INTRODUCTION

When debt holders face a default payment, they have to decide whether or not to exercise their option to liquidate the assets of the distressed firm. By rescheduling their debt, they grant the firm a chance to face its obligations later. Creditors can also forgive due payment or proceed to a deep financial reorganization by swapping debt for equity. Empirically speaking, maturity extension is however a very common form of distressed loans modification (see Asquith, Gertner and Scharfstein (1994) and Mann (1997)). This approach can appear sufficient when the distressed firm is economically viable. Longstaff (1990) also points the avoidance of immediate and significant liquidation costs as a sound incentive for debt holders to grant a delay. Harding and Sirmans (2002) demonstrate that the extension technique align the

Business Risk Targeting and Rescheduling of Distressed Debt

Franck MORAUX*¹, Patrick NAVATTE**

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interests of debtors and creditors better than other methods. Whatever the motivation, all parties benefit from debt rescheduling.

This paper considers the equity holders’ behavior before a default to come. As long as debt holders possess no covenant limiting their creativity, debtors can take initiatives to act in the best of their interests. Because owners of the firm have only a limited liability, common wisdom suggests that the appropriate policy before liquidation is to take maximum risk. The picture is not so clear however when debt rescheduling is possible. The volatility of the firm’s assets belongs indeed to the set of variables used by debt holders to choose the extension maturity of rescheduled debt. So equity holders can be better off by adjusting the business risk appropriately.

This paper examines the volatility adjustment that may occur before the issuance of rescheduled debt. It contrasts with previous works that explore what happens *ex post* a debt issuance or how claimants bargain *during* financial reorganization. From these contributions, it is well understood that equity holders have incentives to behave strategically *ex post* the issuing of a corporate liability and this, in most cases, at the expense of debt holders. Among others, Jensen-Meckling (1976), Leland (1998), Ericsson (2000) and Décamps and Faure-Grimaud (2000) insist on the asset substitution problem. Leland (1994) adds that debtors can decide not to refund the firm after the debt issuance. Anderson and Sundaresan (1996), Mella-Barral and Perraudin (1997) study cases where debtors propose take-it-or-leave-it offers based on the debt service to creditors. Mella-Barral (1999) investigates dynamics of default and debt reorganization. To our knowledge, the period preceding the issuance of rescheduled debt has not been considered.

To keep things tractable, this paper analyses first a parsimonious context and then develops it in different directions. In the simplest setting, anything can happen almost instantaneously at the maturity of the debt contract. Debt holders can reschedule their debt optimally; whereas equity holders can set the firm’s asset volatility in the best of their interests. The challenge here is to solve a simultaneous optimization problem. We show that equity obtained in rescheduling is a quasi-concave function of the firm’s business risk level and that there exists for equity holders an optimal volatility to target. Clearly, this property contrasts with the common wisdom that equity holders are risk loving
in absence of monitoring devices (see Myers (1977), Green (1984)). We also find that debt holders do not necessarily suffer from the implied business risk targeting. Rather, creditors can only face opportunity costs. The precise picture depends on the firm’s contemporaneous asset volatility and the implied shift. We show that, when opportunity costs exist, magnitudes are comparable to agency costs faced by creditors in case of more classical asset substitution.

We then develop the parsimonious framework in a couple of directions. The first one investigates the complex relations existing between the time-to-maturity and the volatility equity holders should target. In a first step, we find that, at any point in time, the equity price is a not-so-simple function of the firm’s asset volatility in terms of convexity. Simulations reveal however that a certain business risk level can still appear more valuable than other alternatives some times before maturity. When time-to-maturity decreases and potential rescheduling become more and more perceptible, equity holders have incentives to shift the business risk of the firm to a finite level. In lines with intuition, as the time-to-maturity lengthens, standard features are recovered and the maximum risk possible should be favoured by equity holders. In a second step, we question the precise timing of business risk shifting. We propose a couple of adjustment schedules that depend on how equity holders understand they can benefit from rescheduling. Differences appear however minor since both imply a regime switch in the firm’s asset volatility some time before contractual maturity. As a final step, we examine effects of an early meeting triggered by creditors that can lead to early financial restructuring.

The second direction addresses influences of stake holders on the other’s decision process since both equity holders and debt holders may affect one another. In case of negotiation, moreover, every one has really a word to say on the decisions to change the business and to extend the maturity of the distressed debt. These influences are embedded in a Nash bargaining game with different levels of bargaining power. The way, we proceed here, is inspired from Fan and Sundaresan (2000). Numerical experiments demonstrate that a business risk targeting is still realistic.

The rest of this paper is organized as follows. Section 1 introduces the parsimonious set up; describes claims created by rescheduling and presents the general model for business risk targeting. Section 2 deve-
lops, in a simple context, the business risk targeting model and its various consequences. There, we study the way the equity price depends on the firm’s asset volatility and reveal the existence of an optimal business risk level. We quantify various effects of such a risk shift and highlight some potential opportunity costs faced by debt holders. Section 3 develops section 2 to investigate the intricate relation between time-to-maturity and business risk shifting. We examine whether equity holders are better off by adjusting the business risk long before the maturity of the debt contract. Section 4 explores the effect of an early meeting triggered by creditors. Section 5 introduces reciprocal influences on the decisions to adjust business and reschedule debt. We solve numerically several asymmetric generalized Nash bargaining problems. Section 6 offers concluding remarks on the opportunity to renew the business risk targeting program.

1. THE FRAMEWORK

Our analysis adopts the continuous time framework of Black, Scholes and Merton (1973, 1974). Financial markets are perfect, complete and trading takes place continuously. There exists a riskless asset paying a known and constant interest rate denoted by $r$. There are neither taxes, nor transaction costs nor, for the moment, liquidation costs. We consider a risky leveraged firm with a simple capital structure. The firm is financed by equity and a single debt whose maturity is $T_1$ and face value $F_1$. We assume that debt holders own a covenant preventing asset sales before debt maturity. The firm’s asset value at time $t$ is denoted by $V_t$ and its process is assumed to be correctly described, under the risk neutral measure, by:

$$dV = rV dt + \sigma_1 V dW$$

(1)

$W$ is a Brownian motion and $\sigma_1$ denotes the firm’s assets volatility. $\sigma_1$ stands for the level of business risk that prevails before maturity. Assuming now that things are going on until the maturity, the payoff at $T_1$ for equity holders is that of a call option ($\max(V_{T_1} - F_1; 0)$). For their part, in absence of liquidation costs, debt holders receive either
the promised face value if \( V_{T_1} \geq F_1 \) or the value of the firm’s assets (for short \( \min(V_{T_1}; F_1) \)).

### 1.1. A parsimonious model for debt rescheduling and its consequences on the pricing of corporate securities

In presence of bankruptcy costs, the liquidated value received by debt holders is strictly lower than the assets’ value. Denoting by \( \beta \in [0,1] \) the realization rate, this value is equal to \( \beta V_{T_1} \). To alleviate bankruptcy costs (equal to \( (1-\beta) V_{T_1} \)), debt holders can accept to reschedule their debt, e.g., by granting a delay. Longstaff (1990) demonstrates that there exists an optimal extension period \( \tau \). His procedure is based on creditors’ wealth which mainly depends on the firm’s assets value at time \( T_1 \) i.e. the default severity. By contrast, our own procedure highlights the key importance of both \( V_{T_1} \) and the volatility at time \( T_1 \). Under the risk neutral measure \( Q \), the net gain function is given by:

\[
H(V_{T_1}, \sigma_1, T_2) = e^{-r(T_2-T_1)} E_{T_1}^Q[\beta V_{T_1} 1_{V_{T_2}<F_1} + F_1 1_{V_{T_2} \geq F_1}] - \beta V_{T_1} \tag{2}
\]

Unnecessary references to the face value and the interest rate have been omitted. This function evaluates, at time \( T_1 \) the swap of the known payoff \( \beta V_{T_1} \) for the new debt that promises at \( T_1 + \tau \) if \( V_{T_1+\tau} \geq F_1 \) and \( \beta V_{T_1+\tau} \) if otherwise. Straightforward computations then yield:

\[
H(V_{T_1}, \sigma_1, T_2) = -\beta V_{T_1} + \beta V_{T_1} N[-d_1, V_{T_1}/F_1(T_2-T_1)] + F_1 e^{-r(T_2-T_1)} N[d_2, V_{T_1}/F_1(T_2-T_1)] \tag{3}
\]

where \( d_{1,x}(t) = \frac{\ln x + (r + \frac{1}{2} \sigma^2) t}{\sigma \sqrt{t}}, d_{2,x}(t) = d_{1,x}(t) - \sigma \sqrt{t} \) and \( N \) is the standard cumulative distribution function. The function \( H \) involves the firm’s asset value, the level of business risk, the default severity and liquidation costs that prevail at the time of default. It is a concave function of \( T_2 \) and may serve as an objective function. The optimal extension maturity is obtained by computing:
\[ T(V_{T_1}, \sigma_1) = \arg \max_{t \in [T_1, \infty]} H(V_{T_1}, \sigma_1, t) \] (4)

