} = \langle\mathbf{k}\rangle + \langle\mathbf{k}.\nabla\mathbf{b}\rangle$$

Flow of a non-Newtonian fluid

Case of Generalized Stokes equation

- **Pore-Scale problem (Re~0)**



$$\nabla \cdot \left[\mu_\beta \left(\dot{\gamma} \right) \left(\nabla \mathbf{v}_\beta + {}^{\mathsf{T}} \left(\nabla \mathbf{v}_\beta \right) \right) \right] - \nabla p_\beta + \rho_\beta \mathbf{g} = 0 \quad \text{in } V_\beta$$

$$\nabla \cdot \mathbf{v}_\beta = 0 \text{ in } V_\beta \ ; \ \mathbf{n}_{\beta\sigma} \cdot \mathbf{v}_\beta = 0 \text{ at } A_{\beta\sigma}$$

Rheology:

$$\mu_\beta = \mu_0 \, \hat{\mu} \left(\dot{\gamma} \right) \qquad \dot{\gamma} = \left\| \frac{1}{2} \left(\nabla \mathbf{v}_\beta + {}^{\mathsf{T}} \left(\nabla \mathbf{v}_\beta \right) \right) : \left(\nabla \mathbf{v}_\beta + {}^{\mathsf{T}} \left(\nabla \mathbf{v}_\beta \right) \right) \right\|$$

- **Upscaling: (vol. aver. $\langle \psi_\beta \rangle = \varepsilon_\beta \langle \psi_\beta \rangle^\beta$ with $\varepsilon_\beta = V_\beta / V$)?**

$$p_\beta = \left\langle p_\beta \right\rangle^\beta + \tilde{p}_\beta \ ; \ \mathbf{v}_\beta = \left\langle \mathbf{v}_\beta \right\rangle^\beta + \tilde{\mathbf{v}}_\beta$$

Typical local (over a REV) features

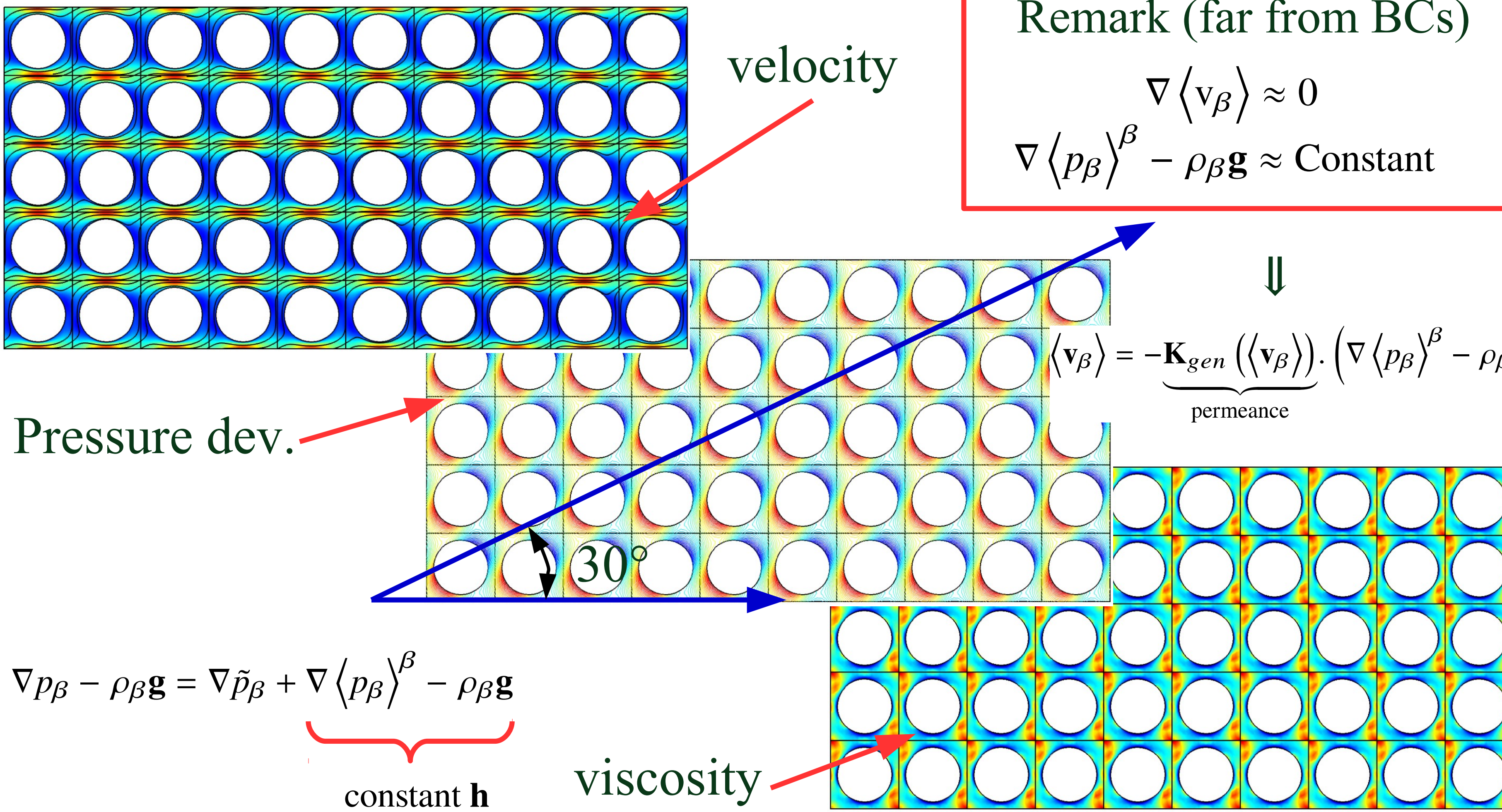
Remark (far from BCs)

$$\nabla \langle \mathrm{v}_\beta \rangle \approx 0$$

$$\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g} \approx \text{Constant}$$

$$\Downarrow$$

$$\langle \mathbf{v}_\beta \rangle = -\underbrace{\mathbf{K}_{gen}\left(\langle \mathbf{v}_\beta \rangle\right)}_{\text{permeance}} . \left(\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g}\right)$$



$$\nabla p_\beta - \rho_\beta \mathbf{g} = \nabla \tilde{p}_\beta + \underbrace{\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g}}_{\text{constant } \mathbf{h}}$$

Upscaling flow of a non-Newtonian fluid

- **Averaging (vol. aver. $\langle\psi_\beta\rangle=\varepsilon_\beta\langle\psi_\beta\rangle^\beta$ with $\varepsilon_\beta=V_\beta/V$)**

$$\text{macro}\left\{\begin{array}{c}\nabla.\left\langle \mathbf{v}_\beta \right\rangle = 0 \\ -\varepsilon_\beta\left(\nabla\left\langle p_\beta\right\rangle^\beta - \rho_\beta \mathbf{g}\right) + \underbrace{\frac{1}{V}\int_{A_{\beta\sigma}} \mathbf{n}_{\beta\sigma}\cdot\left[\mu_\beta\left(\tilde{\gamma}\right)\left(\nabla\tilde{\mathbf{v}}_\beta + {}^{\mathsf{T}}\left(\nabla\tilde{\mathbf{v}}_\beta\right)\right) - \tilde{p}_\beta\right] dA}_{\text{Closure?}} = 0 \quad +\ldots\end{array}\right.$$

$$\text{micro}\left\{\begin{array}{c}0 = -\nabla\tilde{p}_\beta + \nabla\cdot\left(\mu_\beta\left(\nabla\tilde{\mathbf{v}}_\beta + \nabla\tilde{\mathbf{v}}_\beta^T\right)\right) - \left(\nabla\left\langle p_\beta\right\rangle^\beta - \rho_\beta\mathbf{g}\right) \\ \nabla.\left(\tilde{\mathbf{v}}_\beta\right) = 0 \\ \tilde{\mathbf{v}} = -\left\langle \mathbf{v}_\beta\right\rangle^\beta \text{ at } A_{\beta\sigma}\end{array}\right.$$

$\Rightarrow$ **Problem must be solved for each value of $\langle v_\beta\rangle^\beta$!**

"Closure"?

