

Stress and strain in fractured rocks

Assessing effective elastic properties



4th Cargese Summer School: June 25th – July 7th 2018
Flow and Transport in Porous and Fractured Media



UNIVERSITÉ DE
RENNES 1



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Stress and strain in fractured rocks

Introduction

- Why does it matter ?

Characterization and modeling

- Discrete Fracture Network short introduction
- Assumptions
- Isolated Fracture
- DFN scale / rock mass scale
- Interactions and stress fluctuations
- Application example

Fractures weaken rock elastic and strength properties

... How and to which extent ?

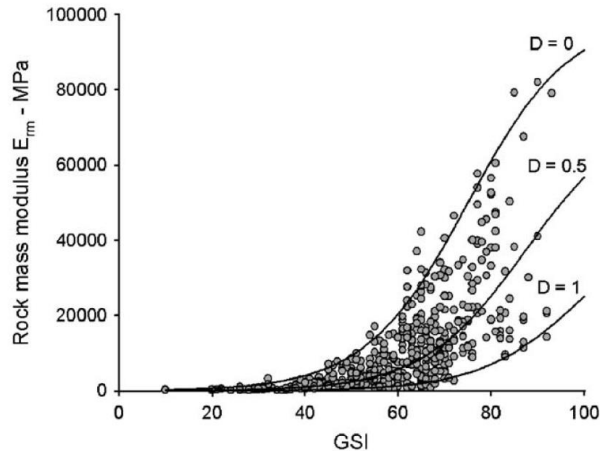


Fig. 9. Plot of Simplified Hoek and Diederichs equation for Chinese and Taiwanese data.

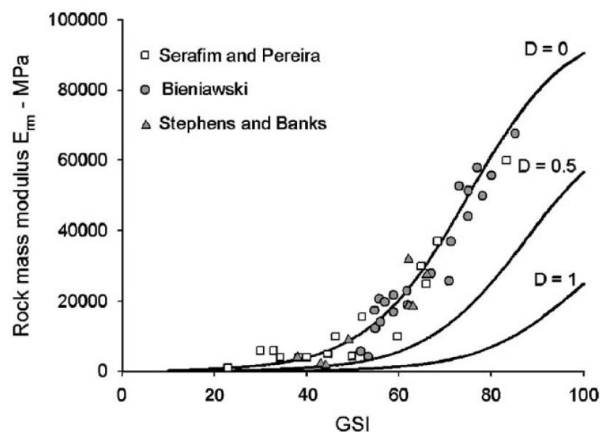
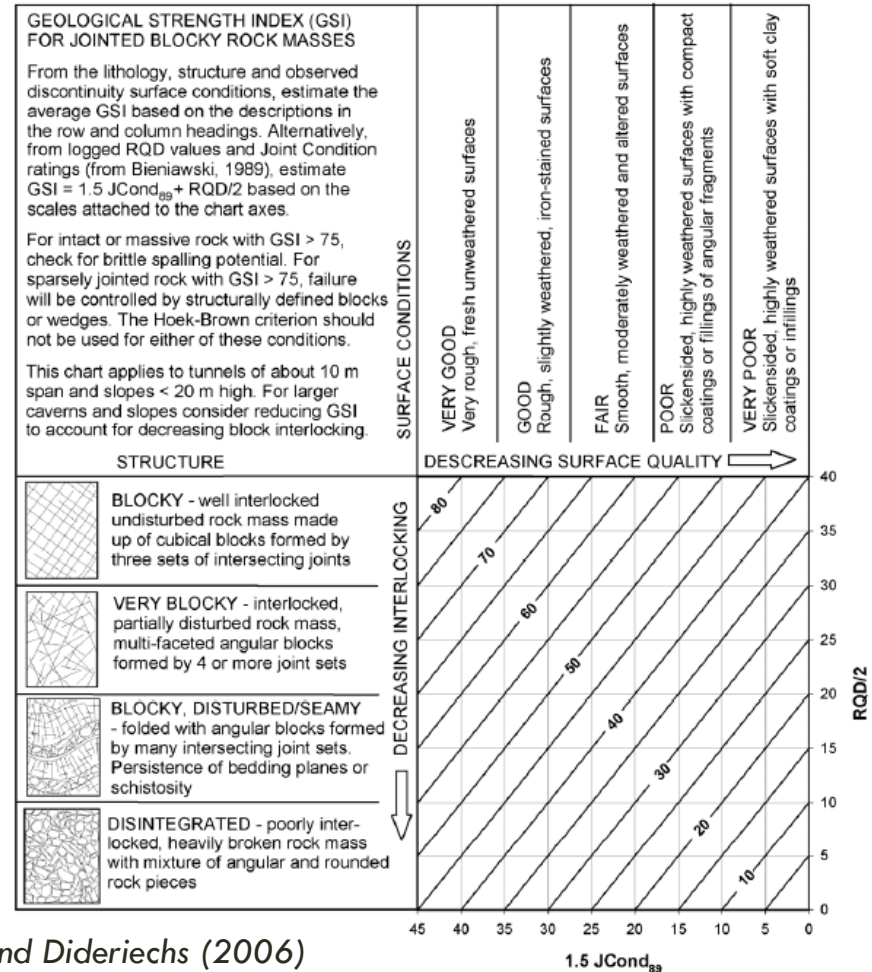


Fig. 10. Plot of in situ rock mass deformation modulus data from Serafim and Pereira [4], Bieniawski [5] and Stephens and Banks [6] against Simplified Hoek and Diederichs equation (2).



Hoek and Dideriechs (2006)

Fractures disrupt wave propagation

... How to define appropriate effective elastic properties to predict wave propagation ?

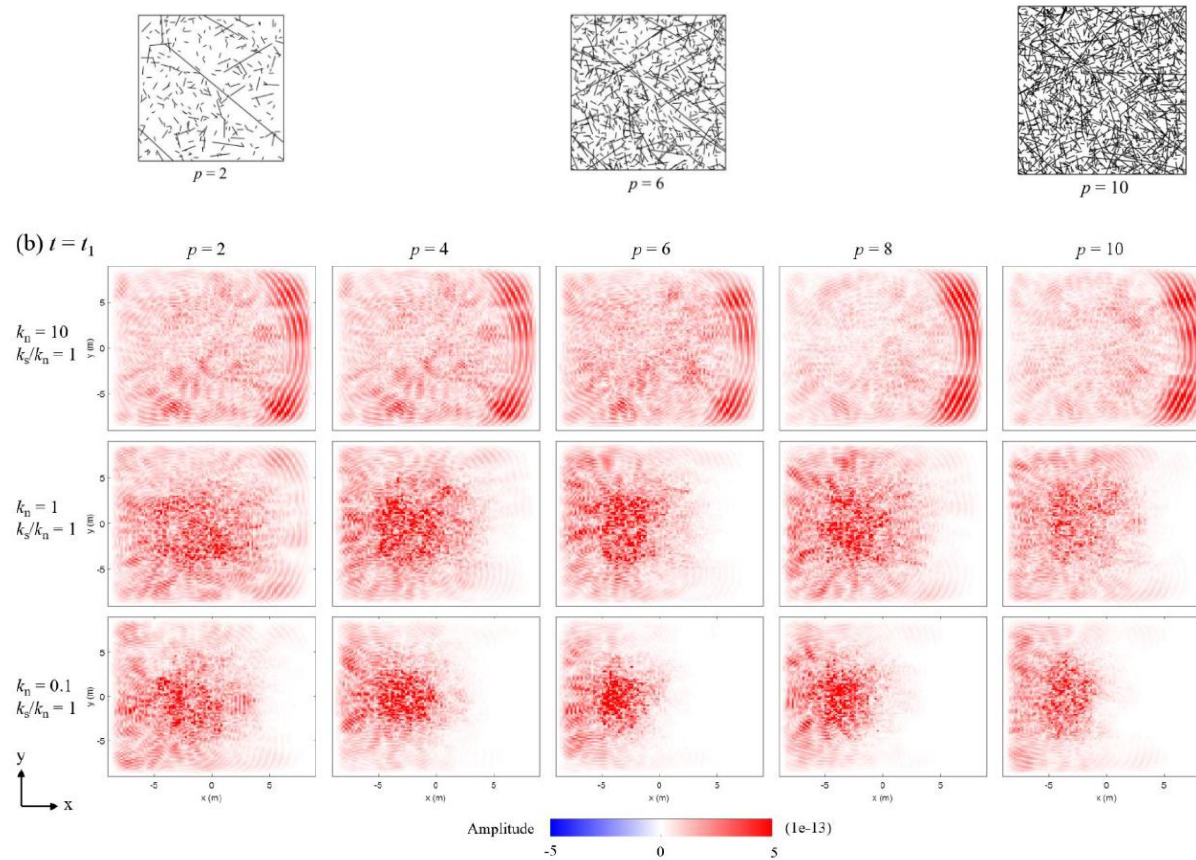
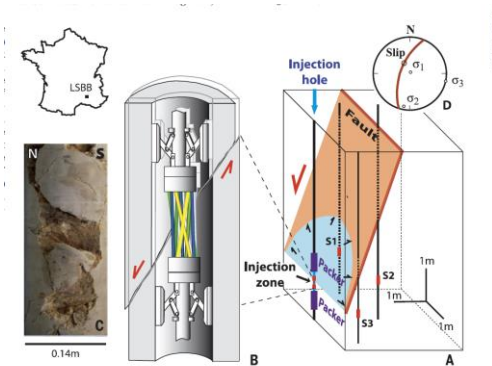


Fig. 5. Wavefield of two different stages (a) $t_0 = 0.0015$ sec and (b) $t_1 = 0.0037$ sec in the fractured media associated with different percolation parameters p and different normal and shear stiffnesses, k_n and k_s .

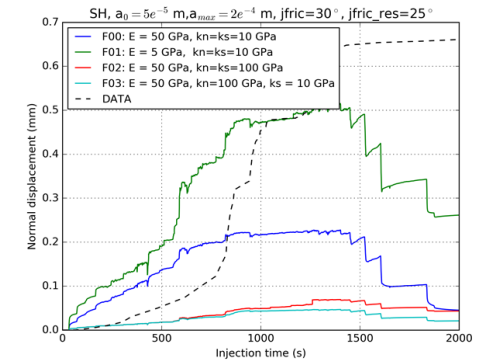
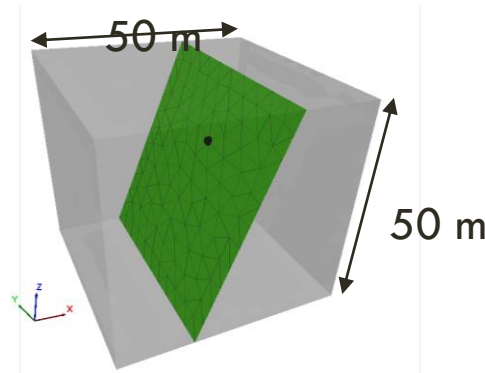
Shi and Lei (2018)

Water injection in a fault - Hydro-mechanical coupling

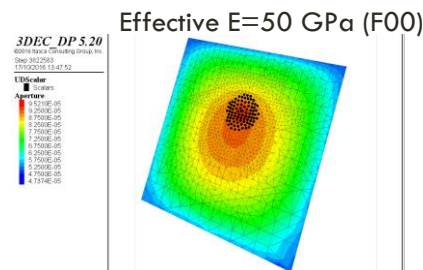
... Equivalent elastic properties for HM modeling



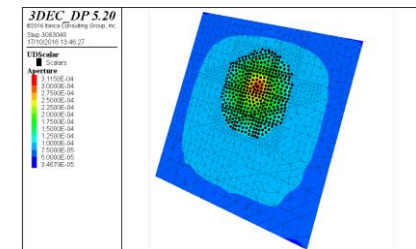
Guglielmi et al (2015)



- Water injection, induced seismicity
- Knowing the surrounding rock mass mechanical properties is critical for modeling the fault behavior and HM process

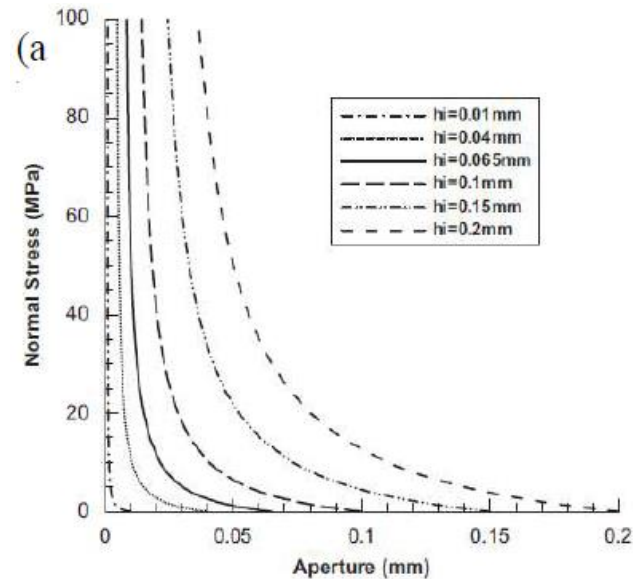
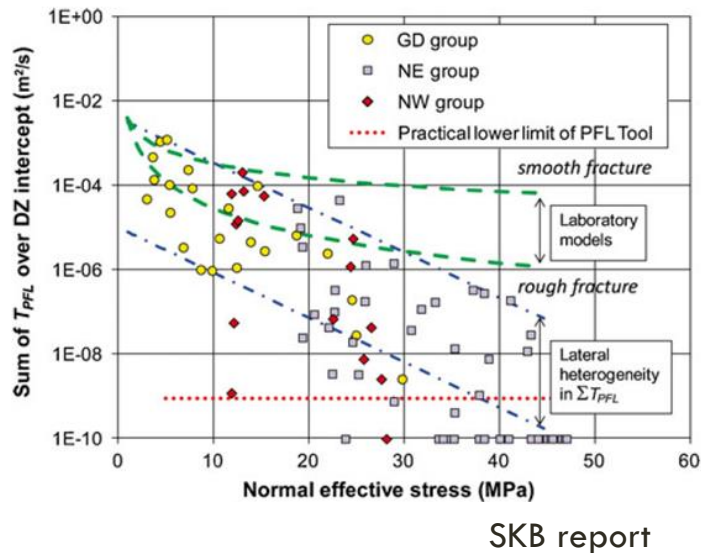


Effective E=5 GPa (F01)



In-situ stress partly control fracture transmissivity

- ... Fracture transmissivity vs fracture hydraulic and mechanical aperture
- ... Predicting normal stress to predict transmissivity



Baghbanan and Jing (2008)

- A decrease of transmissivity with depth, in correlation with an increase of effective normal stress
- High variability
- Non linear decrease of aperture when increasing normal stress

WHAT ARE WE LOOKING AT

How rock mechanical strength and **elastic properties** are weakened by fractures

How Stress and Deformation fields are heterogeneous due the fractures

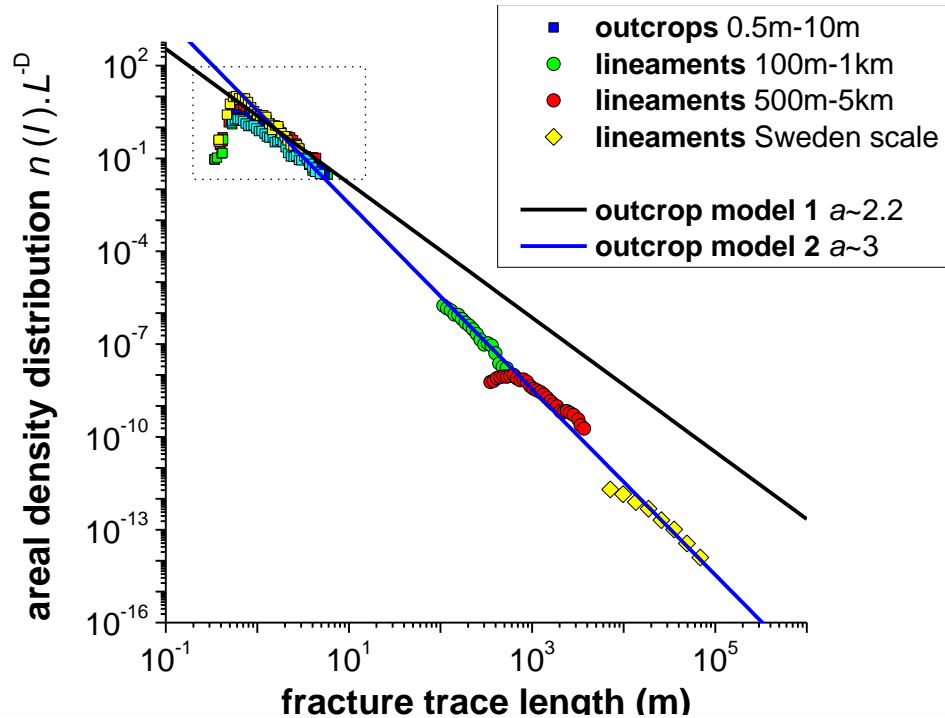
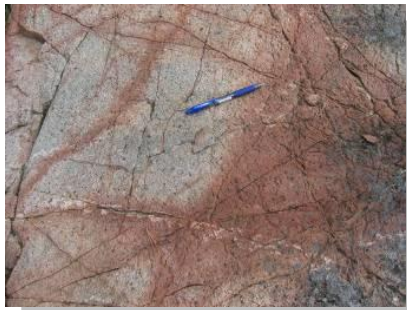
Numerous applications (geotechnical, investigations of water injection induced seismicity, damage etc)

Characterization and modeling

- Discrete Fracture Network short introduction
- Assumptions
- Isolated Fracture
- DFN scale / rock mass scale
- Interactions and stress fluctuations
- Application example



DFN basis: How many small and large fractures?



