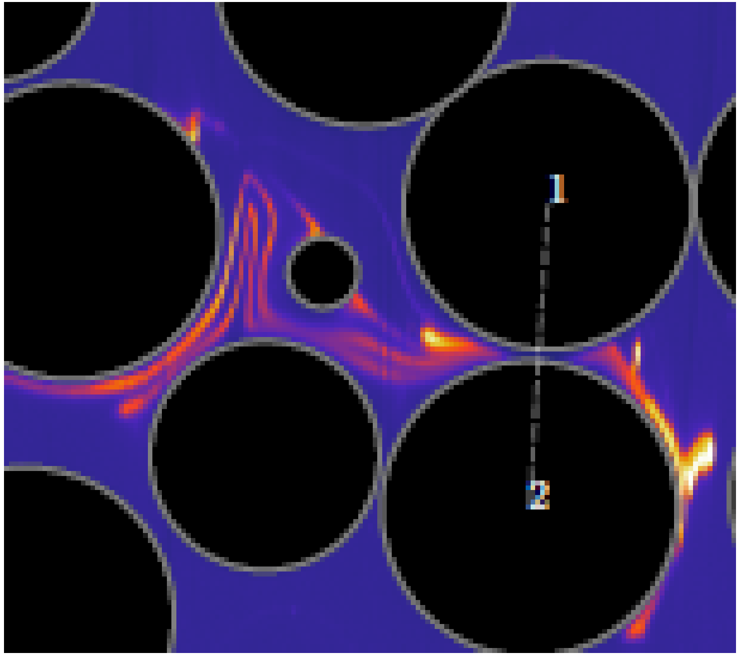
Transport phenomena: mixing

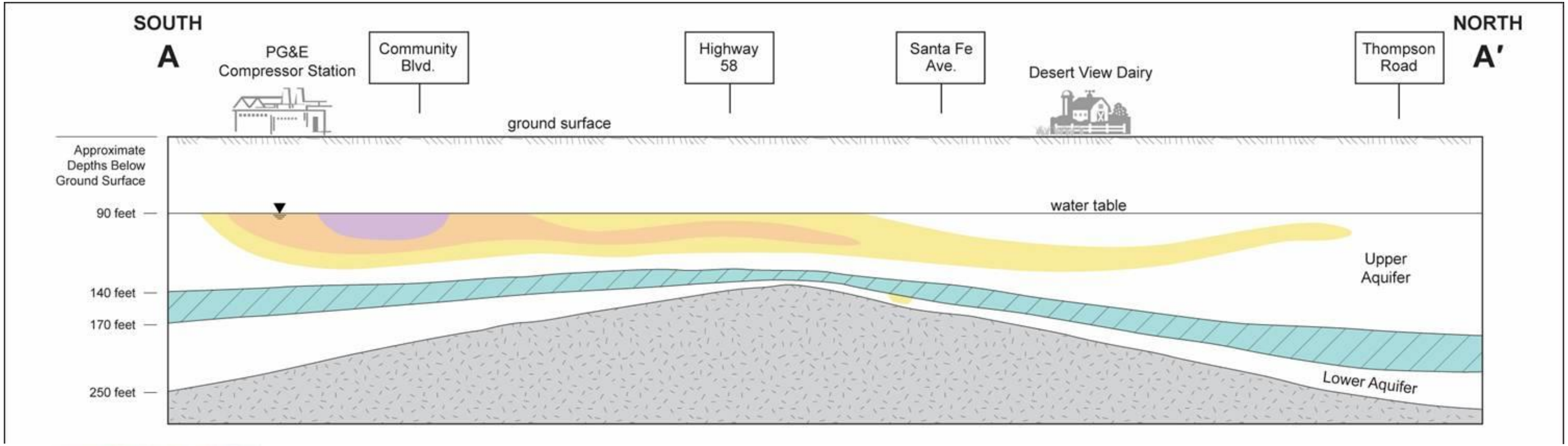


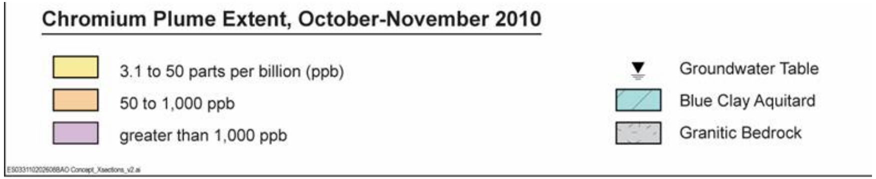
Tanguy Le Borgne

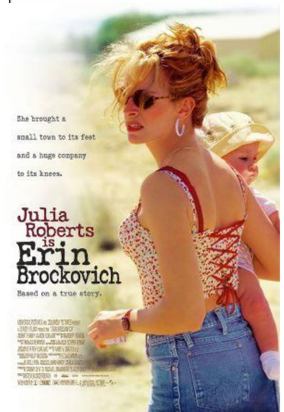
University of Rennes 1, France

Mixing in subsurface flows

Contaminant transport and remediation





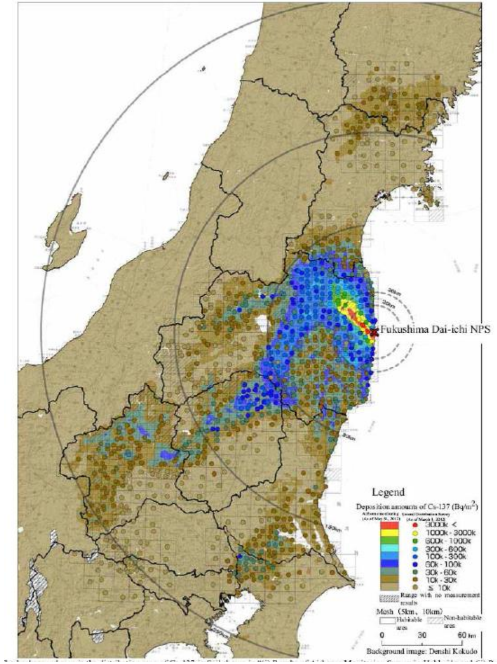


In 2013 Pacific Gas & Electric (PG&E) paid $ 300 million in damage and $700 million in remediation

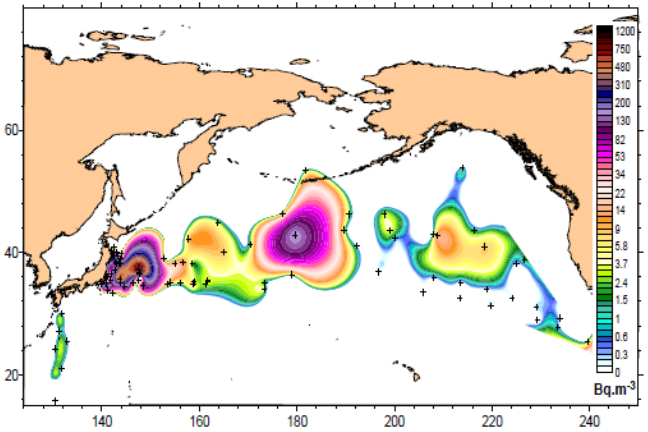
Mixing in subsurface flows

Contaminant transport and remediation

Fukushima



Mesures de concentration en Cesium 137 dans les sols autour de Fukushima en 2012



Mesures de concentration de Cesium 134 en mer en mars 2011 (Aoyama et al. 2012)

Données extraites du rapport IRSN n° PRP-ENV/2013-00044

Mixing in subsurface flows

AGU PUBLICATIONS

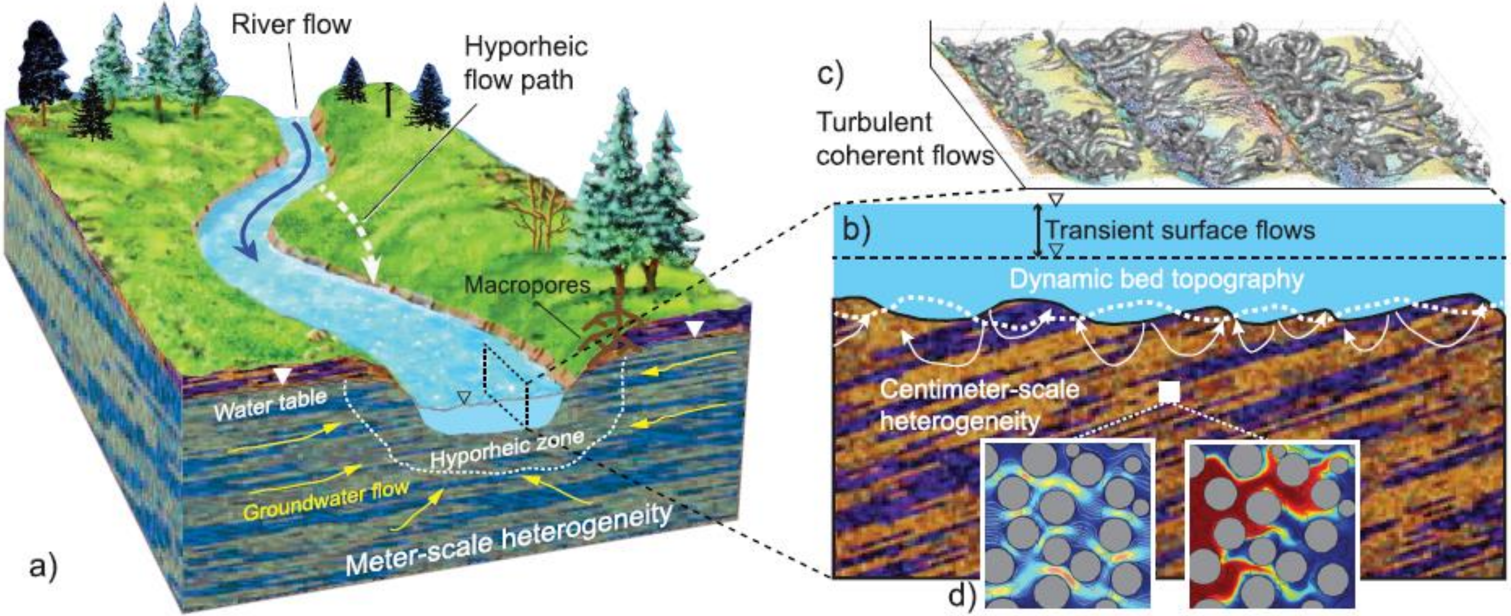
Water Resources Research

COMMENTARY

10.1002/2016WR020005

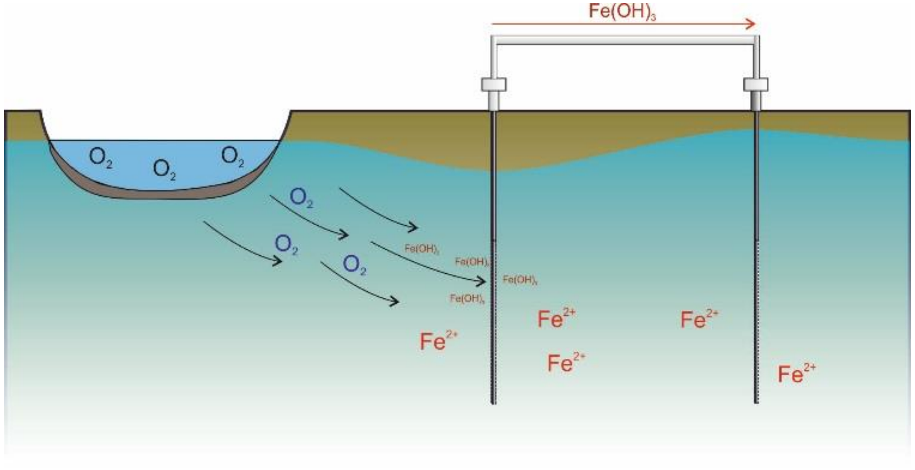
The importance and challenge of hyporheic mixing

