Tracking bedload particle in a steep flume: methods and results.

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ABSTRACT: In this paper, we present an experimental study about the motion of bedload particles in water. Recent developments in the stochastic theory of bedload transport call for precise experimental data to validate the models. We set up the experiment in a tilted narrow flume, where we could control water discharge. The erodible bed was made of natural rounded particles of mean diameter 8 mm. Two high speed cameras, disposed next two each other, recorded the transport process at 200 frames per seconds over an observation window of approximatively 1 meter and during periods of 150 seconds. Contrasting with previous experimental study that aimed to track moving particles, we imposed ourselves three constraints: (i) all moving particles have to be tracked, (ii) tracking should be fully automatic, (iii) bedload transport rates have to be measured independently. The former constraint is mandatory to correctly compare to theoretical predictions while the second is a condition to be able to get large samples of trajectories in relatively short time periods. The latter is obviously needed to validate the algorithm. After briefly introducing the numerical treatment that allow the particle tracking, we present some interesting results about the bedload particles dynamics.

1 INTRODUCTION

Recently, the stochastic approach to the problem of the transport of bedload particles has regained the interest of the scientific community (Parker et al. 2000, Ancey et al. 2008, Singh et al. 2009, Turowski 2010, Ancey 2010, Martin et al. 2012, Furbish et al. 2012, Roseberry et al. 2012, Heyman et al. 2013, Furbish and Schmeckle 2013). This might be due to the apparent failure of the majority of deterministic based formulas in predicting bedload transport rates, both in the laboratory and in the field, within descent precision (Recking 2013). Indeed, the relative error between predicted and observed transport rates rarely drops beyond 5 in laboratories, and often reaches several orders of magnitudes in natural rivers. In the latter, this surprising unpredictability is generally attributed to the lack of precise knowledge of boundary and initial conditions (topography, grain size, shear stress) and to complex effects such as armoring (Chiari and Rickenmann 2010), segregation, dunes and bars migration (Dinehart 1992, Nelson et al. 2010). In the laboratory, that is in a precisely controlled environment, the failure of averaged bedload formulas is harder to justify. Meanwhile, time fluctuations in the transport rates are strongly interfering in the sampling of the average rates (Carey 1985, Bunte and Abt 2005, Ancey et al. 2006). The amplitude of those fluctuations is known to be large, often several time larger than the mean transport rates (Ancey et al. 2006).

The stochastic approach to bedload transport aims thus to take into account the intrinsic fluctuating nature of the transport of particles in order to derive statistically consistent averages as well as their expected fluctuations. In the recent contributions, (Furbish et al. 2012) comes up with a macroscopic definition of the solid flux based on statistical arguments. Contrarily to classical bedload flux definitions, a diffusive flux appear in the macroscopic equations, due to particle velocity fluctuations. They also presented few experiments that confirmed their findings. In the same time, (Ancey and Heyman 2013) generalized a previously published stochastic model (Ancey et al. 2008) to a spatio-temporal stochastic theory of bedload transport. In a similar manner, a diffusive flux emerges from particle velocity fluctuations. The authors showed that the particle activity, defined as the number of moving particles per unit bed area, is responsible for the large fluctuations of the bedload flux. From their stochastic model, (Heyman et al. 2013) showed that correlation in time and space could emerge from particle motions, and that the theoretical predictions compared well with previous experimental studies.

Meanwhile, the refinement of theory calls for new experimental techniques to measure the dynamics of individual particle. Indeed, the knowledge of instantaneous positions and velocities of moving particles is particularly relevant to describe completely the
In bedload transport, particles in motion have generally the same appearance of those resting on the erodible bed. Indeed, no distinction between “moving” or “resting” particles can be a priori made when looking at an individual frame. Thus, we need to consider an avoid light refraction effects) is not possible without perturbing the flow. As a consequence, we took the image sequences by the side transparent wall. The channel width being relatively small (~ 4\(d_{50}\)), moving particles were rarely occluding each other. Experimental conditions are resumed in table 1.

We used 2 cameras Basler\(^\text{a}\) A504k of resolution 1280 × 256 with 8 bits gray scale pixels. The cameras were placed next to each other so that their fields of vision overlap by a few centimetres. We used 28 mm lenses, a fair compromise between image deformation and angle of vision. In total, a one-meter long observation zone of the bed was achieved (Fig. 1). The frame rate was fixed to 200 fps so that the quickest particles could not move further than one diameter away between two frames. This parameter was found to be crucial for the particle trajectories reconstruction. A set of 4 halogen lights and diffusers were placed above the flume so that the camera exposure time could be decreased down to 0.02 seconds for a 4 mm lens aperture. As a consequence, even the fastest particles were sharply delimited on frames (Fig. 2). 328 K\(b\) are required to store each frame, so that, with 20 G\(b\) available random access memory on our computer, 30000 frames (2.5 minutes) could be acquired in once. These video sequences were taken repetitively after saving the frames in the disk.