Multiscale model: the fracture density distribution is a power law

$$n(l_f) = \alpha \cdot l_f^{-a}$$
$$3 < a \leq 4$$

Macroscopic Density descriptors

- $P_{30} (m^{-3})$ Number of fractures by unit of volume
- $P_{21} (m^{-1})$ Total length of fracture trace by unit of surface
- $P_{32} (m^{-1})$ Total surface of fracture by unit of volume
- $P_{10} (m^{-1})$ Number of fracture intercepts by unit of length
- $P (-)$ Percolation parameter: sum of each fracture surrounding volume by unit of volume

Contrary to $(\alpha; a)$ these indicators are dependent on the observation range (ie min and max sizes)

Correspondences are built via stereological analyses

Density P_{32}

Total surface of fracture by unit of volume (m^2/m^3)

$$P_{32}(l_1; l_2) = \int_{l_1}^{l_2} n(l) \cdot \frac{\pi}{4} l^2 \cdot dl$$

$$P_{32}(l_0; l_{max}) = \frac{\pi}{4} \alpha \frac{(l_0^{-a+3} - l_{max}^{-a+3})}{a-3}$$

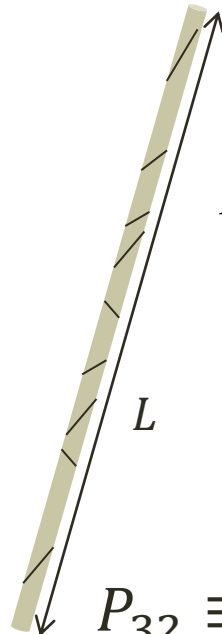
$$P_{32}[l \geq l_0] = \frac{\pi}{4} \alpha l_0^{-a+3}$$

The density of a dataset is dependent on the range of sizes sampled in the dataset

In case of usual power-law model ($a \geq 3$), the smallest fractures contribute more to DFN density (max boundary can be neglected)

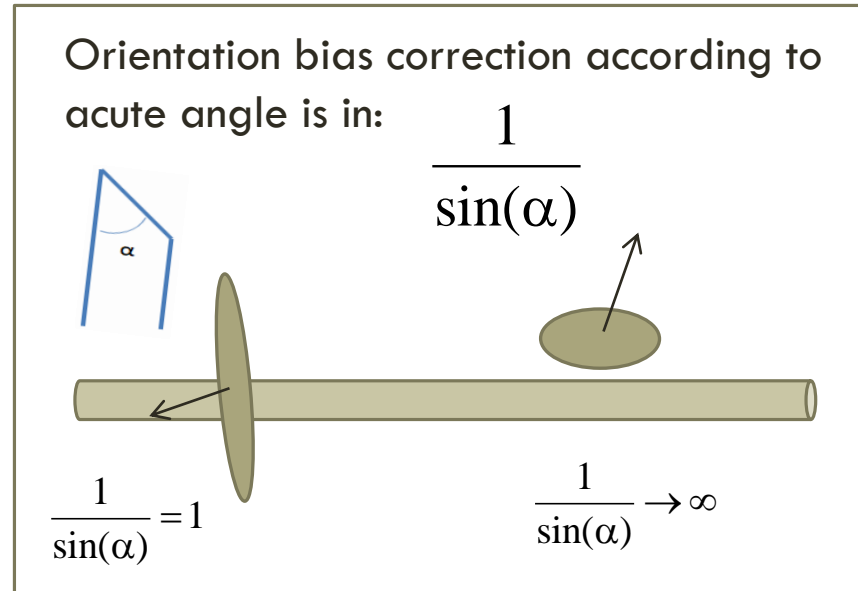
At the SKB Swedish sites, P_{32} estimated from core data is on average between 4 and 5 m^2/m^3

P_{32} RELATION TO CORE LOGGING P_{10} AND SPACING



$$P_{10} = \frac{1}{\langle spacing \rangle}$$

$$P_{32} \equiv \sum_f \frac{1}{\sin(\alpha_f)} \frac{1}{L}$$

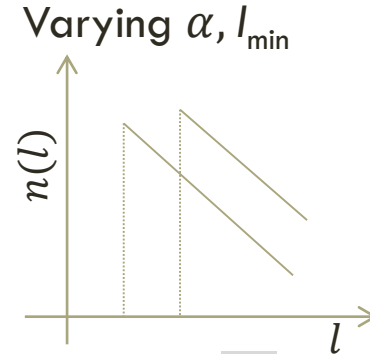


For uniformly distributed pole orientations $P_{32} = 2 \times P_{10}$

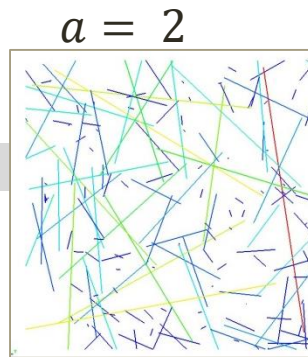
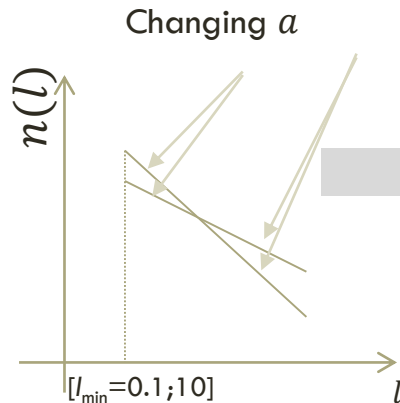
If all fractures are \perp to the borehole $P_{32} = P_{10}$

Examples: models at constant P_{32}

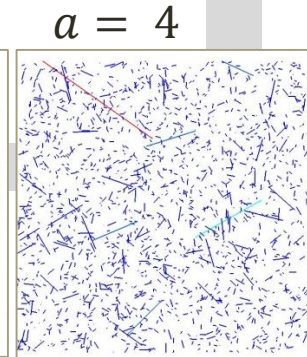
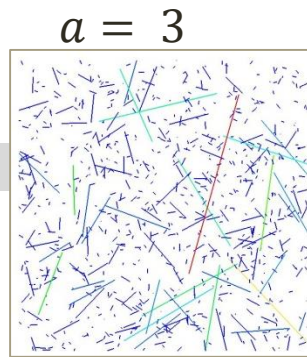
Several DFN models uniformly distributed orientations with $P_{32} = 4$ (and $P_{10} = 2$)



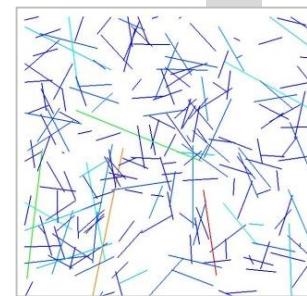
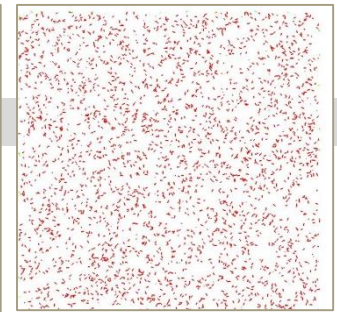
?



2D cuts - side 10m



$l_{\min} = 0.1$



The same borehole fracture frequency can be related to any of these scaling models $n(l)$

Connectivity and percolation parameter

The percolation parameter P controls statistically the DFN connectivity and the size of the largest connected cluster [Bour and Davy, 1998; Dreuzy et al., 2000]:

$$P = \frac{\pi^2}{8} \cdot \int n(l) \cdot l^3 \cdot dl$$

Percolation threshold p_c (~ 2.5): percolation value at which the DFN is connected towards system boundaries by a critical cluster of fractures

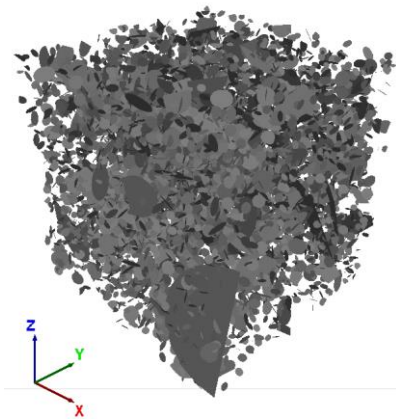
Model with

$a = 4$

$l_{min} = 1$

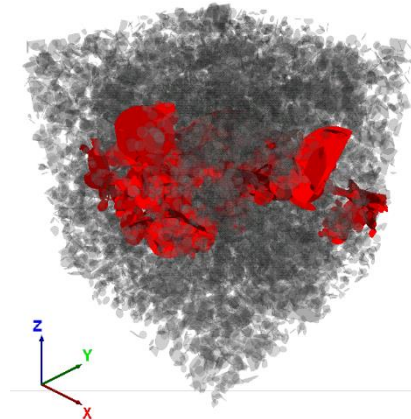
$L = 20$

$p \in [1.5; 2.5; 5]$



$p \ll p_c$

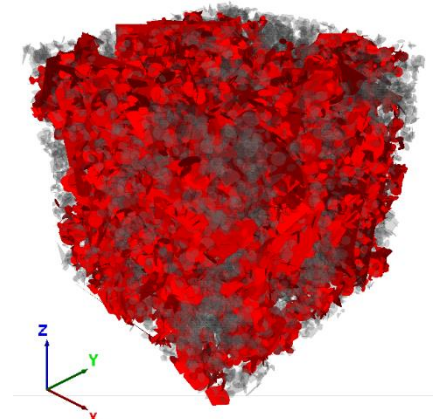
No flow



$p = p_c$

critical flow

With "red" links



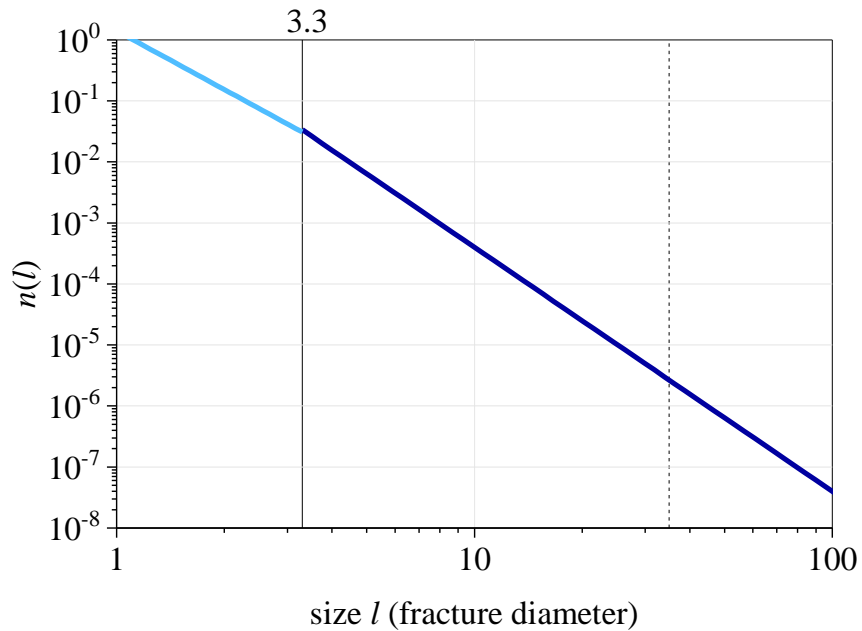
$p \gg p_c$

Many flow paths, flow

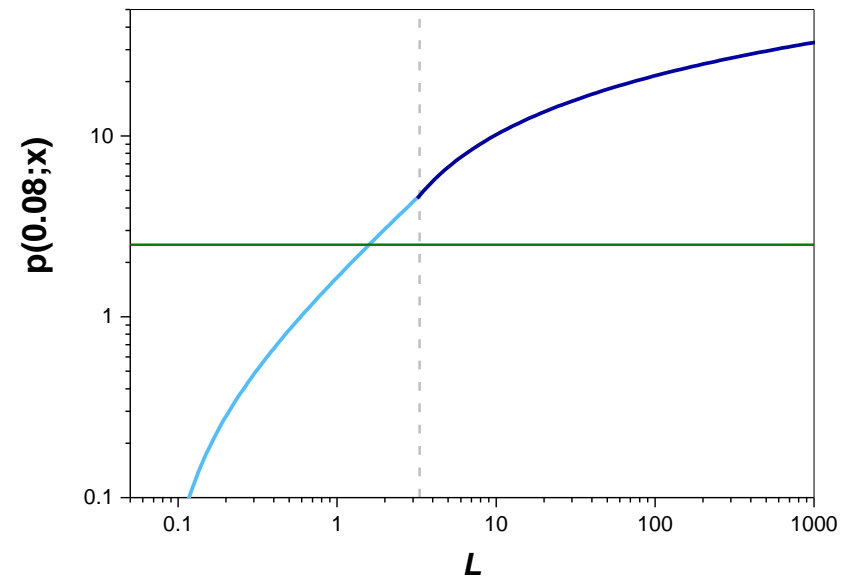
dominated by density effects

Evolution of $p_\theta(L)$

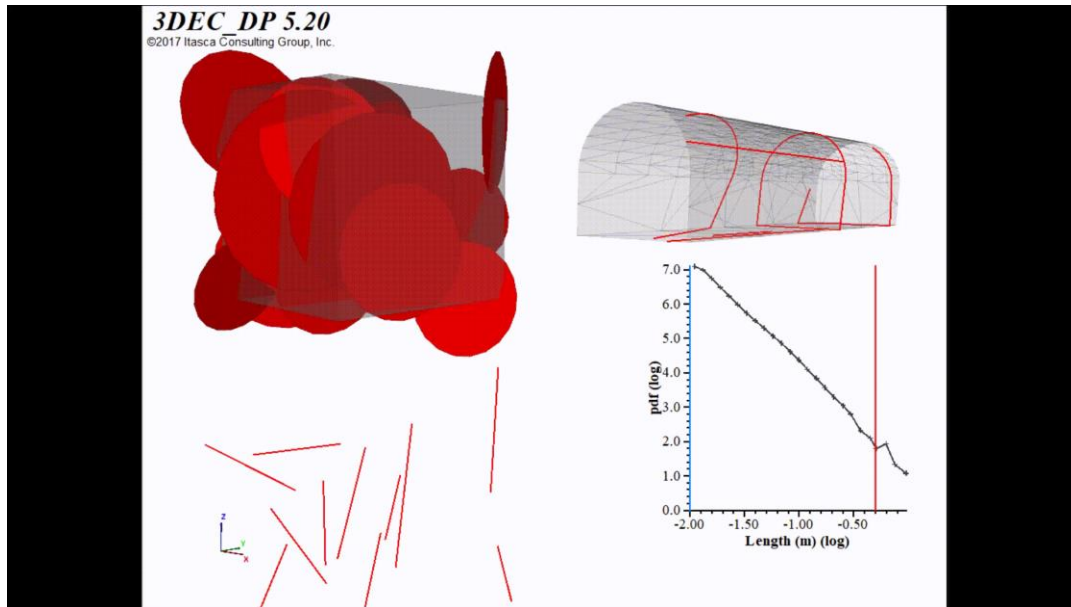
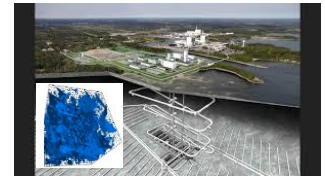
$n(l)$