The price of the rescheduled debt is then evaluated by\(^2\):

\[ D^R_{T_1}(V_{T_1}, \sigma_1) = H(V_{T_1}, \sigma_1, T(V_{T_1}, \sigma_1)) + \beta V_{T_1} \] (5)

The rescheduling also leads to the creation of a new claim received by equity holders at time \( T_1 \). Because our set up is similar to that of Black, Scholes and Merton, this claim resembles a call option except that its expiration is given by Equation 4. Its price at time \( T_1 \) is given by:

\[ E^R_{T_1}(V_{T_1}, \sigma_1) = \text{call}(V_{T_1}, \sigma_1, T(V_{T_1}, \sigma_1) - T_1) \] (6)

Pricing formulae for corporate securities any time before \( T_1 \) can then be derived. If we denote by \( Q_t(V_{T_1}, \sigma_1) \) the risk neutral density of \( V_{T_1} \) conditional on its \( t \)-value \( V_t \), the \( t \)-time price of the equity is:

\[ E_t(V_t, \sigma_1) = \text{call}(V_t, \sigma_1, T_1 - t) + e^{-r(T_1 - t)} \int E^R_{T_1}(V_{T_1}, \sigma_1) 1_{\{V_{T_1} < F_1\}} Q_t(V_{T_1}, \sigma_1) dV_{T_1} \] (7)

and the debt price is:

\[ D_t(V_t, \sigma_1) = F_t e^{r(T_1 - t)} [d_2, V_t, F_1(T_1 - t)] + e^{-r(T_1 - t)} \int D^R_{T_1}(V_{T_1}, \sigma_1) 1_{\{V_{T_1} < F_1\}} Q_t(V_{T_1}, \sigma_1) dV_{T_1} \] (7')

These two equations are important for our analysis. We can note that the first term of the right hand side of equation 7 is the Black, Scholes and Merton’s equity price whose theoretical expiration is time \( T_1 \). The second term is the (discounted) expected value of the claim received at time \( T_1 \) from rescheduling (see also equation (6)). In the same way, the

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\(^2\) the superscript \((R)\) means “rescheduled” whereas the subscript puts some emphasis on time \((T_1)\).
first term of equation 7' stands for repayment in full of the debt contract. The second term replaces the usual expected value of the liquidated asset obtained in case of default$^3$.

Both equations highlight the way rescheduling affects security prices. If the firm is in trouble some time before debt maturity, the standard call option in equation 7 and the first term of equation 7' can be worth little compared to the component implied by rescheduling. They also indicate that the firm’s asset volatility at time $T_1$ is an important variable for share holders’ wealth.

1.2. A general model for business risk targeting driven by potential debt rescheduling

The previous subsection assumes that only rescheduling happens at time $T_1$. This means that share holders are inactive during the period preceding debt maturity. This is not completely satisfactory since equity holders can anticipate problems with full repayment of their liability and the future debt rescheduling$^4$. Because the firm’s asset volatility is a critical variable of their wealth, they can question the opportunity to adjust the business risk level so as to benefit from their position (remind that asset sales are proscribed by the covenant held by creditors). We assume a single adjustment of the business risk because of the financial distress. This volatility targeting may be viewed as being driven by future potential rescheduling. To emphasize the role of the firm’s asset volatility $\sigma$, the equity price and the debt price have been denoted by $E_t(V_t, \sigma)$ and $D_t(V_t, \sigma)$ respectively.

We know that $\sigma$ impacts prices directly and indirectly through, e.g., the extended maturities $T(V_t, \sigma)$. When debtors decide to adjust the volatility at time $t$, they choose the level of business risk so as to maximize the equity price at that time. Properties of the equity price with respect to the volatility must be explored to find the potential optimal level of business risk $\sigma^*_t$. This value satisfies the first-order condition

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3. The equation 7 was first derived by Longstaff (1990) in his study of extendible options. He discussed potential applications to the corporate context somewhat anecdotally. In another paper, we develop the equation 7' and examine both equations in the corporate environment.

4. Creditors can act too but this point is considered later.
\[ \frac{\partial E_t(V_t, \sigma)}{\partial \sigma} \bigg|_{\sigma=\sigma_t^*} = 0 \] and requires that \( E_t(V_t, \sigma) < E_t(V_t, \sigma_t^*) \) for any other admissible volatility \( \sigma \). Whether and to which extent they are worth adjusting the firm’s assets volatility are questions we treat in different environments. Note that we also investigate the role of debt holders’ behaviour given that they possess no covenant to prevent the activism of equity holders on the unobservable business risk of the firm.

It is important to point out that our model for business risk targeting makes several implicit assumptions. First, there is no agency problem between equity holders and the management of the firm so as to put emphasis on the relation between equity holders and creditors. Second, the firm’s management is able to render operational desired changes in asset volatility. Third, adjustment costs are not considered. Note that we discuss this issue in the appendix. In the following sections, we investigate whether equity holders can benefit from their position by finding an appropriate volatility \( \sigma_t^* \) before maturity. We proceed by enriching a simple setting which first considers a business risk adjustment at time \( T_1 - \varepsilon \) for small value of \( \varepsilon \).

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5. Throughout the paper, we discuss equity properties in the range of admissible values for the business risk level of the firm. These values are commonly retained in the quantitative corporate finance literature since the work of Black, Scholes and Merton (1973, 1974).

6. Other questions of interest concern the impact of the remaining time-to-maturity and the appropriate timing of adjustment, this latter being clearly the most delicate issue. Because we consider in this paper the volatility adjustment as being driven by rescheduling, we adopt a pragmatic backward-style approach for decision making. We acknowledge that the timing issue could deserve a more specific paper because the equity holders’ behaviour does not consist in optimizing future rescheduling only. By contrast, they deal with a broad equity price (denoted by \( E_t(V_t, \sigma_t, \sigma) \)) that embeds, among other things, the option to adjust the firm’s business risk level later i.e. at any time before maturity (\( s \in [t, T_1] \)). Finding an optimal adjustment time means finding the optimal exercise of an American-like option. And, the option to delay the adjustment time is probably an important element to account for.
2. BUSINESS RISK TARGETING IN A SIMPLE SETTING

Throughout the paper, we assume that equity holders try to take advantage of their position by adjusting the firm’s asset volatility given that creditors will design the rescheduled debt in the best of their interest (i.e., as explained in subsection 1.1). In the simple setting of this section, equity holders prepare reorganization during the period immediately preceding the default (which in turn implies debt rescheduling). Hence, we consider the period \([ T_1 - \varepsilon, T_1 ]\) with \(\varepsilon\) a rather small value such that \(V_{T_1 - \varepsilon} \approx V_{T_1} < F_1\).

At time \(T_1 - \varepsilon\), by virtue of equations 6 and 7, the equity is theoretically worth:

\[
E_{T_1 - \varepsilon}(V_{T_1 - \varepsilon}, \sigma_1) = \text{call}(V_{T_1 - \varepsilon}, \sigma_1, \varepsilon) + e^{-r\varepsilon} \int \text{call}(V_{T_1}, \sigma_1, T(V_{T_1}, \sigma_1) - T_1) 1\{V_{T_1} < F_1\} Q_{T_1 - \varepsilon}(V_{T_1}, \sigma_1) dV_{T_1} \quad (8)
\]

where \(Q_{T_1 - \varepsilon}(V_{T_1}, \sigma_1)\) is the risk neutral density of \(V_{T_1}\) conditional on its \((T_1 - \varepsilon)\)-value. Since \(\varepsilon\) is rather small, default and rescheduling are certain \((V_{T_1 - \varepsilon} \approx V_{T_1} < F_1)\) and substantial simplifications arise. We find: \(e^{-r\varepsilon} \approx 1\), \(Q_{V_{T_1} < F_1 / V_{T_1 - \varepsilon}} \approx 1\) and:

\[
dQ(V_{T_1}) \approx \delta\{V_{T_1} = V_{T_1 - \varepsilon}\} \quad (9)
\]

where \(\delta\) is the Dirac measure. As a result, the first term of equation 8 becomes negligible:

\[
\text{call}(V_{T_1 - \varepsilon}, \sigma_1, \varepsilon) \quad (10)
\]

while the second term vanishes to:

\[
E_{T_1 - \varepsilon}(V_{T_1 - \varepsilon}, \sigma_1) \approx E^R_{T_1 - \varepsilon}(V_{T_1 - \varepsilon}, \sigma_1) \approx \text{call}(V_{T_1 - \varepsilon}, \sigma_1, T(V_{T_1 - \varepsilon}, \sigma_1) - T_1). \quad (11)
\]
2.1. The existence of the optimal business risk

As explained in section 1, the existence of the optimal business risk level depends on properties of the equity price. Figure 1 plots equity prices described by equation 11 as a function of the business risk for different values of $\beta$. Base case parameters are as follows. The face value, due at time $T_1$, is worth $F_1 = 40$. Without loss of generality, the firm’s asset value is set to $V_{T_1-\varepsilon} = 24$ and the level of the interest rate to $r = 6\%$. The quasi leverage ratio is equal to $F_1 e^{-r\varepsilon} / V_{T_1-\varepsilon} \approx 40/24 = 1.666$. For illustration purposes, seven values for the realization rate $\beta$ are considered between 20% and 80% and volatility ranges from 2.5% to 22.5%.

