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Modeling the business risk of financially weakened firms: A new approach for corporate bond pricing

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Abstract

Most structural models of default risk assume that the firm's asset return is normally distributed, with a constant volatility. By contrast, this article details the properties that the process of assets should have in the case of financially weakened firms. It points out that jump-diffusion processes with time-varying volatility provide a refined and accurate perspective on the business risk dimension of default risk. Representative Arrow–Debreu state price densities (SPD) and term structures of credit spreads are then explored. The credit curves show that the business uncertainties play a major in the pricing of corporate liabilities.

JEL classification: G1; G33
Keywords: Business risk; Credit spreads; Default risk; Jump risk; Bond pricing

1. Introduction

Several approaches exist in the literature for modeling the default risk in corporate bond prices. Initiated by Black and Scholes (1973) and Merton (1974; BSM hereafter), the structural approach explains that a default occurs as the firm's asset value falls below a threshold level. This setting assumes the knowledge of the firm's financing structure as well as the process of its asset value. Avoiding arbitrage opportunities, it provides consistent and stochastic recoveries upon default. This approach is also the only way to price complex corporate debts such as convertibles (see Ingersoll, 1977). However, empirical studies have early concluded that such settings fail to price efficiently corporate

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liabilities (Jones, Mason, & Rosenfeld, 1984). Most critically, it produces short-end spreads that are too low.

The reduced form approach is the pragmatic alternative, where the instantaneous probability of default, termed as hazard rate or intensity, is directly modeled. This setting implicitly assumes that the default is a surprise: The underlying event causes a jump in the bond price. The reduced form approach provides nonzero, very short-term credit spreads. In its original form (see, e.g., Duffie & Singleton, 1999; Jarrow & Turnbull, 1995), the hazard rate process is independent of any information, except the observed term structure of credit spreads. Recent contributions relate the intensity to credit information (Jarrow, Lando, & Turnbull, 1997; Kalemanova & Schmidt, 2002) or firm-specific variables (Hübner, 1996; Madan & Unal, 1999).

The main goal of this paper is to motivate extension of the business risk model specification routinely used in the structural approach. Sudden changes, that is, unpredictable jumps as well as a random time-varying volatility, are shown to be important components of the firm's asset value process for modeling default risk of financially weakened firms.[1] By doing so, one bridges the gap between the two well-known approaches and explains the huge business risk levels that are implicitly anticipated in the short credit spreads. It is important to note that disappointing results of the structural setting have early been attributed to the underlying assumptions[2], but most of improvements rely on the hypothesis that the anticipated return of firm's asset is Gaussian, with a constant volatility. Only very few authors have questioned the underlying distribution assumption.

A notable exception is Bhattacharya and Mason (1981), who have noted the importance of jumps in pricing risky bonds. These authors have suggested a pure jump process with binomial amplitudes for the firm's asset. More recently, Kijima and Suzuki (2001) and Zhou (2001) have claimed that jump-diffusion processes are attractive because they can explain both observed jumps in bond dynamics and the different shapes of risky term structures.[3] Wong and Hodges (2002) have explored credit spreads sensitivity to systematic jumps faced by the firm. It is usually concluded that the way that business uncertainties are designed plays a major role in the pricing of defaultable securities.

This paper considers a Black–Scholes–Merton framework, where the firm's asset value follows the extended process studied by Bakshi, Cao, and Chen (1997, 2000).[4] As will become clear below, this rather general process is appealing for modeling the default risk of financially weakened firms because of its jump component and its time-varying

[1] In what follows, "financial weakness" and "financial distress" will be used interchangeably, although it can be pointed out that, rigorously, the former usually leads to the latter.

[2] Black and Cox (1976) question the bankruptcy signaling process, Shimko, Tejima, and Deventer (1993) the interest rates design, Leland (1994) both the liquidation costs and the endogenous bankruptcy decision, and Anderson and Sundaresan (1996) and Mella-Barral and Perraudin (1997) the strategic behavior.

[3] Upward sloping, flat, or downward sloping term structures have been documented (Johnson, 1967; Jones et al., 1984; Sarig & Warga, 1989), while the structural models that use a diffusion process can only generate smaller and smaller credit spreads as the time to maturity tends to zero. One must then put forward an informational imperfection (Duffie & Lando, 2001) or liquidity premia (Anderson & Renault, 2001) to explain such strictly positive instantaneous credit spreads.

[4] Whereas the above proposition develops the setting of Longstaff and Schwartz (1995).

conditional volatility. It is attractive for bond pricing because strictly positive stochastic interest rates can be considered. This specification avoids the drawbacks of many structural models where a Gaussian model for the instantaneous interest rate is used (Longstaff & Schwartz, 1995; Shimko et al., 1993; Zhou, 2001). And lastly, this process extends the one designed by Bhattacharya and Mason (1981) and nests most previous propositions as special cases.