Under several constraints: scale separation, far from BCs, ... $\Rightarrow$

Tentatively:

$$\begin{aligned} \tilde{\mathbf{v}}_\beta &= \mathbf{B} \cdot \left\langle \mathbf{v}_\beta \right\rangle^\beta + \ldots \\ \tilde{p}_\beta &= \mu_0 \mathbf{b} \cdot \left\langle \mathbf{v}_\beta \right\rangle^\beta + \ldots \end{aligned}$$

$$\left(\nabla \cdot \left[\hat{\mu}\left(\hat{\dot{\gamma}}\right) \left(\nabla \mathbf{B} + {}^{\mathsf{T}}(\nabla \mathbf{B}) \right) \right] - \nabla \mathbf{b} \right) \cdot \left\langle \mathbf{v}_\beta \right\rangle^\beta = \left\langle \nabla \cdot \left[\hat{\mu}\left(\hat{\dot{\gamma}}\right) \left(\nabla \mathbf{B} + {}^{\mathsf{T}}(\nabla \mathbf{B}) \right) \right] - \nabla \mathbf{b} \right\rangle^\beta \cdot \left\langle \mathbf{v}_\beta \right\rangle^\beta$$

$$(\nabla \cdot \mathbf{B}) \cdot \left\langle \mathbf{v}_\beta \right\rangle^\beta = 0 \ ; \ \mathbf{B} \cdot \left\langle \mathbf{v}_\beta \right\rangle^\beta = -\mathbf{I} \cdot \left\langle \mathbf{v}_\beta \right\rangle^\beta \text{ at } A_{\beta\sigma}$$

$$\mathbf{B}(\mathbf{r}) = \mathbf{B}(\mathbf{r} + h\mathbf{e}_i) \text{ and } \mathbf{b}(\mathbf{r}) = \mathbf{b}(\mathbf{r} + h\mathbf{e}_i) \text{ for } i = 1, 2, 3 \text{ (periodicity)}$$

$$\langle \mathbf{b} \rangle = 0$$

with $\hat{\dot{\gamma}} = \left\| \left(\nabla \mathbf{B} + {}^{\mathsf{T}}\nabla \mathbf{B} \right) \cdot \mathbf{e}_\beta \right\| \left\| \left\langle \mathbf{v}_\beta \right\rangle^\beta \right\|$ $\qquad$ $\mathbf{e}_\beta = \left\langle \mathbf{v}_\beta \right\rangle^\beta / \left\| \left\langle \mathbf{v}_\beta \right\rangle^\beta \right\|$

$\Rightarrow$ Problem must be solved for each value of $\langle \mathrm{v}_\beta \rangle^\beta$!

A classical story: the linear case and Darcy's law (see Sanchez-Palencia, Whitaker, ….)

- **Closure (any solution is a linear combination of elementary solutions for $\langle \mathbf{v}_\beta \rangle^\beta = \mathbf{e}_i$ for $i=1,2,3$)**

$$\boxed{\begin{aligned} \tilde{\mathbf{v}}_\beta &= \mathbf{B}\cdot\left\langle \mathbf{v}_\beta \right\rangle^\beta + \ldots \\ \tilde{p}_\beta &= \mu_0 \mathbf{b}\cdot\left\langle \mathbf{v}_\beta \right\rangle^\beta + \ldots \end{aligned}}$$

$$\left(\nabla\cdot\left(\nabla\mathbf{B}+{}^{\mathsf{T}}(\nabla\mathbf{B})\right)-\nabla\mathbf{b}\right)=\left\langle \nabla\cdot\left(\nabla\mathbf{B}+{}^{\mathsf{T}}(\nabla\mathbf{B})\right)-\nabla\mathbf{b}\right\rangle^\beta$$

$$(\nabla\cdot\mathbf{B})=0\ ;\ \mathbf{B}=-\mathbf{I}\text{ at }A_{\beta\sigma}$$

over a UC!

$$\mathbf{B}(\mathbf{r})=\mathbf{B}(\mathbf{r}+h\mathbf{e}_i)\ \text{ and }\ \mathbf{b}(\mathbf{r})=\mathbf{b}(\mathbf{r}+h\mathbf{e}_i)\ \text{ for } i=1,2,3\ \text{(periodicity)}$$

$$\langle \mathbf{b} \rangle = 0$$

- **Macro-Scale equation and effective properties**

Darcy's law:

$$\boxed{\left\langle \mathbf{v}_\beta \right\rangle = -\frac{1}{\mu_0}\mathbf{K}_0.\left(\nabla\left\langle p_\beta\right\rangle^\beta-\rho_\beta\mathbf{g}\right)}$$

Intrinsic permeability: $\mathbf{K}_0^{-1}=\dfrac{1}{V}\displaystyle\int_{V_\beta}\left(\nabla\cdot\left(\nabla\mathbf{B}+{}^{\mathsf{T}}(\nabla\mathbf{B})\right)-\nabla\mathbf{b}\right)dV$

Important: Proof of symmetry of $\mathbf{K}_0$ requires periodicity!

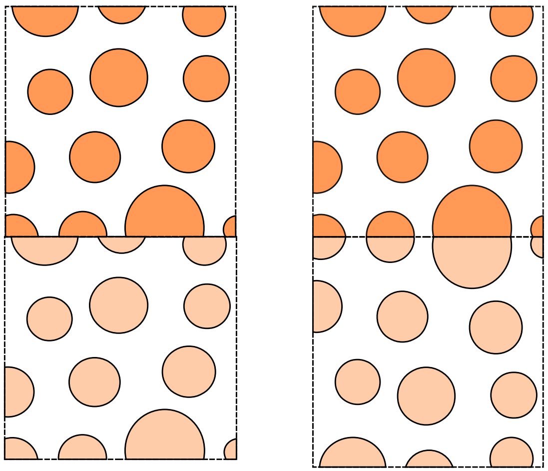
Calculations of the permeability

Case of "diffusion" problem: e.g., permeability, effective diffusion

- 3 possibilities
  - Initial closure problem
  - Transformation of closure problem into ~Stokes with source term and periodic pressure and velocity
  - "permeameters": no-periodicity

See discussion in Guibert et al., 2015

- Making image periodic?
  - I: Percolation problem
  - II: Loss of anisotropy
  - III: potentially various bias

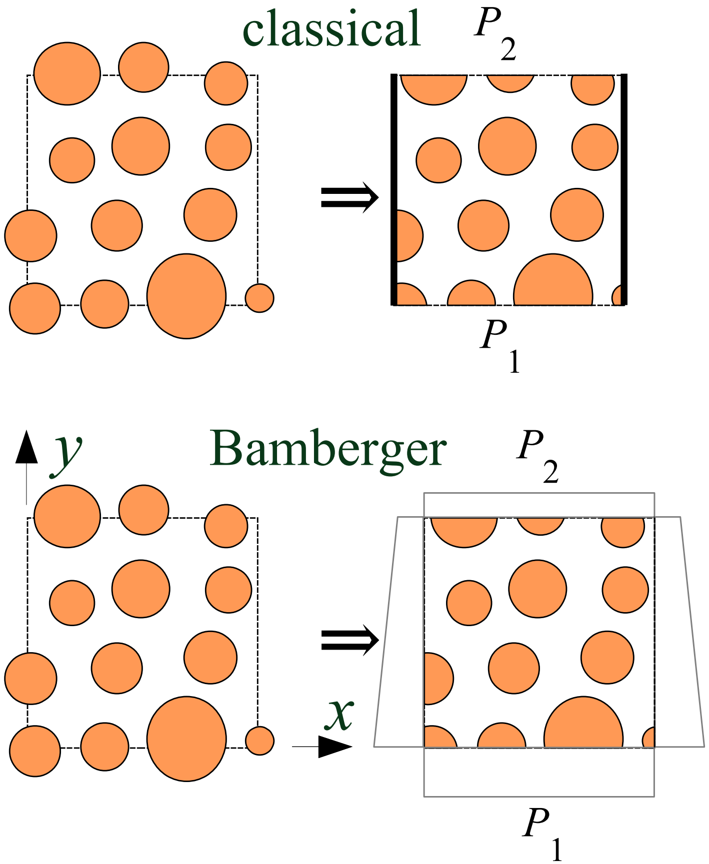


- thin layers + periodicity
- Eff. Medium
- …..

