Energy Spectral Density and Correlation

Correlation

1. Compute the autocorrelation function for $x(t)=e^{-at}u(t)$. Assume that $\tau > 0$.

2. Find the energy content of the above signal by considering the energy spectral density.

(Note that $FT\left[e^{-a\tau}u(\tau)\right]=\dfrac{1}{a+j\omega}$ and $\int \dfrac{1}{1+x^2}dx=\arctan x$)

Discrete Fourier transforms and series

3. Find the discrete Fourier transform of the signal $x[n] = \{2, 0, -1, 3\}$

4. Find the discrete Fourier transform of the signal $x[n] = a^n$ where a is a constant and $0 \le n \le N-1$

5. Consider a sequence $x[n]=\sum_{k=-\infty}^{\infty}\delta[n-4k]$.

a) Sketch several periods of $x[n]$.

b) Find the Fourier coefficients c_k of $x[n]$.

Modulation and signal recovery

Amplitude modulation

1. Suppose that a message signal is given by $m(t) = \frac{t}{1+t^8}$.

a) Plot the signal.

b) Compute $x_{AM}(t)$ for 70% modulation, assuming that the carrier frequency is $f_c = 1$ Hz

c) Show the envelope corresponding to this signal.

2. Consider a system with a percent modulation equal to 50% and a system B with percent modulation equal to 90%. Find the efficiency in each case.

3. Suppose that a message signal is given by $m(t) = 4\cos(\pi t)$ and the carrier wave is $x_c(t) = 2\cos(40\pi t)$. Can envelope detection be used to demodulate the signal?

4. Find the fraction of total power contained in the sidebands if there is 100% modulation.

5. Consider the demodulation of a DSB signal $x(t) = m(t)\cos(\omega_c t)\cos(\omega_c t + \pi/2)$. What is the effect of the phase error on the output?

Signal recovery: lock-in amplifier

6. Independent work: read and summarise the paper entitled "A Frequency-Domain Description of a Lock-in Amplifier" by John H. Scofield, *American Journal of Physics* **62** (2) 129-133 (Feb. 1994). (available for download at the address http://www.oberlin.edu/physics/Scofield/pdf_files/ajp-94.pdf)