Many rationale exist to justify the use of such a process. To summarize, the routine distribution assumption is not suitable for financially weakened firms. Because of the lack of funds, such firms cannot continuously adjust their business so as to compensate for shocks. This has two important consequences. On one hand, these firms are especially sensitive to the business regime. Their asset value is likely to be subject to sudden changes. Some of these jumps may even cause default, as assumed by Bhattacharya and Mason (1981) and Zhou (2001). On the other hand, even if there is no jump, the firm's asset volatility cannot be constant either. Financially weakened firms cannot preserve the constant risk exposure required by the investment policy of stockholders. In addition, deviations from this policy may be caused by conflicts of interest with the manager. A mean-reverting, but random, behavior for the conditional volatility can model this point.

The reminder of this paper is organized as follows. Section 2 discusses the traditional structural framework. Section 3 details the arguments to extend the usual distribution. Section 4 describes in depth the extended process assumed for the firm's asset and provides some representative state price densities (SPD). Finally, Section 5 presents analytical formulae useful for corporate debt pricing and studies the term structures of credit spreads the model can generate.

2. The traditional structural framework

Let us describe the structural approach of Black and Scholes (1973) and Merton (1974) for modeling default risk. Assume a firm whose assets are financed by equity and a single zero-coupon bond promising an amount B at maturity T. This debt has no safety covenant: Its bondholder is not allowed to force bankruptcy before maturity. Markets are continuous, perfect, and complete. As a result, agents behave rationally, and there are neither taxes nor transaction costs. The riskless term structure of interest rates is flat and equal to r. Behavior of the no-pay-out firm asset value V through time is assumed properly described under the true probability by:

$$\frac{\mathrm{d}V}{V} = \mu_{\mathrm{V}}\mathrm{d}t + \sigma_{\mathrm{V}}\mathrm{d}Z^{V} \tag{1}$$

where the constant μ_{V} represents the instantaneous expected rate of return on the firm value, σ_{V} is the volatility, and $Z^{V}=(Z_t^{V})_t$ is a standard Brownian motion. The firm's asset value is independent of the capital structure. Its volatility represents the risk level implicitly required by equity holders in their investment policy.

Any riskless bond promising an amount of B at maturity T is valued, in such a framework, by $P(t,T)=B\mathrm{e}^{-r(T-t)}$. A comparable corporate bond should be worth less.

Denoting H the credit spread required by investors for lending to this specific firm, the default risky bond is priced $B(V_t,t,T)=\mathrm{B}\mathrm{e}^{-r(T-t)-H(V_t,t,T)(T-t)}$. In a risk-neutral world, this margin also represents the securitization cost induced by the default risk exposure. This is Black–Scholes–Merton's critical argument for pricing the risky debt. Because a put option written on the firm's asset secures the considered risky debt, the credit spread is given by:

$$H(V_t,t,T) = -\frac{1}{T-t}\ln\left[\frac{V_t}{B}N(a_1) + \mathrm{e}^{-r(T-t)}N(a_2)\right] - r \tag{2}$$

where $a_i = \frac{(-1)^i a}{\sqrt{U}} - \frac{1}{2}\sqrt{U}$ with $a = \ln\frac{V_t}{\mathrm{B}\mathrm{e}^{-r(T-t)}}$, $U=\sigma_V^2(T-t)$, and N is the Gaussian cumulative density function. Note that the risk neutral probability of default is $[1-N(a_2)]=N(-a_2)$. The credit margin H is a function of the promised face value B of the bond, the current firm value, its constant volatility, the constant interest rate level, and the time-to-maturity.

Considering Eq. (2), a term structure of risky interest rates over the risk-free one is easily constructed. Computing the partial derivative of H relative to the time-to-maturity could illustrate the remark of Fama (1986) that the observed negative slope of the term structure of spreads cannot be explained by "conventional" contingent claims analysis. Some authors have also found that modeling firm value as a stochastic process, while being theoretically tractable, has not produced results consistent with observed short-term credit spreads behavior (e.g., Madan & Unal, 1999). This is the first and obvious theoretical motivation to extend the firm's asset process.

Finally, note that other functions of the underlying firm's asset value could be considered for bond valuation. For example, a corporate bond pricing formula à la Longstaff and Schwartz (1995) leads to $H_{\mathrm{LS}}(V_t,t,T)=-(1/T-t)\ln[1-\omega N(-a_2)]$ under our assumptions (except that ω is an exogenous recovery rate). This kind of formula has already been largely investigated by Briys and de Varenne (1997), Schöbel (1999), and Zhou (2001). Nonetheless, some arguments are undoubtedly in favor of the Black–Scholes–Merton approach compared with those of Longstaff and Schwartz (1995) and Zhou (2001). First, this setting is able to price complex corporate securities as subordinated convertible debt. Second, it provides a stochastic endogenous recovery rate and prevents arbitrage opportunities. Whatever the choice is, what follows can easily be adjusted for this. The arguments for extending the usual specification are now considered.