$p(l)$



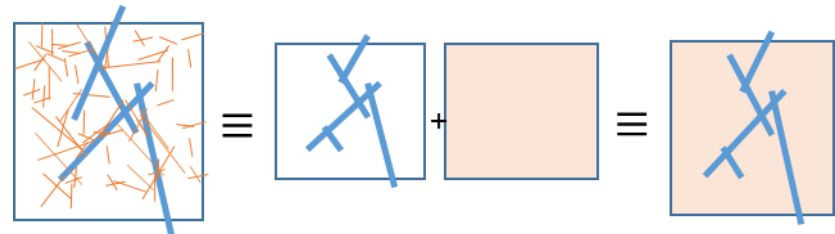
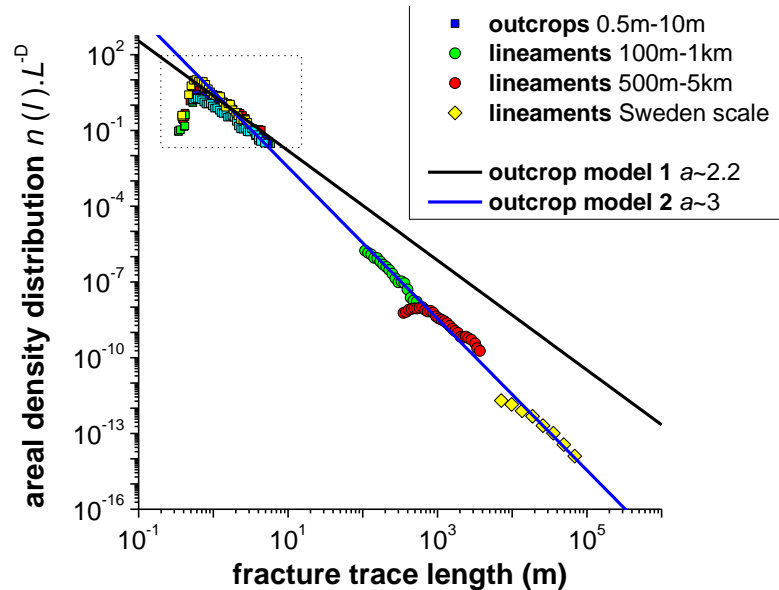
Naturally fractured systems in hard rocks



- Multiscale DFN
- No a priori REV
- P_{32} dominated by small fractures
- Percolation more dominated by large fractures
- Mechanical properties may be dependant on the fracture sizes

Evaluating the extent of potential scale effect is critical

Effective properties and critical fractures



Which fractures are critical (to an application) and which part of the DFN can be simplified

Required for many applications and subsequent numerical models, including rock mass scale equivalent elastic properties assessment

Characterization and modeling

- Discrete Fracture Network short introduction
- Assumptions
- Isolated Fracture
- DFN scale
- Interactions and stress fluctuations
- Analytical solution for effective properties
- Applications

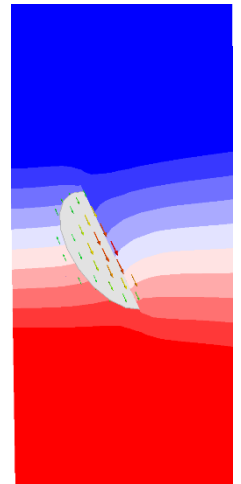
ASSUMPTIONS

Define rock mass scale effective elastic properties with :

- DFN: any set of disc-shaped planar fractures (multi-oriented, multi-scale)
- Elastic conditions (no rock damage)
- Rock matrix: isotropic elastic material, Young's modulus E_m [Gpa] and Poisson ratio ν_m [-]
- Fracture mechanical model based on slip Coulomb model with cohesion (c), friction (friction angle φ), normal stiffness (k_n) and shear stiffness (k_s)
- Emphasis on shear displacement (normal displacement is of second order)
- No in-plane heterogeneity

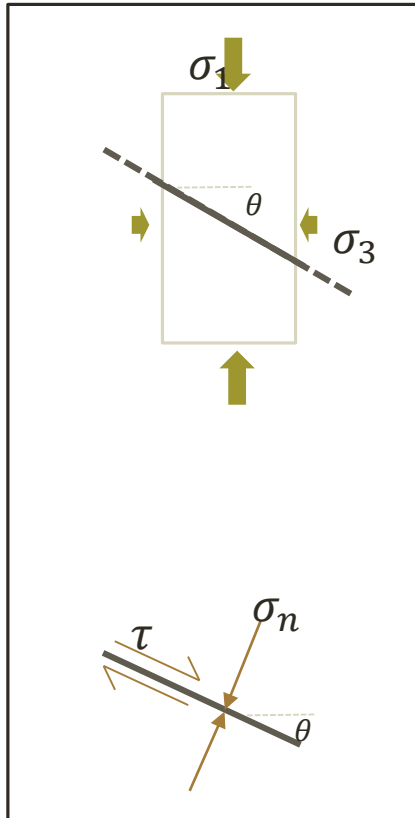
Characterization and modeling

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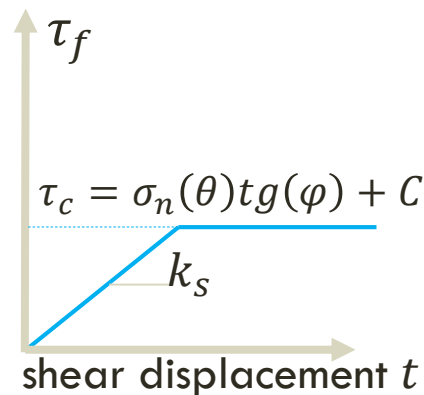
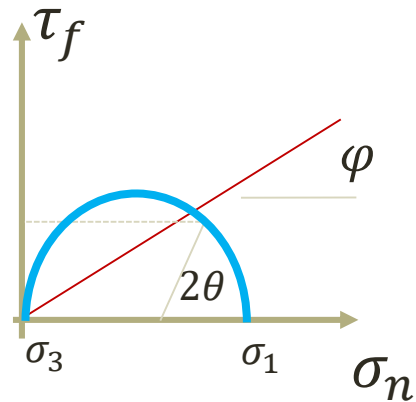


Fracture shear mechanics: Slip Coulomb model

Infinite fracture



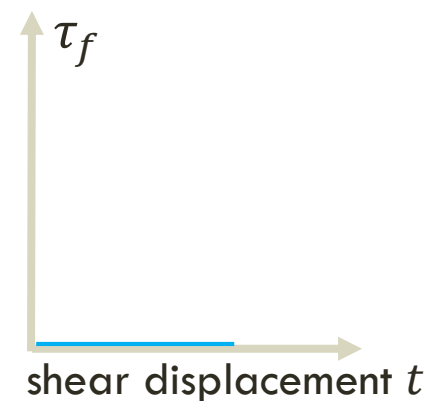
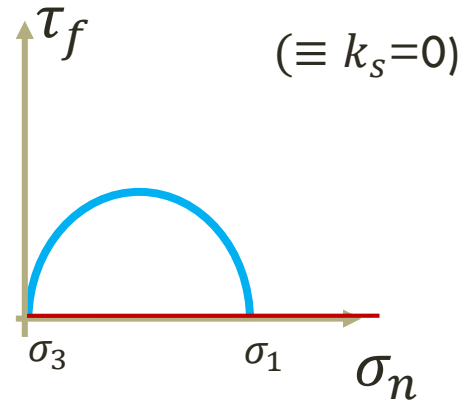
General case



$$\tau_f = k_s \cdot t \text{ if } \tau_f < \tau_c$$

$$\tau_f = \tau_c \text{ if } \tau > \tau_c$$

If $C = 0$ and $\varphi = 0$
 $(\equiv k_s = 0)$

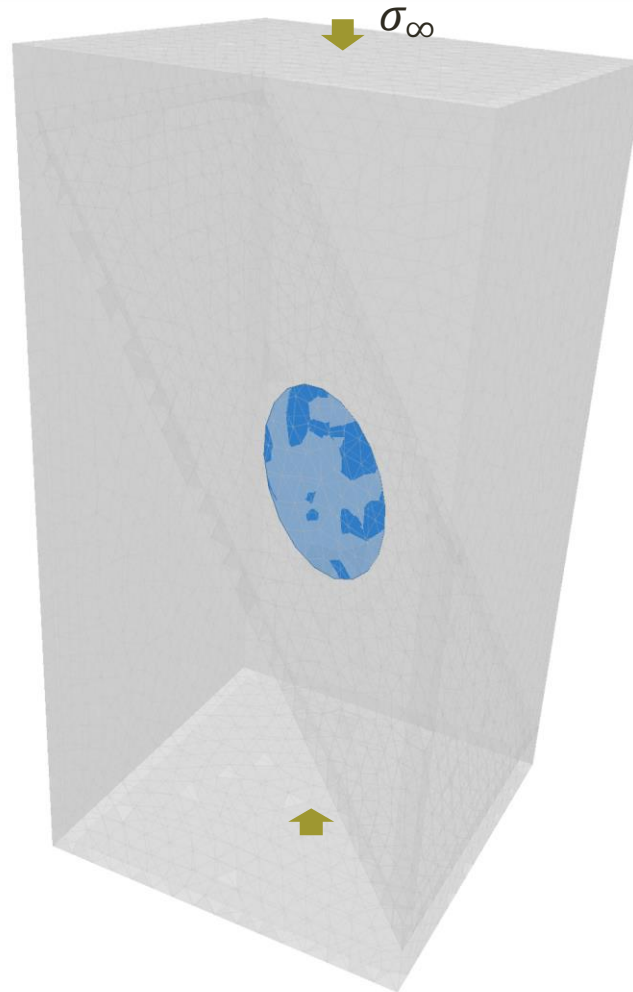


$$\tau_f = 0$$

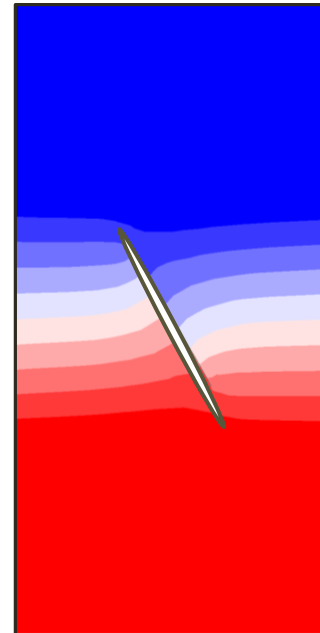
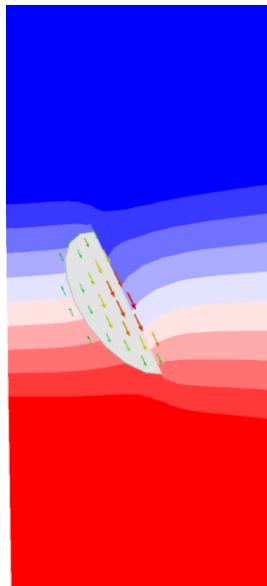
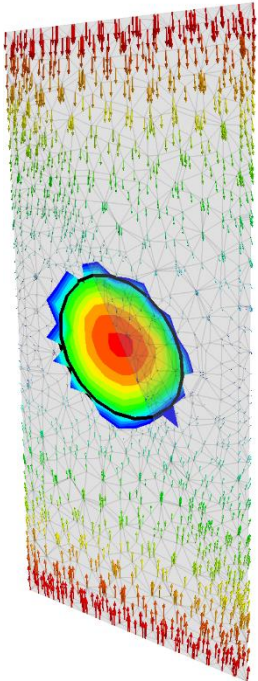
Fracture embedded in the rock

3DEC DP 5.20

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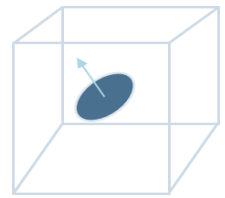


One fracture isolated and loaded

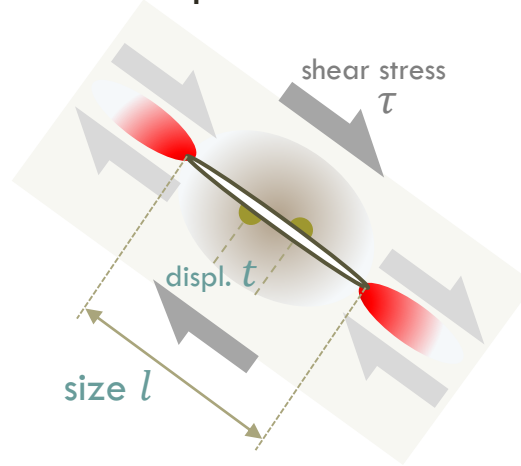
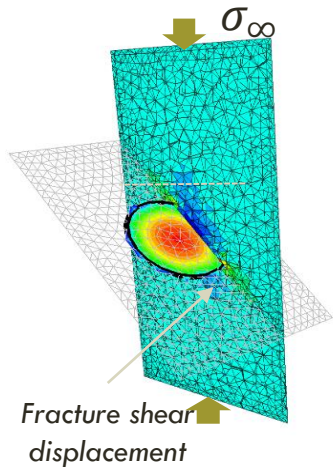


Frictionless isolated fracture

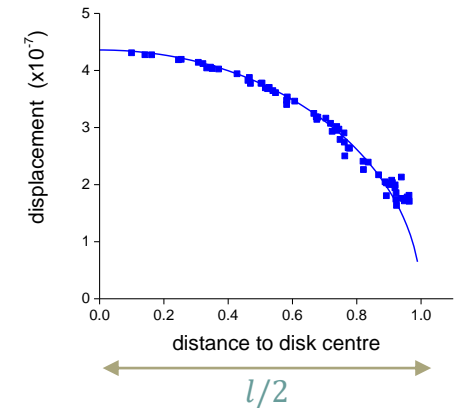
[Fabrikant, 90]



Stress and displacement at fracture



Displacement profile



Average shear displacement

$$t = \frac{\tau}{k_m}$$

k_m : "matrix to fracture stiffness"

$$k_m = \frac{3\pi}{8} \cdot \frac{1-\nu_m/2}{1-\nu_m^2} \cdot \frac{E_m}{l} \sim \frac{E_m}{l}$$

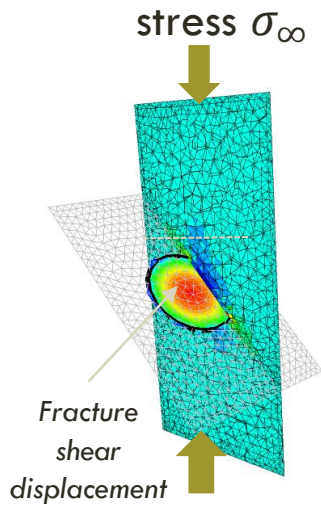
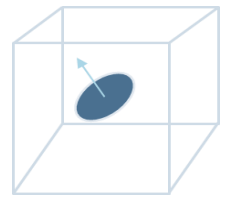
Modulus

Poisson ratio

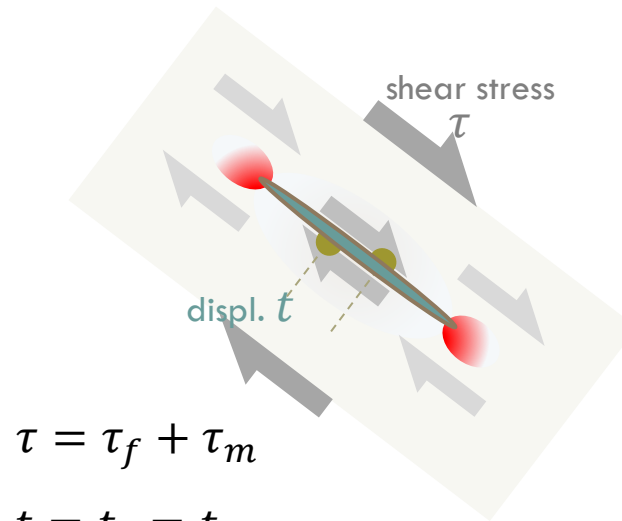
Fracture size

Frictional isolated fracture [Davy et al, 2018]