As expected, rescheduling is a good arrangement for equity holders since they get a strictly positive value whatever the business risk is. We observe that the equity price is lower and lower as the realization rate increases. A reason for this is that the optimal extension maturity is a strictly increasing function of liquidation costs $1 - \beta$. The main point of this figure is that the equity price is not a straightforward function of volatility as usual. The graph displays a quasi concave structure with respect to the firm’s assets volatility. The humped shape is clearly in contradiction with usual conclusions of the Black-Scholes and Merton approach stating that equity is a strictly increasing function of the underlying firm’s assets volatility. Here, equity is affected differently by the level of business risk because volatility also influences the extended maturity $T(V_{T_1}; \sigma_1)$.

A business risk targeting procedure is then equivalent to the calculation:

$$\sigma^* \approx \arg\max_{\sigma \in \mathbb{R}^+} \left[ E_{T_1-\varepsilon}(V_{T_1-\varepsilon}, \sigma) \right]$$  \hspace{1cm} (12)

7. Bris, Welch and Zhu (2006) have recently reported very significant changes in firm’s assets value in liquidation and low recovery for creditors. They write, page 1264, that “Chapter 7 assets drop by at least 20% in mean and 62% in median”: This implies very low values for the realization rate since the corresponding mean and median are respectively 80% and 38%! In the next sentence, they pursue by noting that, “assuming our overly pessimistic reported-only creditor recovery, the median chapter 7 dissipates substantially all its assets even before fee are paid” and find, page 1287, that: “in about half of our 30 Chapter 7 liquidations, secured creditors receive nothing. [...] the mean recovery is 32%”. They finally enlighten, page 1289, that “unsecured creditors receive nothing in 95% of our Chapter 7 cases”. 

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where one must calculate:

\[ T(V_t, \sigma) = \arg\max_{t \in [T_1, \infty]} H(V_{t-}, \sigma, t) \approx T(V_{t-}^{\varepsilon}, \sigma) \]  

(13)

Equity holders can identify the optimal business risk to the extent that a unique solution to the above optimization program exists. The quasi concave feature of \( E_{T_1 - \varepsilon}(V_{T_1 - \varepsilon}, \sigma) \) with respect to \( \sigma \), observed in Figure 1, suggests that the optimal business risk level may be found as the finite value \( \sigma_{T_1 - \varepsilon}^* \) such that:

\[ \left. \frac{\partial E_{T_1 - \varepsilon}(V_{T_1 - \varepsilon}, \sigma)}{\partial \sigma} \right|_{\sigma = \sigma_{T_1 - \varepsilon}^*} = 0 \]  

(14)

As a matter of fact, an analytical expression exists for this derivative, but the dependence of the extension maturity to volatility makes it really complex. It does not permit any closed form solution for \( \sigma_{T_1 - \varepsilon}^* \) either. This is the reason why we favour a numerical approach and

**Figure 1.** - The price of the new equity when the maturity is optimally chosen by debt holders

The graph plots the price of the claim received by equity holders at reorganization as a function of business risk and for different values of \( \beta \). The extension maturity is set optimally by debt holders. \( \beta \) ranges from 20% to 80%. Parameters are \( V_{T_1} = 24 \), \( F_1 = 40 \) and \( r = 6\% \).
conclude the existence of a finite optimal business risk level for all the sets of parameters we investigate.

2.2. Effects of targeting business risk

The target business risk level has a direct impact on the price of rescheduled debt. We now investigate debt price sensitivity to the business risk level and whether business risk targeting is made at the expense of debt holders. Adjustment of the initial business risk profile \( \sigma_1 \) towards the optimal volatility \( \sigma^*_T - \varepsilon \) leads to either an upward or a downward movement. And the magnitude \( |\sigma^*_T - \varepsilon - \sigma_1| \) can be more or less significant.8

Figure 2 is very similar to Figure 1 except that we deal with the price of rescheduled debt. This graph shows that this is a decreasing function of the business risk (everything else being equal). Hence, debt holders will benefit or suffer from the modification of the business risk profile depending on the way the business risk is shifted towards the target level... If the initial level of business risk \( \sigma_1 \) is lower (resp. greater) than \( \sigma^*_T - \varepsilon \), equity holders will shift upward (resp. downward) the current level of risk and debt holders will be worse off (resp. better off).

In view of Figure 2, we can conclude that creditors can either face opportunity costs or benefit from opportunity gains. So the business risk targeting is not necessarily achieved at the expense of debt holders. In some cases, both parties benefit from it.

In our set up, the reason for this is clear. Debt rescheduling is not a zero-sum game. The firm’s total value at time \( T_1 \) is not invariant to the firm’s asset volatility. By neglecting \( \varepsilon \), the firm’s total value \( v_{T_1} \) verifies \( v_{T_1}(V_{T_1}, \sigma_1) = V_{T_1} - BC(V_{T_1}, \sigma_1) \) with bankruptcy costs equal to \( BC(V_{T_1}, \sigma_1) = (1 - \beta)V_{T_1}N[-d_1(V_{T_1}/F_1(T(V_{T_1}, \sigma_1) - T_1)]. \)

\( v_{T_1} \) depends on volatility both directly and indirectly via the optimal

8. As for equity, the impact of the business risk level on the price of rescheduled debt can be studied formally with the derivative of \( D_{T_{1-\varepsilon}}(V_{T_{1-\varepsilon}}, \sigma) \) with respect to \( \sigma \). Here again, an analytical expression exists for this. However, due to the indirect impact of \( \sigma \) on the extension maturity \( T(V_{T_1}, \sigma) = \arg \max_{t \in [T_1, \infty]} H(V_{T_1}, \sigma, t) \), it is not straightforward to analyze. And we favour a numerical illustration again.
extension maturity. An optimal financial rearrangement may be worthwhile for all parties. The next paragraph places more emphasis on the price effects of shifting volatility from $\sigma_1$ to $\sigma^{*}_{T_1 - \epsilon}$.

2.3. Opportunity costs or opportunity gains?

This business risk targeting is worthwhile for equity holders, but it may have some positive and negative effect on the debt price. In order to study the magnitude of these latter opportunity gains or costs, we proceed as follows. We assume that equity holders shift the volatility of the firm’s assets from $\sigma_1$ to $\sigma^{*}_{T_1 - \epsilon}$ and we assess the impacts by means of a couple of measures. The first measure is the difference of prices defined by:

$$S(V_{T_1}, \sigma^{*}_{T_1 - \epsilon}, T(V_{T_1}, \sigma^{*}_{T_1 - \epsilon})) - S(V_{T_1}, \sigma_1, T(V_{T_1}, \sigma_1)),$$

the second one is the relative price difference, computed by

$$\frac{S(V_{T_1}, \sigma^{*}_{T_1 - \epsilon}, T(V_{T_1}, \sigma^{*}_{T_1 - \epsilon})) - S(V_{T_1}, \sigma_1, T(V_{T_1}, \sigma_1))}{S(V_{T_1}, \sigma_1, T(V_{T_1}, \sigma_1))},$$

where $S$ is the price of the considered claim. In both measures, the price prevailing before business risk targeting is the benchmark price.

So, when we apply these measures to the equity price, a positive value
can be directly interpreted as the magnitude of the incentives for equity holders to change the “actual” level of business risk.