3. Arguments for an extended distribution

Arguments for extending the process of the firm's asset value are empirical as well as theoretical. First, the business of the firm depends on many assets whose behavior is rather general. It is well known, for instance, that commodity prices and exchanges rates admit nonconstant volatility, volatility clustering, jumps, leptokurtic density, and so forth. Some of these key variables for the business have even motivated developments of the ARCH class processes and other stochastic volatility models. Jump-diffusion processes are now common for modeling both commodity spot prices and the exchanges rates (see, respectively, Bates, 1996a, 1996b; Hilliard & Reis, 1998). Market indices, which can be used as proxies for the business of the firm, may have some complex behavior too. Bakshi et al. (1997), Bakshi and Chen (1997), and Scott (1997) provide evidences on the S&P 500. Firms therefore face both diversifiable and systematic risks that justify diversification or hedging strategies. One will argue below that the financially weakened firms cannot support the associated costs.

In addition to such direct observations, common stock returns may have complex dynamics, irreconcilable with a simple Gaussian distribution for the underlying firm's asset returns. Documented characteristics of individual stock returns are fat-tailed densities and volatility that change randomly over time; significant jumps have also been documented by Ball and Torous (1983) and Jarrow and Rosenfeld (1984). These evidences have been widely examined in the theory of option pricing (see Carr, Geman, Madan, & Yor, 2002; Merton, 1976 and the references herein). No consequence of these has yet been deduced for the behavior of firm's assets. Bensoussan, Crouhy, and Galay (1994) have already remarked that in the BSM framework, the variance of equity is time varying even if the underlying firm's asset volatility is constant. If jumps are considered in stocks returns, the probability density function of equity could have several modes. One can then demonstrate that this feature is incompatible with simultaneous assumptions of (1) normality for the underlying assets returns and (2) the existence of a one-to-one relation. Appendix A gives a detailed demonstration for this claim.

Other theoretical-oriented arguments can also motivate an extended distribution for a firm's asset. Succinctly, the assumption of a constant volatility is simply not realistic for financially weakened firms. First, one may suspect the financial distress to be caused by some extreme exogenous shocks. This may justify lots of nonstandard models for the business risk. Second, the financial weakness may be linked to a structural inability of the firm to face (nonextreme) exogenous market conditions. It is important to understand that constant volatility implicitly assumes that the firm can continuously rebalance and hedge its portfolio of exposures (e.g., business units) in a timely fashion. In financial distress, however, money is a scarce resource. The capacity to invest is largely diminished, if not completely destroyed. The portfolio-based management of assets cannot be as efficient as it should be. (1) The firm is unable to diversify efficiently its activity and therefore faces some diversifiable risks. (2) The firm exposure to its business cannot be timely adjusted to compensate the random business environment (even if it is accurately anticipated). (3) The firm is unable to pay any costly hedging strategy.

Hence, the expected behavior of the firm's asset value particularly depends on the specific behavior of the firm's business. On one hand, the firm will not be able to compensate sudden shocks nor avoid any jumpy fluctuations it faces. Ideally, the value process should admit a jump component. On the other hand, the random behavior of the environment (commodity prices, changes) prevents the distressed or weakened firms from having a constant risk exposure to its business as required by the equity holders through their investment policy (recall that the volatility is assumed to be chosen by them). The volatility is likely to fluctuate around the "target" level due the scarcity of fund, and this is so even if managers are willing to reach the defined objective (in terms of risk exposure). It is clear that, in addition, conflicts of interest between stockholders and managers could reinforce this deviation. Financial distress is suspected of giving strong incentives to managers to deviate from the stockholders' interests.[5] This last argument concludes that a structural model of the behavior of the firm's asset is required.

To conclude this section, the firm's asset dynamics must allow some random jumps in its intertemporal behavior both for empirical and theoretical considerations. Even in absence of such jumps, deviations from the strict level of risk exposure required by the equity holders are desired.

4. An extended process for the firm's asset

Ideally, any distribution considered for firm asset returns should be as flexible as the Gaussian one introduced by BSM. It should permit direct financial interpretations too. The process chosen here is that which Bakshi et al. (1997, 2000), Bakshi and Chen (1997), and Bates (1996a, 1996b) have used in another context. It is a geometric jump-diffusion process whose instantaneous variance, conditional on no-jump, follows a mean-reverting square-root process. The jump contribution is modeled through a Poisson shock arrival and a random lognormal amplitude. All other BSM hypotheses hold, except that several different equivalent martingale measures (EMM) can exist if the market is incomplete. The uniqueness of the EMM is no longer guaranteed without additional assumptions. For simplicity, it is assumed that the firm-jump and the volatility risks are diversifiable.[6]