I II III

Calculations over non-periodic images

- "permeameters"
  - All methods have bias
  - $\langle v_x \rangle^{\beta} \neq 0$
  - $K_{xy} \neq K_{yx}$

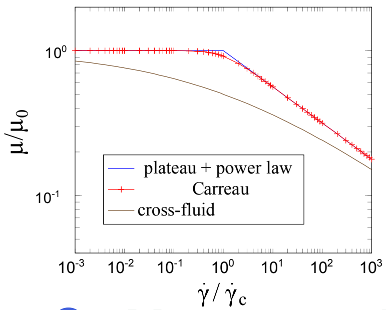


Note: minimal bias if large sample and anisotropy along the axis

See discussion in: Manwart et al. 2002; Piller et al. 2009; Guibert et al., 2015; …

Non-Linear Case: Non-Newtonian Fluid

- **Fluid rheology**



PLCO

$$\mu = \begin{cases} \mu_0 & \text{if } \dot{\gamma} < \dot{\gamma}_c, \\ \mu_0 \left(\frac{\dot{\gamma}}{\dot{\gamma}_c}\right)^{n-1} & \text{else.} \end{cases}$$

- **No generic closure independent of fluid velocity! Generic macro-scale law:**

$$\langle \mathbf{v}_\beta \rangle = \underbrace{-\mathbf{K}_{gen}\left(\langle \mathbf{v}_\beta \rangle\right)}_{\text{permeance}} . \left(\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g} \right)$$

- **Representation as a deviation from Darcy's law**

$$\langle \mathbf{v}_\beta \rangle = -\frac{1}{\mu_0} k_n \mathbf{P} . \underbrace{\mathbf{K}_0 . \left(\nabla \langle p_\beta \rangle^\beta - \rho_\beta \mathbf{g} \right)}_{\text{newtonian velocity}}$$

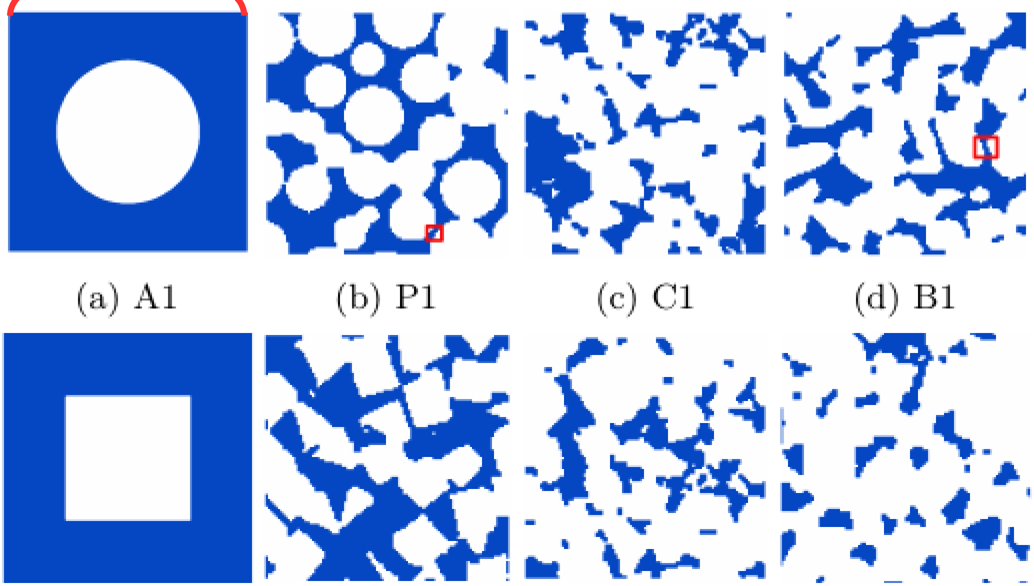
  – k_n, **P** (rotation "matrix"): depend on $\langle \mathbf{v}_\beta \rangle^\beta$ (modulus and orientation)

Test cases

Needs **very** fine grid!