Figure 3 gathers together two different graphs which focus on a specific measure. The upper graph plots price differences (in, say, euros) whereas the lower graph considers relative price differences by percentage. The abscissa displays different magnitudes for risk shifting.

The graphs plot the price difference and the relative price difference for equity and debt as a function of the initial volatility expressed as a percentage of the target business risk. The initial volatility ranges from $\frac{1}{3}\sigma^*$ to $\frac{5}{3}\sigma^*$. For the claim $S$, the price difference is computed as $S(\sigma^*) - S(\sigma_1)$ and the relative price difference by $(S(\sigma^*) - S(\sigma_1))/S(\sigma_1)$ (this latter being expressed in percentage). Parameters are $V_{T_1} = 24$, $F_1 = 40$, $\beta = 60\%$ and $r = 6\%$, the extension maturity being set optimally by debt holders.

*Figure 3. – Impact of the business risk adjustment and potential opportunity costs*
The shift is defined with the help of a variable $u$ ranging from $-\frac{2}{3}$ to $+\frac{2}{3}$ (expressed by percentage in the graphs). We set $\sigma_1 \equiv \sigma_{T_1-\epsilon}^*(1 + u)$ to center the abscissa on the target risk level. A negative value for $u$, say $-10\%$, means that the value is actually 10%-lower than the optimal level and that equity holders will shift the volatility upward. A positive value for $u$, say $+10\%$, means that the value is actually 10%-greater than the optimal level and that equity holders will shift downward the business risk of the firm.

Figure 3 offers different insights. We can first check that when the initial firm’s asset volatility $\sigma_1$ is equal to the optimal $\sigma_{T_1-\epsilon}^*$ (i.e. $u = 0$), equity holders have no incentive to make any changes in the firm’s business and that, in all the other cases, equity holders benefit from changing the business risk level. When the initial volatility is lower ($\sigma_1 < \sigma_{T_1-\epsilon}^* \iff u < 0$), debtors are better off increasing the business risk level. This is especially perceptible on the lower graph that indicates an increase of more than 40% of the initial equity price. This benefit decreases as the modification tends to be moderate, i.e. as the initial $\sigma_1$ is initially closer to $\sigma_{T_1-\epsilon}^*$. Almost symmetrically, debt holders suffer from such risk shifting. The negative values observed on both graphs are measures of the opportunity costs faced by creditors.

When the initial volatility is 66% lower than the optimal one, absolute costs are about 4 times higher than the equity holders’ gain. In relative values, however, risk shifting appears less detrimental for debt holders than it is beneficial for equity holders. When the initial volatility is greater than the optimal level of business risk ($\sigma_1 > \sigma_{T_1-\epsilon}^* \iff u > 0$), the situation is rather different since both parties benefit from business risk targeting. Compared to equity holders, debt holders are even better off in absolute values! In relative values, however, the gain is comparable. To conclude, we retain that business risk targeting can induce opportunity costs but it can also align interests of both parties. A natural question arising now is the economic significance of opportunity costs.

2.4. Do opportunity costs really matter?

To answer whether opportunity costs matter, we compare them to costs resulting from the asset substitution problem ex post reorganiza-
tion. From standard analysis of corporate finance, unexpected post-default asset substitution leads to some agency costs for lenders. This context involves some degree of information asymmetry. It is well known that such risk shifting is beneficial for equity holders at the expense of debt holders. Of course, agency costs can be avoided indirectly by an appropriate covenant. Bhanot and Mello (2006), e.g., have recently suggested a trigger clause to monitor the rating of the debt. They serve however our purpose by providing benchmarks.

In view of previous sections, $\sigma_{T_1-\epsilon}^*$ is the level of business risk used by debt holders to compute extension maturity, i.e. to design the rescheduled debt. This level may or may not be equal to the pre-default level of a firm’s asset volatility $\sigma_1$. When $\sigma_1 < \sigma_{T_1-\epsilon}^*$, debt holders face some opportunity costs, that can be measured by relative price differences. After debt rescheduling, equity holders can (once again) increase the volatility of the firm’s assets (say, to $\sigma_2$). We assume that they act immediately after reorganization. The price of the new claims may be rewritten $E(V_{T_1},\sigma_2,T(V_{T_1};\sigma_{T_1-\epsilon}^*))$ and $D(V_{T_1},\sigma_2,T(V_{T_1};\sigma_{T_1-\epsilon}^*))$ to highlight the different volatilities. Agency costs implied by the upward shift of the business risk can then be assessed by a relative difference of debt prices. We compute

$$\frac{D(V_{T_1},\sigma_2,T(V_{T_1};\sigma_{T_1-\epsilon}^*)) - D(V_{T_1},\sigma_{T_1-\epsilon}^*,T(V_{T_1};\sigma_{T_1-\epsilon}^*))}{D(V_{T_1},\sigma_{T_1-\epsilon}^*,T(V_{T_1};\sigma_{T_1-\epsilon}^*))}$$

where $D(V_{T_1},\sigma_{T_1-\epsilon}^*,T(V_{T_1};\sigma_{T_1-\epsilon}^*)) = D^R(V_{T_1},\sigma_{T_1-\epsilon}^*)$.

Table 1 provides and compares opportunity costs versus agency costs. Opportunity costs are implied by business risk targeting while agency costs are caused by the classical ex post asset substitution problem. Both consider comparable shifts in business risk. Shifts are computed by $(\sigma_{T_1-\epsilon}^* - \sigma_1)/\sigma_{T_1-\epsilon}^*$ and $(\sigma_2 - \sigma_{T_1-\epsilon}^*)/\sigma_{T_1-\epsilon}^*$ as percentage of the optimal business risk. Other parameters are $V_{T_1} = 34$, $F_1 = 40$ and $r = 6\%$.

Table 1 shows magnitudes of respective costs. As expected, we find negative values for every relative price difference we compute. We can observe in both cases, that the larger the risk shifting is, the more significant costs are. The same is true for liquidation costs. The main point is that opportunity costs appear larger than agency costs. In view of this, we can conclude that opportunity costs are economically significant.
The above simple setting can be extended in different ways. We proceed in successive steps. The first and second steps question whether it is profitable for equity holders to adjust business risk long before the maturity of the debt contract. The next step addresses the reciprocal influence of stakeholders on the other’s decision process. The final one addresses whether rescheduling can be granted several times. Adjustment costs are discussed in an appendix.

3. BUSINESS RISK ADJUSTMENT AND THE REMAINING TIME TO MATURITY

We know from the above section that a business risk adjustment (just before maturity) may lead to a better position for equity holders during the rescheduling process. This section examines the intricate relation existing between business risk adjustment and the remaining time to maturity. Different questions emerge about the profitability for
equity holders to adjust the volatility some time before the contractual expiration of the corporate debt, and on a potential “time to maturity” effect on the desirable volatility. The first sub-section explores potential effects of the remaining time to maturity on the existence and the magnitude of the target volatility. The second sub-section discusses the timing of the adjustment. Hereafter, we denote by $\theta_{AT}$ the adjustment time with $\theta_{AT} < T_1$; the business risk adjustment is no longer contemporaneous to rescheduling\(^9\).

3.1. **Time to maturity effects on the equity price**

This sub-section investigates potential “time to maturity” effects on the price of equity. By virtue of equations 7, the price of equity at time $\theta_{AT} < T_1$ is described by:

$$E_{\theta_{AT}}(V_{\theta_{AT}}, \sigma) = \text{call}(V_{\theta_{AT}}, \sigma, T_1 - \theta_{AT})$$

$$+ e^{-r(T_1 - \theta_{AT})} \int_0^T E^R_{T_1}(V_{T_1}, \sigma) 1_{V_{T_1} < F_1} Q_{\theta_{AT}}(V_{T_1}, \sigma) dV_{T_1} \quad (15)$$

where $Q_{\theta_{AT}}(V_{T_1}, \sigma)$ is the risk neutral probability density function of $V_{T_1}$ conditional on its value $V_{\theta_{AT}}$ at time $\theta_{AT}$.

Equation 15 highlights two distinct components in the wealth of equity holders. The first term corresponds to the standard call option they hold (even) in absence of rescheduling. The second one is the (discounted) weighted sum of claims they receive at time $T_1$ if the debt is rescheduled ($E^R_{T_1}(V_{T_1}, \sigma)$). It is well known from option theory that the first component (the standard call option) is a strictly increasing function of the level of business risk $\sigma$. By contrast, section 2 suggests that, individually, each claim of the weighted sum (i.e. each $E^R_{T_1}(V_{T_1}, \sigma)$) is a quasi-concave function of $\sigma$ in the sense that it first increases, admits a maximum and then strictly decreases. Overall, the global property of equity is not easy to appreciate, so we experiment simulations of equation 15.