Fracture mechanics: friction, cohesion and stiffness terms



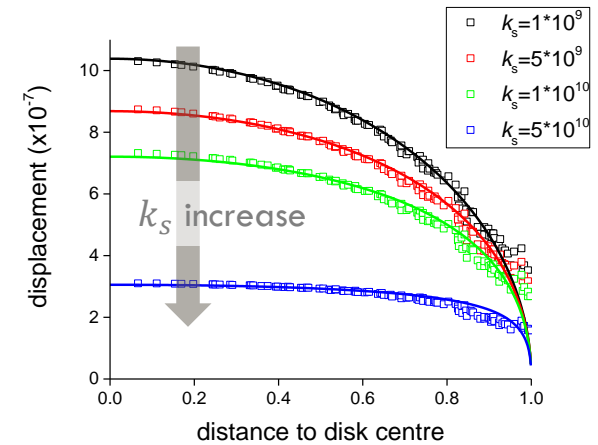
Stress and displacement at fracture



$$\tau = \tau_f + \tau_m$$

$$t = t_f = t_m$$

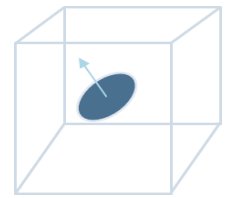
Displacement profile



Average shear displacement

$$t = \frac{\tau}{k_m + k_s}$$

$$k_m \sim \frac{E_m}{l}$$



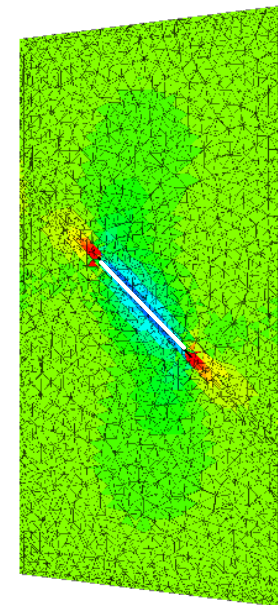
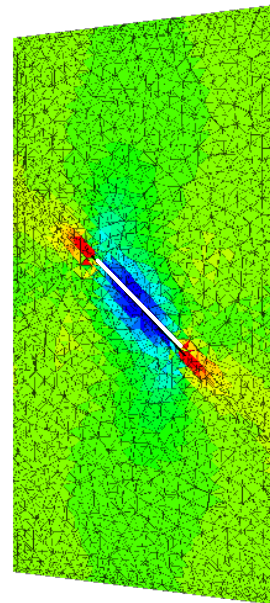
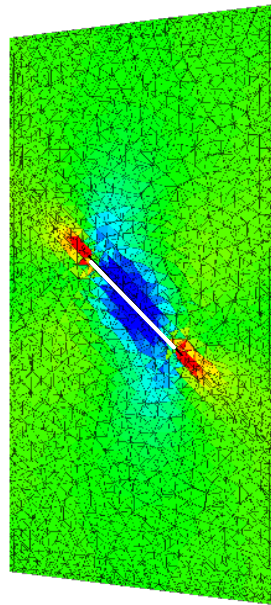
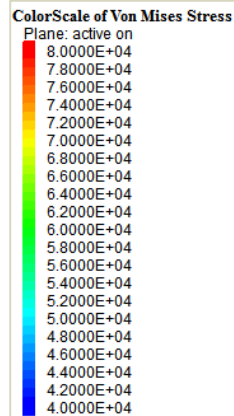
Stress perturbation around the fracture

With $k_m \approx \frac{E_m}{l}$

$$k_s = 0$$

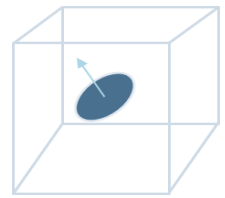
$$\frac{k_s}{k_m} = 0.5$$

$$\frac{k_s}{k_m} = 1$$



Increasing k_s relatively to k_m decreases the stress perturbation

THE TYPICAL “STIFFNESS SIZE” l_m



We define a *stiffness size* $l_s = \frac{E_m}{k_s}$

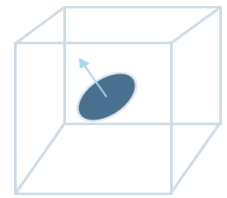
From which two distinct regimes can be defined

If $l \gg l_s$ $t = \frac{\tau}{k_s + k_m} \approx \frac{\tau}{k_s}$



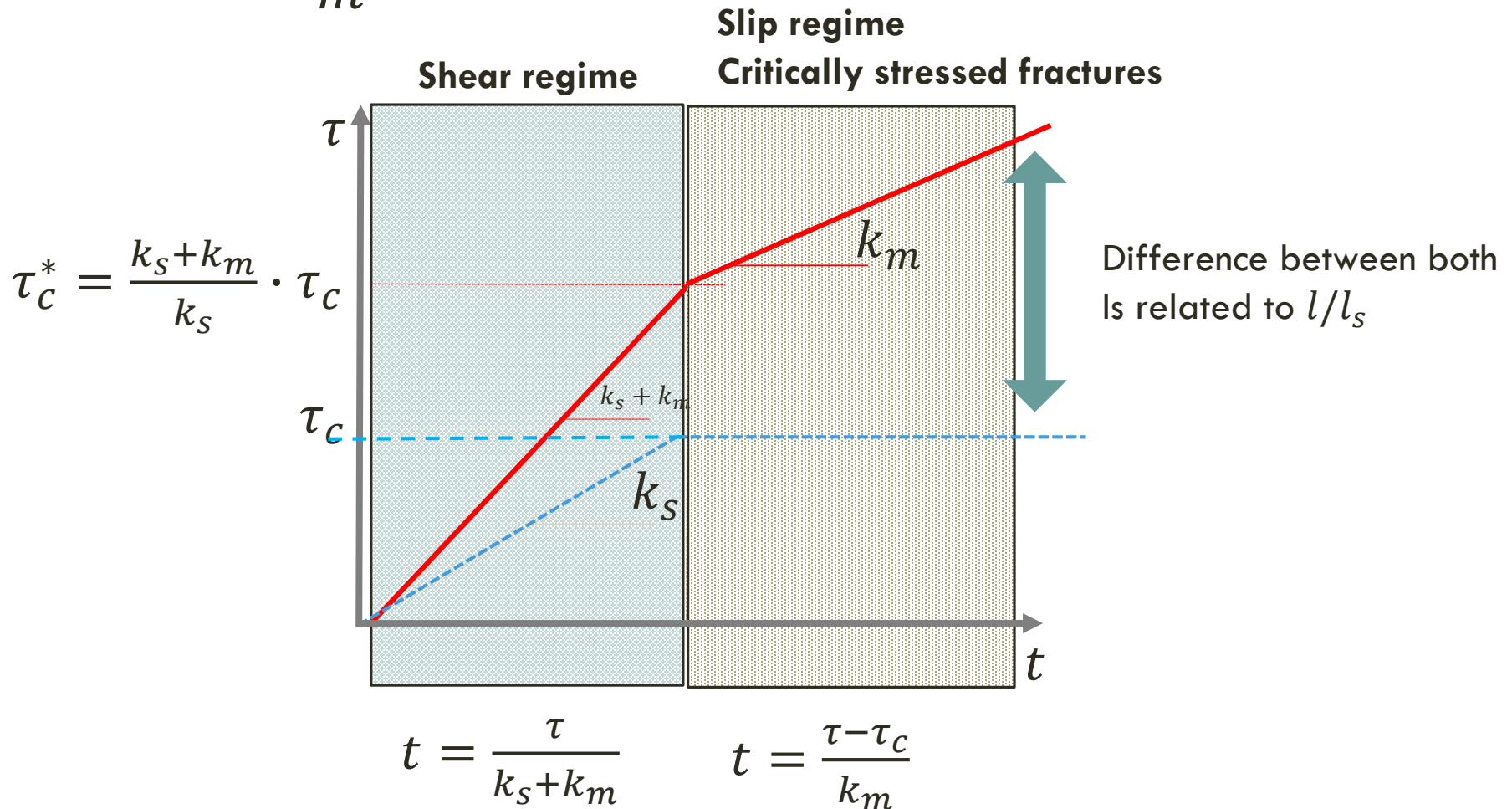
If $l \ll l_s$ $t = \frac{\tau}{k_s + k_m} \approx \frac{\tau}{k_m} \propto l$



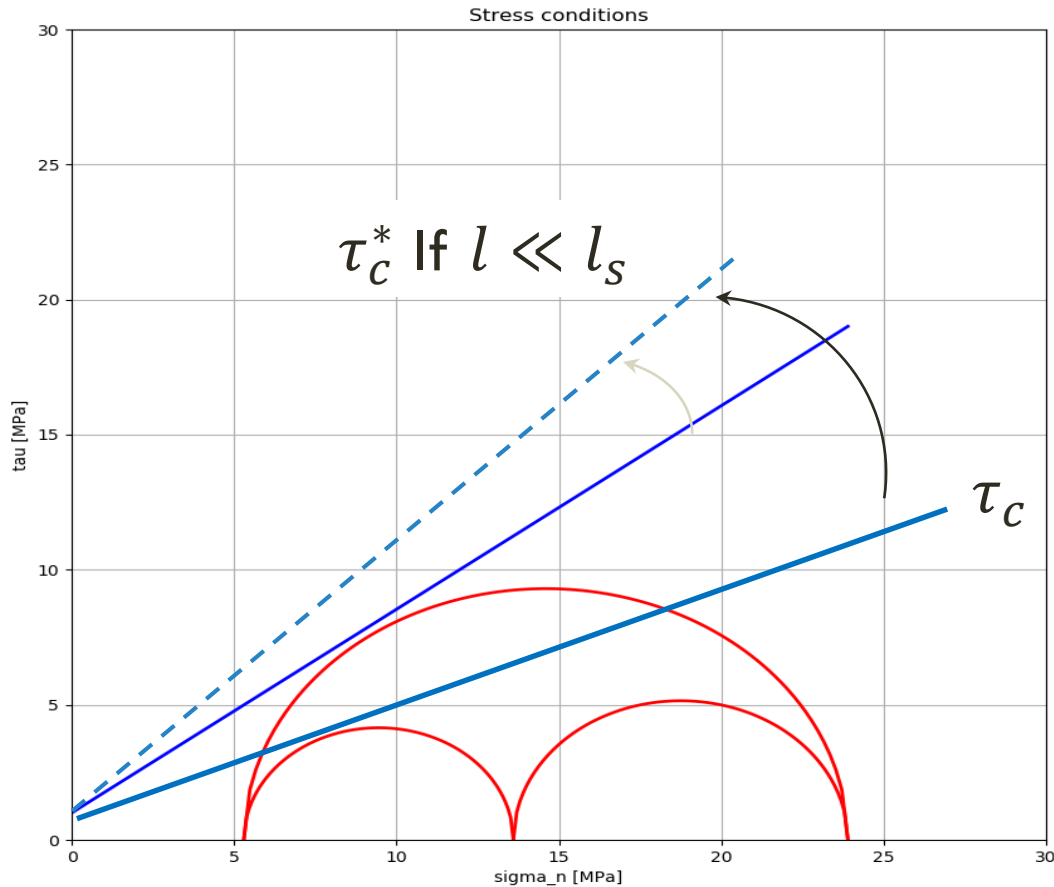


SHEAR AND SLIPPING REGIME

If $l \ll l_m$



CRITICALLY STRESSED FRACTURES



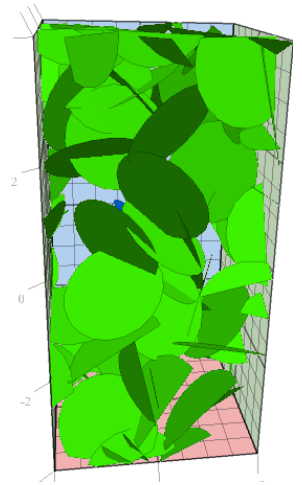
$$\text{With } \tau^* = \frac{k_m + k_s}{k_s} \tau_c$$

If $\gg l_s$

Proportion of critically stressed fractures will change with size l

Characterization and modeling

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NUMERICAL MODELING FOR DERIVING EFFECTIVE PROPERTIES

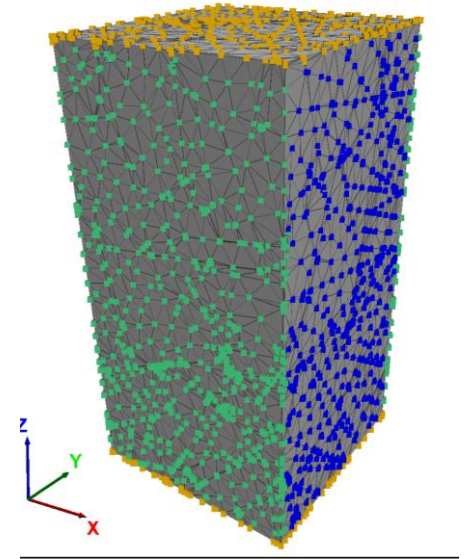
- Numerical simulations of the fractures rock (3DEC)
 - Load a Synthetic Rock Mass Sample
 - Measure the resulting strain
 - Define the equivalent elastic component like $E = \sigma/\epsilon$

3DEC DP 5.20
©2017 Itasca Consulting Group, Inc.
Step 33314
04/07/2018 16:47:26

Block
Colorby: Block
DFN dfn name Label
Shrink Factor: 1
Fractures (578)
■ keep_real
Components: X Y Z

Applied velocity vectors
Maximum: 8.11834e-07
Scale: 301723

8.1183E-07
8.0000E-07
7.5000E-07
7.0000E-07
6.5000E-07
6.0000E-07
5.5000E-07
5.0000E-07
4.5000E-07
4.0000E-07
3.5000E-07
3.0000E-07
2.5000E-07
2.0000E-07



NUMERICAL ROCK MASSES WITH DFN

Multiple realizations of DFN models:

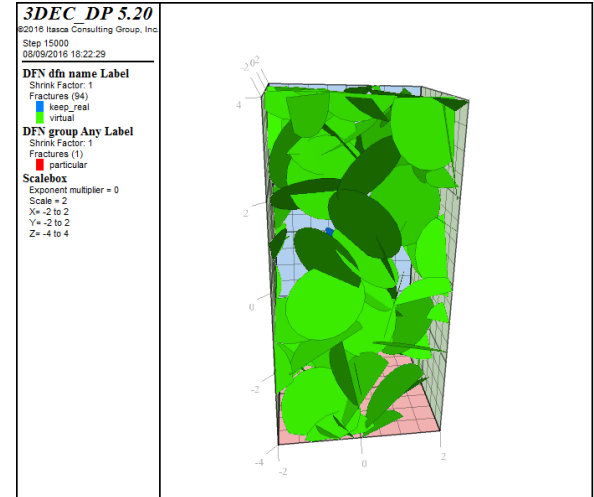
- Constant sizes : $l \in [0.5; 1; 2]$
- Isotropic random orientations
- Densities d and p values
 $d \in [1; 2; 3; 5]$
 $p \in [0.7; 15]$

$$p_{\theta} = \frac{1}{V} \sum_f (l_f S_f \cos^2 \theta \sin^2 \theta)$$

- $k_s = 0; 12; 72$ GPa/m
- $E_m = 53$ GPa, $\nu_m = 0.25$

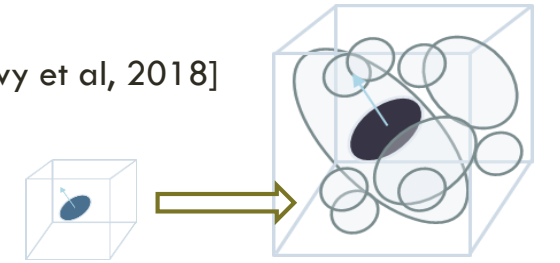


Compute the evolution of apparent Young's Modulus as function of the DFN densities and p values