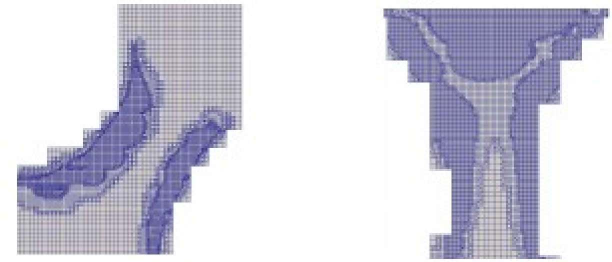
2D

Clashach Bentheimer

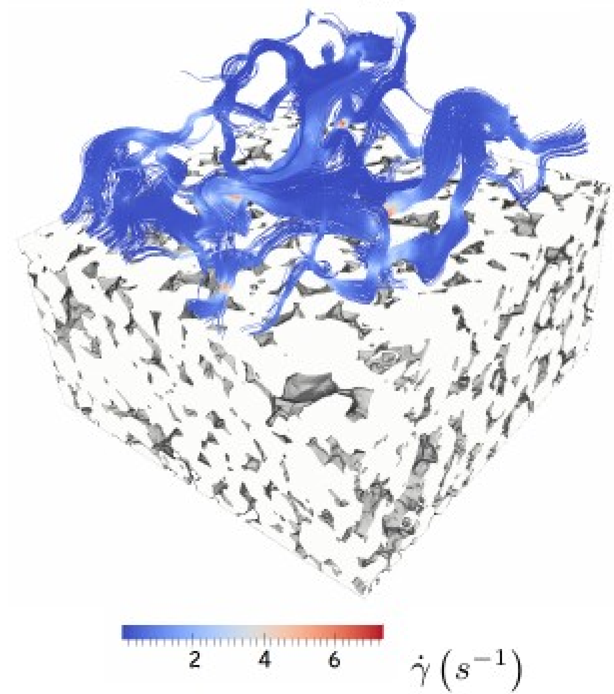


(a) A1 (b) P1 (c) C1 (d) B1

(f) A2 (g) P2 (h) C2 (i) B2



(e) Mesh P1 (j) Mesh B1



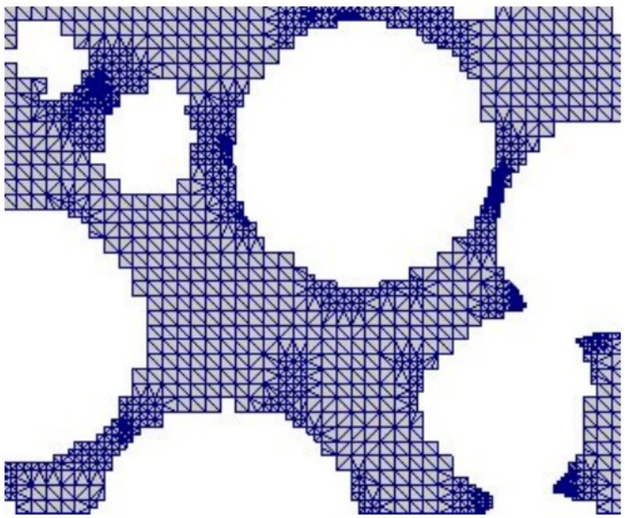
(k) B1

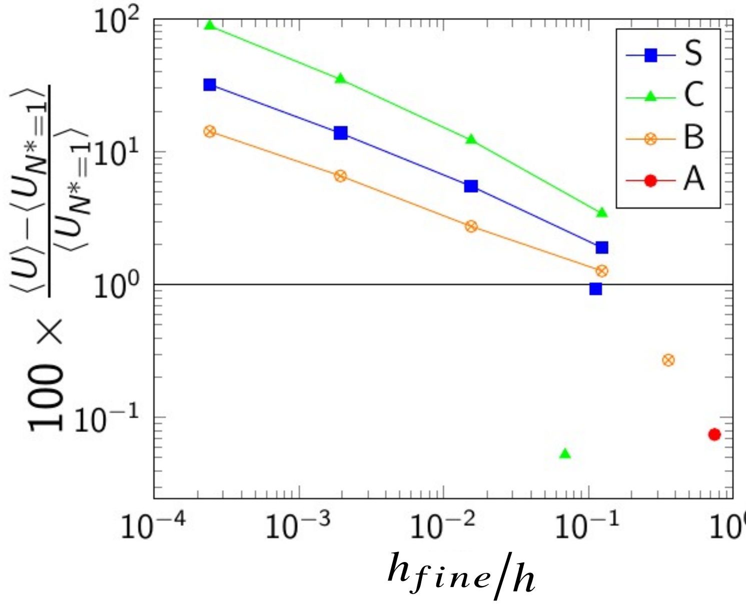
HPC center EOS-Calmip:
Typically: 10^8 mesh cells
10^5 cores×hours

often limited to $\sim$ mm^3!

Resolution with OpenFoam

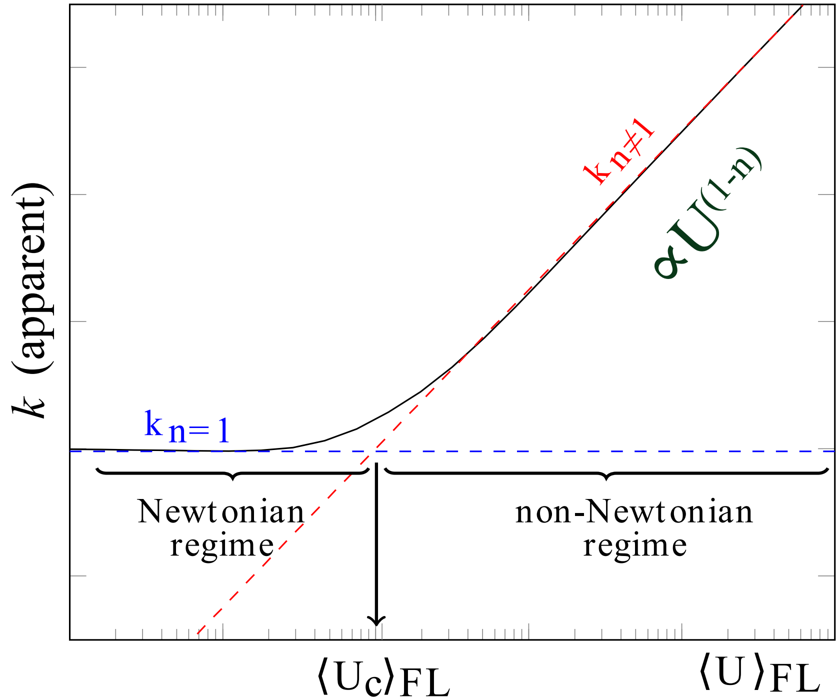
- FVM with OpenFOAM (SIMPLE, second-order scheme)
- Use of HPC, calculations up to 100 millions mesh elements
- a total of 100000 hours of CPU time.
- Conform orthogonal hexahedral elements.
- Multi-criteria grid convergence study = OK.





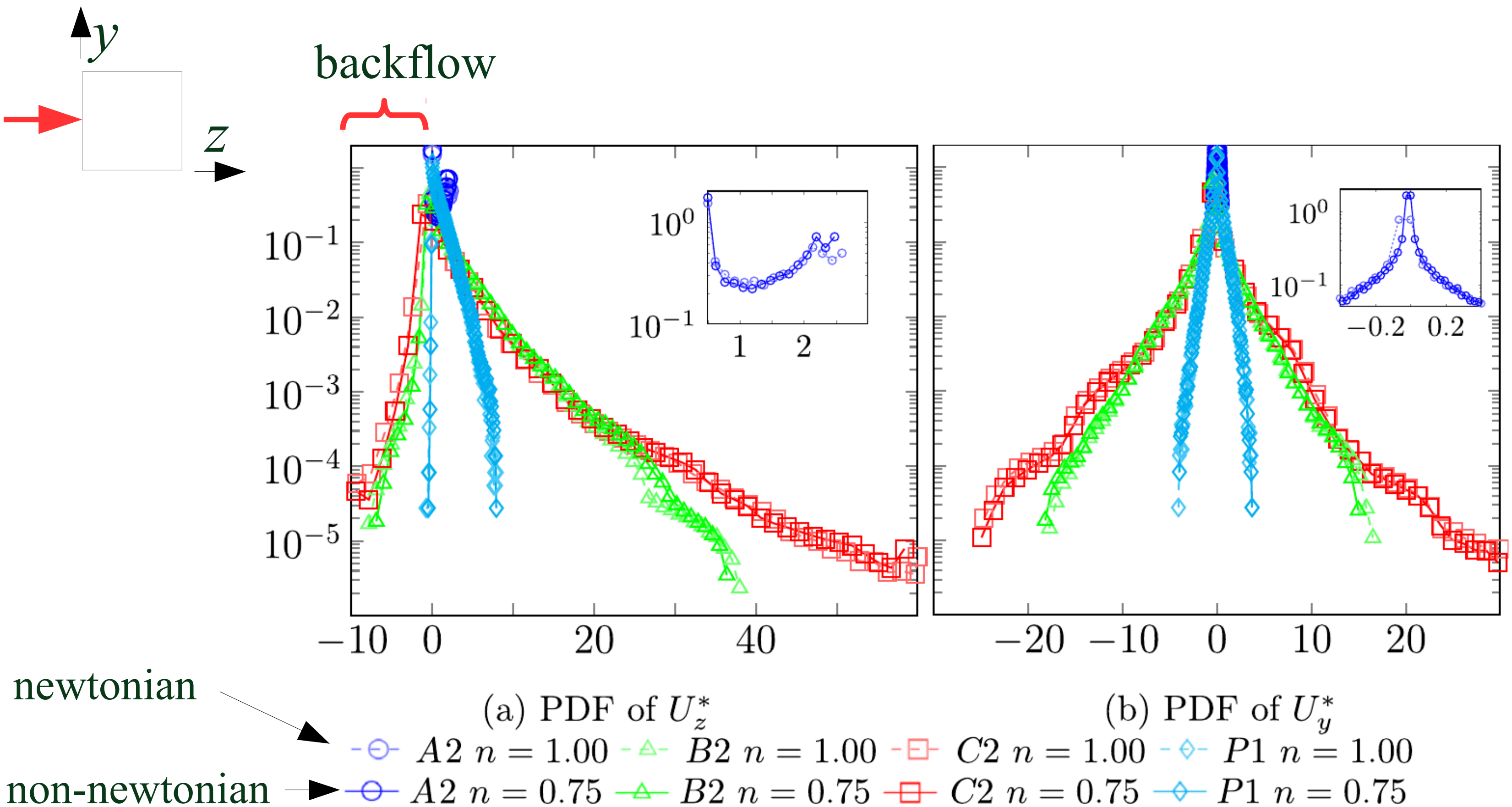
Results

- Computations allows to analyze various features:
  - Properties of pore-scale fields (PDFs)
  - Transition:
    - Starts in a few narrow constrictions
    - Scaling for transition?