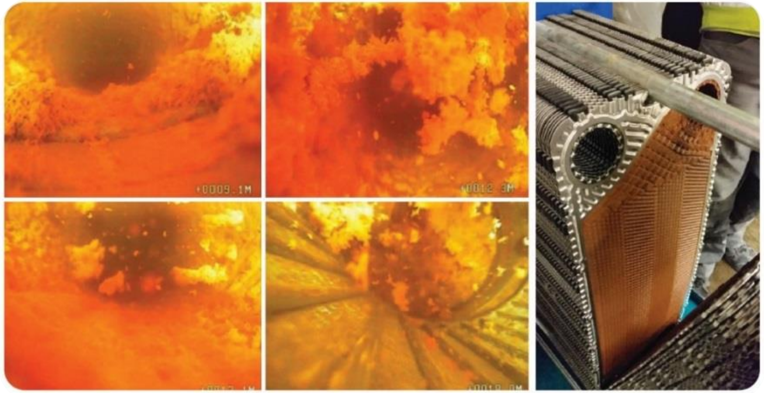
Erich T. Hester[1], M. Bayani Cardenas[2], Roy Haggerty[3], and Sourabh V. Apte[4]

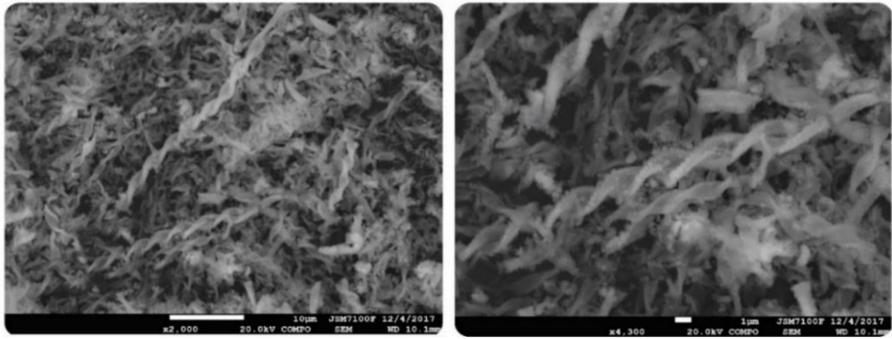


Mixing in subsurface flows

Clogging in Geothermal energy or artificial recharge)

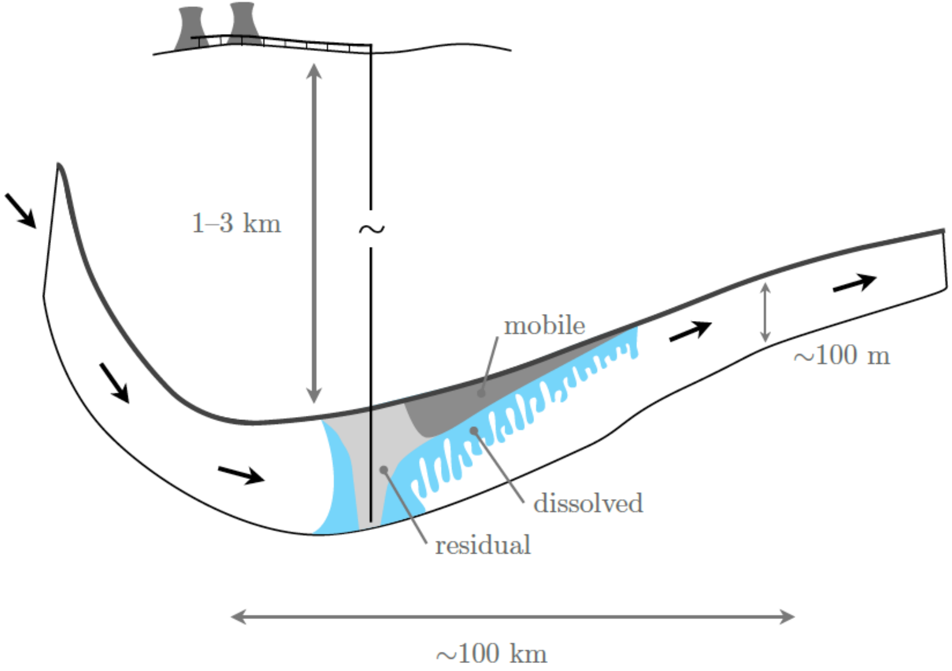






Luc Burté, PhD thesis, University of Rennes, Antea group

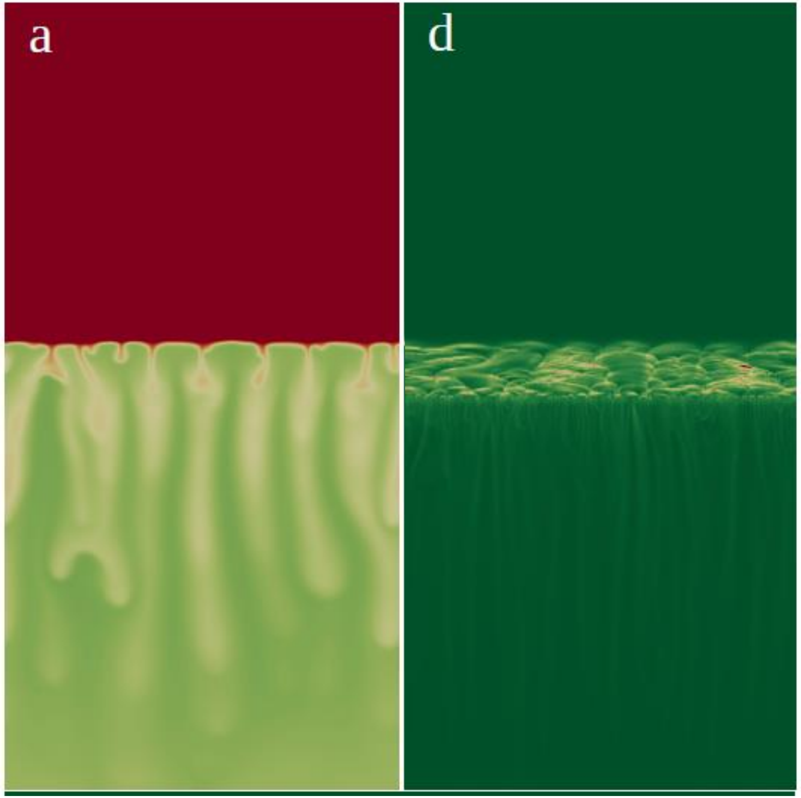
Mixing in subsurface flows



Macminn et al., J. of Fluid Mech.., 2011

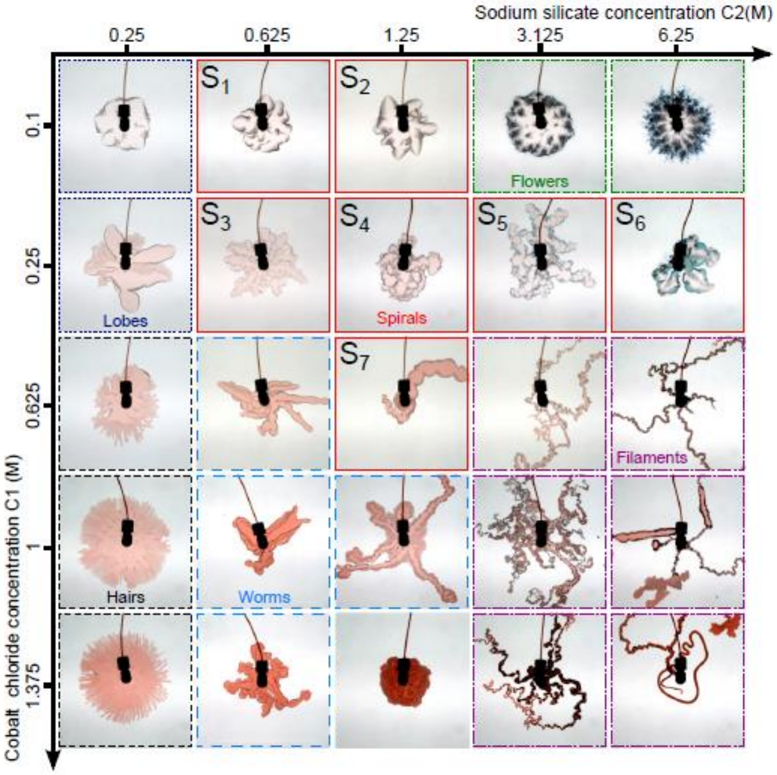
Dissolved CO2

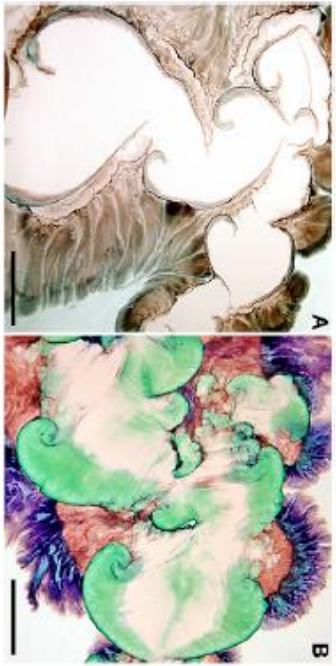
Porosity changes



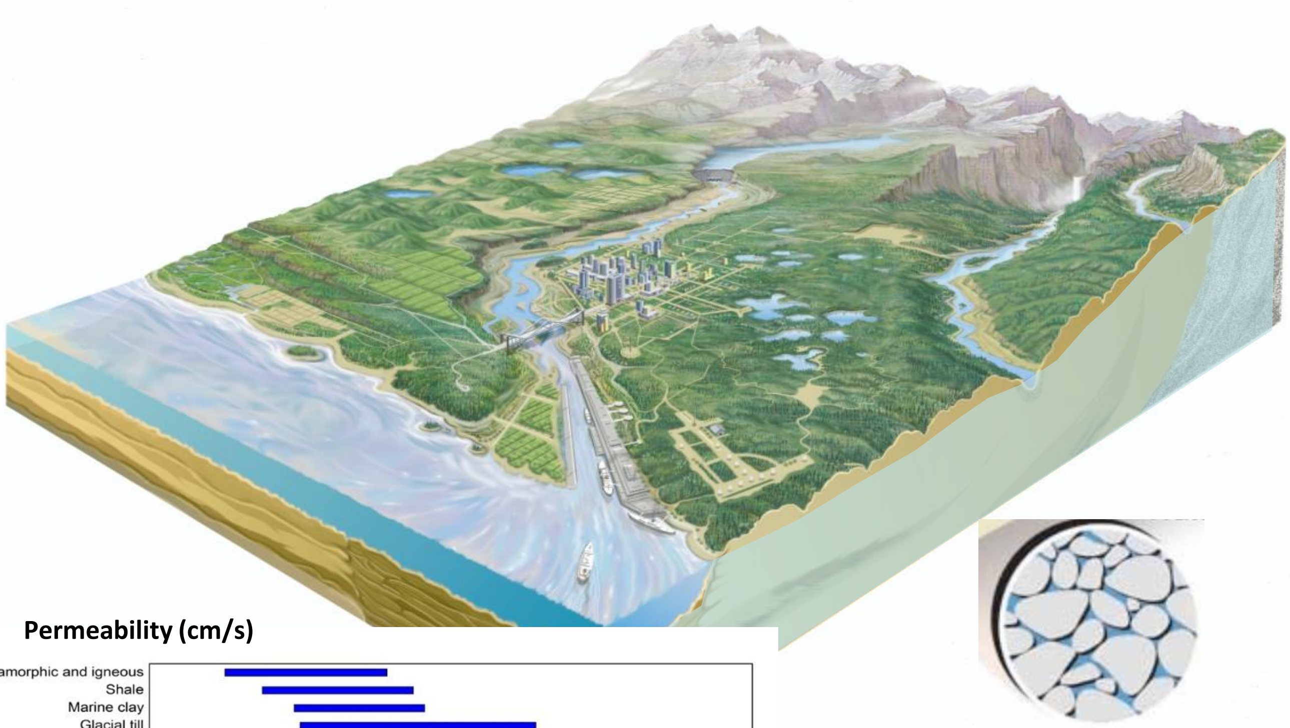
Hidalgo et al., Geophys. Res. Lett., 2015