WHAT IS EXPECTED

[Davy et al, 2018]



Rock mass with DFN

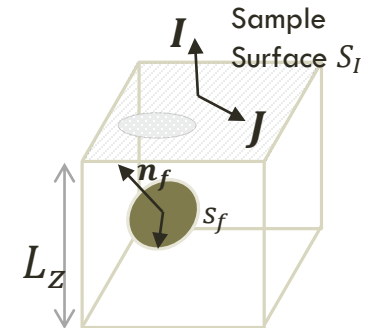
DFN contribution to rock mass strain tensor $\bar{\bar{\epsilon}}$:

Sum the contribution of each fracture f and intact rock m

$$\epsilon_{ij} = \sum_f (\epsilon_{ij})_f + (\epsilon_{ij})_m$$

Fracture f contribution to rock mass strain

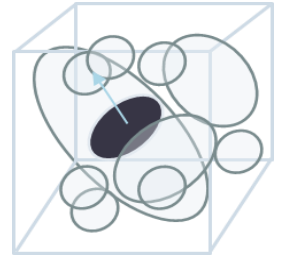
$$(\epsilon_{ij})_f = \frac{S_f (\mathbf{n}_f \cdot \mathbf{I})}{S_I} \cdot \frac{t_f (\mathbf{s}_t \cdot \mathbf{J})}{L_z} = \frac{S_f t_f}{V} \cdot (\mathbf{n} \cdot \mathbf{I})(\mathbf{s} \cdot \mathbf{J})$$



If $t_f \sim l_f$ the contribution is in l_f^3
 If not, the contribution is in l_f^2

WHAT IS EXPECTED

[Davy et al, 2018]



Rock mass with DFN

DFN contribution to rock mass strain tensor $\bar{\bar{\epsilon}}$:

Sum the contribution of each fracture f and intact rock m

$$\epsilon_{ij} = \sum_f (\epsilon_{ij})_f + (\epsilon_{ij})_m$$

Derive effective compliance tensor components C_{ijkl} :

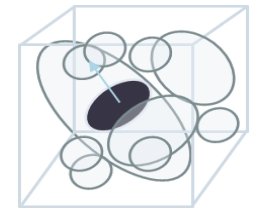
$$\epsilon_{ij} = C_{ijkl} \sigma_{kl}$$

General case conditions :

- **Shear displacement** (k_s)
- Change of regime for critically stressed fractures (slipping, dilation)
- Normal displacement (normal stiffness k_n)

ANALYTICAL SOLUTIONS FOR SIMPLE CASES

preliminary investigations

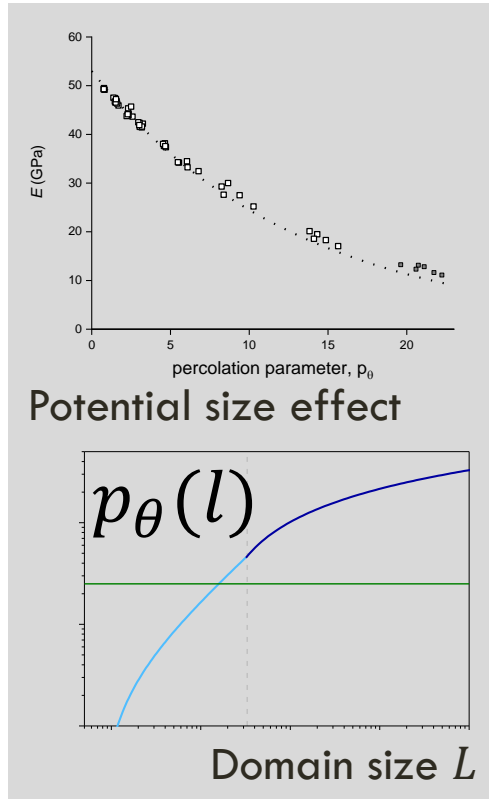


- If all the fractures of a DFN are such that $l \ll l_s$

$$E_{eff} = \frac{E_0}{1+c \cdot p(\theta)}$$

DFN percolation parameter: ratio between the total volume surrounding fractures and the sample volume

$$p(\theta) = \frac{1}{V} \sum_f (l_f^3 \cos^2 \theta_f \sin^2 \theta_f)$$



- If all the fractures of a DFN are such that $l \gg l_s$

$$E_{eff} = \frac{k_s}{p_{32}(\theta) + k_s/E_0}$$

DFN density p_{32} The total fracture surface per unit volume

$$p_{32}(\theta) \sim \frac{1}{V} \sum_f l_f^2 \cos^2 \theta_f \sin^2 \theta_f$$

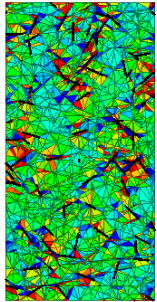


No size effect

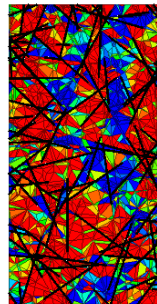


NUMERICAL RESULTS – FRICTIONLESS FRACTURES ($\varphi = 0$)

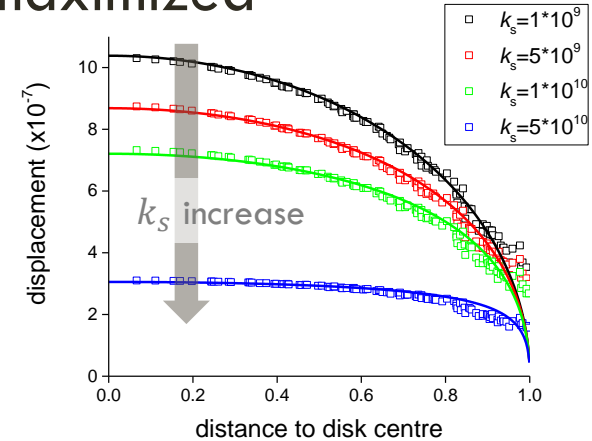
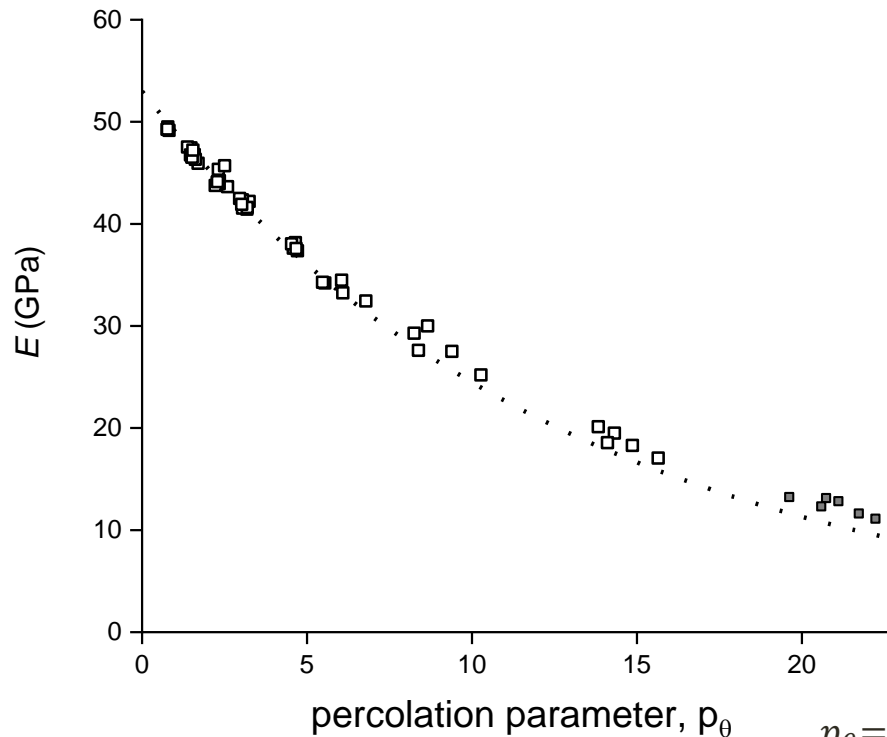
$p=0.785$



$p=15.7$



Shear displacement is maximized

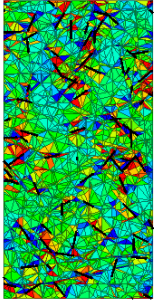


$$p_\theta = \frac{1}{V} \sum_f (l_f S_f \cos^2 \theta \sin^2 \theta)$$

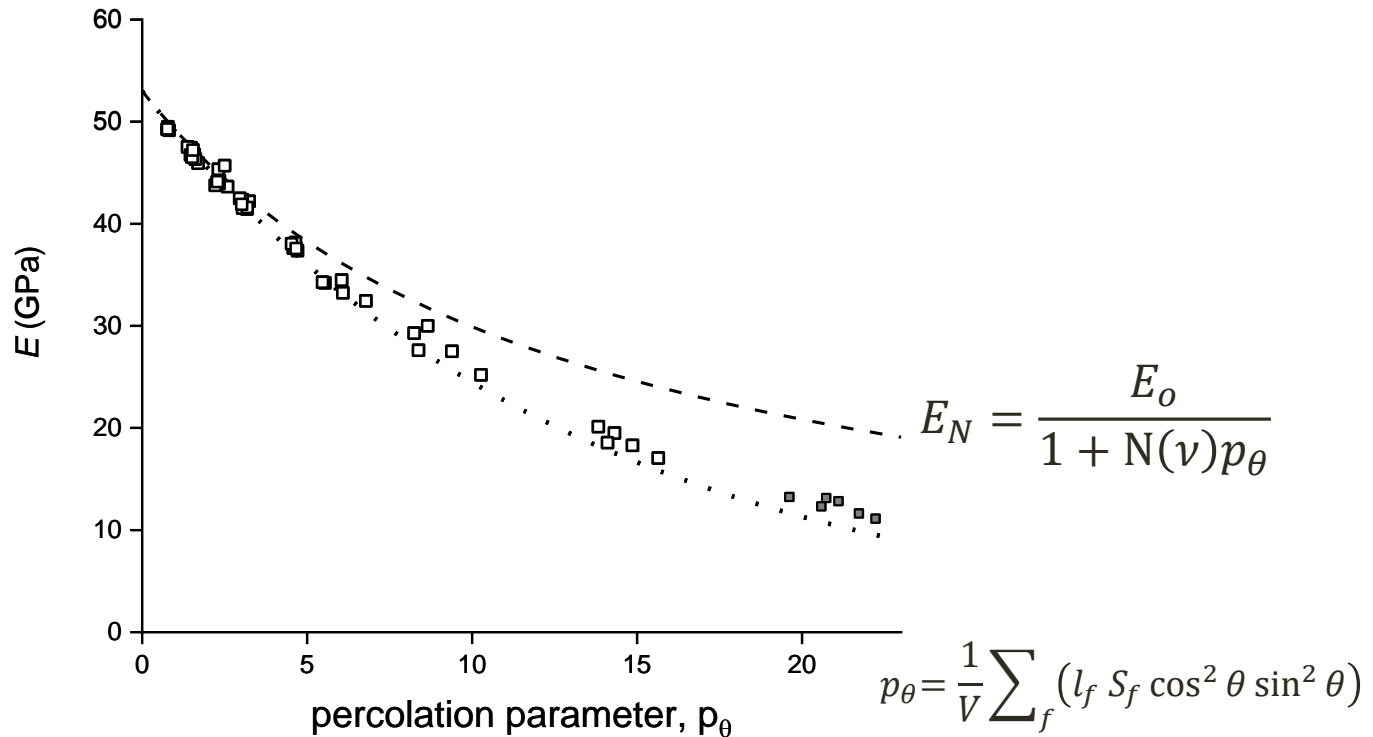
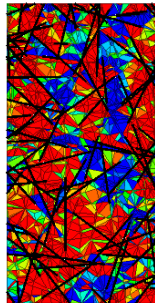
Decrease consistent with a P_θ dependency

NUMERICAL RESULTS — FRICTIONLESS FRACTURES

$p=0.785$



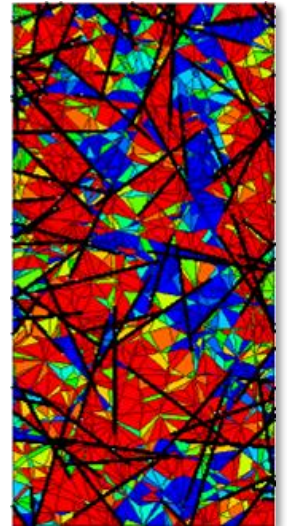
$p=15.7$



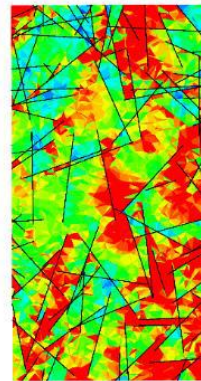
Decrease consistent with a P_θ dependency
 ... But different from the initial expectation!

Characterization and modeling

- Discrete Fracture Network short introduction
- Assumptions
- Isolated Fracture
- DFN scale / rock mass scale
- Interactions and stress fluctuations
- Application example



Stress and strain fluctuations in a network due to mechanical interactions between fractures

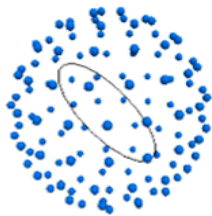


Fracture stress and displacement

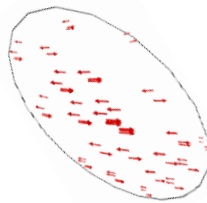
$$\bar{t} = \frac{\bar{\tau}}{k_s + k_m}$$

Stress applying on the fracture
Remote ($\tau = \tau_\infty$) or perturbed

Matrix properties around fracture
Intact ($k_m \sim \frac{E_m}{l}$) or damaged

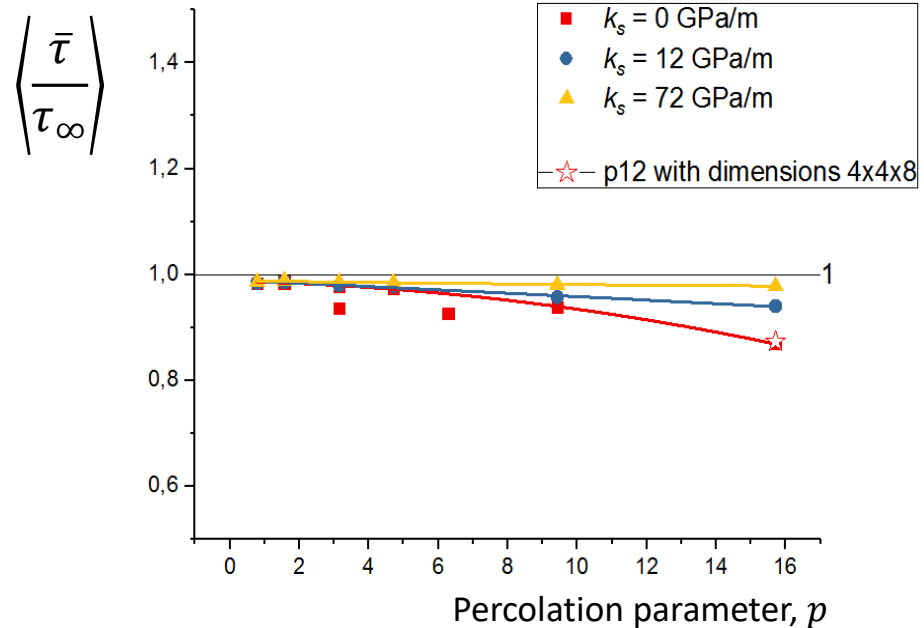
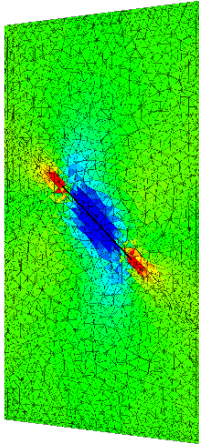


$\bar{\tau}$ is the average stresses surrounding the fracture.
We measure it on a sphere of diameter = $1.5 * l$