\(^9\) In section 2, $\theta_{AT}$ was more or less assimilated to maturity in the sense that $\theta_{AT} = T_1 - \epsilon \approx T_1$ ($\epsilon$ was assumed rather small). In the next section, creditors will hasten reorganization in response to such an adjustment.
We can note that the debt price is similarly given by:

\[
D_{\theta AT}(V_{\theta AT}, \sigma) = F_1 e^{-r(T_1 - \theta_{AT})} N\left[d_2, \frac{V_{\theta AT}}{F_1} (T_1 - \theta_{AT})\right] \\
+ e^{-r(T_1 - \theta_{AT})} \int D_{T_1}^R (V_{T_1}, \sigma) 1_{\{V_{T_1} < F_1\}} Q(\theta_{AT}(V_{T_1}, \sigma)) dV_{T_1} \quad (15')
\]

Simulations of this equation can illustrate consequences of a business risk targeting on the creditors’ wealth.

Graphs of Figure 4 plot the equity price (on the left) and the debt price (on the right) as functions of the underlying business risk level for different times to maturity. Here debt holders extend the maturity of their debt at time \(T_1\) only if necessary. Structural parameters are those of Figure 1. The time to maturity \(T_1 - \theta_{AT}\) ranges from a quarter (i.e. 0.25) to 2 years. In both graphs, the continuous line depicts the case \(T_1 = \theta_{AT}\) i.e. that of Figure 1.

Figure 4 demonstrates that prices of corporate securities are more complex functions of the firm’s volatility, in the sense that their convexity clearly depends on the remaining time period \(T_1 - \theta_{AT}\). Observed shapes are direct consequences of the two components highlighted in equations 15 and 15’. We mainly comment hereafter those of the equity price; the right graph highlighting situations where debt holders can suffer from shifting.

The upper graph mainly shows a couple of things. First of all, for moderate volatilities, prices are not much different from one another and approximate those computed in section 2. Second, when the remaining time to maturity \(T_1 - \theta_{AT}\) is not zero, equity prices are locally concave with respect to volatility. Hence, the volatility revealed by the hump is not a global optimal business risk level (anymore) but only a local solution. The reason for this is unambiguous. The standard call is a strictly increasing function of volatility and, for \(T_1 - \theta_{AT} \gg \varepsilon > 0\), it cannot be neglected in equation 15. So there always exists a high volatility that implies a higher equity price than the one observed at the hump. For many \(\theta_{AT} - T_1\), however, this local solution appears plausible because alternative levels of business risk are (operationally) questionable. As times to maturity get short, the business risk level revealed by the hump becomes effectively valuable. For longer times to maturity, the standard call induces a rather pronounced positive slope (with respect to volatility) for large volatilities.
The equity price conforms here with standard implications of the Black, Scholes and Merton framework: equity holders are better off increasing the business risk level as much as possible\textsuperscript{10}. Overall, simulations suggest that there is, ceteris paribus, no significant “time to maturity” effect on the volatility to target before rescheduling but that the timing of business risk targeting is a key decision to make.

\textbf{Figure 4. – The price of liabilities at adjustment time}

These graphs plot prices of the different corporate liabilities as a function of the business risk for different values of time to maturity \( T_1 - \theta_{AT} \). Extension maturity is set optimally by debt holders at time \( T_1 \). \( T_1 - \theta_{AT} \) ranges from 2 years to a quarter (0.25). Parameters are: \( V_{\theta_{AT}} = 24 \), \( F_1 = 40 \), \( \beta = 60\% \) and \( r = 6\% \).

\textsuperscript{10} For very long time-to-maturity, equity recovers standard properties in term of convexity because the second component of equation 15 vanishes.
3.2. On the timing of business risk targeting

Whether there exists an optimal timing for business risk targeting is now the natural question. To answer this precisely, it is necessary to model and price the American option held by equity holders and solve the associated optimal exercise problem. Clearly, this is challenging. A simpler approach consists in finding *ad hoc* criteria to answer slightly different questions such as “how long before maturity could it be profitable for equity holders to adjust their business?” or “is current time a suitable time to adjust business?” Such a pragmatic approach essentially neglects the value of delaying the volatility adjustment. We now suggest a couple of methods in this vein.

The first method we consider is a two-step approach. It consists in a) questioning if the current time is appropriate for a volatility targeting and then b) finding the appropriate volatility. Given a contemporaneous volatility \( \sigma_1 \), equity holders can conclude that it is time to adjust business if their current wealth (defined by the equation 7) rests mainly on potential rescheduling rather than full debt repayment. The adjustment time \( \tilde{\theta}_{AT} \) then verifies:

\[
\text{call}(V_{\tilde{\theta}_{AT}}, \sigma_1, T_1 - \tilde{\theta}_{AT}) < e^{-r(T_1 - \tilde{\theta}_{AT})} \int_{E_{T_1}}^{} E_{T_1}^R(V_{T_1}, \sigma_1) 1_{\{V_{T_1} < F_1\}} Q_{\tilde{\theta}_{AT}}(V_{T_1}, \sigma_1) dV_{T_1} \quad (16)
\]

This criterion involves the contemporaneous firm’s asset volatility \( \sigma_1 \) so that the adjustment time is a function of it (\( \tilde{\theta}_{AT} = \tilde{\theta}_{AT}(\sigma_1) \)). It applies before any business risk adjustment and defends the idea that equity holders decide to make the adjustment as soon as they understand that they are better off with rescheduling (even without being in an optimal context of business risk). By denoting \( E_{\tilde{\theta}_{AT}}(V_{\tilde{\theta}_{AT}}, \sigma_1) \) the total wealth of equity holders, the previous condition is equivalent to:

\[
e^{-r(T_1 - \tilde{\theta}_{AT})} \int_{E_{T_1}}^{} E_{T_1}^R(V_{T_1}, \sigma_1) 1_{\{V_{T_1} < F_1\}} Q_{\tilde{\theta}_{AT}}(V_{T_1}, \sigma_1) dV_{T_1} > \frac{1}{2} E_{\tilde{\theta}_{AT}}(V_{\tilde{\theta}_{AT}}, \sigma_1).
\]

Once \( \tilde{\theta}_{AT} \) given, equity holders must find an optimal volatility \( \sigma^*(\tilde{\theta}_{AT}) \).
The second method we propose exploits findings of Figure 4 and characterizes $\tilde{\theta}_{AT}$ and $\sigma^*_{\tilde{\theta}_{AT}}$ simultaneously. To this end, we consider a specific firm whose operational activities imply a certain maximum level for business risk $\sigma_H$ as in Leland (1998) and Ericsson (2000). By virtue of equation 7 and Figure 4, equity holders have chosen $\sigma_H$ as proper business risk level long before maturity. As the time-to-maturity decreases, equity holders question the opportunity of a business risk targeting. They can think that a shift from $\sigma_H$ to the optimal business risk level $\sigma^*_{\tilde{\theta}_{AT}}$ is worthwhile at time $\tilde{\theta}_{AT}$ if the following equation is verified:

$$E_{\tilde{\theta}_{AT}}(V_{\tilde{\theta}_{AT}}, \sigma^*_{\tilde{\theta}_{AT}}) = E_{\tilde{\theta}_{AT}}(V_{\tilde{\theta}_{AT}}, \sigma_H)$$

Equation 17 states that equity holders understand at time $\tilde{\theta}_{AT}$ that they should not maximize the firm’s risk as suggested by the standard approach.

The way rescheduling is taken into account in equation 17 may appear less explicit than in equation 16. However, both approaches lead to a regime switch in volatility before maturity and both are driven by the perspective of rescheduling. We illustrate in Table 2 the different timing implied by the above two criteria. This table supports the intuition that the more severe the financial distress is, the earlier the adjustment is. It is interesting to note that adjustment times appear rather similar and that no criterion is systematically earlier than the other.

<table>
<thead>
<tr>
<th>Firm Value</th>
<th>First approach</th>
<th>Second approach</th>
</tr>
</thead>
<tbody>
<tr>
<td>$\sigma_H = 20%$</td>
<td>$\sigma_H = 30%$</td>
<td>$\sigma_H = 20%$</td>
</tr>
<tr>
<td>$V = 34$</td>
<td>1.11</td>
<td>0.56</td>
</tr>
<tr>
<td>$V = 30$</td>
<td>1.98</td>
<td>1.05</td>
</tr>
<tr>
<td>$V = 24$</td>
<td>3.64</td>
<td>1.98</td>
</tr>
</tbody>
</table>

This table compares the timing for business risk adjustment implied by the criteria defined by equations 16 and 17. Other parameters are $F_1 = 40$, $\beta = 60\%$ and $r = 6\%$. 

