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Valuing callable convertible bonds: a reduced approach

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This paper analyses the pricing of corporate callable convertible bonds. It reconciles the applicability of the reduced form approach with optimal strategies usually discussed in the structural approach. One demonstrates that some conditions causing rational voluntary conversions may be effectively neglected. The main contribution of the paper is to show that adjusted American Capped Call options well duplicate 'classical' optimal strategies. Numerical experiments are then conducted.

I. INTRODUCTION

A convertible bond is a risky and hybrid financing vehicle that gives the holder the right to convert his bond to a given number of stocks. The investor-bondholder is protected from a decrease in the firm's value while he, as a potential stockholder, may benefit from an increase in the stock value. In addition, convertible bonds usually include additional features, *callable* convertible bonds (CCB hereafter) being nevertheless the most common ones. Such bonds allow the issuer to call back his debt. The investor must then choose either the cash redemption or the conversion of his bond. Strategic behaviours of both agents must therefore be taken into account to price consistently the CCBs.

The pricing of convertible bonds may be divided in two distinct frameworks. The structural approach (Ingersoll, 1977b; Brennan and Schwartz, 1977, 1980) adopts the Black–Scholes–Merton framework and relates the convertible bonds to the corporate structure, the firm value and its assets volatility. As a result, both the default risk and the implied dilution are *naturally* taken into account. This setting does not however permit strong practical applications. The reduced form approach exploits directly available information on stocks and bonds. In this way, Ho-Pfeffer (1996) have suggested a two-factor arbitrage-free discrete time valuation model *à la* Ho and Lee (1986) that uses a constant credit margin. More recently, Tsiveriotis and Fernandes (1998) has shown that credit risky convertible bonds, in a flat term structure environment, are solutions of a system of two-dimensional Black–Scholes stochastic differential equations. Nyborg (1996) presents a more general survey.

In this paper, the corporate CCBs are considered in a continuous reduced form framework. The optimal strategies (from both investors and the issuer's viewpoints) are duplicated by an adjusted American Capped Call option. This contract has been originally studied in Boyle and Lau (1994), Broadie and Detemple (1995) and Boyle and Turnbull (1989). The rest is organized as follows. Section II recalls classical optimal strategies. Section III presents the key result of the paper. Section IV details the implementation. Section V explores computational results.

II. OPTIMAL CONVERSION AND CALL POLICIES

Optimal conversion and call policies have been extensively studied since the seminal works of Ingersoll (1977a,b) and Brennan and Schwartz (1977).

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First, consider the investor viewpoint. Ingersoll (1977a,b), Brennan and Schwartz (1977) have demonstrated that it will never be optimal to convert a CCB except

- immediately prior to a dividend date,
- immediately prior to an adverse change in the conversion terms,[1]
- when constrained by an early exercise of the call feature or
- at maturity.

In addition, the voluntary conversion prior to a dividend payment requires that:

- the difference between the dividend (to be received) and the coupon is higher than the actual conversion premium,[2]
- the conversion process is immediate,
- transaction and informational costs are neglectible.

Bond covenants however precise that investors cannot benefit from both the dividend and the coupon during the same accounting period. Roughly speaking, delivered stocks will receive dividends only the following period. The key point is therefore to anticipate the future dividend level. Consequently, it is challenging to compute the probability that the dividend will exceed a coupon plus transactions costs given that the conversion premium is low. In other words, effective arbitrage between coupon and dividend (conjectured by the theory) is operationally difficult.[3]

Let's now turn to the issuer viewpoint. According to Ingersoll (1977a,b), Brennan and Schwartz (1977), the call policy for the issuer is optimal as soon as the convertible's market price reaches the call price. It's important to note, however, that many studies find widespread violations of this policy. Firms have been empirically shown not to call as soon as the contractual call price is reached.[4] Many researchers have then advanced many plausible reasons why management might rationally delay calling convertible bonds. First, Asquith and Mullins (1991), Campbell *et al.* (1991) pointed out that firms may enjoy a cash-flow advantage from not calling. Second, according to Harris and Raviv (1985), the delayed call results from an information-signalling hypothesis. Third, as noted by Ingersoll (1977a), Asquith and Mullins (1991) and by Asquith (1995), the existence of a notice period may induced a rational call delay.