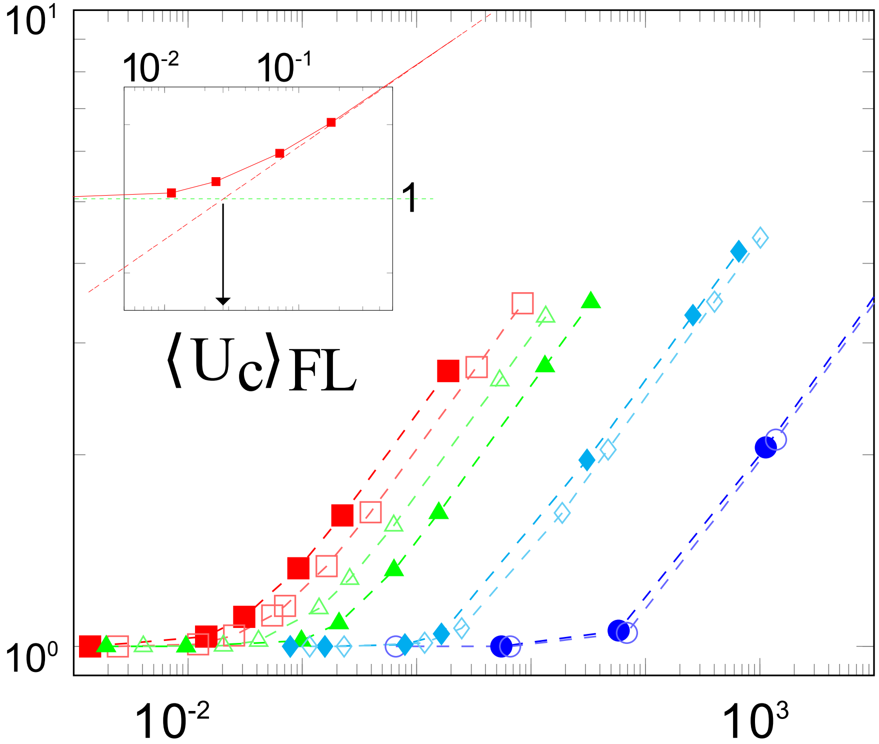
$\langle . \rangle_{FL}$ = intrinsic fluid average

Structure of the Velocity Field



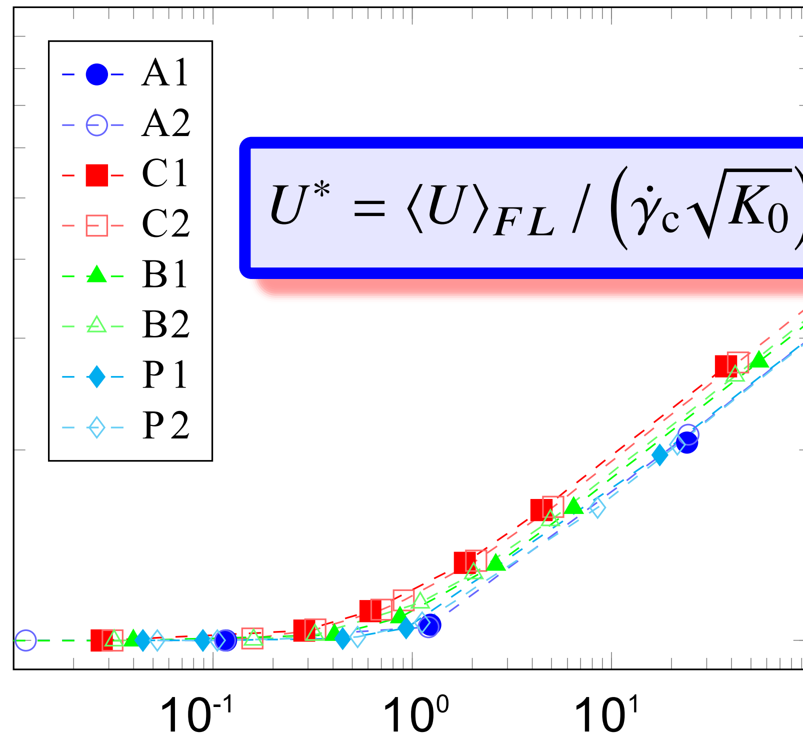
Normalized pdf ~similar between Newtonian and non-Newtonian flow! Not valid for pdf of $\nabla \langle p_\beta \rangle^\beta$

Transition Scaling



(a) k^* vs $\langle U \rangle_{FL}$ $\mu m.s^{-1}$

$$k^* = K_{app}/K_0$$



$$U^* = \langle U \rangle_{FL} / \left(\dot{\gamma}_c \sqrt{K_0}\right)$$

(b) k^* vs U^*

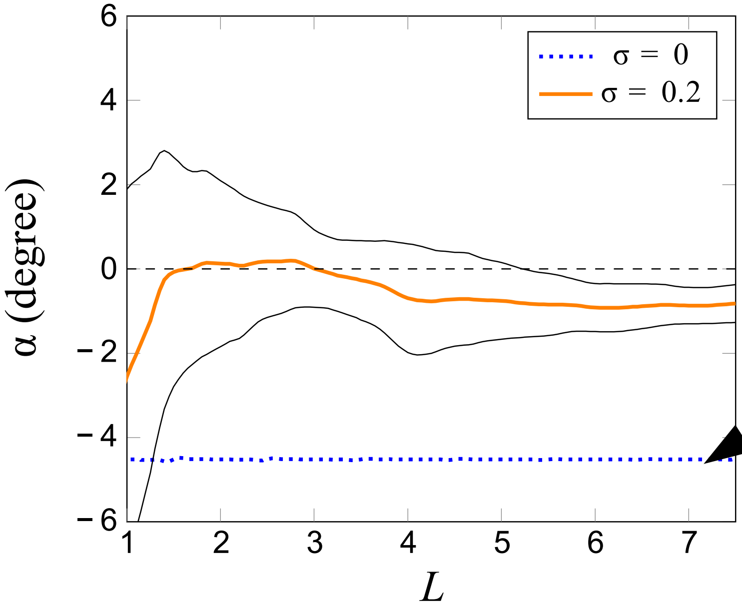
$$\dot{\gamma}_c = 1\mathrm{s}^{-1} \;;\; n = 0.75$$