4. THE EFFECT OF AN EARLY MEETING

In the setting of section 3, equity holders adjust the business risk level before debt maturity with no debt holders’ reaction. This section weakens this assumption by letting creditors trigger an early meeting after the adjustment time. We denote by \( \theta_{RT} \) the time of early meeting triggered by creditors \((\theta_{AT} \leq \theta_{RT} \leq T_1)\) and make assumptions for sake of tractability. If at time \( \theta_{RT} \), \( V_{\theta_{RT}} < F_1 \), creditors conclude to default, trigger a financial reorganization and reschedule their debt earlier than \( T_1 \). Otherwise, i.e. if \( V_{\theta_{RT}} \geq F_1 \), nothing happens and creditors cannot trigger any other meeting until maturity.

Creditors are more or less rapidly aware of the volatility adjustment. A couple of arguments may explain the delay measured by \( \Delta \theta = \theta_{RT} - \theta_{AT} \). On the one hand, debt holders don’t know exactly how and when debtors will proceed. Section 3 illustrates that there is no straightforward timing and that equity holders may consider an extraneous criterion to solve this challenge (see e.g. equations 16 and 17). On the other hand, nor the firm’s assets volatility nor its adjustment is immediately observable by debt holders. Creditors can monitor the firm’s asset value but the value effects of the volatility shift is not necessarily instantaneously visible. Overall, the difference between \( \theta_{AT} \) and \( \theta_{RT} \) is related to the existing information asymmetry between parties and the ability of creditors to monitor the firm’s activity.

By virtue of this discussion, the equity price \( E^{\theta_{AT}, \theta_{RT}} \) verifies at time \( \theta_{AT} \):

\[
E^{\theta_{AT}, \theta_{RT}}(V_{\theta_{AT}}, \sigma) = e^{-r(\theta_{RT} - \theta_{AT})} \int E^{\theta_{AT}}(V_{\theta_{RT}}, \sigma) 1_{\{V_{\theta_{RT}} > F_1\}} Q^{\theta_{AT}}(V_{\theta_{RT}}, \sigma) dV_{\theta_{RT}}
\]

\[
+ e^{-r(\theta_{RT} - \theta_{AT})} \int E^{R}(V_{\theta_{RT}}, \sigma) 1_{\{V_{\theta_{RT}} < F_1\}} Q^{\theta_{AT}}(V_{\theta_{RT}}, \sigma) dV_{\theta_{RT}}
\]

\[
= e^{-r(\theta_{RT} - \theta_{AT})} \times \left[ \text{call}(V_{\theta_{RT}}, \sigma, T_1 - \theta_{RT}) \right.
\]

\[
+ e^{-r(T_1 - \theta_{RT})} \int E^{R}(V_{T_1}, \sigma) 1_{\{V_{T_1} < F_1\}} Q^{\theta_{RT}}(V_{T_1}, \sigma) dV_{T_1}
\]

\[
\times 1_{\{V_{\theta_{RT}} > F_1\}} Q^{\theta_{AT}}(V_{\theta_{RT}}, \sigma) dV_{\theta_{RT}}
\]

\[
+ e^{-r(\theta_{RT} - \theta_{AT})} \int E^{R}(V_{\theta_{RT}}, \sigma) 1_{\{V_{\theta_{RT}} < F_1\}} Q^{\theta_{AT}}(V_{\theta_{RT}}, \sigma) dV_{\theta_{RT}}.
\]
σ is the level of business risk at time θ_{AT} and \( Q_{θ_{AT}}(V_{θ_{RT}}, σ) \) the risk neutral probability density function of \( V_{θ_{RT}} \) conditional on its value \( V_{θ_{AT}} \) at time \( θ_{AT} \). The first equality states that the wealth of the equity holders is the discounted weighted sum of the equity prices observed at time \( θ_{RT} \). The second one simply brings in the equation 7. At time \( θ_{RT} \), debt holders detect a default (\( V_{θ_{RT}} < F_1 \)) or not. In the former case, an early rescheduling is triggered and equity holders immediately receive a new claim (\( E_{θ_{RT}} \)). These equations highlight that the equity price is a function of both \( Δθ = θ_{RT} - θ_{AT} \) and the period of time between \( θ_{RT} \) and \( T_1 \). Similarly, the debt price is given by:

\[
D_{θ_{AT}}(V_{θ_{RT}}, σ) = e^{-r(θ_{RT} - θ_{AT})} \int D_{θ_{RT}}(V_{θ_{RT}}, σ)1_{\{V_{θ_{RT}} > F_1\}} Q_t(V_{θ_{RT}}, σ) dV_{θ_{RT}} + e^{-r(θ_{RT} - θ_{AT})} \int D_{θ_{RT}}^R(V_{θ_{RT}}, σ)1_{\{V_{θ_{RT}} ≤ F_1\}} Q_t(V_{θ_{RT}}, σ) dV_{θ_{RT}}
\]

where \( D_{θ_{RT}}(V_{θ_{RT}}, σ) \) is described by equation 7’.

To explore consequences of the creditors’ reaction, we provide in Figure 5 two series of graphs placed in three different contexts of times to maturity \( T_1 - θ_{AT} \) (2 years, 1 year and 9 months). We plot equity prices as functions of the delay \( Δθ = θ_{RT} - θ_{AT} \) (expected by equity holders) and the firm’s assets volatility σ. All graphs exploit base case parameters except that \( V_{θ_{AT}} = 30 \) in lower ones. The volatility ranges from 0 to 50% which is supposed to be a maximum for the considered firm’s activity (\( σ_H \)). Note that we consider different values of \( Δθ \) in upper and lower graphs for illustration purpose.

Graphs of Figure 5 essentially expose what is going on in-between a couple of extreme scenarios. Interestingly, these two cases are either similar or equivalent to what have already been considered in previous sections. First of all, equity holders can presume \( Δθ = T_1 - θ_{AT} \) and \( θ_{RT} = T_1 \) i.e. a so large information asymmetry that debt holders react only at the debt maturity. This situation is nothing else than the one considered in Figure 4 where no reaction from creditors was allowed. At the other extreme, there is no information asymmetry and debt holders react immediately to the business risk adjustment: \( Δθ = 0 \) and
Figure 5. – The impact of an early meeting

These graphs plot equity price as a function of the volatility $\sigma$ and $\Delta \theta = \theta_{RT} - \theta_{AT}$ the interval between the adjustment time and the renegotiation time. A couple of distress situations ($V_{\theta_{AT}} = 24$ or 30) and three different values for $T_1 - \theta_{AT}$ are considered (2 years, 1 year and 9 months). $\sigma$ ranges from 0 to 50%. Rescheduling is designed by debt holders either at $\theta_{RT}$ or at $T_1$. Other parameters are: $F_1 = 40$, $\beta = 60\%$ and $r = 6\%$. 
\[ \theta_{RT} = \theta_{AT}. \] Because \( V_{\theta_{AT}} < F_1 \), the equity price \( E^{\theta_{AT}, \theta_{RT}}_{\theta_{AT}}(V_{\theta_{AT}}, \sigma) \) vanishes to \( E^R_{\theta_{AT}}(V_{\theta_{AT}}, \sigma) \). Consequently, the situation appears equivalent to that analyzed in the simple setting of section 2, in the sense that there exists a business risk level to target. A major difference is that rescheduling takes place at time \( \theta_{RT} = \theta_{AT} \) instead of \( T_1 (V_{\theta_{AT}} < F_1) \).

In a certain way, equity holders should decide here to adjust their business in order to force a valuable rescheduling earlier than maturity. The way we can use Figure 5 greatly depends on the distress severity, the contemporaneous volatility of the firm, the time-to-maturity and the expected delay of creditors’ reaction. Let’s however consider, for illustration, a long time before the debt maturity (say two years) and a current firm’s assets volatility lower than 50% (say 40%). In view of the sole Figure 4, equity holders would have shifted the business risk upward. With a possible creditors’ reaction, things are different. Left graphs show that there is an incentive for equity holders to change the volatility upward, only if they expect a very large delay in creditors’ reaction. If an early rescheduling is triggered unexpectedly, their position is suboptimal in terms of volatility. The possible early meeting acts here as a threat because the rescheduled equity is worth less than its current price. Right and middle upper graphs display different situations where an early rescheduling is worthwhile for equity holders. We can add that the level of volatility to target is mainly not affected by the delay \( \Delta \theta = \theta_{RT} - \theta_{AT} \).