Mixing in subsurface flows

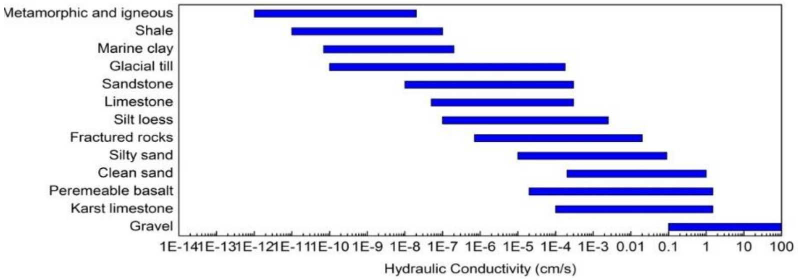




Haudin et al., PNAS 2014

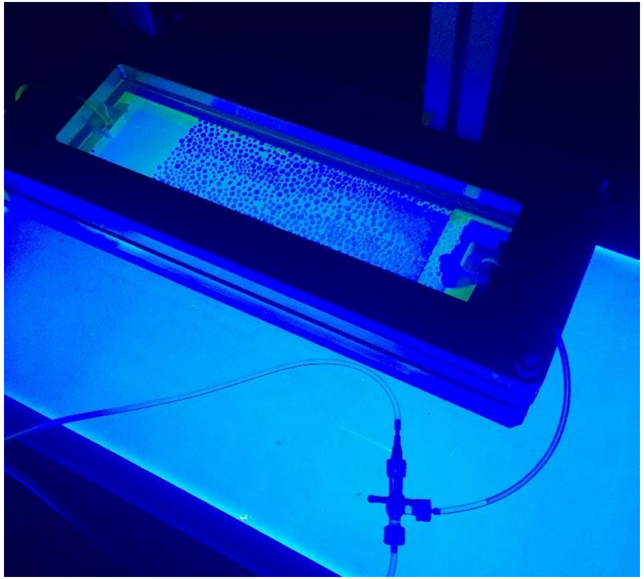


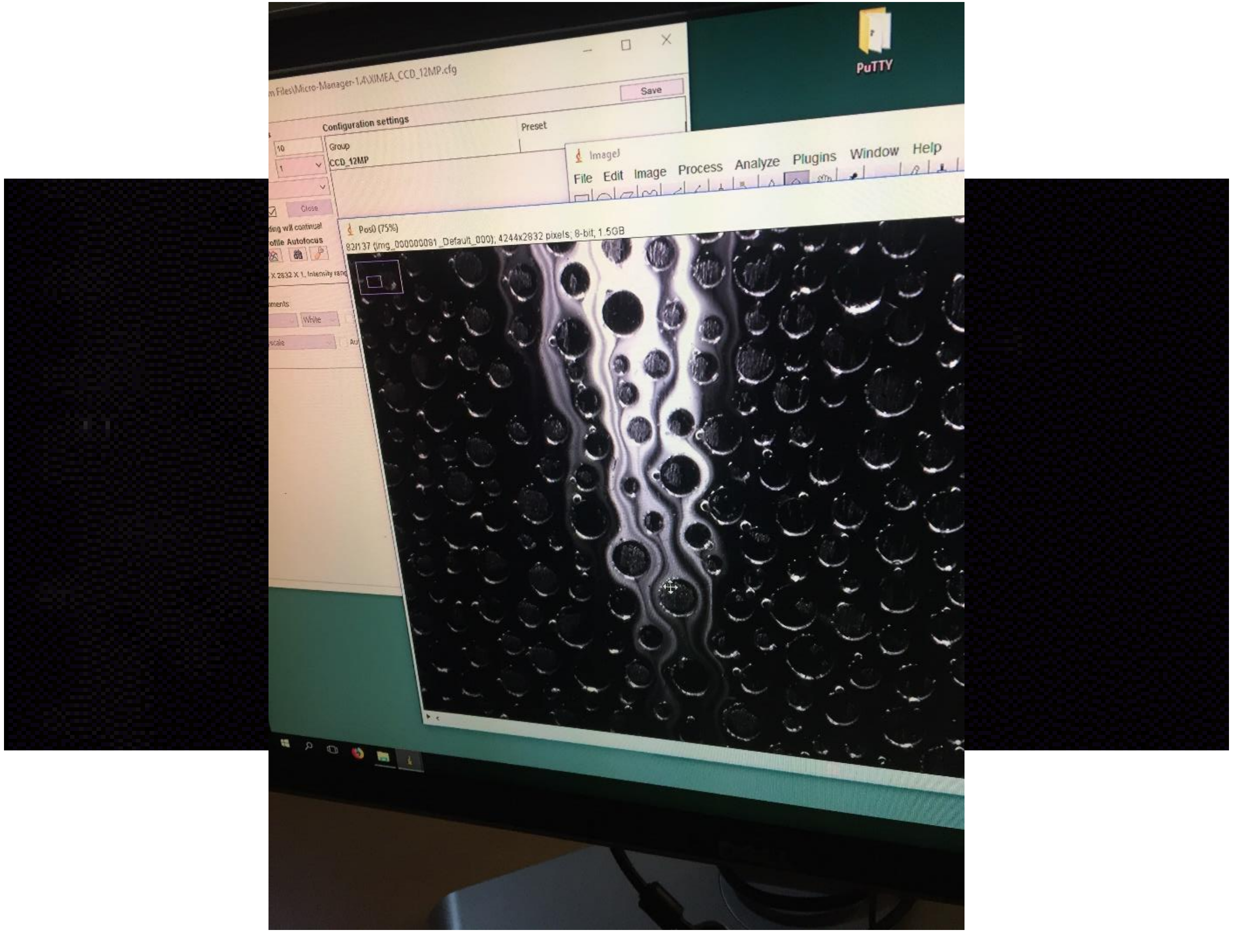
Permeability (cm/s)



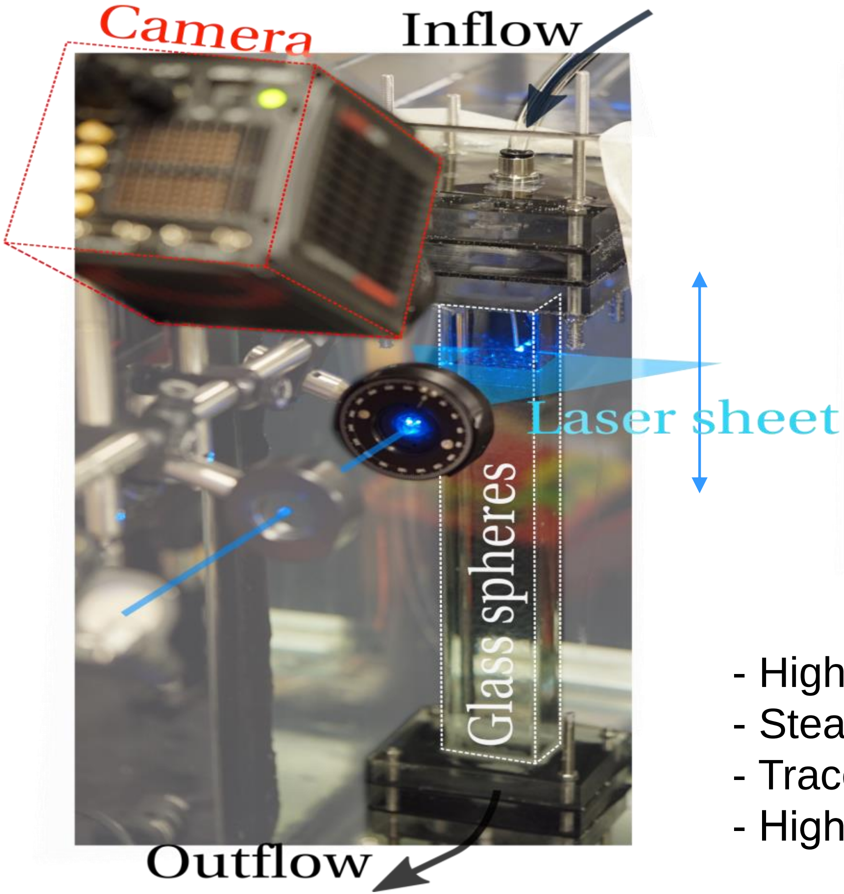
Freeze and Cherry, 1979

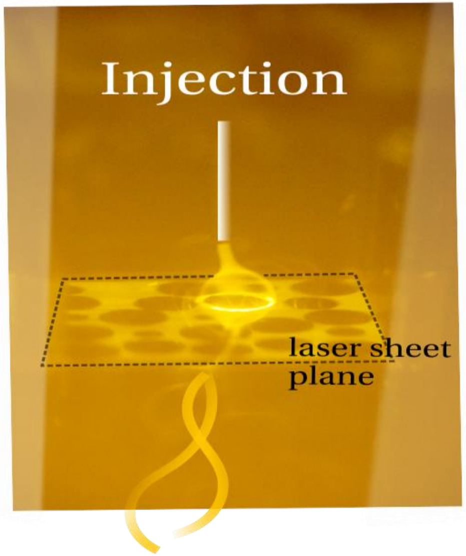
Experimental observations of mixing in porous media



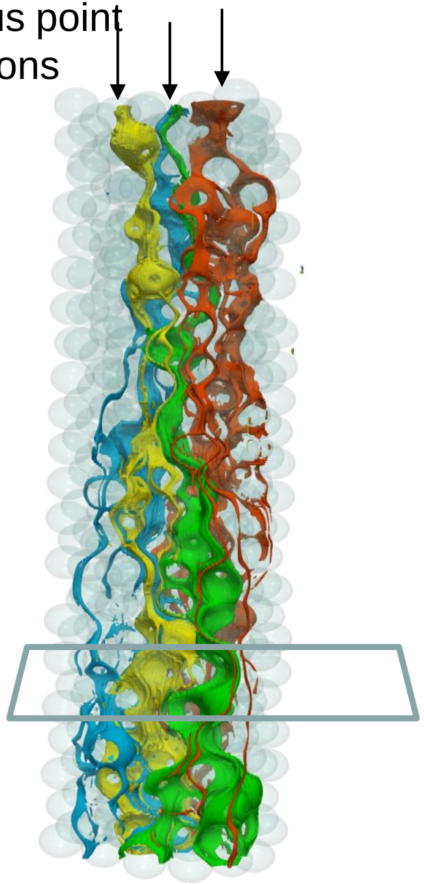


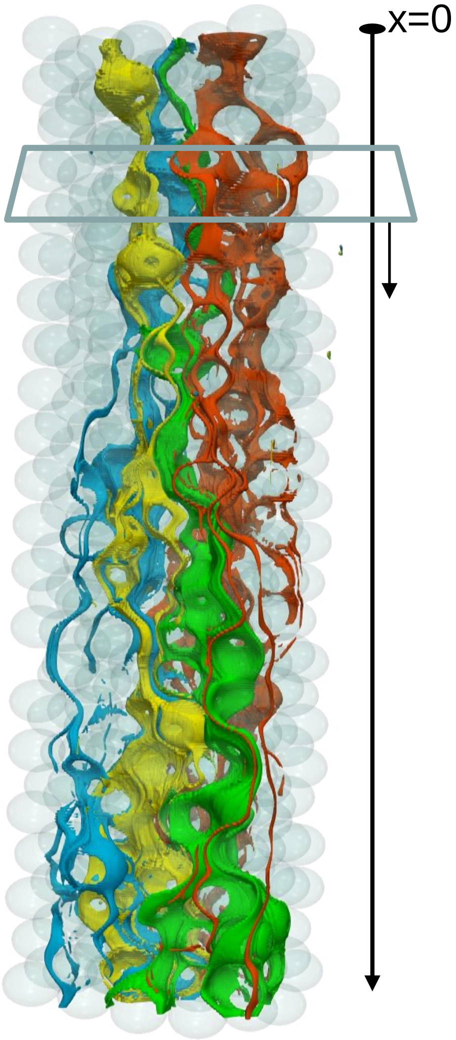
Experimental observations of mixing in porous media

