Curve-rational functions

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Abstract. (As in the version published online in Math. Annalen 2017) Let W be a subset of the set of real points of a real algebraic variety X. We investigate which functions $f : W \to \mathbb{R}$ are the restrictions of rational functions on X. We introduce two new notions: *curve-rational functions* (i.e., continuous rational on algebraic curves) and *arc-rational functions* (i.e., continuous rational on arcs of algebraic curves). We prove that under mild assumptions the following classes of functions coincide: continuous hereditarily rational (introduced recently by the first named author), curve-rational and arc-rational. In particular, if W is semialgebraic and f is arc-rational, then f is continuous and semialgebraic. We also show that an arc-rational function defined on an open set is arc-analytic (i.e., analytic on analytic arcs). Furthermore, we study rational functions on products of varieties. As an application we obtain a characterization of regular functions. Finally, we get analogous results in the framework of complex algebraic varieties.

I propose to explain some aspects of this work, namely that any function which is continuous rational on arcs of algebraic curves is in fact semialgebraic (and a posteriori rational). This is quite subtle since there is a continuous function on $\mathbb{R}^2$ which is semialgebraic on semialgebraic arcs but is not semialgebraic. The main argument uses a deep result from complex analysis of Siciak (in a version of Błocki), which is a real version the classical Hartog theorem about separate analyticity and somehow "forgotten" results of Bochner and Martin about separate rationality.