5. RECIPROCAL INFLUENCES OF CLAIM HOLDERS BEFORE AND AT REORGANIZATION

Until now, our parsimonious set up has neglected reciprocal influences of parties on the other’s decision. It was assumed that equity holders have no say on the design of extended debt and that debt holders have no influence on the choice of the business risk level. This may be regarded as inappropriate because reorganization may lead to a real agreement between parties. We now relax these assumptions.
5.1. Equity holders’ influence on rescheduling

If equity holders can influence the financial reorganization at time $T_1$, they will affect the objective function to maximize. Their relative bargaining power denoted by $\eta$ hereafter verifies $0 \leq \eta < 1$. $1 - \eta$ represents the debt holders “remaining” bargaining power. $\eta$ is of course less than one, otherwise equity holders can design rescheduling alone; in simulations, it does not exceed 0.5. The objective function to consider for rescheduling is given by:

$$ H_{total}(VT_1, \sigma_1, T_2; \eta) = H_{debt}^{1-\eta}(VT_1, \sigma_1, T_2) H_{eq}(VT_1, \sigma_1, T_2) $$

(18)

where $H_{debt}(VT_1, \sigma_1, T_2) = H(VT_1, \sigma_1, T_2)$ and $H_{eq}(VT_1, \sigma_1, T_2) = E^R_{T_1}(VT_1, \sigma_1) = \text{call}(VT_1, \sigma_1, T_2 - T_1)$ are respective net gain functions. The corresponding optimal extension maturity is then computed by:

$$ T(VT_1, \sigma_1; \eta) = \arg\max_{t \in [T_1, \infty]} H_{total}(VT_1, \sigma_1, t; \eta) $$

(19)

subject to the constraints that both $H_{debt}$ and $H_{eq}$ remain positive. Since $H_{eq}$ is, by definition, positive for all parameter values, $H_{debt}$ should be positive. Otherwise this means that debtors could swindle creditors. As long as the extension maturity described by equation 19 exists, share holders obtain new rescheduled claims denoted by $E^R_{T_1}(VT_1, \sigma_1; \eta)$ and $D^R_{T_1}(VT_1, \sigma_1; \eta)$. These claims are similar to $E^R_{T_1}(VT_1, \sigma_1)$ and $D^R_{T_1}(VT_1, \sigma_1)$ except that their (common) expiration date is now $T(VT_1, \sigma_1; \eta)$. Of course, when $\eta = 0$, i.e. when creditors have all the bargaining power, the above rescheduling design drops to the standard one, $H_{total}(VT_1, \sigma_1, T_2; 0) \equiv H(VT_1, \sigma_1, T_2)$ and the extension maturity vanishes to $T(VT_1, \sigma_1; 0) = T(VT_1, \sigma_1)$.

5.2. Creditors’ influence on business risk targeting

If debt holders can influence the choice of a business risk profile, they will modify the objective function to consider. Their intervention

11. The equation (18) forms an asymmetric generalized Nash bargaining problem.
power, denoted by $\gamma$, verifies $0 \leq \gamma < 1$. $1 - \gamma$ is associated to the equity holders’ position. $\gamma$ is less than one, otherwise creditors would control the firm. If the design of rescheduled claims remains at the creditors’ discretion, then the new objective function to consider for business risk targeting is:

$$G(V_t, \sigma_1; \gamma) = D_t^\gamma (V_t, \sigma_1) E_t^{1-\gamma} (V_t, \sigma_1)$$ (20)

$E_t(V_t, \sigma_1)$ and $D_t(V_t, \sigma_1)$ are described in equations 7 and 7’ respectively and we know that both involve $T(V_{T_1}, \sigma_1) = \arg\max_{t \in [T_1, \infty]} H(V_{T_1}, \sigma_1, t)$.

The corresponding optimal business risk is then computed by:

$$\sigma^* = \arg\max_{\sigma \in \mathbb{R}^+} G(V_t, \sigma; \gamma)$$ (21)

Of course this equation nests our previous analysis. For $\gamma = 0$ and $t = T_1 - \varepsilon$ with $\varepsilon$ rather small; we find:

$$G(V_{T_1-\varepsilon}, \sigma_1; 0) = E_{T_1-\varepsilon} (V_{T_1-\varepsilon}, \sigma_1) \approx E_{T_1-\varepsilon}^R (V_{T_1-\varepsilon}, \sigma_1)$$

$$= \text{call}(V_{T_1-\varepsilon}, \sigma_1, T(V_{T_1-\varepsilon}, \sigma_1) - T_1)$$ (22)

5.3. Reciprocal influences in the general context

In the most general context, equity holders influence rescheduling and creditors influence the business risk targeting. The complete picture is then obtained when both influences are considered in the same framework. The general objective function for business risk targeting is defined by:

$$G(V_t, \sigma_1; \gamma, \eta) = D_t^\gamma (V_t, \sigma_1, \eta) E_t^{1-\gamma} (V_t, \sigma_1, \eta)$$ (23a)

12. It is important to understand that the creditors’ influence on business risk targeting is not at all symmetric to the equity holders’ influence on the rescheduling. Equity holders take always benefit from rescheduling (whatever its design) while debt holders can face opportunity costs from the volatility shifting. In our framework, debt holders cannot prevent business risk targeting but they can try to influence the choice of the targeted volatility.
where $E_t(V_t, \sigma_1, \eta)$ and $D_t(V_t, \sigma_1, \eta)$ are respectively given by:

$$E_t(V_t, \sigma_1, \eta) = \text{call}(V_t, \sigma_1, T_1 - t)$$

$$+ e^{-r(T_1 - t)} \int E^{R}_{T_1}(V_{T_1}, \sigma_1; \eta) 1_{(V_{T_1} < F_1)} Q_t(V_{T_1}) dV_{T_1} \quad (23b)$$

and

$$D_t(V_t, \sigma_1; \eta) = F_1 e^{-r(T_1 - t)} N[d_2; V_t / F_1(T_1 - t)]$$

$$+ e^{-r(T_1 - t)} \int D^{R}_{T_1}(V_{T_1}, \sigma_1; \eta) 1_{(V_{T_1} < F_1)} Q_t(V_{T_1}) dV_{T_1} \quad (23c)$$

Both claims $E^{R}_{T_1}(V_{T_1}, \sigma_1; \eta)$ and $D^{R}_{T_1}(V_{T_1}, \sigma_1; \eta)$ depend on

$$T(V_t, \sigma_1; \eta) = \arg\max_{t \in [T_1, \infty]} H_{\text{total}}(V_t, \sigma_1, t; \eta) \quad (23d)$$

where

$$H_{\text{total}}(V_t, \sigma_1, T_2; \eta) = H_{\text{debt}}^{l-\eta}(V_t, \sigma_1, T_2) H_{\text{eq}}^{\eta}(V_t, \sigma_1, T_2) \quad (23e)$$

as described in equation 18. Business risk targeting is then solved when stakeholders can find a business risk level $\sigma_t^*$ such that:

$$\sigma_t^* = \arg\max_{\sigma \in \mathbb{R}^+} G(V_t, \sigma; \gamma, \eta) \quad (24)$$

Equations 23 and 24 form a system that must be solved simultaneously. In what follows, we extend the setting of section 2 to undertake the analysis of this system.

### 5.4. Reciprocal influences in the simple setting

This part questions whether the optimal business risk level defined by equation 24 exists and aims to appreciate quantitatively the impact of reciprocal influences. We restrict our attention to a setting that permits direct comparison with previous investigation. As in section 2, we...
consider $t = T_1 - \varepsilon$ with $\varepsilon$ a rather small value. Equation 23a then becomes:

$$G(V_{T_1-\varepsilon}, \sigma_1; \gamma, \eta) = D_1^R(V_{T_1-\varepsilon}, \sigma_1, \gamma) E_1^{1-\gamma}(V_{T_1-\varepsilon}, \sigma_1, \eta)$$

$$\approx (D_1^R(V_{T_1-\varepsilon}, \sigma_1, \eta))^\gamma (E_1^R(V_{T_1-\varepsilon}, \sigma_1, \eta))^{1-\gamma}$$

$$\approx (D_1^R(V_{T_1}, \sigma_1, \eta))^\gamma (E_1^R(V_{T_1}, \sigma_1, \eta))^{1-\gamma}$$

Equation 25

$E_1^R(V_{T_1}, \sigma_1; \eta)$ and $D_1^R(V_{T_1}, \sigma_1; \eta)$ are similar to $E_1^R(V_{T_1}, \sigma_1)$ and $D_1^R(V_{T_1}, \sigma_1)$ except that their common expiration date $T(V_{T_1}, \sigma_1; \eta)$ satisfies:

$$T(V_{T_1}, \sigma_1; \eta) = \arg\max_{t \in [T_1, \infty]} H_{total}(V_{T_1}, \sigma_1, \gamma, \eta)$$

where $H_{total}(V_{T_1}, \sigma_1, T_2; \eta) = H_{debt}^{1-\eta}(V_{T_1}, \sigma_1, T_2) H_{eq}^{\eta}(V_{T_1}, \sigma_1, T_2)$, see equation 18. Figure 6 explores whether $G(V_{T_1-\varepsilon}, \sigma; \gamma, \eta)$ admits a maximum value with respect to $\sigma$ and effects of the reciprocal influence on business risk targeting.