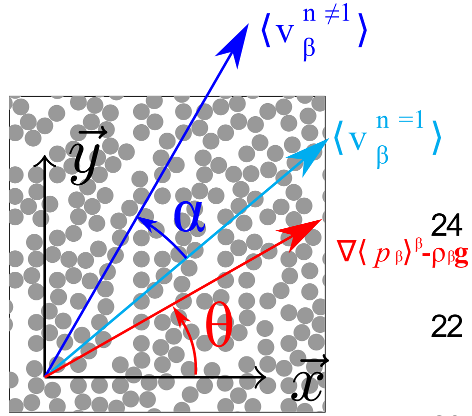
Zami-Pierre et al., 2015

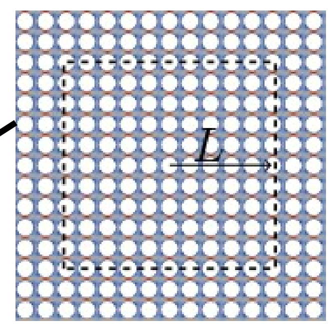
Impact of Domain Size

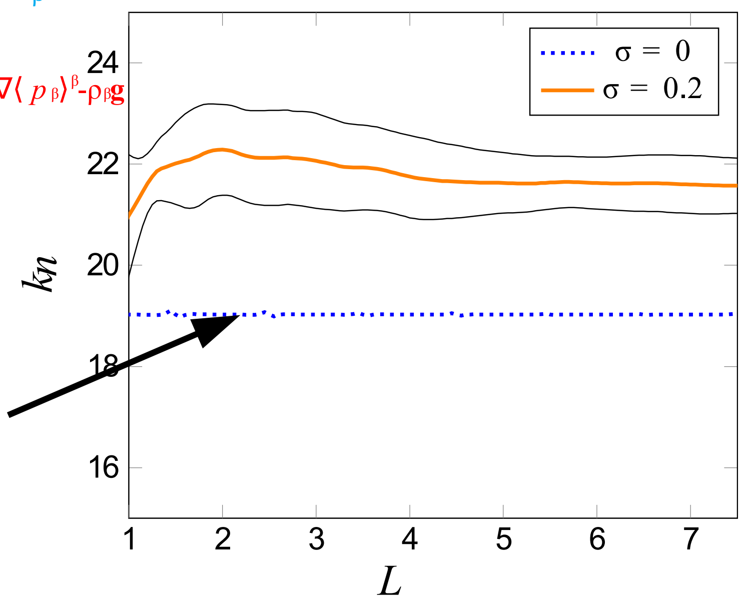
$\dot{\gamma}_c = 1\mathrm{s}^{-1}$; $n = 0.70$

$\theta = 22°$ $\quad U^* \gg 1$



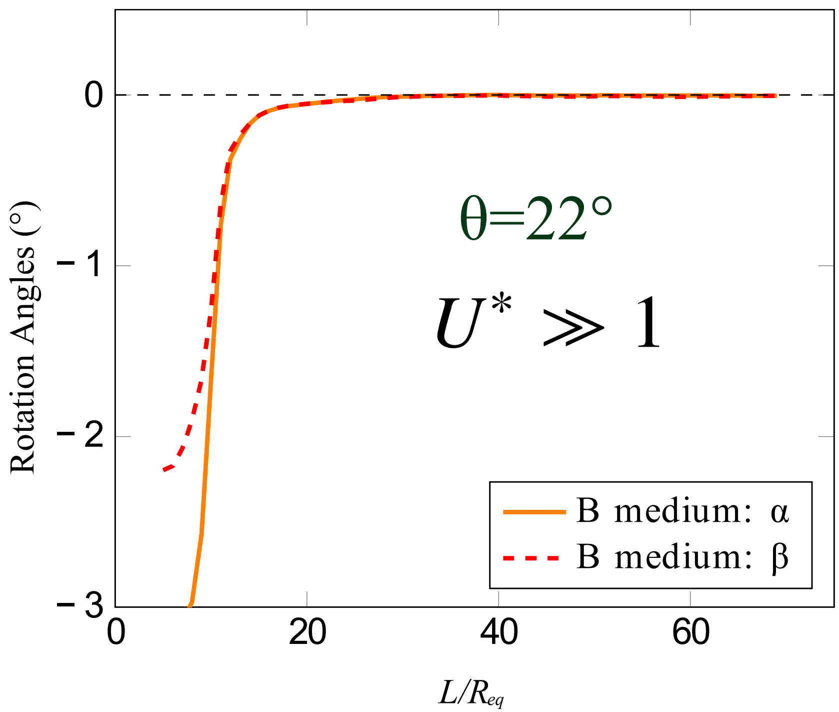


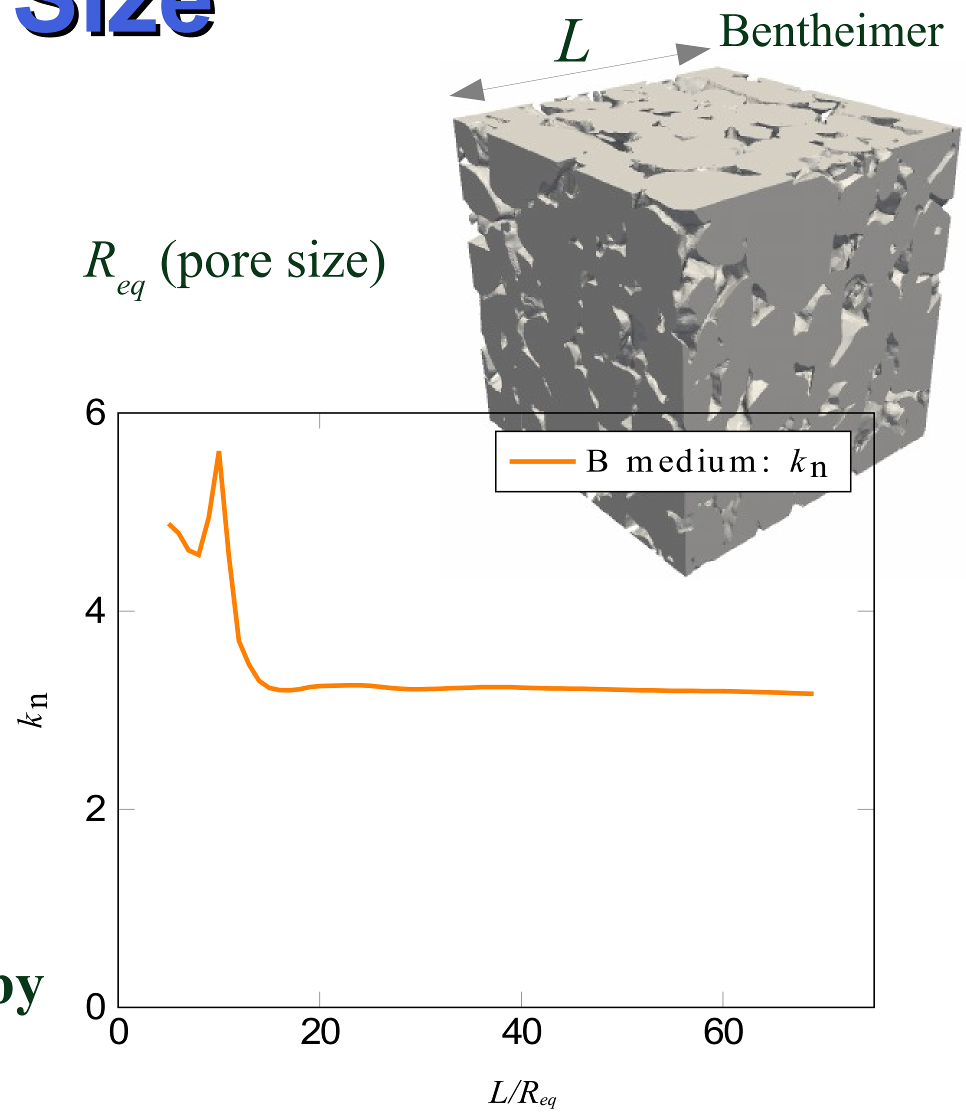




- Anisotropy induced by non-linear behavior decreases with $\nearrow L$ for disordered media
- Effective property variance decreases with $\nearrow L$

