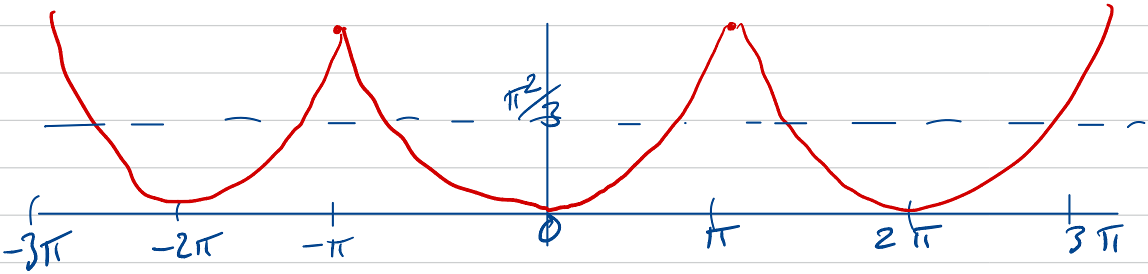
$x(t) = t^2 \quad -\pi \leqslant t \leqslant \pi$ duplicated along the time line



$T_0 = 2\pi \qquad t^2$ is an even function ie $f(x) = f(-x)$

$\sin(t)$ – odd function $\qquad \sin(t) = -\sin(-t)$

$\cos(t)$ – even " $\qquad \cos(t) = \cos(-t)$

so here $\forall n \; b_n = 0$

$$a_0 = \frac{1}{2\pi}\int_{-\pi}^{\pi} t^2 \, dt = \frac{1}{2\pi}\left[\frac{t^3}{3}\right]_{-\pi}^{\pi}$$

$$\underline{a_0} = \frac{\pi^3}{6\pi} - \frac{(-\pi)^3}{6\pi} = \boxed{\frac{\pi^2}{3}}$$

$$a_n = \frac{2}{2\pi}\int_{-\pi}^{\pi} t^2 \cos\left(\frac{2\pi n t}{2\pi}\right) dt = \frac{1}{\pi}\int_{-\pi}^{\pi} t^2 \cos(nt)\, dt$$

Integration by parts $\int f G \, dt = [FG] - \int F g \, dt$

set $f = \cos(nt) \qquad G = t^2$

$F = \frac{1}{n}\sin(nt) \qquad g = 2t$

$$\Rightarrow a_n = \underbrace{\frac{1}{\pi}\left[\frac{t^2 \sin nt}{n}\right]_{-\pi}^{\pi}}_{=0} - \frac{1}{\pi}\int \frac{\sin(nt)}{n} \cdot 2t \, dt$$

Parts again $F = -\cos(nt)$ $g = 1$

$$a_n = \frac{-2}{\pi n}\left[\frac{-t\cos nt}{n}\right]_{-\pi}^{\pi} + \underbrace{\frac{2}{\pi n}\int_{-\pi}^{\pi} \frac{-\cos nt}{n} \, dt}_{=0}$$

$$a_n = \frac{2}{\pi n^2}\left[t\cos nt\right]_{-\pi}^{\pi} = \frac{2}{\pi n^2}\left[\pi \cos n\pi - (-\pi)\cos(-n\pi)\right]$$

$$a_n = \frac{2}{\pi n^2} \times 2\pi(-1)^n = \frac{4}{n^2}(-1)^n$$

$$\boxed{x(t) = \frac{\pi^2}{3} + 4\sum_{n=1}^{\infty} \frac{(-1)^n}{n^2}\cos(nt)}$$