Simultaneously, we also monitored the bedload discharge at the outlet of the flume by an indirect acoustic technique described in (Heyman et al. 2013). This technique allows for a high temporal resolution (from 0.1 seconds to days) and was systematically controlled by weighting the cumulative sediment collected. This simultaneous measurements were necessary to infer the accuracy of the processing algorithm used on video frames as shown in §2.3.

### 2.2 Image processing

The image processing consists in three main tasks: (i) Background subtraction (ii) Centroid detection and (iii) Trajectory reconstruction. Each of these tasks can be obtained by different techniques. Without pretending to be exhaustive, we will try to review the different options that offered to us.

#### 2.2.1 Background subtraction

In bedload transport, particles in motion have generally the same appearance of those resting on the erodible bed. Indeed, no distinction between “moving” or “resting” particles can be a priori made when looking at an individual frame. Thus, we need to consider an

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Table 1: \(\theta\), slope in degrees; Fr, Froude number; Re, Reynolds number; \(\tau\), Shields stress; \(\bar{h}\), mean water depth (m); \(\bar{R}_h\), Hydraulic radius; \(\bar{u}\), mean velocity (m/s); \(\bar{q}_s\), mean bedload flux (particle/s).
algorithm to split the image into moving and still regions. This is what we call “Background subtraction” (or its complementary “Foreground selection”). There are (at least) 3 methods to achieve this.

The simplest way is to subtract pixel to pixel the current frame from the immediate previous one. The resulting image is thus formed of only the pixels values that change in time (particles that moved). Although being extremely fast, this basic method raises issues when particles travels less than their own diameter, resulting in “crescent moon” particle shapes.

A better result is obtained by the technique of median filtering. A background image $B$ is updated sequentially by comparing it to the current frame $I$ according to the rules:

$$
\begin{align*}
B_{ij} & > I_{ij} \quad \text{then} \quad B_{ij} \leftarrow B_{ij} - 1, \\
B_{ij} & < I_{ij} \quad \text{then} \quad B_{ij} \leftarrow B_{ij} + 1.
\end{align*}
$$

These simple rules insure that the background image converge to the median image, that is an image where only the resting particles would appear. Furthermore, as the background of the scene evolves through time (because of erosion and deposition of particles), the median background automatically adapts itself to the changes. The foreground image is simply obtained by subtracting the median background to the current frame. The last possibility, more technical, is called mixture-Gaussian background subtraction. It consists in representing the value of each pixel as a superposition of $n$ Gaussian distributions. At each frame, a pixel is set to foreground if the probability to observe its value fall beyond a certain threshold. The gain in performance of this method compared to median filtering is not obvious, so that we chose the latter for our purposes.

2.2.2 Centroids detection

Once the foreground image has been appropriately separated from the background image, positions of the particle centroids have to be determined. One expect the foreground picture to be noisy so that image filtering has often to be apply. Two options are described in the following.

The first one consists in thresholding the foreground image and successively apply morphological operations on pixel (erosion, dilatation, closing...) to remove isolated pixels. At the end, only groups of pixels delimiting moving particles should remain. By studying the properties of these regions, the center of mass, the area, the eccentricity and orientation of the particles can be obtained. This option assumes the choice of a threshold, that has to be retrospectively adjusted. The disadvantage of this method is that when two particles collide, the thresholding operation may result in a large unique region, precluding the detection of two particles. This drawback is partially solved by using the alternative method presented below.

The second option involves a convolution filter. Recall that the convolution product is a “comparator” of two functions such that:

$$
(f \star g) = \int f(x)g(y-x)dy.
$$

In our case, $f$ is the two-dimensional foreground image while $g$ is a two-dimensional kernel that has approximately the shape of a particle. The Laplacian of Gaussian kernel is particularly suited for our purpose (Fig 2(d)). It is symmetric (rounded particles), it allows the definition of a mean (mean particle diameter) as well as a variance (variance in particle diameter). Particle centroids are obtained by finding local maxima in the convolution product. In other words, the maxima are located where the kernel fits particularly well the foreground picture, that is when a moving particle is present. The advantage of the Laplacian of Gaussian kernel (compared to a simple gaussian for instance) lies in the presence of negative values on the edges of the filter, that result in a negative “halo”
over the boundary of particles. Thus, when two particles collide, the halo prevents them to appear as a unique particle, so that two local maximums are still detected. Note that, unlike the thresholding method, none of the other properties (area, eccentricity...) can be obtain from the convolution product so that the best method would be a combination of both.