4.1. The model

The risk neutral process of the firm's asset value is given by:

$$\frac{\mathrm{d}V_t}{V_t} = (r - \lambda m_{\mathrm{J}})\mathrm{d}t + \sigma_t \mathrm{d}Z^V + J_t \mathrm{d}q_t \tag{3}$$

$$\mathrm{d}\sigma_t^2 = \kappa_\sigma(\theta_\sigma - \sigma_t^2)\mathrm{d}t + \sigma_\sigma\sqrt{\sigma_t^2}\mathrm{d}Z^\sigma \tag{4}$$

$$\ln(1+J) \sim N[(\ln(1+m_{\mathrm{J}}) - \frac{1}{2}\sigma_{\mathrm{J}}^2; \sigma_{\mathrm{J}}^2] \tag{5}$$

where Z^V and Z^σ are standard uncorrelated Brownian motions and dq a Poisson counter with a constant intensity parameter λ.[7] λ, Which is positive, may be interpreted as the annual frequency of jumps. J is the jump amplitude conditional on a jump. This jump amplitude (expressed in percentage and conditional on a jump occurring) is supposed lognormal. The jump shocks and its amplitude are uncorrelated with the two Brownian

[5] This point has not been studied in detail in the valuation literature. It deserves a more detailed study.

[6] Hence, one implicitly assumes here that the firm cannot blow these risks away because of its financial weakness. In the case of systematic jumps, Wong and Hodges (2002) conclude that a jump-diffusion specification preserves its mathematical structure. See also Bates (1991) and Zhou (2001, p. 2021) for related discussion.

[7] The probability to see more than one jump in a very small period is assumed to be null. Otherwise, $\Pr[\mathrm{d}q=1]=\lambda \mathrm{d}t$, $\Pr[\mathrm{d}q=0]=1-\lambda \mathrm{d}t$.

motions. In views of this assumption, the total variance of asset return is simply the sum of variances. It should be noted, finally, that one of the key properties of the chosen process is its asymmetry.

Contrasting with BSM, the business risk has now several distinct sources of uncertainty. The jump contribution models the sudden changes of the firm's asset value. In our setting, it represents unanticipated news about industry specific events or the firm specific activity (see Footnote 6). It can also be used to reflect anticipated large moves in returns, which are caused by the fact that the firm is unable to avoid future outcomes by efficiently diversifying or adjusting its business. The values chosen for the intensity and amplitude of the jumps must be consistent with the current state of the firm. The value of λm_{J} has a direct impact on the deterministic behavior of the firm's value. Depending on the sign of the jump amplitude, it lowers or increases the probability of being below a given threshold in the future, that is, lowers or increases the probability of default. In the case of a financial distressed form, jumps are typically numerous (high intensity) and of dramatic negative amplitude. The volatility σ_{J} reflects the structural profile of jumps. Although technically unpredictable, the jumps' effect becomes clear as σ_{J} goes to zero. If $\sigma_{\mathrm{J}}=0$, the jump amplitude is known, as in Bhattacharya and Mason (1981).

The diffusion reflects the variety of "usual" time-varying conditions (sales, prices, financial and real market conditions, etc.) that the firm must face. The instantaneous variance of the diffusion is assumed to be stochastic. Nevertheless, it is assumed that there are only limited departures from a known "target" volatility level $-\theta_\sigma$: The variance process is chosen to be mean reverting, with a variance warranting its positiveness. In light of the discussion in the previous section, the long-run mean of the diffusion variance θ_σ stands for the true level of exposure required by stockholders to their investment policy. As a first step, it is the value routinely used in the BSM framework. $\theta_\sigma-\sigma^2$ is then interpreted as the current distance to this policy. κ_σ is the speed of adjustment, the force to converge to θ_σ. Depending on the modeling choice, κ_σ can have many different interpretations. For instance, let us assume that the manager strictly applies the demanded investment policy. Deviations from the investment policy can only be attributed to the financial weakness. Then, κ_σ stands for the ability of the firm to attain its investment objectives in the absence of external unanticipated shocks. As a result, the parameter κ_σ^{-1} measures the structural weakness, that is, the structural inability to reach the required level. If, instead, one supposes that the manager intentionally deviates from his instructions, κ_σ^{-1} quantitatively represents his own discretionary power.

4.2. Some representative Arrow–Debreu SPD

The risk neutral probability density function (p.d.f.) chosen for the firm's asset return is expected to mimic some specific properties. In particular, it must be able to generate different levels of skewness, kurtosis, and even some multimodality (see the previous section). Following Bakshi et al. (2000), one uses Arrow–Debreu SPD to illustrate the possible shapes for the firm's asset p.d.f. To quote them: "Each parametric model is associated with an Arrow–Debreu SPD. [...] A way to examine alternative models is to compare their implicit SPD." To this end, one exploits the relation existing between the p.d.f. and the characteristic function defined by $\Psi(u)=E_t[e^{iur_{\mathrm{V}}}]$ where r_{V} denotes the firm