Figure 6 gathers together six different graphs. The left graphs plot $G(V_{T_1-\varepsilon}, \sigma; \gamma, \eta)$ of equation 25 as a function of the business risk and the influence of equity holders on financial reorganization ($\eta$ ranges from 0 to 0.25). Three levels of debt holders’ influence on the business risk targeting are considered: $\gamma = 0$, 0.25 and 0.5. The right graphs plot $G(V_{T_1-\varepsilon}, \sigma; \gamma, \eta)$ as a function of the business risk and the influence of creditors on business reorganization ($\gamma$ ranges from 0 to 0.25). Three levels of equity holders’ influence on rescheduling are considered ($\eta = 0$, 0.25 and 0.5). In upper graphs, either $\gamma$ or $\eta$ is null meaning that the influence is one-sided. In the lowest left graph, $\gamma$ is set to 0.5 meaning an identical bargaining power during the business reorganization. In the lowest right graph, $\eta$ is set to 0.5 meaning an identical bargaining power during the financial reorganization. Middle graphs consider intermediate situations with either $\gamma$ or $\eta$ equal to 0.25.

All these graphs reveal unambiguously that $G$ admits a maximum with respect to the volatility whatever the influences are. Hence, reciprocal influences have no effect on the pertinence of a business risk targeting before rescheduling. There exists an optimal volatility to tar-
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Left graphs plot $G(V_T, \sigma; \gamma, \eta)$ (see equation 25) as a function of the business risk and the debtors’ influence on rescheduling ($\eta$); three different levels being considered for $\gamma$ (0, 0.25, 0.50). Right graphs plot $G(V_T, \sigma; \gamma, \eta)$ as a function of the business risk and the creditors’ influence on business targeting ($\gamma$); three different levels being considered for $\eta$ (0, 0.25, 0.50). Other parameters are $V_T = 34$, $F_1 = 40$, $\beta = 40\%$ and $r = 10\%$. The extension maturity is chosen by maximizing $H_{total}(V_T, \sigma, T_2; \eta)$.

Figure 6. – Reciprocal influences and the G-function
get in the sense given by the objective function. Very interestingly, the
function \( g(\gamma, \eta) = \max_{\sigma} G(V_{T-\varepsilon}, \sigma; \gamma, \eta) \) is increasing with both para-
meters \( \eta \) and \( \gamma \) meaning that negotiations with significant \( \eta \) and \( \gamma \) are
valuable. Stakeholders have therefore incentives to design both reor-
ganizations with some degree of consensus. We can finally observe
that the debt holders’ influence on the business reorganization lowers
the business risk level to target. This is in lines with Figure 2 where
debt prices increase as the firm’s assets volatility decreases.

6. CONCLUDING REMARKS

To conclude the paper, we shortly discuss a couple of additional
questions related to our framework. Clearly, more definitive and detai-
led answers deserve further investigation. Is business risk targeting driven
by rescheduling a sequential program? Do our results imply a
dynamic program for business risk adjustment? These questions are of
importance due to the severe consequences of a possible dynamic
approach (see Hennessy and Tserluyeich (2004)). In our context,
however, this should not be the case for several reasons.

The business reorganization can hardly be a sequential issue in our
context. First of all, business adjustments are particularly costly and
irreversible in distressed firms. As a matter of facts, commercial and
industrial partners are rapidly aware of problems and become rather
suspicious. If not, both operational ability and feasibility are limited
because financial constraints are binding. Secondly, we know that debt
holders face opportunity costs when the firm’s assets volatility is
increased. So creditors can be dissatisfied by sequential upward target-
ing and may refuse as punishment the debt rescheduling expected by
equity holders. Thirdly, business adjustment can imply an early meee-
ting with creditors, as in our section 4. So these latter can refuse mul-
tiple renegotiation to avoid the increasing associated transaction costs.
Finally, business reorganization should also lead to a real renegotiation
between parties, as in section 5. This implies that both support trans-
action costs. Omitting the fact that creditors can always refuse such
complication, the marginal interest of equity holders clearly lowers as
the number of sequences increase.

Simple repetition of the debt rescheduling is not probable either.
First, this would require recurrent concessions from creditors. Second,
not repeating the game contributes to control the way the firm is managed. One the one hand, a first rescheduling implies, in our setting, a last due payment date that can be understood as a last recourse. On the other hand, the optimization program for debt rescheduling prevents maturities longer than necessary. It is well known that limiting the maturity of the capital structure has potential to discipline managers. Finally, if ever debt holders accept such reiteration, there is very little chance that debt rescheduling is granted identically. Rather, the new debt will be designed with significant guarantees. Debt holders can add some characteristics (such as covenants, put options, etc.) to limit the equity holders’ creativity or to threaten them. Clearly, this suggests the introduction of a monitoring device or a kind of default threshold. Debt holders can also require some financial efforts from the firm as partial reimbursement or cash infusion from debtors. Note that we have already studied these issues in an earlier paper.

Overall, this paper illustrates how valuable debt rescheduling is for both parties but also that, viewed as an exit option, it should not be neglected by stakeholders and financial analysts. Options to escape from financial distress may dramatically change the way stake holders are expected to behave.

REFERENCES


1. APPENDIX: A FURTHER INSIGHT ON ADJUSTMENT COSTS

This appendix shortly explores adjustment costs that were, for sake of tractability, neglected in the core text. We can conjecture that the more the underlying volatility changes, the more costly the shift should be. The other key point is the financing issue. Costs induced by business risk adjustment may be either supported by equity holders (as a lump sum) or covered by a sale of assets i.e. all stakeholders of the firm. These scenarios in turn generate different kinds of analysis. We reconsider now the parsimonious setting of section 2 that we use in the article.

Assuming that costs are proportional to the firm’s assets value at the adjustment time, the above discussion suggests that costs-adjusted equity is worth:

\[
E^c_{T_1-\varepsilon}(V_{T_1-\varepsilon}, \sigma^*) = E_{T_1-\varepsilon}(V_{T_1-\varepsilon}, \sigma^*) - C(\sigma_1, \sigma^*, \varepsilon) \times V_{T_1-\varepsilon}.
\]

\(\varepsilon\) is the remaining time-to-maturity, \(E_{T_1-\varepsilon}(V_{T_1-\varepsilon}, \sigma^*)\) is given by equation 7 and \(C(\sigma_1, \sigma^*, \varepsilon)\) is an increasing function of the shifting effort measured by \(|\sigma_1 - \sigma^*|\). The way \(\varepsilon\) influence the cost function depends on both the nature of the activity (industry versus utilities) and the time at which costs are paid. The sign of \(\sigma_1 - \sigma^*\) could matter too. For illustration, one can write:

\[
C(\sigma_1, \sigma^*, \varepsilon) = \int_{T_1-\varepsilon}^{T_1} c(\sigma_1, \sigma^*, \varepsilon) \exp(-rs) ds
\]

with \(c\) the instantaneous adjustment cost function and \(\sigma_s\) the time \(s\) volatility towards the target business risk level.

A complete specification of the cost function is beyond the scope of the present article. But we can nevertheless stress a couple of features with respect to the remaining time-to-maturity. When \(\varepsilon\) is significant, costs could be spread over a longer period of time and the business adjustment may be easier to achieve too. And this could lower overall adjustment costs. By contrast, for very low \(\varepsilon\), adjustments are time

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13. We thank referees for suggesting this discussion.
constrained and costs optimization is impossible, so that we can expect the shift to be more costly.

The above discussion alters the optimization program we consider in the core text since equity holders should make their decision on the basis of $E_{T_1 - \varepsilon}(V_{T_1 - \varepsilon}, \sigma^*)$ instead of $E_{T_1 - \varepsilon}(V_{T_1 - \varepsilon}, \sigma^*)$. Implied modifications on our analysis depend however critically on the considered cost function.