2.2.3 Trajectories reconstruction

Once a list of particles centroids through space and time is obtained, the last task is to link them between two frames to construct a unique trajectory. The issues are numerous:
- Several particles have to be tracked at the same time.
- The number of particles to track is varying in time, due to erosion, deposition and migration of particles in and out the image.
- Two particles can collide or/and overlap for a few images, leading to only one centroid detection. Although this phenomena is mitigated by using the Laplacian of Gaussian convolution kernel, it is still likely to happen. This is even more probable when the images are taken from the side wall, as in our application.
- The observation of the true position of centroids is not guaranteed over all frame, as some measurement noise can interfere in the process.
- The motion of particles is chaotic due to repeated impacts and turbulent forces.

Several tools have to be used conjointly to reduce at a minimum the error in trajectory reconstruction. We present some of them in the following. The simplest way to reconstruct a trajectory is to link a centroid to its nearest neighbour in the next frame. However, when multiple trajectories are simultaneously tracked, this does not guaranty the best combination. One would prefer an optimisation algorithm (also called Hungarian algorithm) that minimize a global cost function, defined as the sum of the distance between present particle positions and their potential candidates in the next frame. This insure that the constructed trajectories are the best in average.

To tackle the issue of errors or failures in centroid detections, the use of a Kalman filter is opportune. The Kalman filter consists in two successive steps: a prediction step where the new positions of particles are predicted according to some kinematic model, and a correction step where these predictions are compared to the observations. The kinematic model can be made as complex as wanted, but in the absence of precise knowledge about the forces involved, the simplest model is to assume a constant acceleration (constant forces acting on particles). That is, given the position $x_i^t$ of the $i$-th particle at frame $t$, the predicted position at frame $t + 1$ is given by:

$$x_i^{t+1} = x_i^t + v_i^t \Delta t + a_i^t \Delta t^2,$$

where $v_i^t$ and $a_i^t$ are particle velocity and acceleration respectively. We then compare the centroids detected in frame $t + 1$ with the previous estimations, for instance by mean of the Hungarian algorithm mentioned above. Finally the corrected position $x_i^{t+1}$ of particle $i$ is obtained by a weighted combination of the observed and the predicted position. The relative weight is given as a function of the noise expected in measurements and the expected departure from the kinematic model. This prediction-correction algorithm allows the continuous tracking of particles. When a particle centroid is missing for a few frames, the kinematic model continues to estimate its position, and eventually reconnects it with a future observation.

More general methods use Bayesian statistics to predict and update the particle positions (Schikora et al. 2011). The kinematic model is now defined as a prior probability function. The ultimate position is then obtained by maximizing the posterior function (maximum likelihood).

Fig. 3 shows a sample of particles trajectories obtained by the algorithm. Each trajectory consists in a continuous line. Some trajectories appear to be broken in several pieces, when the Kalman filter couldn’t predict accurately enough their positions. Note that, at this point, still no manual treatment has been involved.

2.3 Validation

A detailed validation of the detection algorithm is difficult. A visual check of each single trajectories is the only way to insure the accuracy of our algorithm. Unfortunately, it is infeasible when a large number of trajectories is involved. However, visual check on a few video samples can help to get an idea of the obtained accuracy. To that end, graphic interfaces like the MtrackJ plug-in for the open-source imageJ software are convenient.

As we could measure the bedload flux at the outlet of the flume at the same time as acquiring images, we can also compare the resulting bedload flux obtained
with both techniques. The instantaneous bedload flux defined as a volume average reads:

\[ q_s(t) = \frac{1}{\Delta x} \sum_{i=1}^{N} v_i \text{ [Particles/s]} \]

where \( \Delta x \) is the length of the observation window (Ancey and Heyman 2013). The cumulative transported load between two times is thus:

\[ Q = \int_{t_1}^{t_2} q_s(t) \, dt, \]

and can be directly compared to the cumulative flux obtained at the outlet of the flume (Fig. 4). Note that we do not expect a perfect agreement between the curves because the observation window was not directly touching the outlet of the flume, but was ending approximately 10 cm before. Still, the mean flux is well described and similitude in instantaneous fluctuations can be observable in both techniques.

3 RESULTS

3.1 Flux

The Exner equation, or mass conservation equation, involves the spatial derivative of the solid flux. Thus, its knowledge is of great importance for any morphodynamic model. The volume average instantaneous solid flux in an observation window of length \( \Delta x \) was defined in equation (4). It is tempting to decompose the bedload flux as:

\[ q_s = \frac{1}{\Delta x} n v \]

where \( n \) and \( v \) are two random variables for the number of moving particles in the windows and their velocities. In a similar manner as a Reynolds decomposition in turbulence, we define \( n = \bar{n} + n' \) and \( v = \bar{v} + v' \) so that

\[ q_s = \bar{n}\bar{v} + n'\bar{v} + v'\bar{n} + v'n'. \]

3.2 Velocity distribution

The velocity distribution of particles is shown in figure 6. It is fairly well described by a Gaussian probability law, of mean \( \bar{v} = 0.34 \text{ m/s} \) and standard deviation \( \sigma_v = 0.26 \text{ m/s} \). This is in agreement with the model proposed in (Ancey and Heyman 2013). Other models suggest an exponential distribution for the particle velocities (Roseberry et al. 2012, Furbish and Schmeeckle 2013). Apart from the negative velocities and the high positive velocities, the experimental probability density function does not show an evident exponential shape (a line in semi-log plot).