To conclude, in what follows, the conversion policy for investors is assumed optimal only at maturity or when constrained by an early exercise of the call feature. In addition, one retains the classical optimal call policy exposed above.

III. FROM THE ANALYSIS TO THE PROPOSITION

Ingersoll (1977b) has shown that the CCB is worth more than a straight bond but less than a convertible one. The CCB is moreover expected to be duplicated in a risky bond (called the investment value) and a non-standard option. To this end, some assumptions are needed. Continuous markets are supposed to be free of limitation and restriction, there is neither tax nor exchange cost. Agents behave rationally and the conversion factor is held constant through time and fixed at unity. Hence, the stock price and the conversion value are the same.

The so-called investment value of the CCB is a comparable corporate bond which is not convertible nor callable. It promises the same coupons c, the same principal amount F at the same dates. It admits the same underlying credit status $\Theta = \{\rho, \delta\}$. Here, ρ stands for the initial rating of the bond and δ is the expected recovery upon default. $w = 1 - \delta$ is therefore the expected write down faced by the bondholder in case of default. Following Jarrow *et al.* (1997), it is assumed constant: no corresponding risk premium is needed. This assumption is well known to be reasonable for investment grade bonds. Denoting r the interest rates process, the price of a corporate zero coupon bond is then given by:

$$P(r, t, T; \Theta) = \delta B(r, t, T) + (1 - \delta)\,\mathrm{B}(r, t, T)(1 - \tilde{Q}_t^T) \quad (1)$$

where B is the price of an equivalent riskless bond and $\tilde{Q}_t^T$ is the risk-neutral default probability. In what follows, risky coupon bonds are evaluated as portfolios of risky zero-coupon bonds. Denoted $P(F, c, r, t, T; \Theta)$ with F the face value and c the coupon rate, its price at τ is given by

$$P(F, c, r, \tau, T; \Theta) = \sum_{t=t_1}^{t_n} cP(r, \tau, t; \Theta) + FP(r, \tau, T; \Theta)$$

where t_1 (*resp.*t_n) is the first (*resp.* the last) coupon date after τ (resp. before T).

Before demonstrating that an adjusted American Capped Call option duplicates the optimal policies, let's recall standard results. An European Capped Call option is a standard call option whose payoff is nonetheless limited to a specified amount H. When the investor is allowed to exercise the option earlier (in the case of American

[1] Neglected in this study.
[2] The conversion premium being defined as the difference between convertible bond price and conversion value.
[3] For a more complete discussion of this point, refer to André and Moraux (2003).
[4] For instance, Ingersoll (1977a,b) has provided evidence that firms wait, on average until the conversion value exceeds the call price by 43.9%. See also Mikkelson (1981), Asquith (1995).

Table 1. *The CCB pay-offs*

Before maturity S_t	No early conversion (0;F]	]F;H[	Early conversion H
Investment value + $(C_{UO} + R) =$ Duplicating portfolio	$P(t)$	$P(t)$	$P(t)$
	$ACC(t)$	$ACC(t)$	I
	$P(t) + ACC(t)$	$P(t) + ACC(t)$	$P(t) + I$
CCB	$P(t) + Opt(t)$	$P(t) + Opt(t)$	H
At maturity S_T	Redeem (0;F]	Conversion]F;H[	Conversion H
Investment value + $(C_{UO} + R) =$ Duplicating portfolio	F	F	F
	0	$S_T - F$	I
	F	S_T	$F + I$
CCB	F	S_T	H