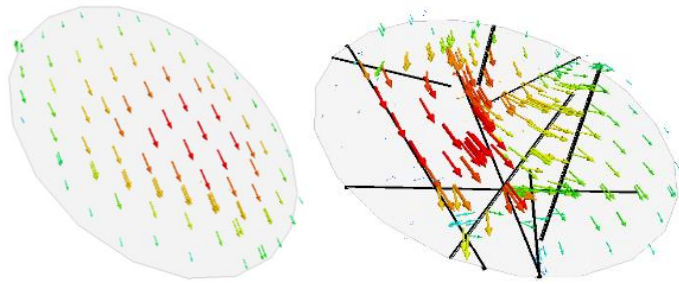
\bar{t} is the average displacement
on the fracture plane

Stress fluctuations at the fracture scale

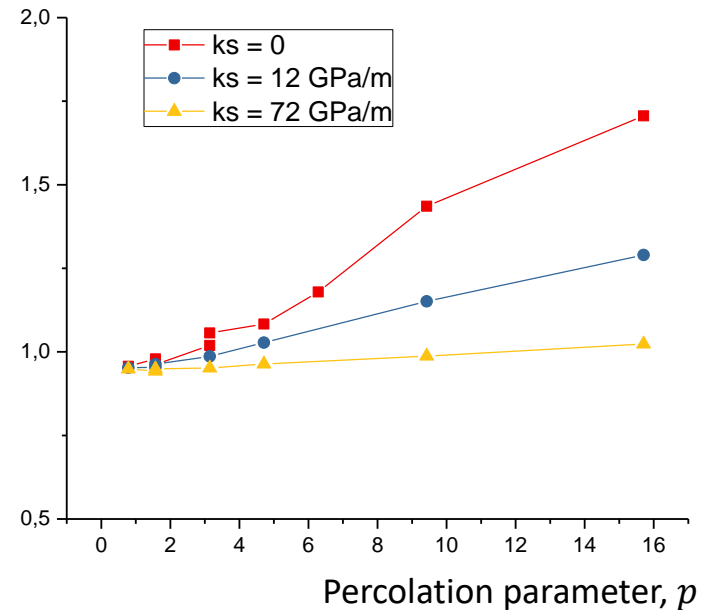


- The stress measured at the fracture scale slightly decreases with the density of fractures by 15% max ($p=15$; $k_s=0$)
- This is consistent with a prevailing shadow effect

DISPLACEMENT FLUCTUATIONS AT THE FRACTURE SCALE



$$\left\langle \frac{t_f}{t_f^{\text{th}}} \right\rangle$$



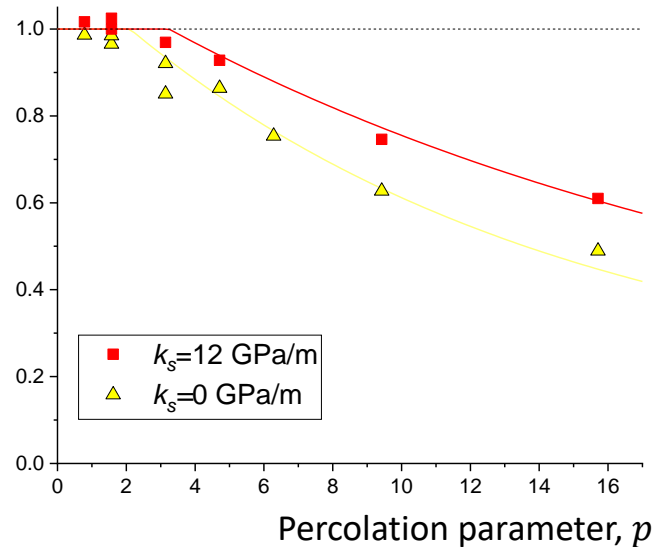
- Fracture interactions tend to increase the average displacement on the fracture
- This is consistent with a “softer” damaged matrix

ACTUAL MATRIX STIFFNESS

Equivalent matrix stiffness

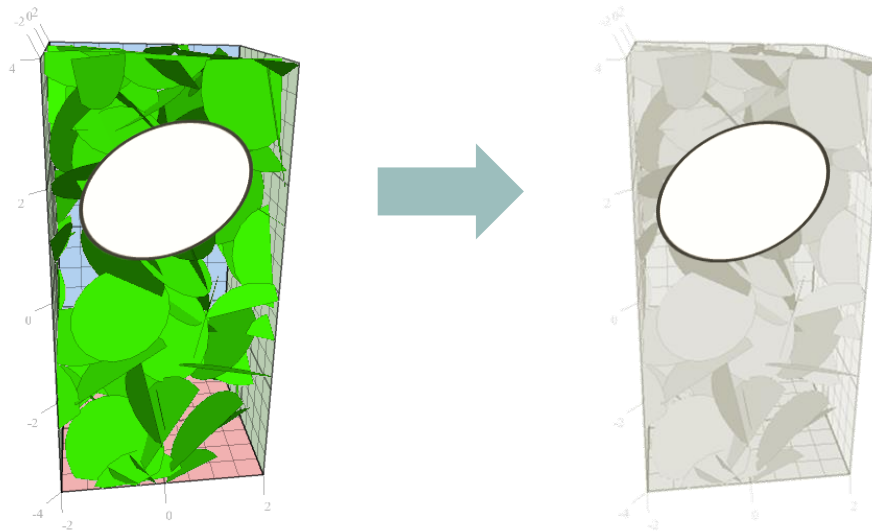
$$k_m^* = \frac{\bar{\tau}}{\langle t_f \rangle} - k_s$$

$$\left\langle \frac{k_m^*}{k_m} \right\rangle$$



- The equivalent matrix stiffness surrounding fractures k_m^* tends to decrease with damage
- The variations of k_m^* are mostly due to displacement rather than stress fluctuations

EFFECTIVE THEORY




Method

The contribution of an additional fracture is calculated from the effective elastic properties of the damaged system

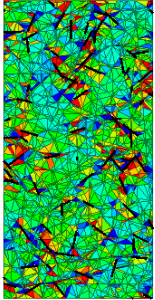
$$t = \frac{\tau}{k_s + \frac{E_m^{\text{eff}}}{l}}$$

with E_m^{eff} the effective elastic properties of the damaged system

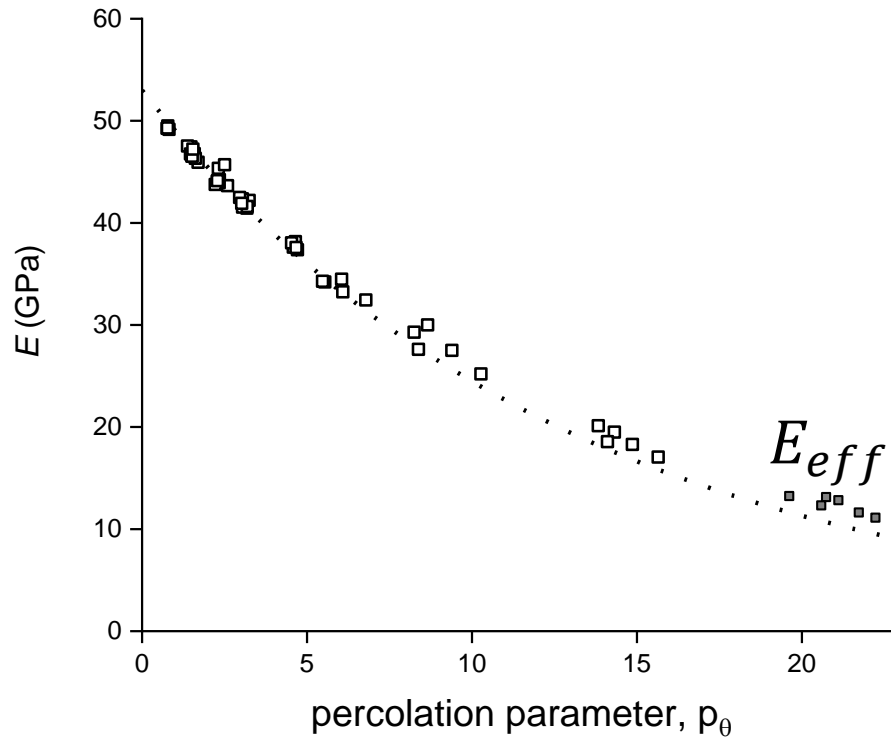
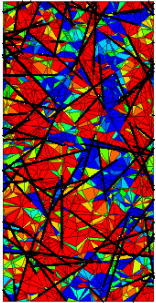

$$E_{eff} = E_0 \exp(-c \cdot p(\theta))$$

BACK TO SIMULATIONS RESULTS

$p=0.785$

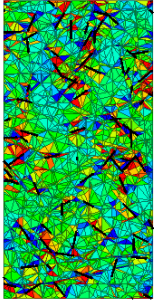


$p=15.7$

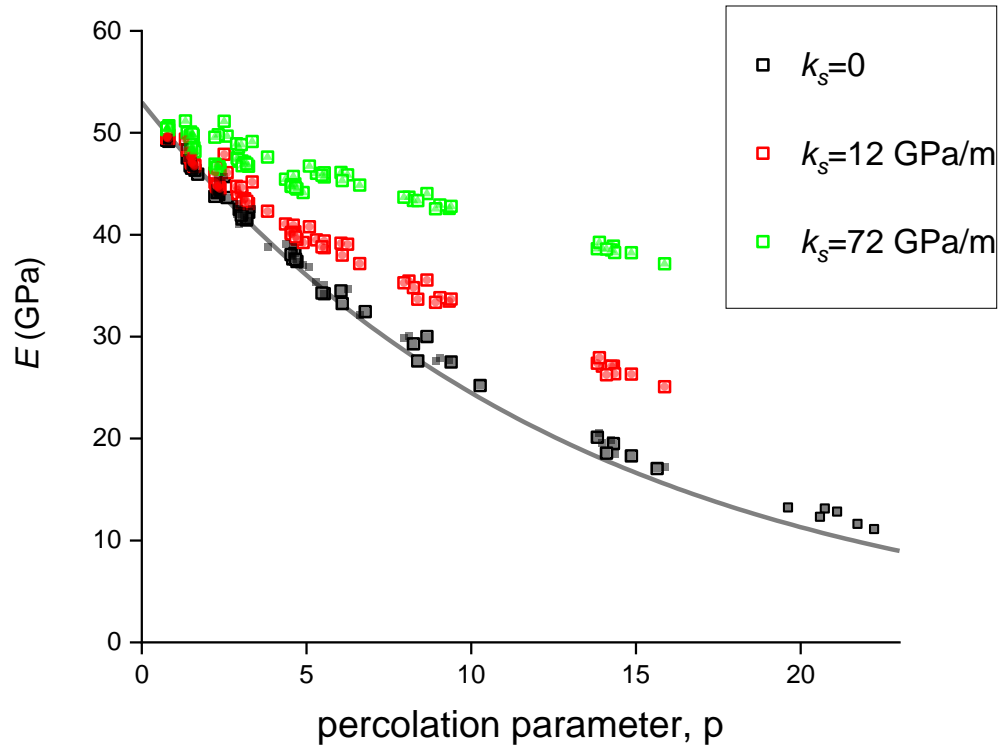
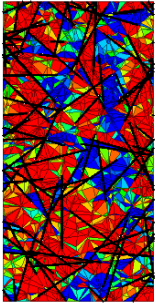


BACK TO SIMULATIONS RESULTS

$p=0.785$

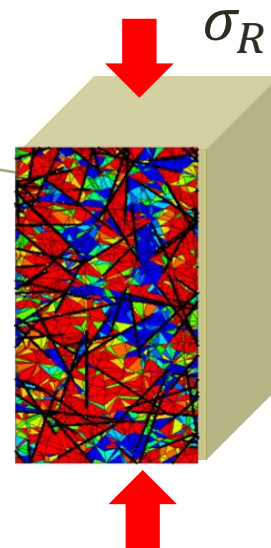
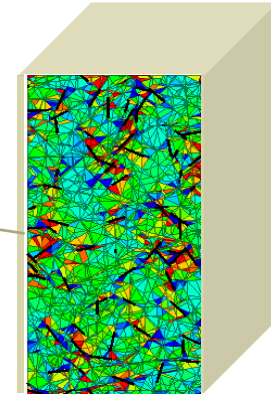
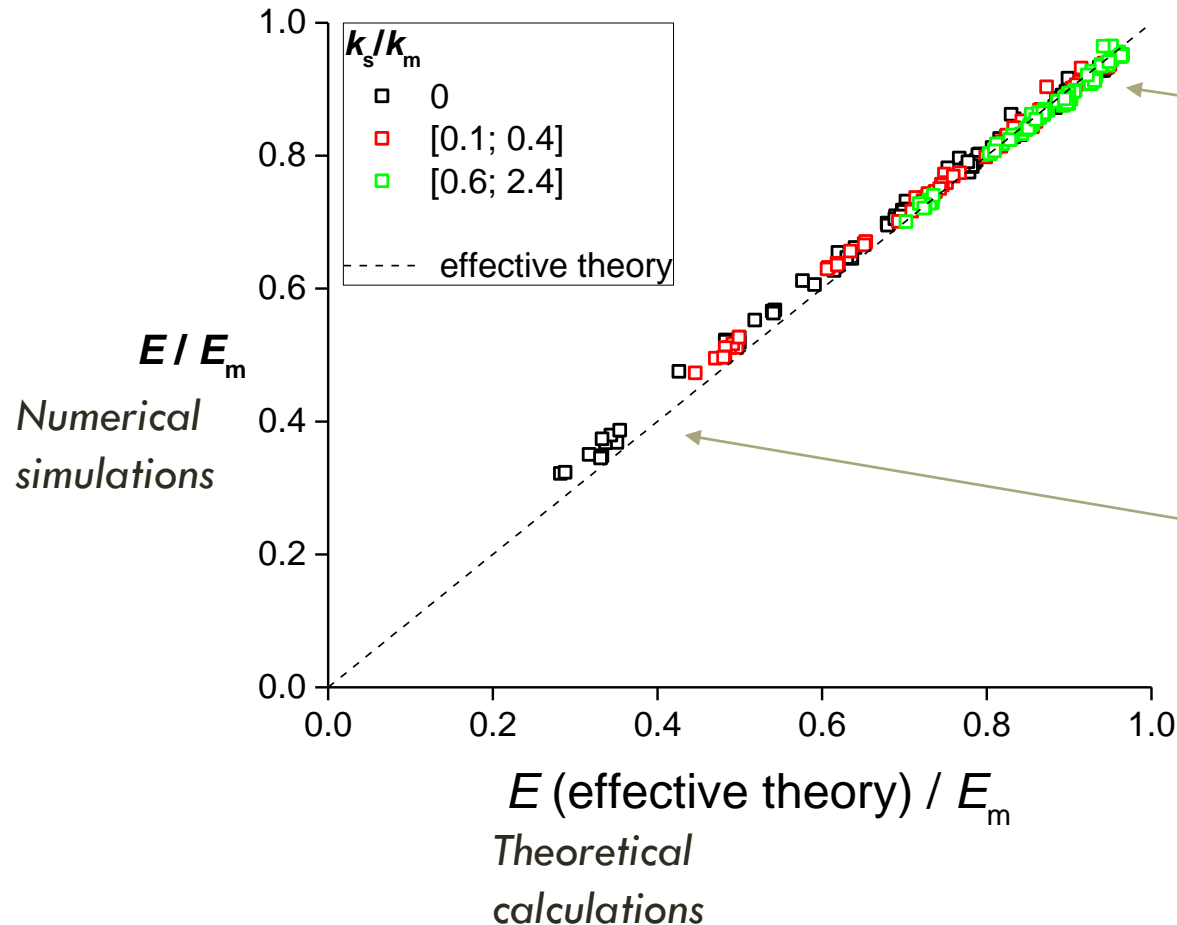
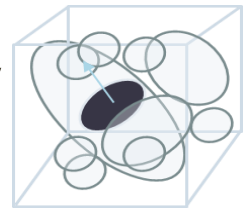


$p=15.7$

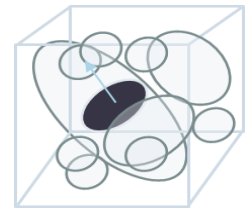


RESULTS WITH INTERACTIONS AND EFFECTIVE THEORY

[Davy et al, 2018]