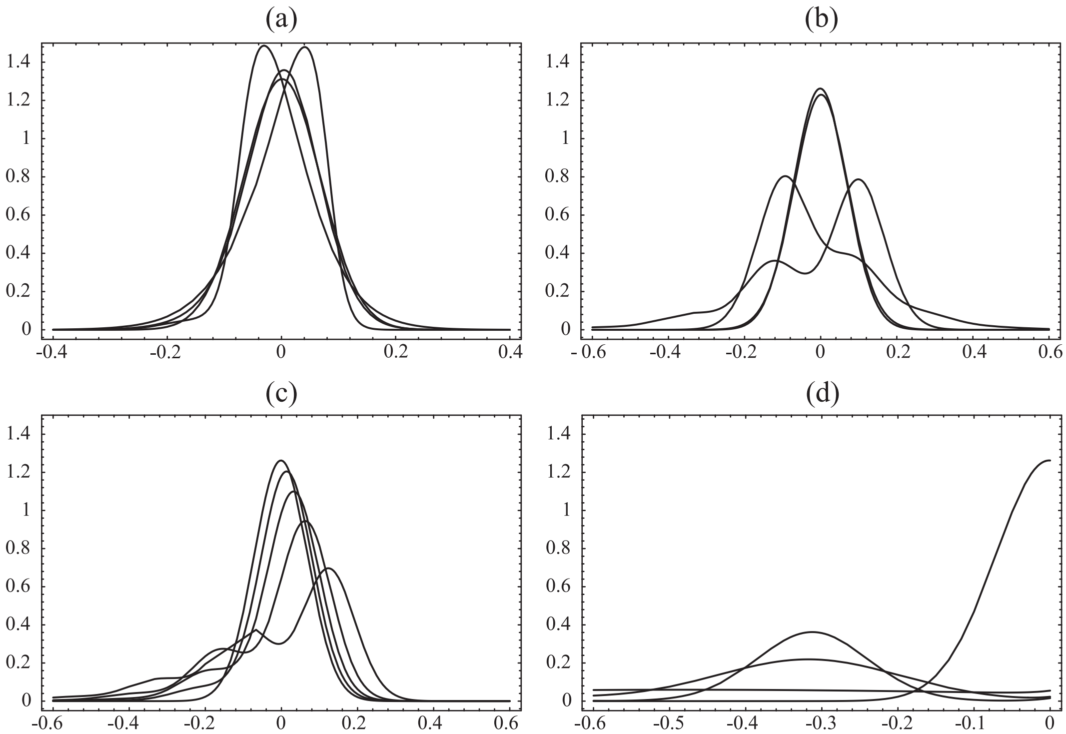

Fig. 1. State price densities for the firm's assets with comparison to a strict Gaussian one. (a) The correlation effects ($\rho = -0.8 \mid 0 \mid 0.8$). $\rho = 0$ in our business risk model. (b) The jump size effects ($\mu_J = -0.2 \mid 0 \mid 0.2$). (c) The jump frequency effects when $\mu_J = -0.2 (\lambda = 0.5 \mid 1.25 \mid 2.5 \mid 5)$. (d) The jump volatility effects on the left tail ($\sigma_J = 0.25 \mid 0.5 \mid 1 \mid 2$). For checking and comparison, parameters of Panel (a) are the ones of Bakshi et al. (2000, p. 289), note that a pay-out rate is included. Base case parameters for Panels (b), (c), and (d) are $r = 3\%$, $V = 450$, $\sigma_V = 20\%$, $\kappa_\sigma = 1$, $\theta_\sigma = 0.2^2$, $\sigma_\sigma = 20\%$, $\lambda = 4$, $\sigma_J = 3\%$, and $\tau = 45/345$.

asset return. Once this latter is available, the corresponding p.d.f. is uniquely defined by the Inverse Fourier Transform of $\Psi(u)$, that is, $\frac{1}{2\pi}\int_{-\infty}^{\infty}\mathrm{Re}[\mathrm{e}^{-izu}\Psi(u)]\mathrm{d}u$. A standard reference for such a numerical inversion is Abate and Whitt (1992). Technically, one computes:

$$\frac{1}{\pi}\int_{0}^{\infty}\mathrm{Re}[\mathrm{e}^{-(iu+1)\ln S_\tau}\Psi_2(V_0,r_0,\sigma_0^2,0,\tau;u)]\mathrm{d}u \tag{6}$$

where Ψ_2 is given in Appendix B.

Fig. 1 plots SPD obtained for different sets of parameters and the benchmark density coming from the usual Gaussian framework of Black–Scholes–Merton. These sets are chosen to illustrate the interest and flexibility of the considered dynamics (interest rates are therefore held constant). Parameters are inspired from Bakshi et al. (2000) to allow comparison and checking. The graphs contain much of interest, but one focuses here on several remarks. Graph (a) plots the SPD obtained for different levels of correlation between the assets diffusion and its stochastic volatility. This graph illustrates that the correlation effect, which is zero in our specification, has anyway a minor impact. Graph (b) plots two SPD that have the same absolute jump size but with different signs, in addition to the benchmark and the "zero-jump amplitude" SPD. Graph (c) considers the different jump frequencies with a negative mean jump size. Graph (d) modifies the jump size volatility and illustrates the possible dispersion around the second mode. These graphs clearly illustrate the way the chosen dynamics can generate skewed, leptokurtic, and multimodale SPD.