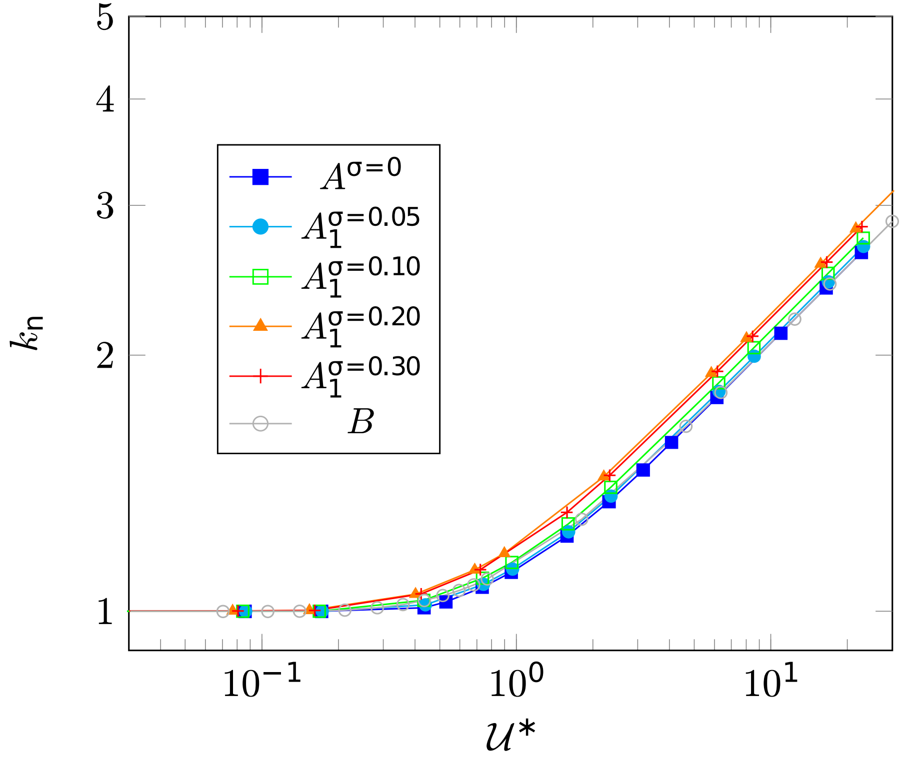
Impact of Domain Size

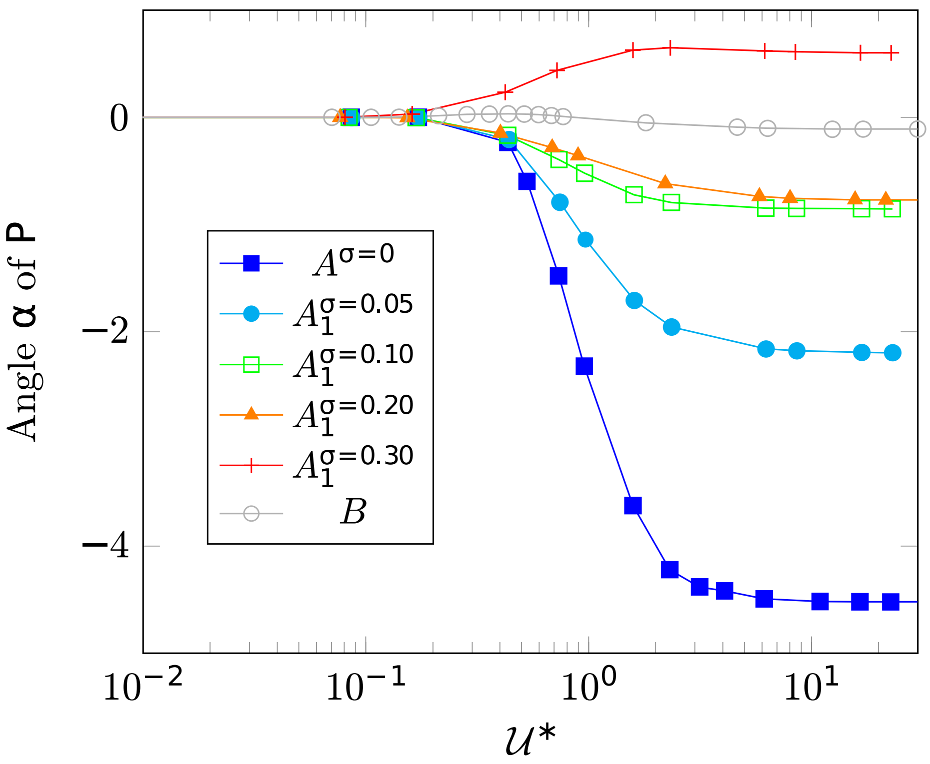




Disorder → no anisotropy induced by non-linearity if *L* large enough!

Impact of disorder and velocity





Practical Consequences

- Eng. Practice: apparent Darcy's law

$$\left\langle \mathbf{v}_\beta \right\rangle = -\frac{1}{\mu\left(\dot{\gamma}_{eq}\right)}\mathbf{K}_0 . \left(\nabla \left\langle p_\beta \right\rangle^\beta - \rho_\beta \mathbf{g}\right) \qquad \dot{\gamma}_{eq} = 4\,\alpha \frac{\left\| \left\langle \mathbf{v}_\beta \right\rangle \right\| / \varepsilon_\beta}{\sqrt{8K_0/\varepsilon_\beta}}$$

Fitting parameter (rock dependent)

- Discussion:
  - P=I for all $\langle v_\beta \rangle^\beta$ if isotropic disordered media and REV ($\rightarrow$ need tests for various sizes)!
  - Apparent permeability ~ scales with $(K_0)^{1/2}$ $\rightarrow$ classical scaling "may" introduce artificial dependence upon parameters such as porosity:

$$\langle U_c \rangle_{FL} = \alpha' \, \dot{\gamma}_c \sqrt{K_0} \qquad \text{versus} \qquad \langle U_c \rangle_{FL} = \frac{1}{\alpha\sqrt{2\varepsilon_\beta}} \dot{\gamma}_c \sqrt{K_0}$$

  - Description of transition near the critical velocity may not be well described by an apparent viscosity (no observed angle in the apparent permeability in the case of PLCO)

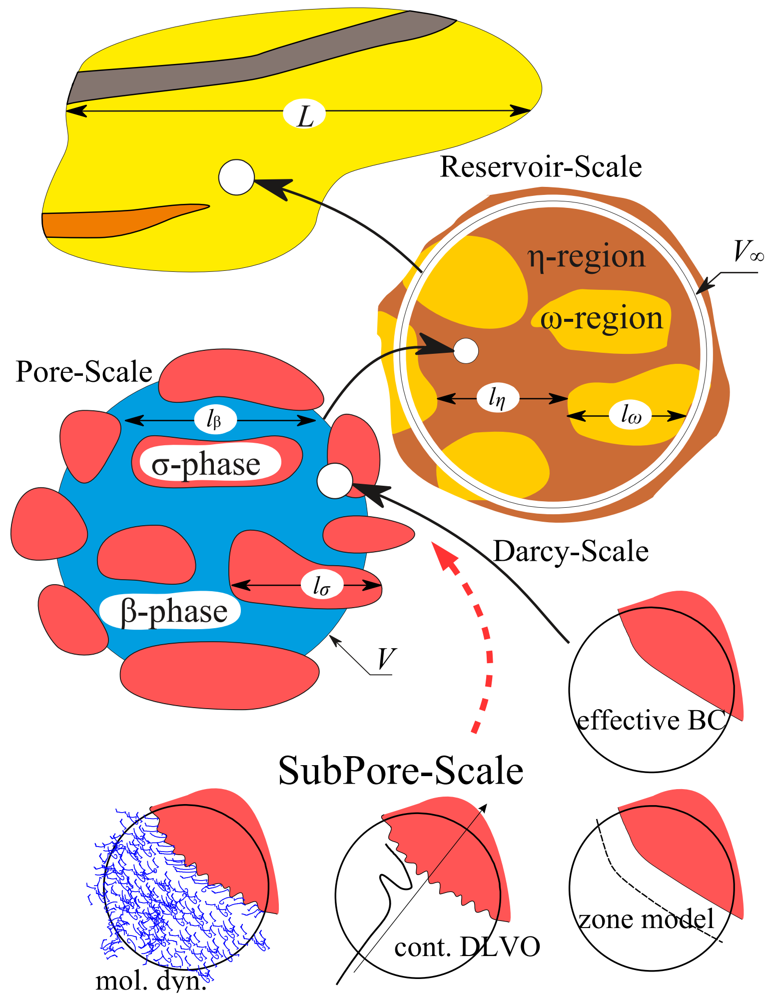
Further upscaling

- **Depletion layer treated as an effective BC**

$$\mathbf{v}_\beta = -\ell\,\mathbf{n}\cdot\left(\nabla\mathbf{v}_\beta + (\nabla\mathbf{v}_\beta)^T\right)\cdot(\mathbf{I}-\mathbf{nn})$$