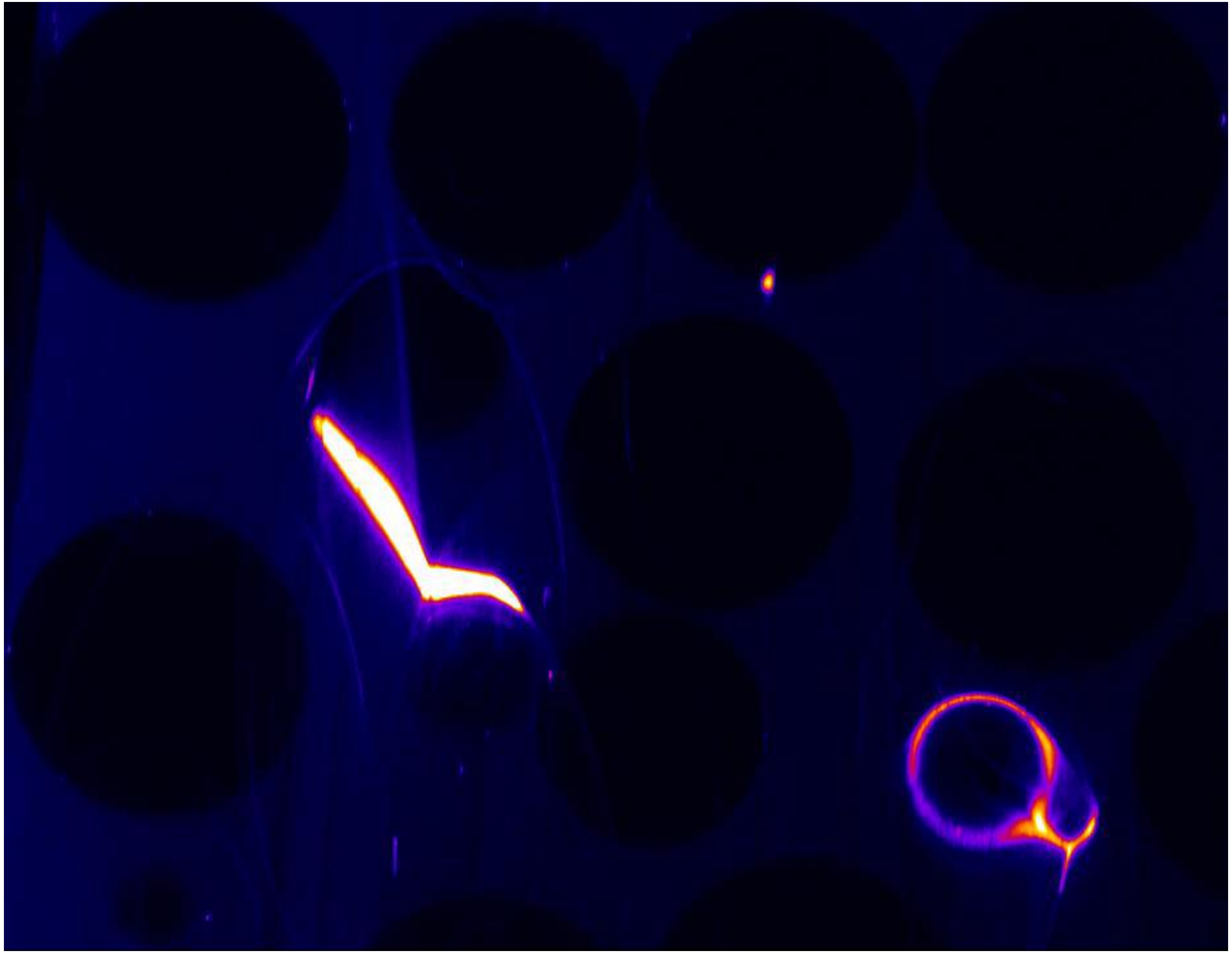




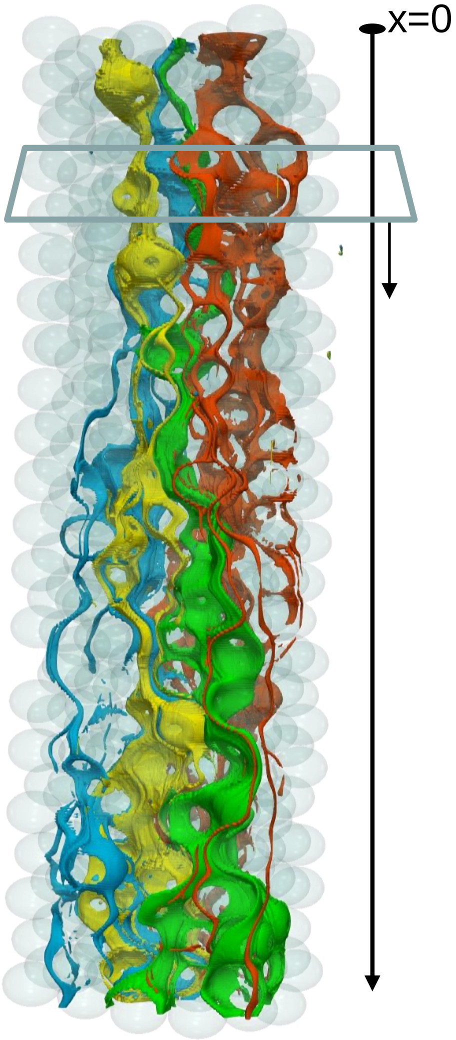
- High Péclet number
- Steady Stokes Flow
- Tracer point source
- High resolution 5cm→ 50 μm

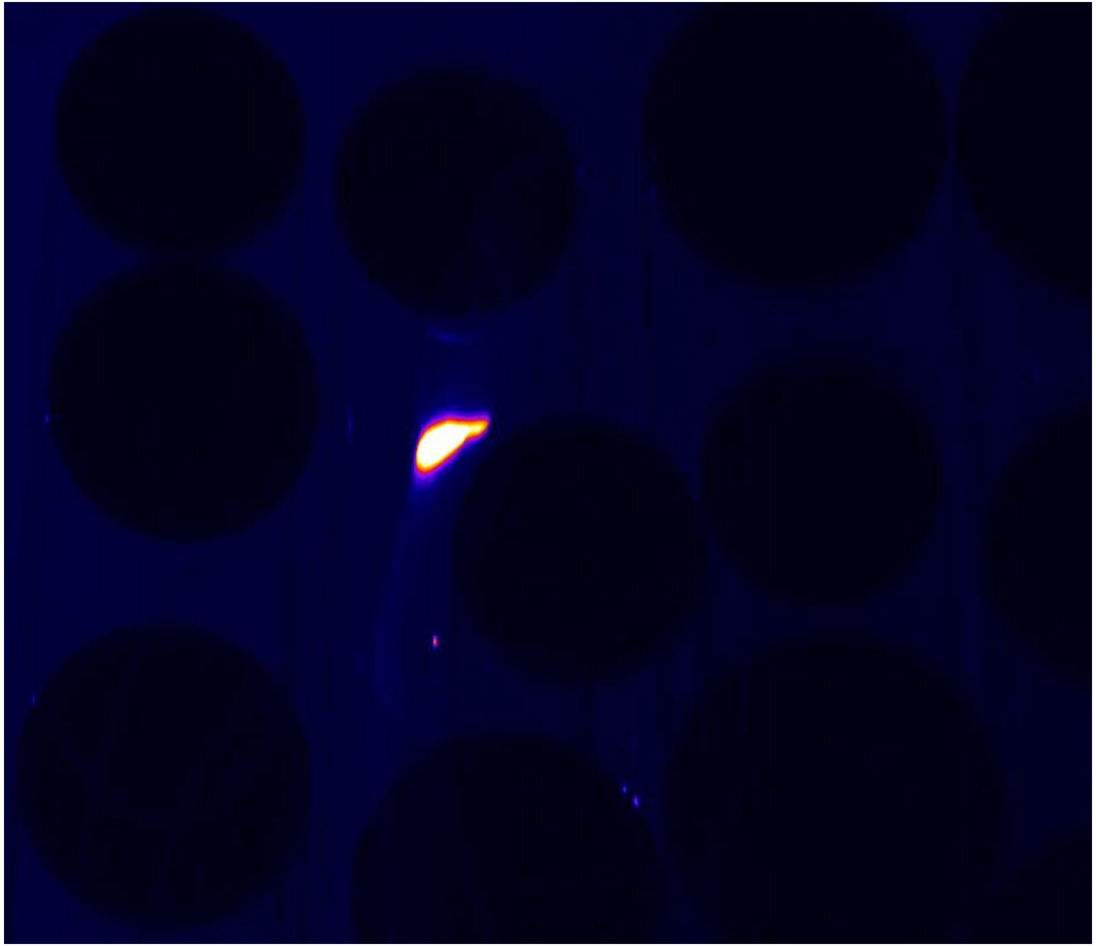






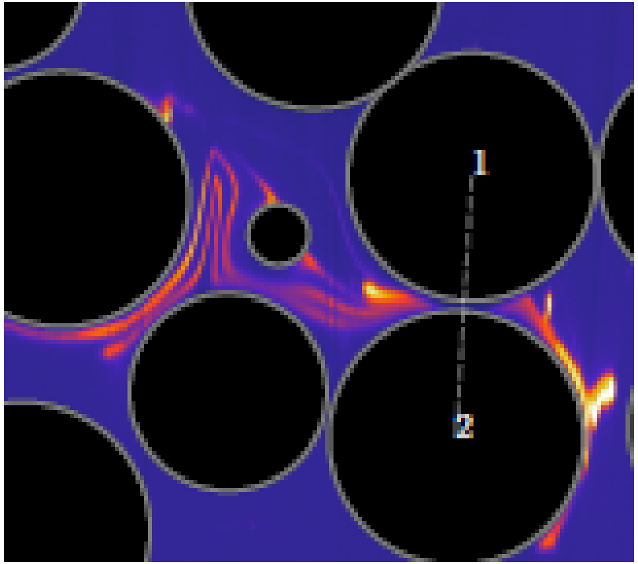
Heyman et al. in prep.