ANALYTICAL SOLUTIONS FOR SIMPLE CASES with interactions

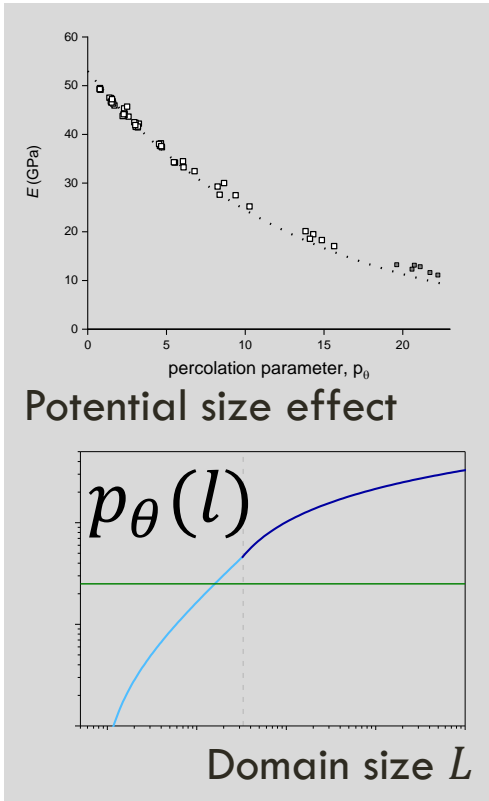


- If all the fractures of a DFN are such that $l \ll l_m$

$$E_{eff} = E_0 \exp(-c \cdot p(\theta))$$

DFN percolation parameter: ratio between the total volume surrounding fractures and the sample volume

$$p(\theta) = \frac{1}{V} \sum_f (l_f^3 \cos^2 \theta_f \sin^2 \theta_f)$$



- If all the fractures of a DFN are such that $l \gg l_m$

$$E_{eff} = \frac{k_s}{p_{32}(\theta) + k_s/E_0}$$

DFN density p_{32} The total fracture surface per unit volume

$$p_{32}(\theta) \sim \frac{1}{V} \sum_f l_f^2 \cos^2 \theta_f \sin^2 \theta_f$$

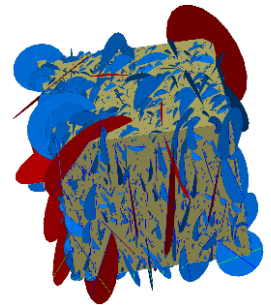
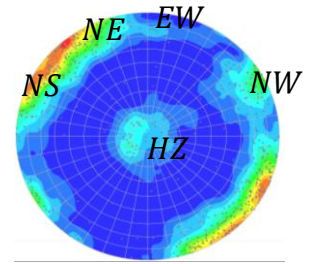


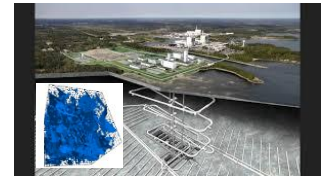
No size effect



Characterization and modeling

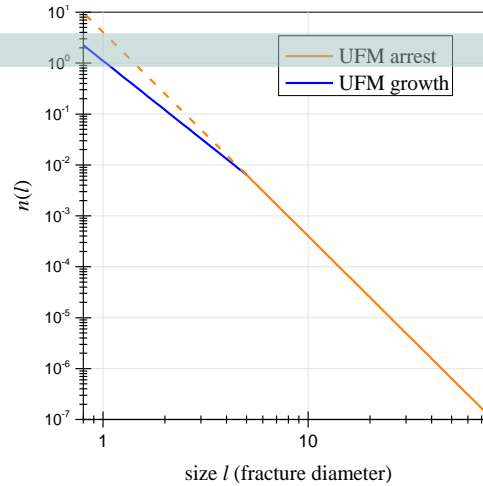
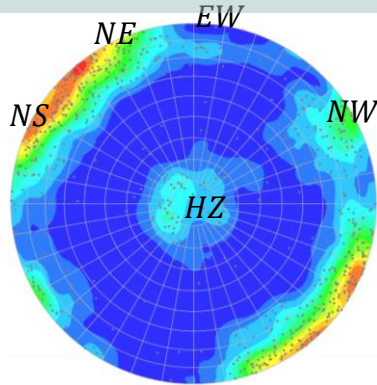
- Discrete Fracture Network short introduction
- Assumptions
- Isolated Fracture
- DFN scale / rock mass scale
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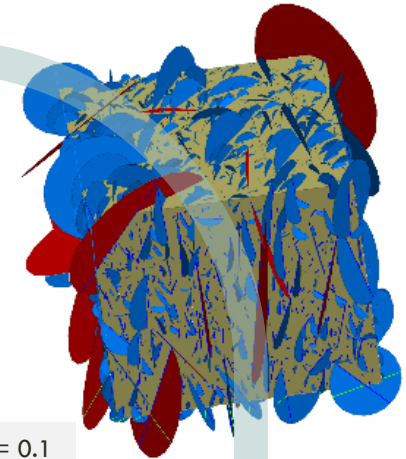
Application - DFN and rock conditions SKB Forsmark site, Sweden

DFN (FFM01 unit)

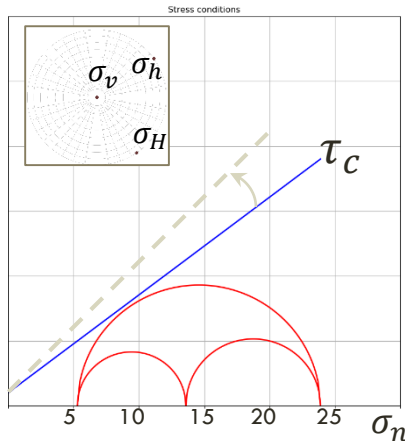


$$\frac{l_{min}}{L} = 0.1$$

$$L = 20$$



Mechanical properties



No critically stressed fractures

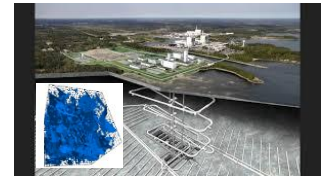
Intact Rock
 $E_m = 76 \text{ GPa}$
 $\nu_m = 0.23$

Fractures
 $k_s(\sigma_n) = 46.55 \times \sigma_n^{0.4039} \times 10^6$
 $k_n > 100k_s$

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \epsilon_{yz} \\ \epsilon_{xz} \\ \epsilon_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_{xx}} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_{xz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{xy}} \end{bmatrix} \times \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix}$$

Compliance tensor $\bar{\bar{C}}$

Anisotropy



$$\epsilon_{if} = C_{ijkl} \sigma_{kl}$$

Depending on the conditions the results are expressed according to

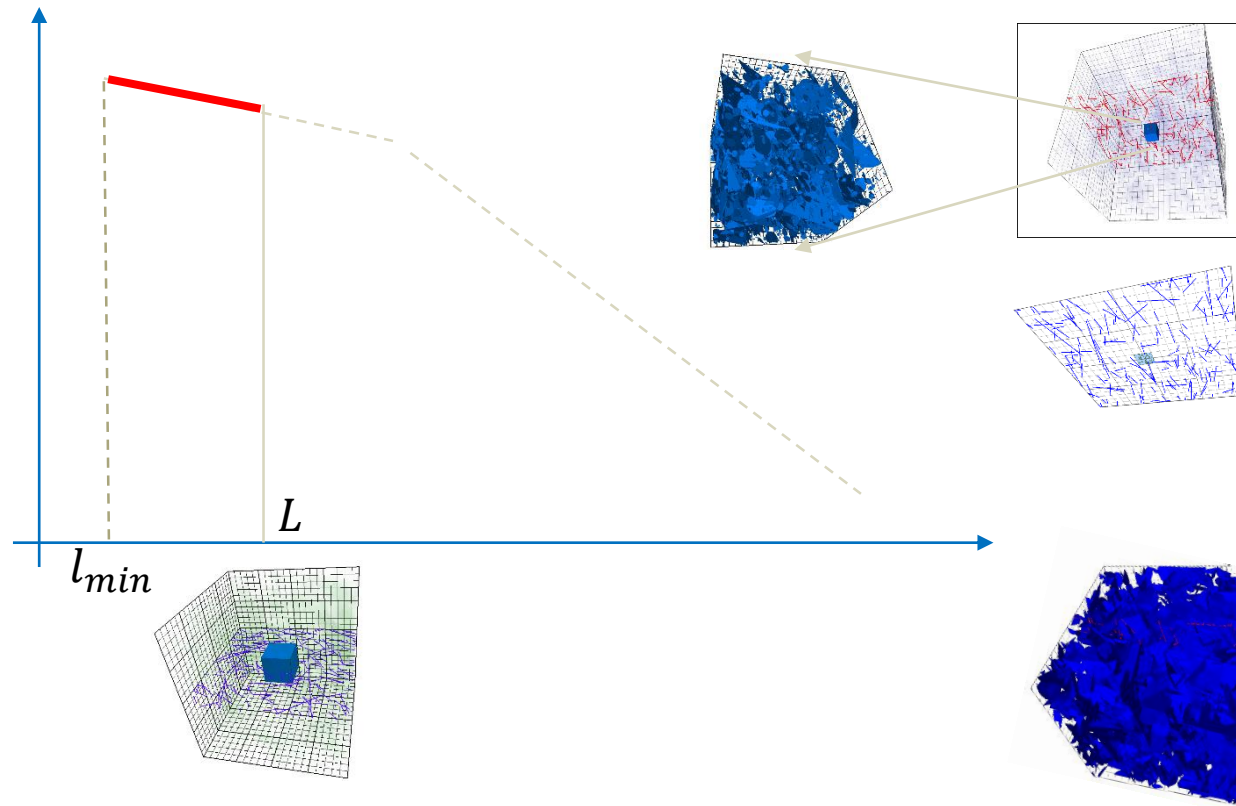
- General case: C_{ijkl}
- Orthotropic equivalent: $E_{xx}, E_{yy}, E_{zz}, G_{yz}, G_{xz}, G_{xy}, \nu_{yz}, \nu_{xz}, \nu_{xy}$
- Isotropic equivalent: E, ν, G

$$\begin{bmatrix} \epsilon_x \\ \epsilon_y \\ \epsilon_z \\ \epsilon_{yz} \\ \epsilon_{xz} \\ \epsilon_{xy} \end{bmatrix} = \begin{bmatrix} \frac{1}{E_{xx}} & -\frac{\nu_{yx}}{E_y} & -\frac{\nu_{zx}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xy}}{E_x} & \frac{1}{E_y} & -\frac{\nu_{zy}}{E_z} & 0 & 0 & 0 \\ -\frac{\nu_{xz}}{E_x} & -\frac{\nu_{yz}}{E_y} & \frac{1}{E_z} & 0 & 0 & 0 \\ 0 & 0 & 0 & \frac{1}{2G_{yz}} & 0 & 0 \\ 0 & 0 & 0 & 0 & \frac{1}{2G_{xz}} & 0 \\ 0 & 0 & 0 & 0 & 0 & \frac{1}{2G_{xy}} \end{bmatrix} \times \begin{bmatrix} \sigma_x \\ \sigma_y \\ \sigma_z \\ \tau_{yz} \\ \tau_{xz} \\ \tau_{xy} \end{bmatrix}$$

Scaling study

$n(l)$ exists from borehole diameter size ($l_{min} = 0.08$) up to site scale (or distance to DZ) with a total $P_{32}(l \geq l_{min}) = 4.76 m^2/m^3$

Sub domains are considered: $L < L_{max}$ and $n(l)$ truncated to L

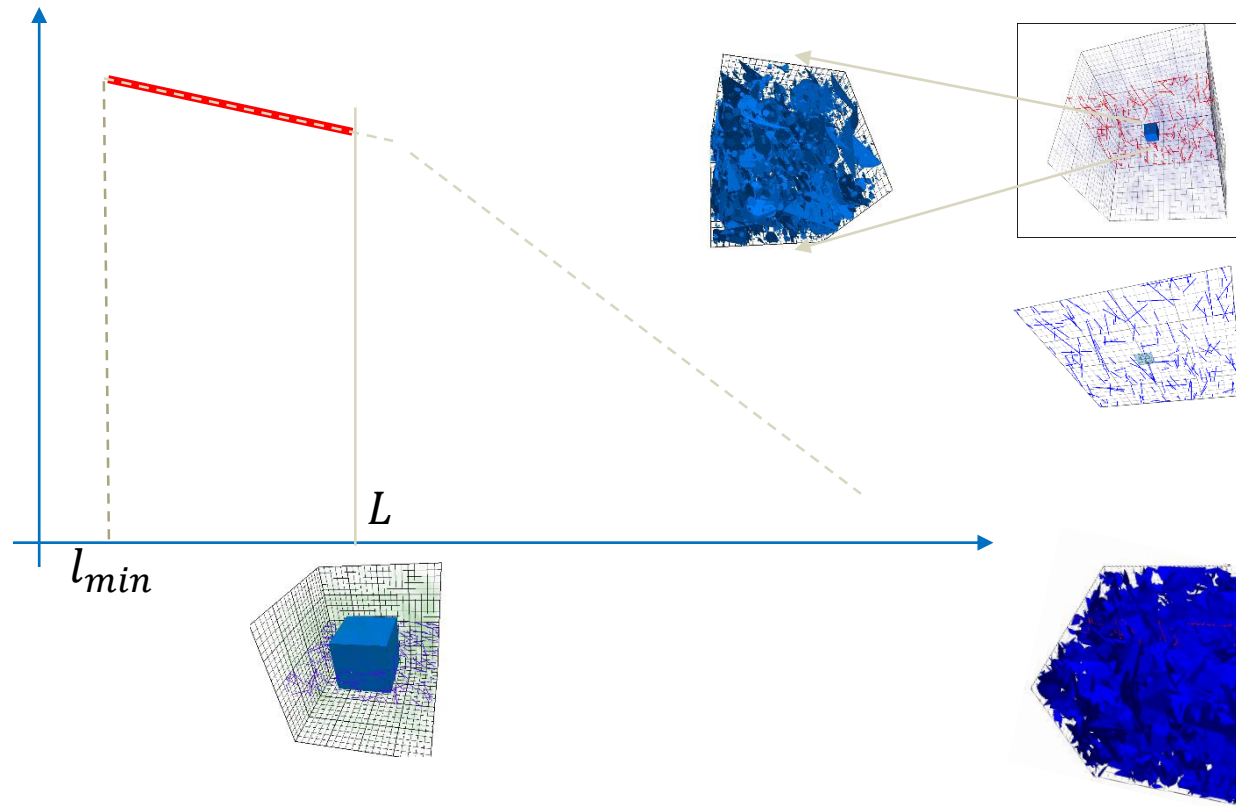


Scaling study

$n(l)$ exists from borehole diameter size ($l_{min} = 0.08$) up to site scale (or distance to DZ)