Zami-Pierre et al., 2017

see Chauveteau (1982),
Sorbie & Huang (1991)
(double-layer model)



Further upscaling

- **Viscoelastic fluids**

$$\rho_l \frac{\partial \mathbf{v}_l}{\partial t} + \rho_l \mathbf{v}_l \cdot \nabla \mathbf{v}_l = -\nabla p_l + \rho_l \mathbf{g} + \underbrace{\nabla \cdot \left(\mu_s \left(\nabla \mathbf{v}_l + \nabla \mathbf{v}_l^T \right) \right)}_{solvent} + \nabla \cdot \boldsymbol{\tau}_v$$

$$\overset{\nabla}{\boldsymbol{\tau}}_v = \frac{\partial \boldsymbol{\tau}_v}{\partial t} + \mathbf{v}_l \cdot \nabla \boldsymbol{\tau}_v - \nabla \mathbf{v}_l^T \cdot \boldsymbol{\tau}_v - \boldsymbol{\tau}_v \cdot \nabla \mathbf{v}_l \qquad \text{upper convected Derivative}$$

- **Rheological models**

FENE-P:

$$f\left(\boldsymbol{\tau}_v\right) \boldsymbol{\tau}_v + \lambda \overset{\nabla}{\boldsymbol{\tau}}_v = 2\, a\, \mu_p \frac{1}{2} \left(\nabla \mathbf{v}_l + \nabla \mathbf{v}_l^T \right)$$

$$f\left(\boldsymbol{\tau}_v\right) = 1 + \frac{3\,a + \left(\lambda / \mu_p \right) \operatorname{tr}\left(\boldsymbol{\tau}_v \right)}{L^2} \; ; \; a = \frac{L^2}{L^2 - 3}$$

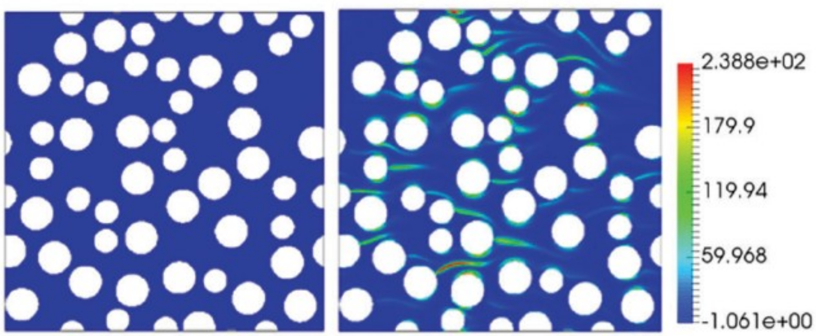
$L^2 \rightarrow \infty$ gives Oldroyd-B model (no-elongation limit)

Example of results: De et al., soft matter, 2018

Steady-state!

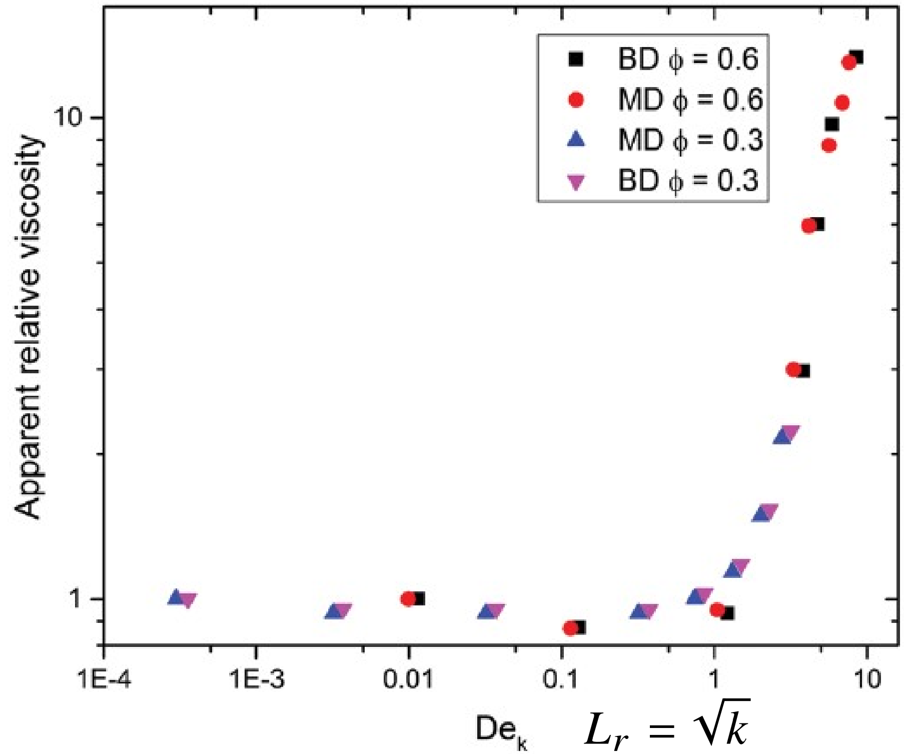
Deborah number: $\mathrm{De} = \dfrac{\lambda U_r}{L_r}$

...also Weissenberg number ☺



De= 0.001 0.1

Normal stress along average flow direction

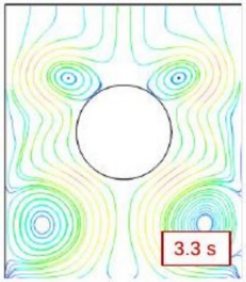


$L_r = \sqrt{k}$

-see previous discussion on “apparent permeability”, etc…

- elastic turbulence?

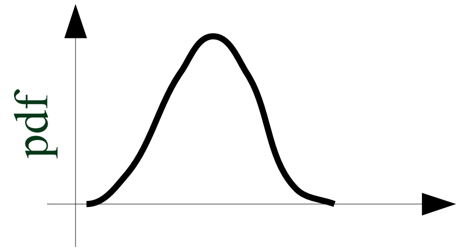
Further perspectives: N-momentum equations, multi-component aspects, ...



- Superfluid: 2 momentum equations → complex behavior → macro-scale model?

  see Allain et al. (2010, 2013, 2015), Soulaine et al. (2015, 2017)

- Polymer solution as multi-component systems:
  - Mechanical segregation, degradation (bio., mech.)
  - Model?
    - Momentum balances:
      - diffusion theory or
      - N-momentum equations
    - Composition:
      - Continuous models or
      - PBM (population balance model), ...



Conclusions

- Upscaling tells that this is not always possible to separate in an apparent Darcy's law permeability and viscosity
- Specific anisotropy effects
- Simplifications arise for disordered media
- Various results published in the literature for various rheology: power-law (...), Ellis and Herschel–Bulkley fluids (Sochi & Blunt, 2008), Yield-Stress Fluids (Sochi, 2008), etc…
- Additional problems: retention effects, Inaccessible Pore Volume (IPV), mobile/immobile effects
- Perspectives: viscoelastic, multicomponent, coupling with other transport problems (transport of species, heat transfer, etc…), ...