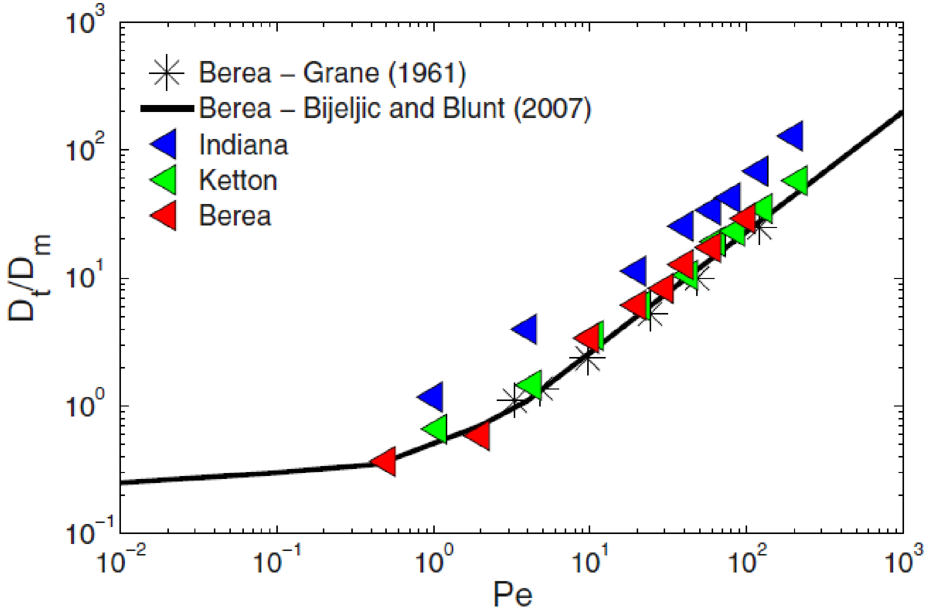


Heyman et al. in prep.

Dispersion

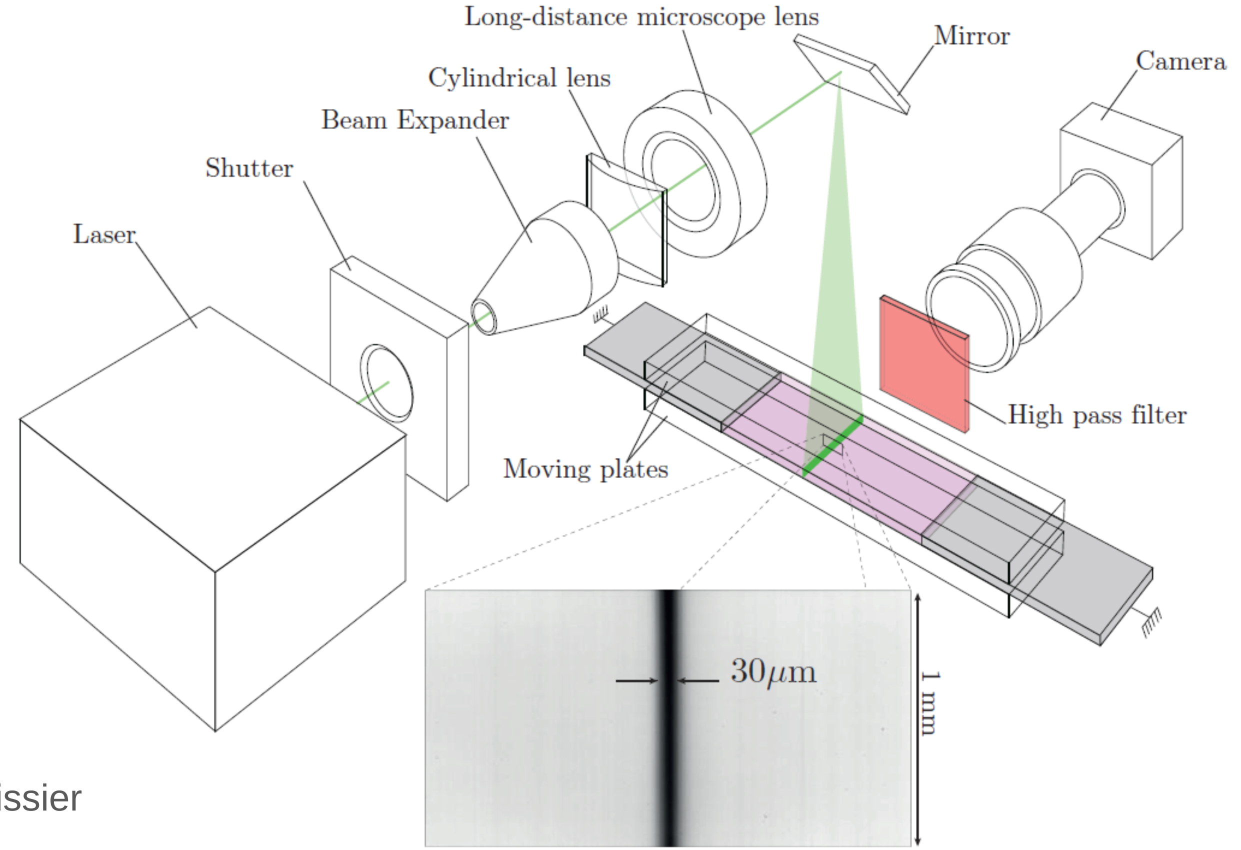


$\sqrt{Dt}$



Boon et al. WRR 2017

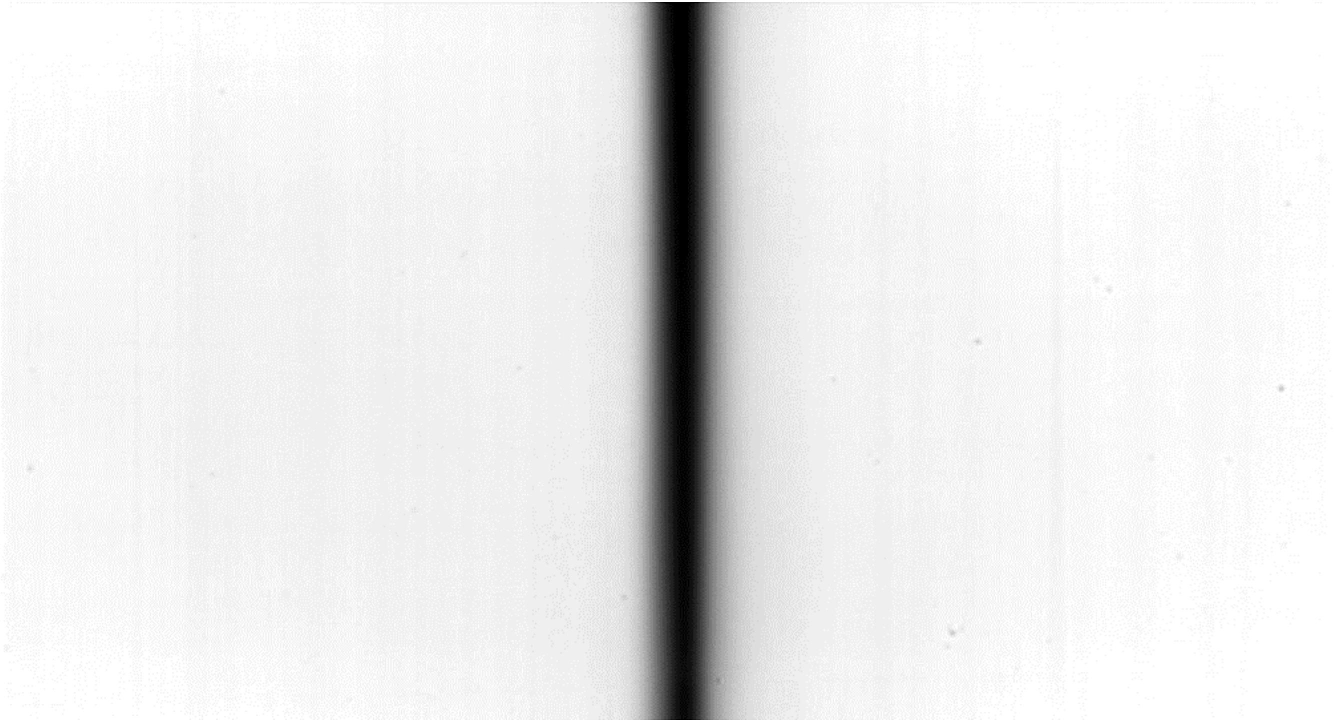
Mixing scale



IUSTI Metzger, Lhuissier

Souzy et al. JFM 2018

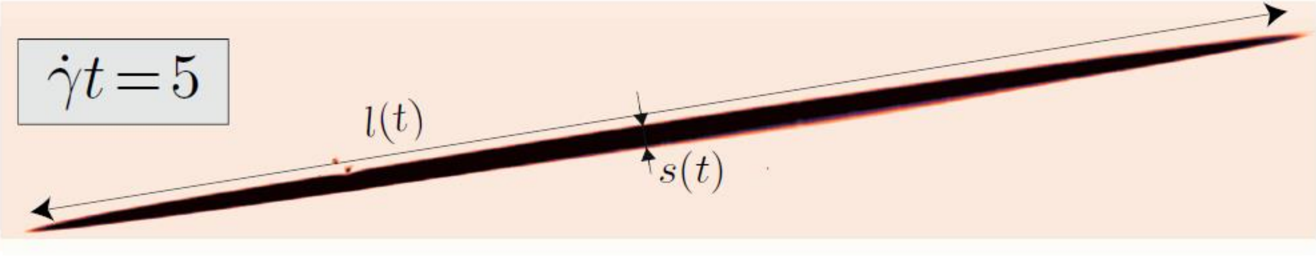
Mixing scale

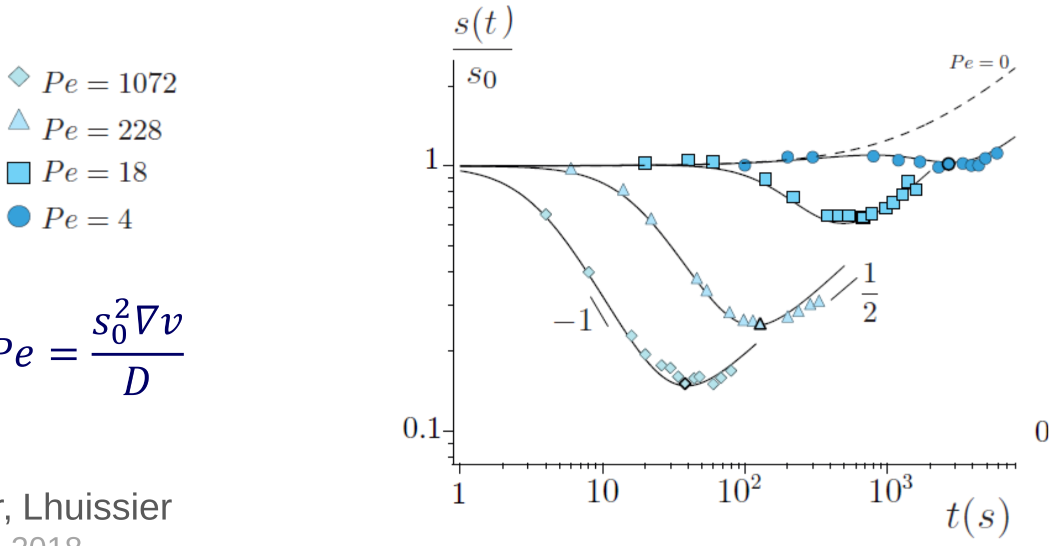


IUSTI Metzger, Lhuissier
Souzy et al. JFM 2018

- $s(t_B) = 15\mu m$
- $1 \text{ pixel} = 1\mu m$

Mixing scale



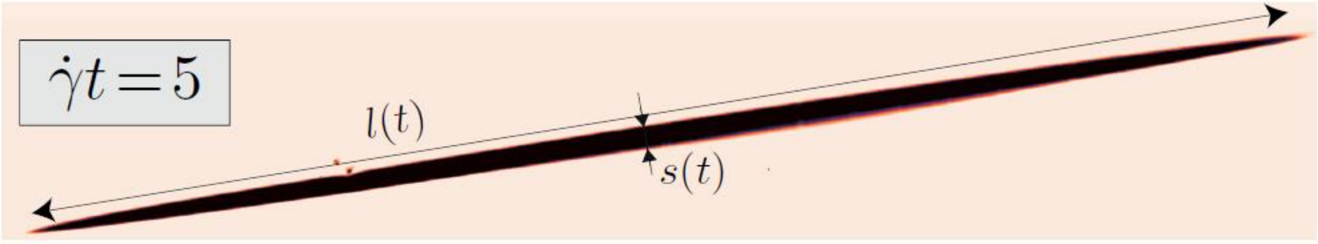


$$Pe = \frac{s_0^2 \nabla v}{D}$$

IUSTI Metzger, Lhuissier

Souzy et al. JFM 2018

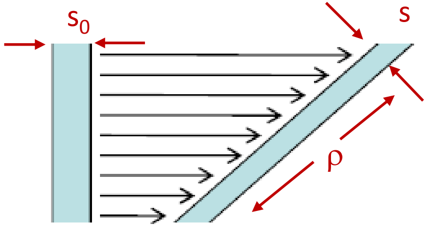
Mixing scale



Elongation

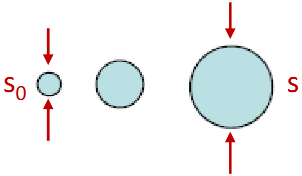
$$\rho = \frac{l}{l_0} = \sqrt{1 + \nabla v^2 t^2}$$