5. Applications to the corporate debt valuation

The firm's asset value has been assumed to follow a jump-diffusion process with, on one hand, an instantaneous conditional (on no jump) variance following a mean-reverting square-root process and, on the other hand, a lognormal jump amplitude. This distribution appears sufficiently flexible to price risky debt, notably because interest rates may also be assumed stochastic and constrained to strict positive values.

5.1. An analytical pricing formula for corporate bonds

Following BSM's line, the key point for pricing any corporate liability is valuing options written on the underlying firm's asset value. In addition to the previous flexible density for firm's asset value, a square-root process, à la Cox, Ingersoll, and Ross (1985), is chosen to warranty nonnegativity. Under such a specification, the instantaneous interest rate is given by:

$$\mathrm{d}r = \kappa_r(\theta_r - r)\mathrm{d}t + \sigma_r\sqrt{r}\mathrm{d}Z^r \tag{7}$$

where the Brownian motion is uncorrelated to the previous ones. The firm's asset process (Eq. (4)) is not, however, independent from the interest rate level because of its drift. The interest rate level converges instantaneously to its long-run mean θ_r with a coefficient speed κ_r. Its instantaneous volatility is proportional to the square root of its level. The condition $2\kappa_r\theta_r \geq \sigma_r^2$ ensures zero not to be attainable. Denoting $D(a,\tau)$=$((1-e^{-a\tau})/a)$, the riskless bond promising an amount of one dollar at T is then valued by:

$$P(r_t, t, T) = e^{C(T-t)-D(T-t)r_t} \tag{8}$$

where $C(\tau) = -(\kappa_r\theta_r/\sigma_r^2)[(\zeta-\kappa_r)\tau + 2\ln W(\zeta,\kappa_r,\tau)]$, $D(\tau) = D(\zeta,\tau)W^{-1}(\zeta,\kappa_r,\tau)$, $W(\zeta,\kappa_r,\tau) = 1 - 1/2(\zeta-\kappa_r)D(\zeta,\tau)$, and $\zeta^2 = \kappa_r^2 + 2\sigma_r^2$.

Bakshi et al. (1997, 2000) provide quasi-explicit formulae for European option prices in such a framework. In our structural setting, the price of a risky debt is straightforwardly given by:

$$B(V_t, r_t, \sigma_t^2, t, T) = V_t Q_1 + BP(r_t, t, T)Q_2 \tag{9}$$

where:

$$Q_j = \frac{1}{2} + (-1)^j \frac{1}{\pi} \int_0^\infty \mathrm{Re}\left[\frac{e^{-iu\ln B}\Psi_j(V_t, r_t, \sigma_t^2, t, T; u)}{iu}\right] du.$$

For ease of exposition, Ψ_j is given in Appendix B. $1 - Q_2$ is the risk-neutral probability of default in our setting.

5.2. Associated term structures of credit spreads

This point investigates the large set of term structures of credit spreads (both in level and in shapes) that the rich business risk specification is expected to generate.[8] By extending Eq. (1), the credit spread may be computed by the following formula:

$$H(V_t, r_t, \sigma_t^2, t, T) = -\frac{1}{T-t}\ln\left[\frac{V_t}{B}Q_1 + P(r_t, t, T)Q_2\right] - R(t, T) \tag{10}$$

where $R(t,T)$ is the continuously compounded interest rate at time t for a term of $T-t$ in the Cox et al. (1985) framework. Term structures of credit spreads are plotted in Fig. 2.

In Fig. 2, one focuses on the jump parameters with regard to the financial risk (defined as pseudo debt ratios). As noted in Footnote 5, the study of the effect of the deviations from investment policy deserves its own investigation; it is left for future research. Each graph plots term structures of credit spreads for a given pseudodebt ratio, a given mean jump size, and various jump frequencies. The very short end of term structure is one day, whereas the longest maturity is 20 years. One explores three different corporate financing structures. Pseudodebt ratios are 90%, 70%, and 50% respectively in Panels (a) and (b), (c)

[8] A companion paper, available upon request, explores implications of the chosen specification on the usual risk management tools.