Sub domains properties are analysed and related to L

L is increased

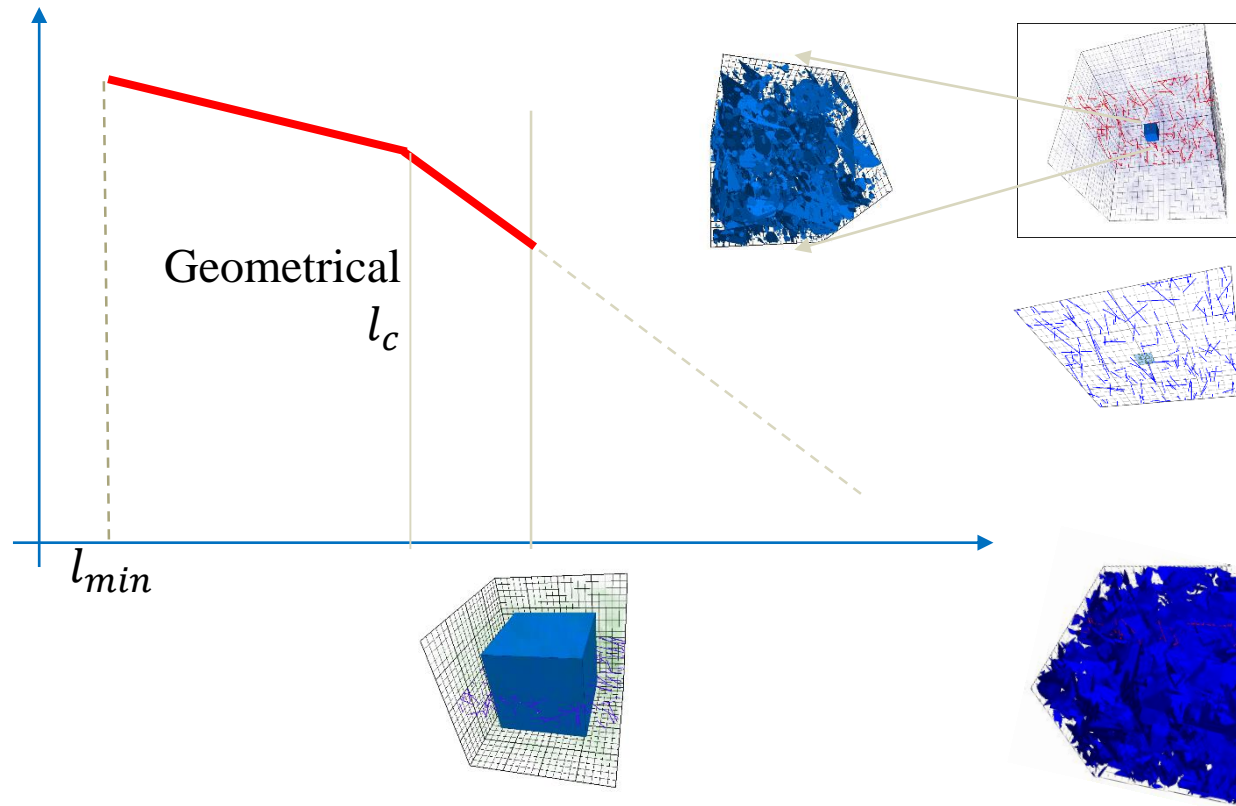


Scaling study

$n(l)$ exists from borehole diameter size ($l_{min} = 0.08$) up to site scale (or distance to DZ)

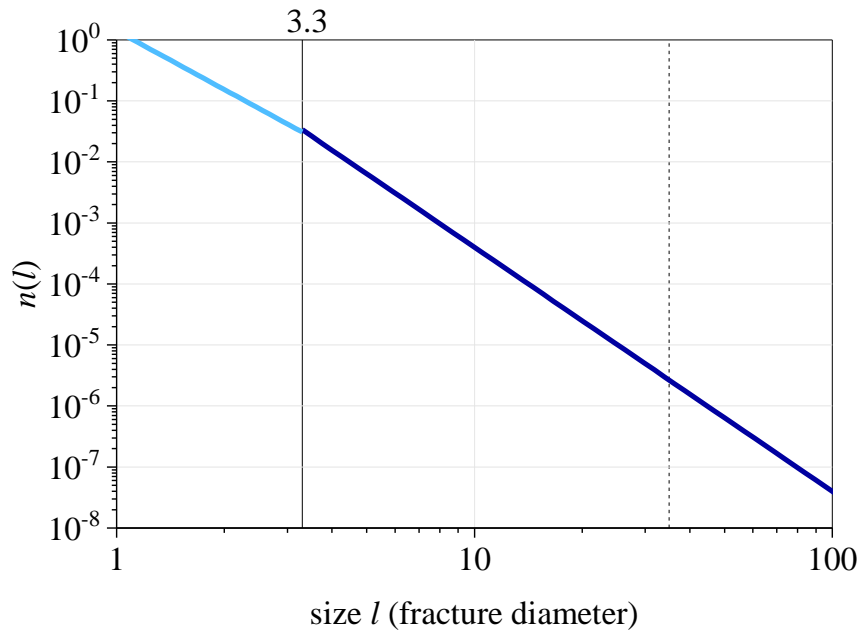
Sub domains properties are analysed and related to L

L is increased ... up to L larger than geometrical and mechanical characteristic length scales

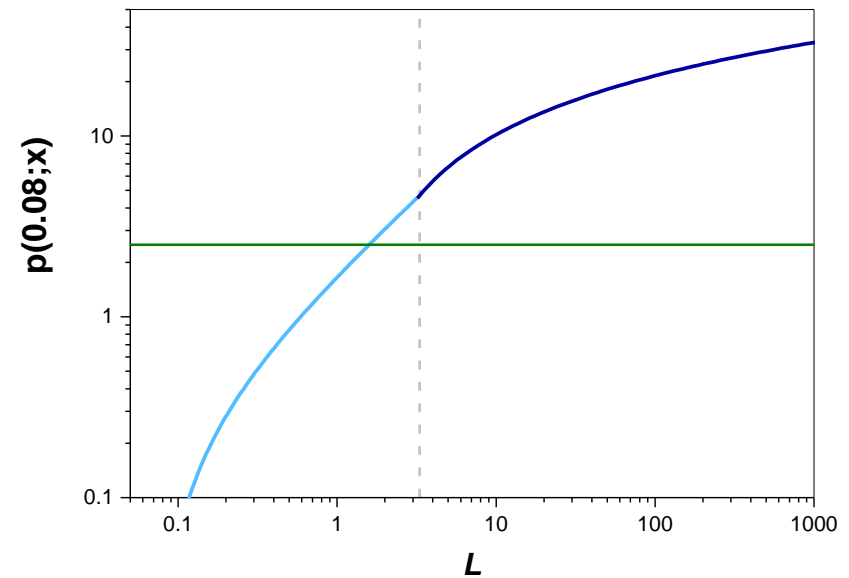


Evolution of $p_\theta(L)$

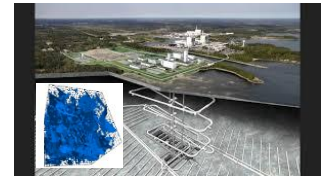
$n(l)$



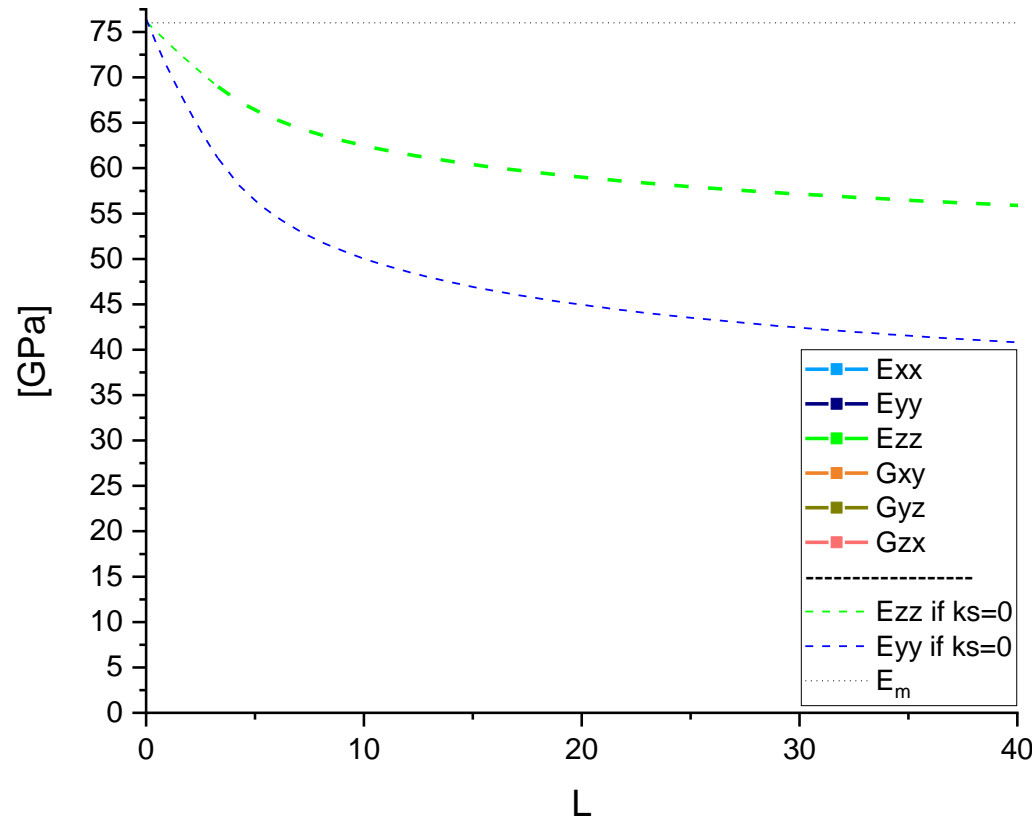
$p_\theta(l)$



Effective properties depend on $\exp(-p_\theta)$ as long as fractures are below the mechanical length ($l_M \sim E_0/k_s$)



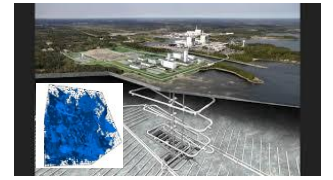
Evolution of E_{ii} and G_{ij} with L



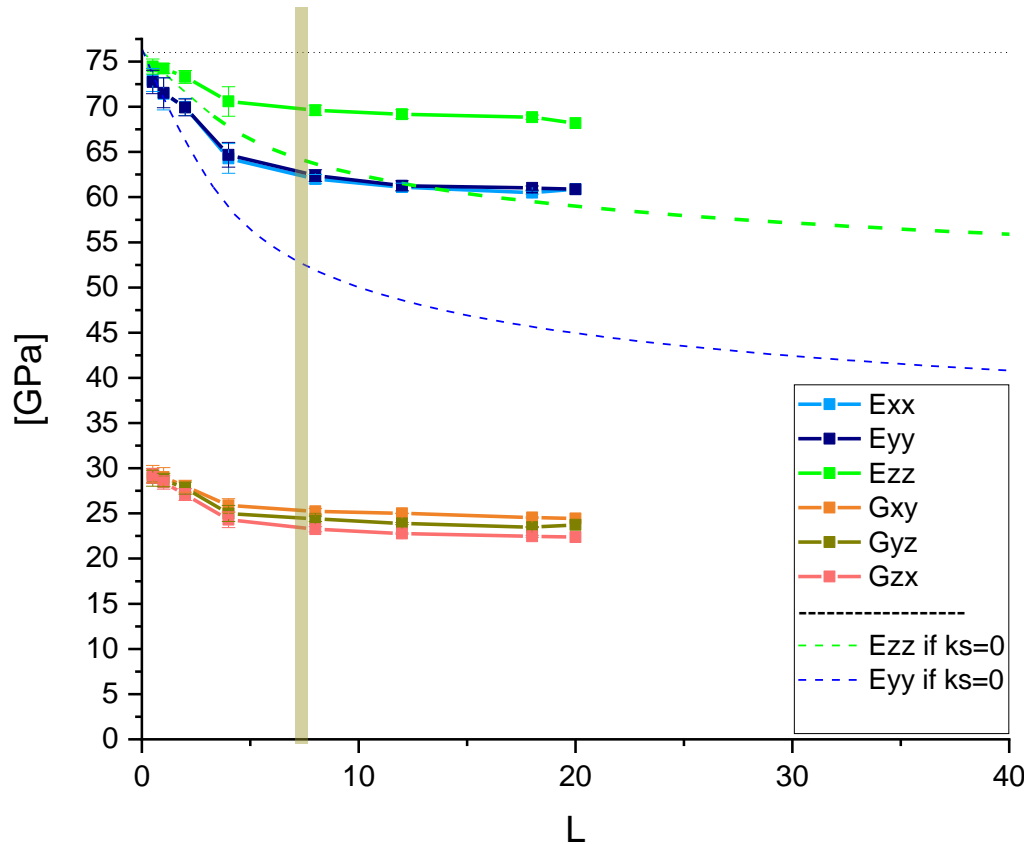
Increasing domain size L tend to put more large fractures without significantly changing P_{32}

Given the DFN conditions:

- If k_s such that $l \ll l_s \rightarrow$ maximise the scaling effect



Evolution of E_{ii} and G_{ij} with L

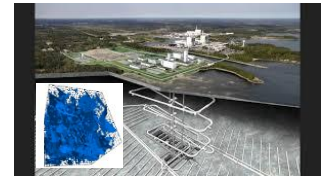


Given the DFN conditions:

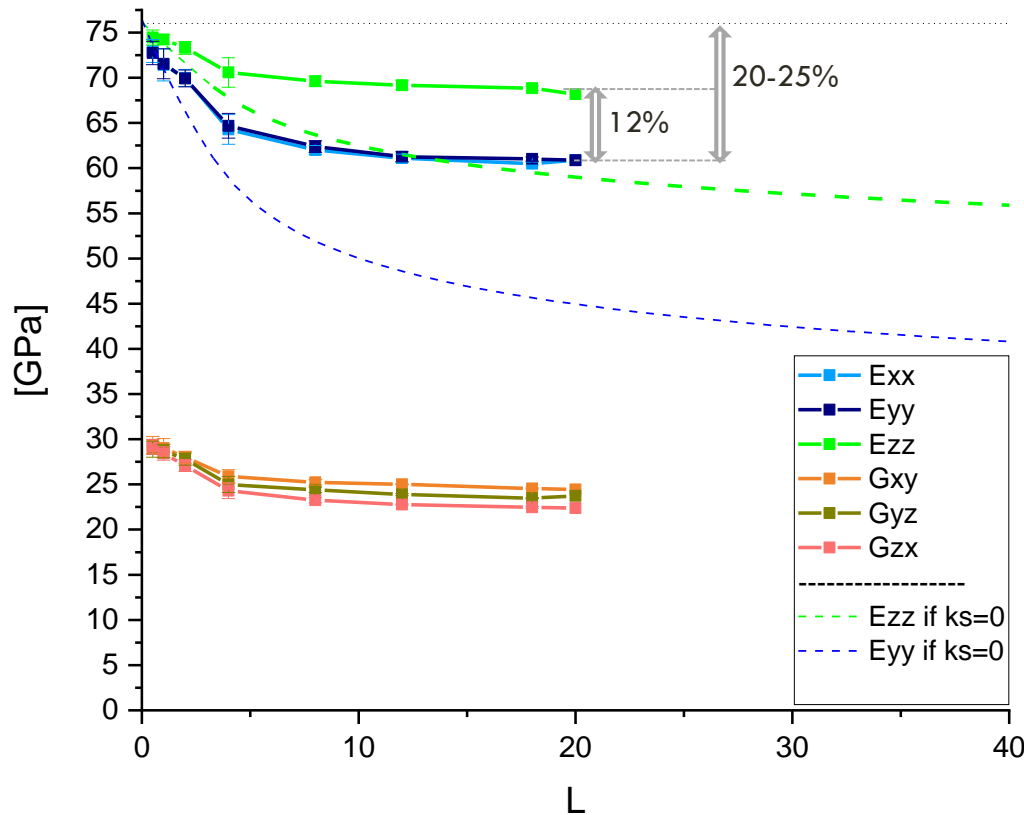
- If k_s such that $l \ll l_m \rightarrow$ maximise the scaling effect

With current mechanical properties

- $\langle k_s \rangle = 3.4e10 \text{ GPa} \cdot \text{m}^{-1}$
- $1.5 \text{ m} \leq l_M \leq 3.5 \text{ m}$
- Decrease of E_{ii} with L up to $\sim 10\text{m}$.
- Same extent for shear terms G_{ij}
- E_{xx} decrease from 76 GPa to about 62 GPa, i.e. about 25%.
- Same proportion for G_{ij}



1 DFN	
l_{min}	= 0.1
L	= 20
Ori	= FFM01
k_s	(σ_n)
k_n	$\approx \infty$



- E_{ii} variations : 60 to 70 Gpa (about 12%)
- E_{zz} less affected by fractures than horizontal E_{hz}
- (Horizontal directional E_{hz} consistent with fracture sets NE and NW, less affected by fracture shearing are at trend 45°)

SUMMARY

- Effective elastic properties of a fractured rock can be assessed by a method combining individual fractures contributions and interactions
- Multiscale DFN potentially display endless scale effect on the elastic properties

But

- $l_s = k_s/E_m$ is critical for the extent of scale effect
- Depending on l/l_s the effective properties are driven by P or P_{32} DFN indicators



Thank you