Compression due to elongation

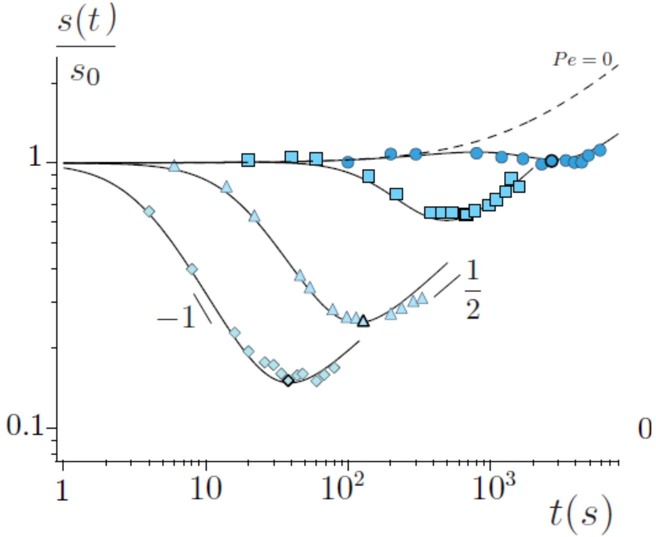


$$s = \frac{s_0}{\rho} \sim \frac{s_0}{\nabla v t}$$

Diffusion

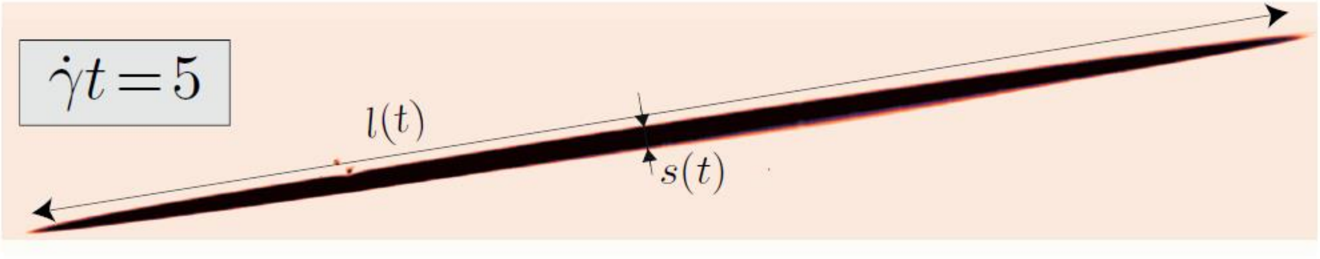


$$s \sim \sqrt{2Dt}$$



Souzy et al. JFM 2018

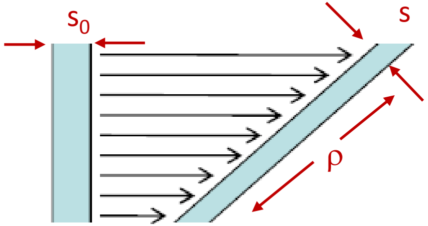
Mixing scale



Elongation

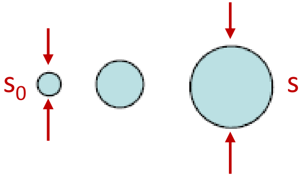
$$\rho = \frac{l}{l_0} = \sqrt{1 + \nabla v^2 t^2}$$

Compression due to elongation



$$s = \frac{s_0}{\rho} \sim \frac{s_0}{\nabla v t}$$

Diffusion



$$s \sim \sqrt{2Dt}$$

$$\frac{s_0}{\nabla v \tau_B} = \sqrt{D\tau_B}$$

$$\tau_B = \nabla v^{-1} \left(\frac{{s_0}^2 \nabla v}{D}\right)^{1/3} = \nabla v^{-1} Pe^{1/3}$$

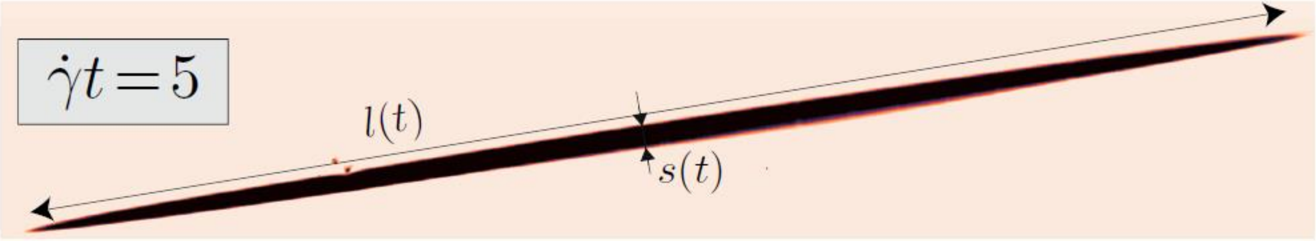
$$\tau_B = \nabla v^{-1} Pe^{1/3}$$

$$Pe = \frac{s_0^2 \nabla v}{D}$$

$$s_B = \frac{s_0}{\nabla v \tau_B} = s_0 Pe^{-1/3}$$

Souzy et al. JFM 2018

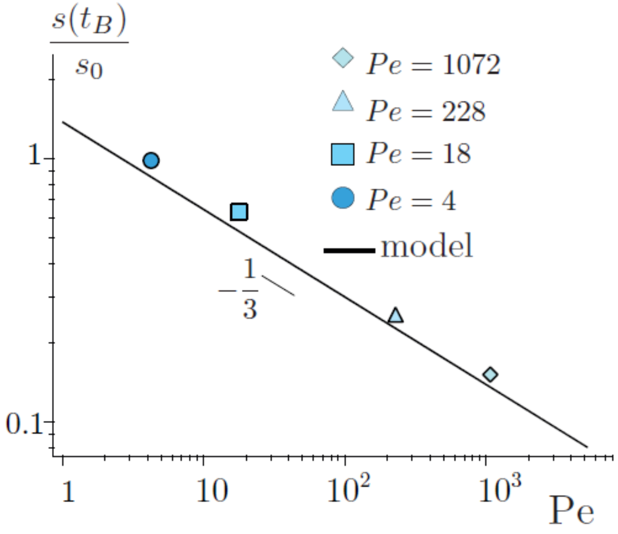
Mixing scale



Elongation

$$\rho = \frac{l}{l_0} = \sqrt{1 + \nabla v^2 t^2}$$

Pe = 1072
Pe = 228
Pe = 18
Pe = 4



$$\frac{s_0}{\nabla v \tau_B} = \sqrt{D \tau_B}$$

$$\tau_B = \nabla v^{-1} \left(\frac{{s_0}^2 \nabla v}{D} \right)^{1/3} = \nabla v^{-1} Pe^{1/3}$$