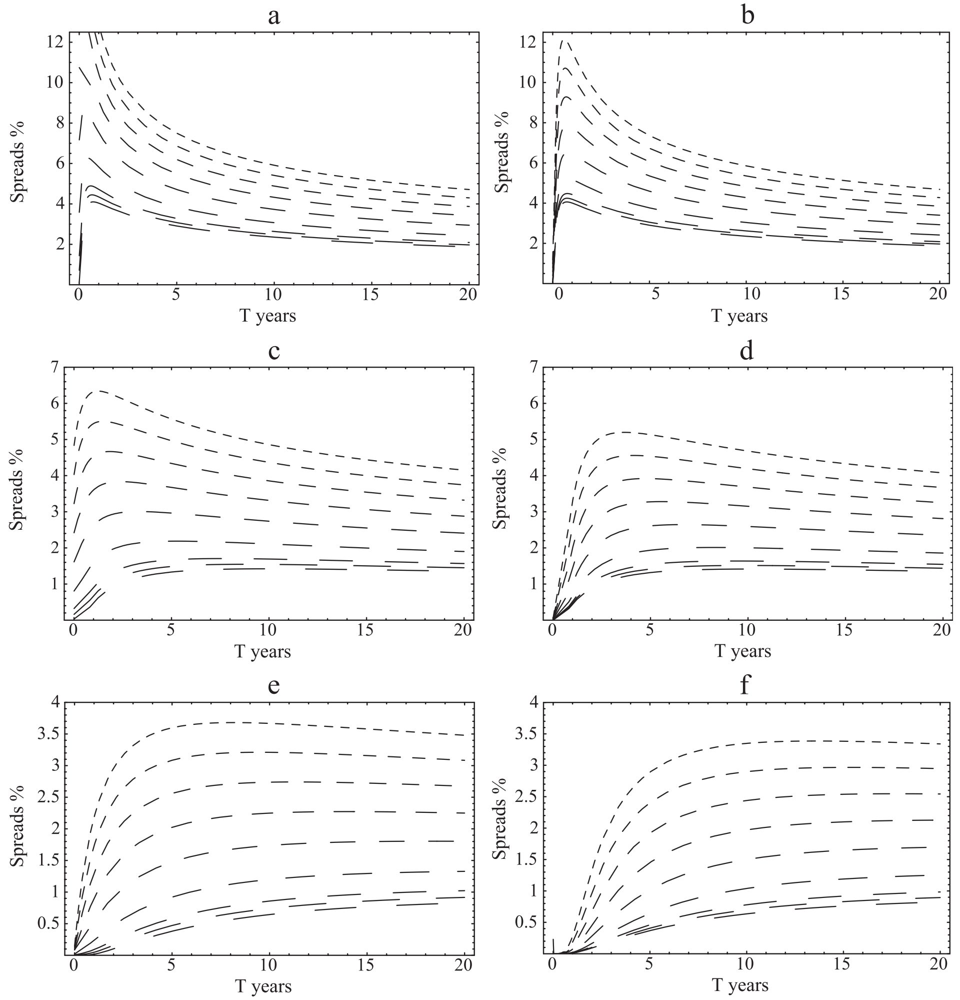

Fig. 2. Term structures of credit spreads. Each graph plots term structures of credit spreads for a given pseudo debt ratio, a given mean jump size and various jump frequencies. Panels (a) and (b), (c) and (d), and (e) and (f) illustrate credit spreads for pseudo debt ratios that equal 90%, 70%, and 50%, respectively. On Panels (a), (c), and (e), that is, the graphs on the left, the mean jump size is negative ($\mu_J = -0.2$), it is positive ($\mu_J = 0.2$) on the right (Panels b, d, and f). The nine jump frequencies (from the dashed lines to the dotted one) are 1%, 5%, 10%, 25%, 50%, 75% and then 1%, 1.25%, 1.5%. The very short end of the term structures is one day, whereas the longest maturity is 20 years. Other base case parameters are $r = 3\%$, $\kappa_r = 1$, $\sigma_r = 4\%$, $\theta_r = 0.2^2$, $\kappa_\sigma = 1$, $\theta_\sigma = 0.2^2$, $\sigma_\sigma = 20\%$, and $\sigma_J = 20\%$.

and (d), and (e) and (f). These are different levels of financial risk. For graphs on the left (Panels a, c, and e), the mean jump size is negative, while it is positive on the right (Panels b, d, and f), but its absolute value is the same. The nine different jump frequencies range between 1% and 1.5%. Corresponding term structures are represented by successive dashed lines, the dotted one being associated to the most frequent jump.

It is worth noting some of the crucial features of Fig. 2. First, as the jumps (in business) become more frequent, the credit spreads increase (from the dashed lines to the dotted term structures in each graph). Second, as the firm becomes financially more weakened (from the bottom graphs to the top ones), credit spreads get larger. Third, and as expected, credit spreads are greater when the jumps have a negative impact rather than a positive one (left graphs compared with the right ones). Fourth, comparing Panels (a) and (b) to Panels (e) and (f) allows us to conclude that the jump component plays a more critical role when firms are financially weakened. If one considers, for instance, 5-year credit spreads, the lowest one on the graphs (Panels a and b; i.e., the one implied by the smallest frequency) is larger than the greatest one on the graphs (Panels e and f). Finally, Panel (a) shows that term structures may be strictly decreasing for some jump frequencies, given the shortest maturity of one day. Such a downward sloping shape is known to be impossible in BSM structural models for "reasonable" parameters (Sarig & Warga, 1989). To conclude, introducing a jump component in the firm's asset dynamics increases the level of credit spreads (and this, whatever its contribution is) and provides some shapes of credit curves that are unattainable in the usual BSM for reasonable values of structural parameters.