3.3 Diffusivity

Unlike the movement of Brownian particles that undergo uncorrelated motion, the velocities of particles shows some non-vanishing correlation over time. Moreover, as particles start from rest and eventually return to rest after some time, the velocity of a single particle exhibits periodicity. According to (Furbish et al. 2012), the effective diffusivity can be ob-
tained by calculating the variance of the particle velocity as well as the integral of its auto-correlation. However, because of the periodicity, the integral does not grow monotonically towards a constant value over long time scales.

An other method to determine the diffusivity makes use of the fact that the mean squared displacement should grow linearly with time $\langle X^2 \rangle \propto 2Dt$ (Taylor 1922). Finding $D$ is thus equivalent to fitting the particle’s mean squared displacement through time with a linear regression curve. For short time scales, the mean squared displacement shows a strong $t^2$ dependence, confirming the super-diffusive behavior due to particle velocity correlations. For scales greater than 0.5s, the mean squared displacement depends linearly on time, such that the diffusivity coefficient can be computed.

3.4 Clustering

The last interesting feature that can be highlight by our algorithm concerns spatial patterns in particles positions. It has been shown by several authors that, close to the threshold of motion, the transport of particle occurs intermittently. Indeed, (Drake et al. 1988) observed that “bursts” in particle activity were happening less than 9% of the time but were concentrating not less than 90% of the total load. He pointed that moving clusters of particles were responsible of those bursts in solid discharge.

Mathematically, this translates into correlations in particle positions. When moving particles are randomly distributed in the space, no correlation should exist between them. However, when particle clusters are present, this indicates a non-zero correlation between particles. The correlation originates from different processes, such that turbulent coherent structures or collective entrainment (Heyman et al. 2013).

A simple way to quantify this apparent correlation is the $K-$function (Ripley 1976), where $K(x)$ represents the expected number of moving particles found in a ball of radius $x$ centred at a particle centroid divided by the mean process rate. If the process is uncorrelated, we expect $K(x)$ to grow linearly with $x$. If the process is positively correlated, $K(x)$ grows quicker than linearly (Fig. 8).

As expected, the experimental $K$-function shows a positive correlation for spatial scales smaller than 30 cm. Its behavior seems to become linear at longer scales, proving that long range correlations are not involved (e.g. the integral of the spatial correlation function exists).

4 CONCLUSIONS

In this paper, we presented an automatic algorithm to track the motion of bedload particles in water. Motion tracking is a vast research area and various methods have been proposed. Their performances depend strongly on the application, such that methods performing well for a soccer ball wont automatically apply to bedload transport. The latter presents several inherent difficulties (see 2.2), such as variations in the number of particles, particle occlusions... Some of them can be avoided by filming the transport process from above, if the free surface has no influence on the flow dynamics. For extremely shallow water flow, such as steep slope flows, filming from the side wall is unavoidable.

We showed that Kalman filters, and more generally all Bayesian filters, are or great help to deal with image noise and measurements error. An a priori knowledge of some fundamental kinematics of the particle motion, like the constant acceleration model mentioned, allows a better reconstruction of particle trajectories. However, it is worst warning that those filters can also modify the apparent nature of particle motion (for instance, by smoothing strong variations) and thus, their impact on the statistical results have to be carefully assessed.

As an example, we presented an experiment carried out in a steep narrow flume. The whole observation window covered about 1 m. To the authors knowledge, this is the largest motion-picture experiment existing. Particle flux calculated from particle trajectories compared well with the flux measured at the outlet of the flume. This validates partially the tracking algorithm. The major limitation arise from the fact that particle trajectories are often broken in several pieces, so that estimation of the rate of erosion and deposition is difficult. Still, a lot of characteristics of the motion of particles can be extracted by our method such as particle flux, particle diffusivity, particle clustering...

Improving the algorithm is certainly possible, and even desirable. The use of advanced methods, such as optical flow or more complex Bayesian filters (Schikora et al. 2011) have to be considered in the future to overcome the present limitations.
Figure 7: Dispersion of particles. Deviation from the mean particle displacement for a sample of trajectories (grey). Variance of the mean particle displacement (red).

REFERENCES