$$\tau_B = \nabla v^{-1} Pe^{1/3}$$

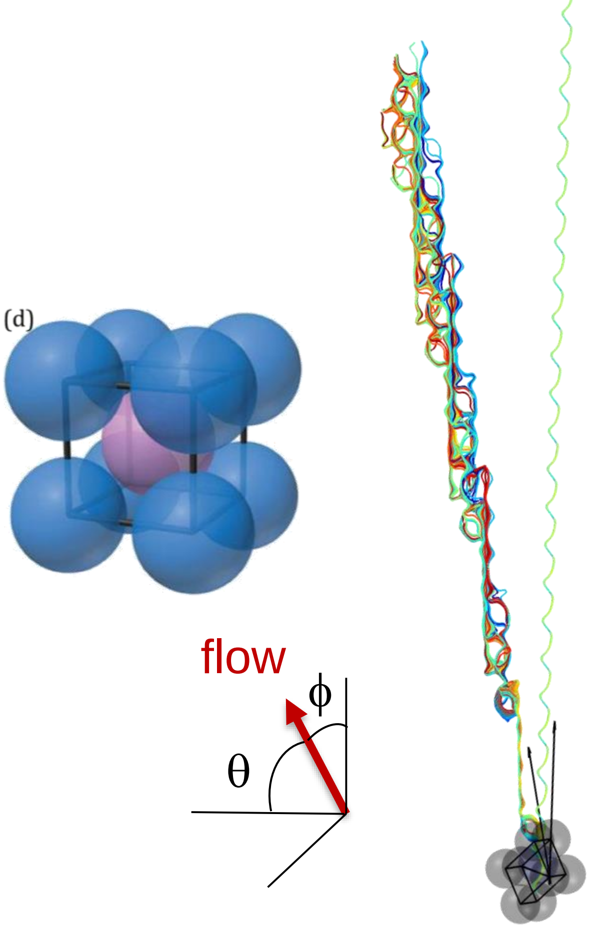
$$Pe = \frac{s_0^2 \nabla v}{D}$$

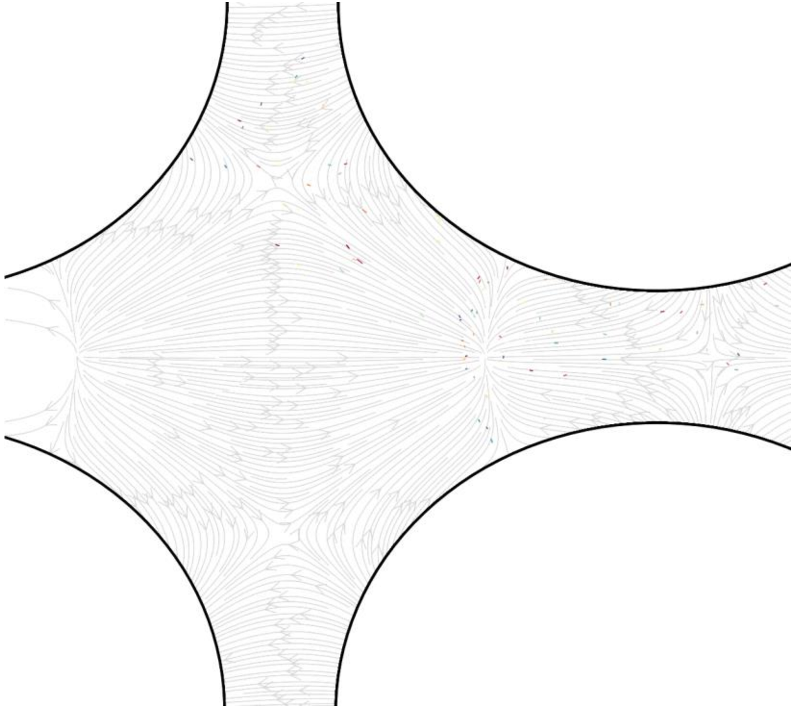
$$Pe = \frac{s_0^2 \nabla v}{D}$$

$$s_B = \frac{s_0}{\nabla v \tau_B} = s_0 Pe^{-1/3}$$

Souzy et al. JFM 2018

Chaotic mixing in granular media

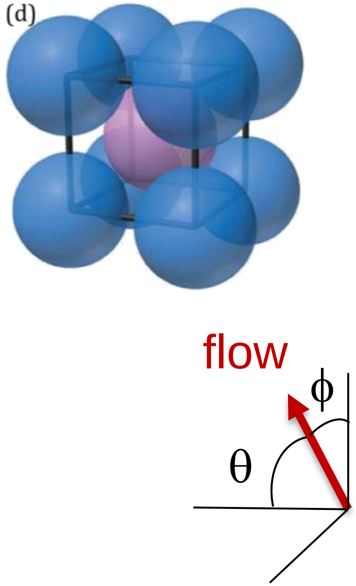


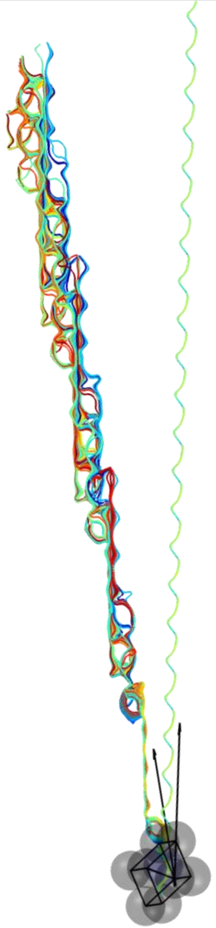


Turuban et al. PRL 2018

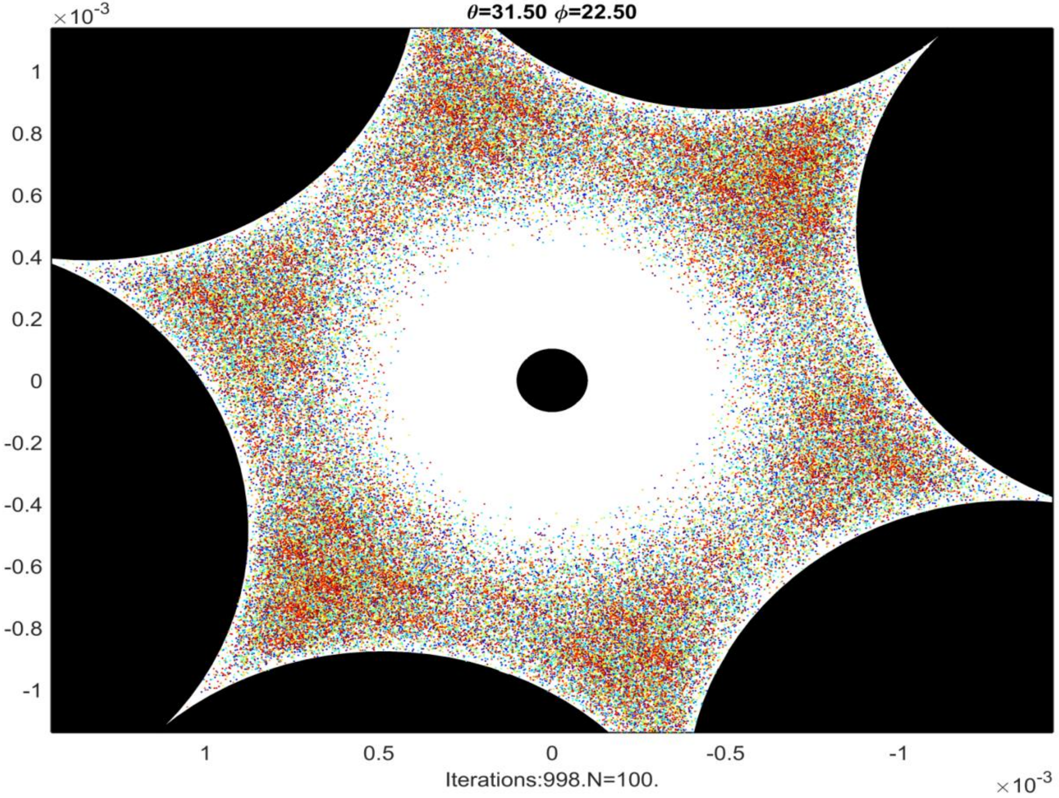
Heyman et al. In prep.

Evidence of chaotic mixing in crystalline porous media





Poincaré sections



Chaotic stirring

A simple estimation: doubling length every grain size

$$\rho(d) = 2\rho(0)$$

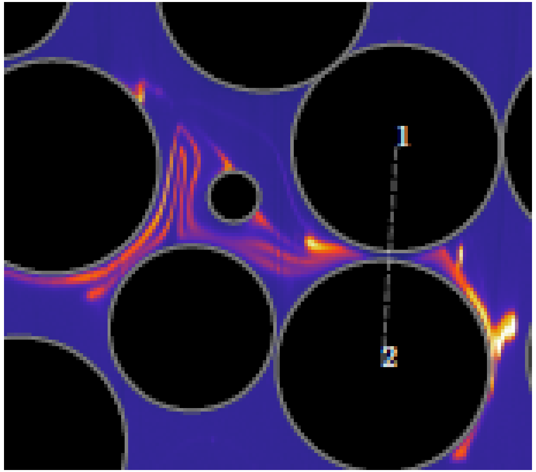
$$\rho(2d) = 4\rho(0)$$

$$\rho(3d) = 8\rho(0)$$

$$\rho(nd) = 2^n\rho(0)$$

$$\rho(nd) = e^{log2\, n}\rho(0) \qquad n = x/d$$

$$\boxed{\rho(x) = e^{\gamma_x x}\rho(0)} \qquad \gamma_x = log2/d$$



Heyman et al. in prep.

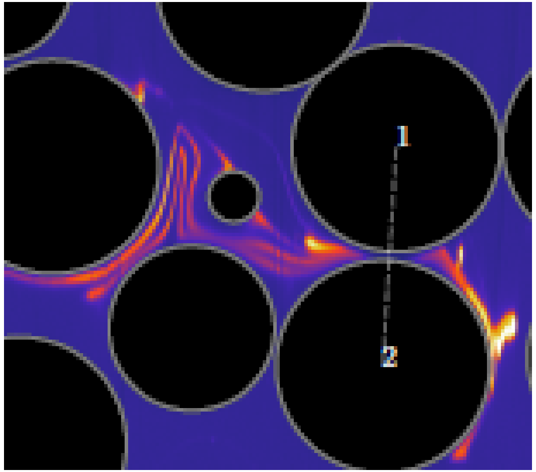
Mixing scale under chaotic stiring

Elongation

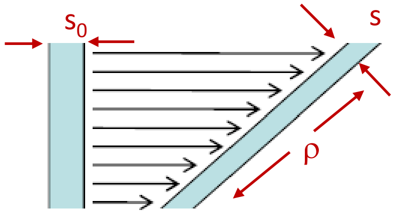
$$\rho = \frac{l}{l_0} = e^{\gamma_t t}$$

$$\gamma_t = \gamma_x v$$

$$\frac{1}{\rho}\frac{d\rho}{dt} = \gamma_t$$



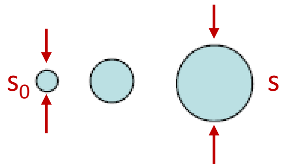
Compression due to elongation



$$\frac{1}{s}\frac{ds}{dt} = -\gamma_t$$

Diffusion

$$s \sim \sqrt{2Dt}$$



$$\frac{1}{s}\frac{ds}{dt} = \frac{D}{s^2}$$

$$\gamma_t = \frac{D}{{s_B}^2}$$

$$s_B = \sqrt{\frac{D}{\gamma_t}}$$

Heyman et al. in prep.

Batchelor scale in turbulent flows

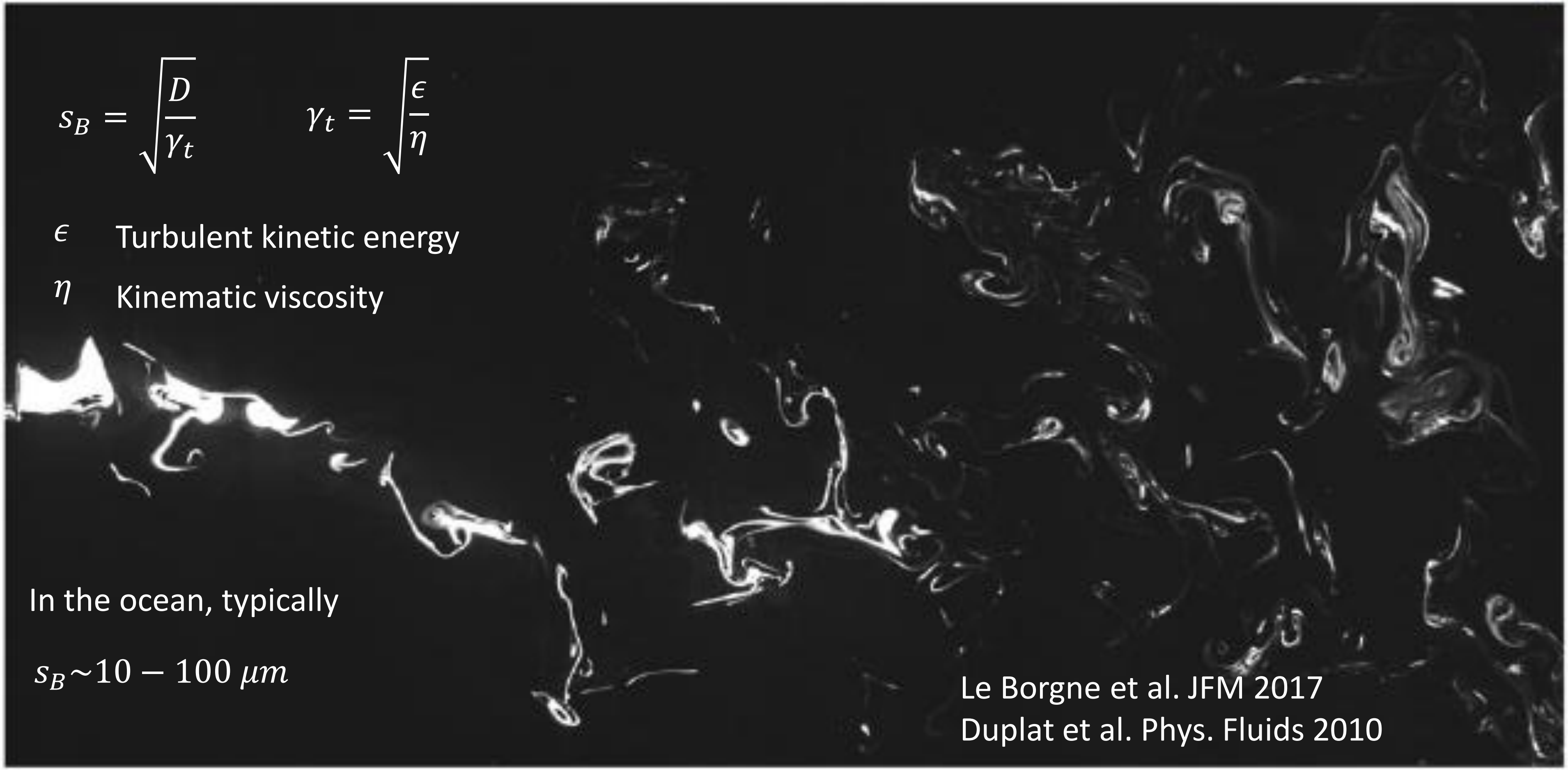
$$s_B = \sqrt{\frac{D}{\gamma_t}} \qquad \gamma_t = \sqrt{\frac{\epsilon}{\eta}}$$

