Laplace and *z*-transforms

Laplace transforms

1. Find the Laplace transform of $x(t) = te^{-at}u(t)$.

2. Find the Laplace transform $X(s)$ and sketch the pole-zero plot with the ROC for the following signals $x(t)$:

a. $x(t) = e^{-2t}u(t) + e^{-3t}u(t)$

b. $x(t) = e^{-3t}u(t) + e^{2t}u(-t)$

c. $x(t) = e^{2t}u(t) + e^{-3t}u(-t)$

3. Find the inverse Laplace transform of the following $X(s)$:

a. $X(s) = \dfrac{2s+4}{s^2+4s+3}, \mathrm{Re}(s) > -1$

b. $X(s) = \dfrac{2s+4}{s^2+4s+3}, \mathrm{Re}(s) < -3$

c. $X(s) = \dfrac{2s+4}{s^2+4s+3}, -3 < \mathrm{Re}(s) < -1$

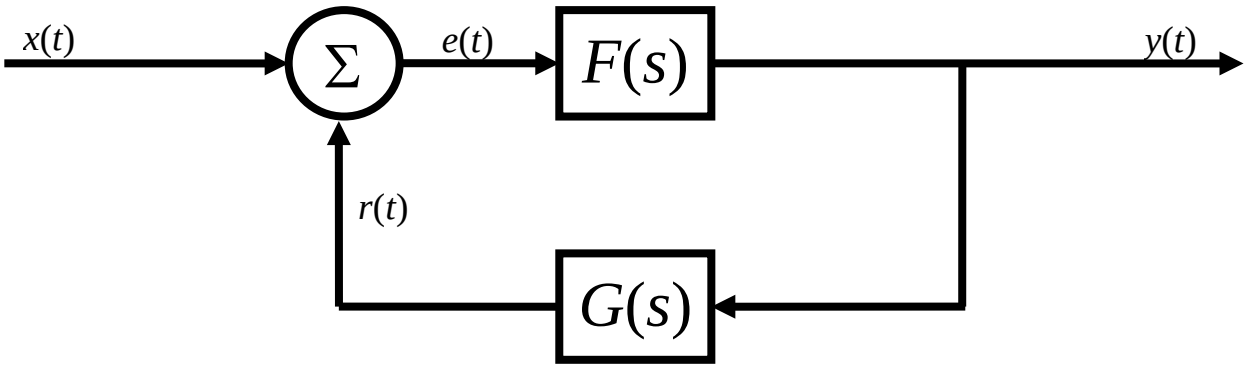
4. Consider a continuous time LTI system for which the input $x(t)$ and the output $y(t)$ are related by

$$y''(t) + y'(t) - 2y(t) = x(t)$$

a. Find the system function $H(s)$.

b. Determine the impulse response $h(t)$ for each of the following three cases : (*i*) the system is causal, (*ii*) the system is stable, (*iii*) the system is neither causal nor stable.

5. The feedback interconnection of two causal subsystems with system functions $F(s)$ and $G(s)$ is shown below. Find the overall system function $H(s)$ for this feedback system.



The element labelled Σ gives an output signal equal to the sum of the two input signals.

***z*-transforms**

6. Find the *z*-transform and the associated ROC for each of the following sequences:

a. $x[n] = \delta[n - n_0]$

b. $x[n] = na^n u[n]$

7. Find the inverse *z*-transform of.

a. $X(z) = \dfrac{z}{2z^2 - 3z + 1}, |z| > 1$

b. $X(z) = \ln\left(\dfrac{1}{1 - az^{-1}}\right), |z| > |a|$

8. A discrete time LTI system has an impulse response given by $h[n] = \{1,1,1\}$. Evaluate the output sequence $y[n]$ when the input is given by $x[n] = \{1,1,1,1\}$.