6. Conclusion

The structural approach for modeling default risk initiated by Black and Scholes (1973) and Merton (1974) has already been generalized to account for stochastic interest rates, liquidation costs, early default signaling, and so on. In this paper, business risk is recalled to be an important determinant of corporate debt valuation. Both empirical and theoretical motivations are provided to relax the usual Gaussian distribution framework. An extended business risk model is then suggested. Focusing on the corporate debt pricing, analytical formulae are derived and computed, thanks to standard Inverse Fourier Transforms. Numerical simulations show that a jump component significantly impacts on the price of corporate debt issued by financially weakened firms.

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Appendix A. Empirical constraints for the theoretical distribution of firm's asset

In the core text, one claims that jumps are regularly observed in stocks returns. Their probability density function (p.d.f. hereafter) may therefore have several modes. It can be shown, then, that this is necessarily the case for the underlying firm asset in a BSM framework. The key point is that the likelihood function of a transformed variable must have the same number of modes than the one of the underlying variable when the transformation is differentiable and bijective.

To show this, let us denote $(f_\theta^V)_{\theta\in\Theta}$ [resp.$(f_\theta^E]_{\theta\in\Theta}$ the parameterized p.d.f. of the underlying firm's asset V (resp. is the firm equity E), θ stands for the set of parameters. Both p.d.f.'s are assumed continuously differentiable with respect to their variable. The structural approach of BSM implies that the observed equity price is related to the underlying firm's asset by:

$$E = \mathrm{BSM}_\theta(V) \Leftrightarrow V = \mathrm{BSM}_\theta^{-1}(E)$$

where BSM is the price of a vanilla call option. This price defines a one-to-one relation between unobserved variable (the firm's asset value) and the transformed one (the firm equity) for every set of parameters $\theta\in\Theta$. Let us denote J_{BSM}=$(\mathrm{dBSM}_\theta/\mathrm{d}V)$ the Jacobian of this transformation. This hedging ratio J_{BSM} is known to be increasing and bounded, it is therefore bijective. The change of variable formula then tells us that f_θ^V and f_θ^E are related to each other by:

$$f_\theta^V(v) = |J_{\mathrm{BSM}}(v)| f_\theta^E[\mathrm{BSM}_\theta(v)]$$

which, in turn, implies that:

$$f_\theta^E(u) = f_\theta^{\mathrm{BSM}_\theta(V)}(u) = |J_{\mathrm{BSM}}[\mathrm{BSM}_\theta^{-1}(u)]|^{-1} f_\theta^V[\mathrm{BSM}_\theta^{-1}(u)]$$

Hence, one obtains a simple relation between the p.d.f. of the observed and unobservable variables: Their shapes are closely related to each other. If multimodale densities (jumps) are appropriately suggested for individual equities, one must also consider some for the underlying firm asset value.

Appendix B. The analytical formula for the characteristic function

Here is the expression for Ψ_j needed to compute Eqs. (6) and (10):

$$\begin{aligned}\ln\Psi_j(V,r,\sigma^2,t,T;u) = {} & iu\ln V - 1_{j=2}\ln P(r,t,T) - iu\lambda m_{\mathrm{J}}(T-t)\\ & +\lambda(T-t)(1+1_{j=1}m_{\mathrm{J}})[\mathrm{e}^{iu\ln(1+m_{\mathrm{J}})+\frac{iu}{2}(iu-(-1)^j)\sigma_{\mathrm{J}}^2}-1]\\ & -\frac{\kappa_r\theta_r}{\sigma_r^2}\Pi(r,j,t,T) + r(iu-1_{j=2})\Gamma(r,j,t,T)\\ & -\frac{\kappa_\sigma\theta_\sigma}{\sigma_\sigma^2}\Pi(\sigma,j,t,T) + \frac{1}{2}\sigma^2[iu-(-1)^j]\Gamma(\sigma,j,t,T)\end{aligned}$$

Where

$$\Gamma(z,j,t,T) = D(\zeta_{z,j},T-t)[1-\xi_{z,j}D(\zeta_{z,j},T-t)/2]^{-1}$$

$$\Pi(z,j,t,T) = \xi_{z,j}(T-t) + 2\ \ln[1-\xi_{z,j}D(\zeta_{z,j},T-t)/2]$$

with $\zeta_{\sigma,j}^2 = \gamma_{\sigma,j}^2 - iu[iu-(-1)^j]\sigma_\sigma^2$, $\zeta_{r,j}^2 = \kappa_r^2 - 2(iu-1_{j=2})\sigma_r^2$, $\xi_{\sigma,j} = \zeta_{\sigma,j} - \gamma_{\sigma,j}$, $\xi_{r,j} = \zeta_{r,j} - \kappa_r$, and $\gamma_{\sigma,j} = \kappa_\sigma - (1_{j=1} + iu)\rho_{V\sigma}\sigma_\sigma$.