ϵ Turbulent kinetic energy

η Kinematic viscosity

In the ocean, typically

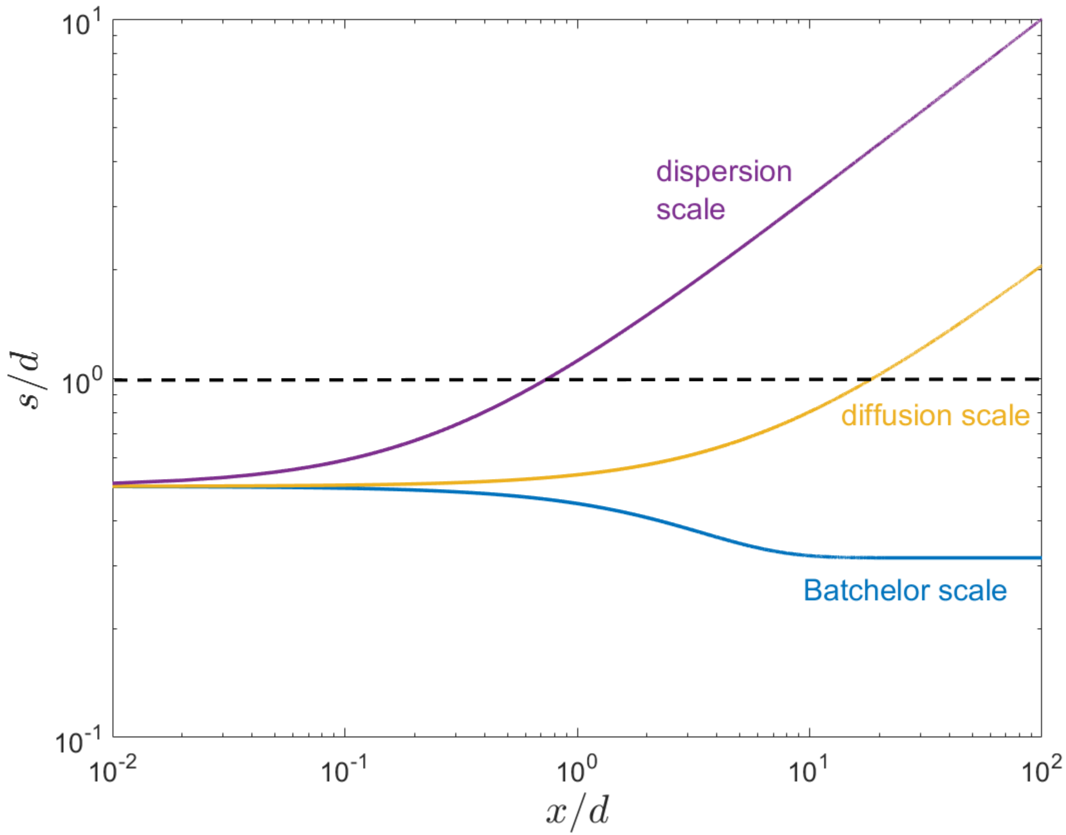
$s_B \sim 10 - 100\ \mu m$

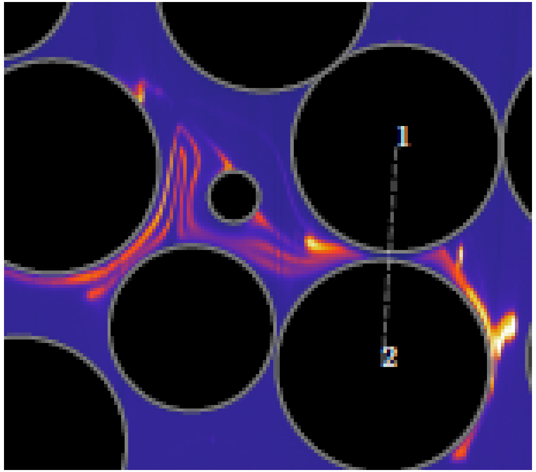


Le Borgne et al. JFM 2017

Duplat et al. Phys. Fluids 2010

Batchelor scale





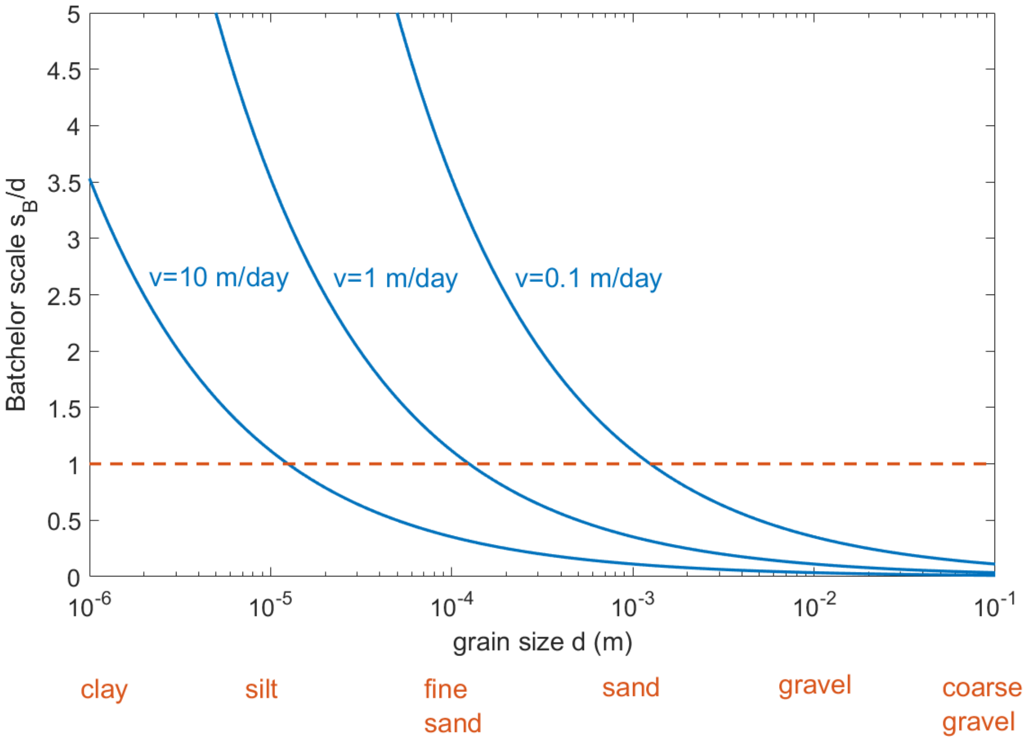
$$s_B = \sqrt{\frac{D}{\gamma_t}} = \sqrt{\frac{D}{\log 2\ v/d}}$$

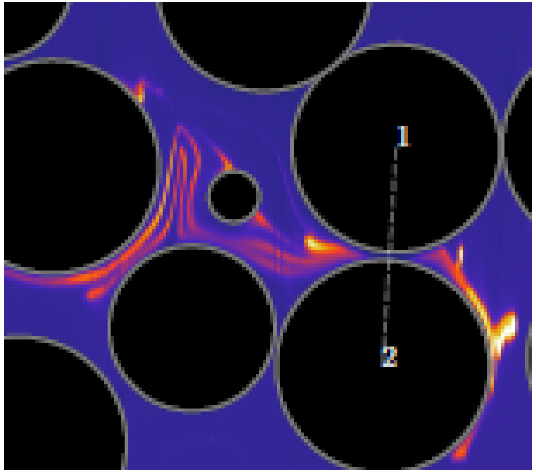
$$\gamma_t = \gamma_x v$$

$$\gamma_x = \log 2\ /d$$

Heyman et al. in prep.

Batchelor scale





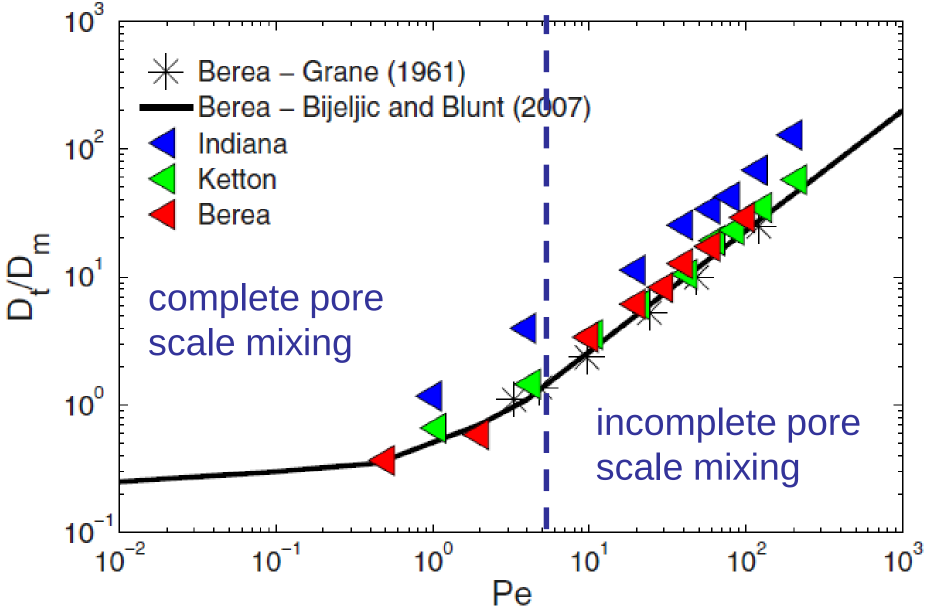
$$s_B = \sqrt{\frac{D}{\gamma_t}} = \sqrt{\frac{D}{\log 2\; v/d}}$$

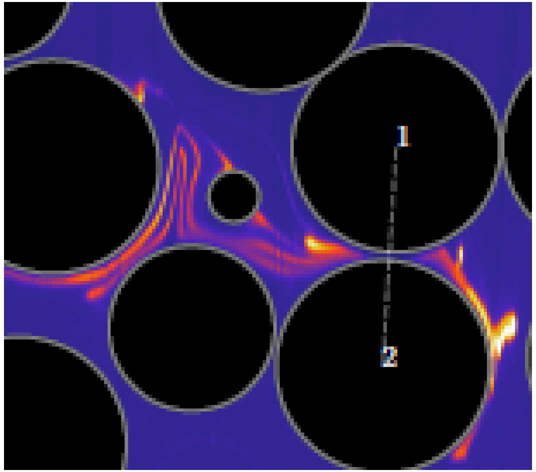
$$\gamma_t = \gamma_x v$$

$$\gamma_x = \log 2\;/d$$

Heyman et al. in prep.

Incomplete mixing





$$s_B = \sqrt{\frac{D}{\gamma_t}} = \sqrt{\frac{D}{\log 2 \; v/d}}$$

$$\gamma_t = \gamma_x v$$

$$\gamma_x = \log 2 \; /d$$

Boon et al. WRR 2017

Heyman et al. in prep.