Capped Call options (*ACC* hereafter)), Boyle and Turnbull (1989) have demonstrated that it's better to do so as soon as the maximum amount is reached (see also Boyle and Lau (1994), Broadie-Detemple (1995)). This maximum amount H may thus be viewed as a threshold level. Denoting S the underlying price process and F the exercise price, the price of an American Capped Call option is then given by:

$$ACC(S, F, H) = C_{UO}(S, F, H) + R \tag{2}$$

where C_{UO} is an *Up&Out* barrier option that disappears as soon as S passes through H and R is the attached rebate limiting the loss incured if the option disappears.[5] R is totally defined by I the amount received for compensation and h the first passage density of the price process through the barrier H. If the compensation amount is time-dependent, the rebate is the total weighted discounted flow delivered, i.e. $\int_0^T I(\tau)\mathrm{e}^{-r\tau}h(\tau)\mathrm{d}\tau$. If not, it equals I times a discounting term either $\mathrm{e}^{-rT}Pr[\tau_H \leq T]$ when paid at maturity or $\int_0^T \mathrm{e}^{-r\tau}h(\tau)\mathrm{d}\tau$ when the threshold is reached (see Reiner-Rubinstein [1991]).

The American Capped Call option is useful to price the contingent claim embedded in a CCB. As the conversion value reaches the *call price*, say at time τ, the issuer is allowed to call back its debt. In such a case, investors receive, against their claims, a fixed amount H equalling both the cash redemption and the conversion value. They therefore give up the remaining value of the investment value and his conversion right at maturity disappears. Let's therefore identify S to the conversion value, F to the principal of the investment value and H to the *contractual* call price. Table 1 then shows that the pay-offs of a CCB are equivalent to the cash flows of a portfolio which contains a corporate bond and an adjusted American capped call whose rebate must be suitably designed.

In our framework, the rebate equals the present value of the total amount received in case of early call of the debt by the issuer. Thanks to Table 1, this amount I is found to be:

$$I = H - P(F, c, r, t, T; \Theta)$$

The adjusted ACC that duplicates the option embedded in the CCB is:

$$Opt(S, F, H, P) = C_{UO}(S, F, H) + \int_t^T (H - P(F, c, r, \tau, T; \Theta))\mathrm{e}^{-r(\tau - t)}h(\tau)\mathrm{d}\tau$$

One finally obtains:

Proposition 1. A Callable Convertible Bond is priced by:

$$G(S, P, H) = P(F, c, r, t, T; \Theta) + Opt(S, F, H, P) \tag{3}$$

where P is the corporate coupons bearing bond.

This proposition implicitly assumes that $S_t < H$. One may observe on real data that when $S_t > H$, the CCB price is approximately worth the conversion value. This special 'In-the-Money' behaviour is illustrated in Fig. 1.

[5] Refer to Rich (1994) and Reiner and Rubinstein (1991) for a complete description of barrier options.

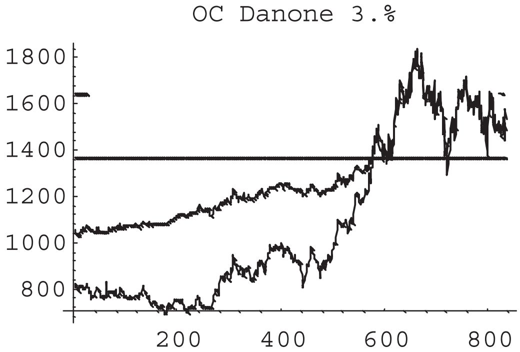


Figure 1. *Danone's '3%-01/01/02' convertible bond price and its conversion value. Daily prices are from 01/01/96 to 17/03/99. The call price is 1369. Its nominal value is 1015 FF and the coupon 30.45 FF the conversion ratio is one to one.*

IV. IMPLEMENTATION

The purpose of this section is to expose as clear as possible a very simple setting to implement the previous proposition. As claimed by Kijima (1998), it is now common to use a Markov chain model to describe the dynamics of a firm's credit rating.[6] Jarrow *et al.* (1997) provide the complete theoretical framework to exploit information on credit delivered by rating agencies (Standard Poor, 1998). One denotes the suitable one-year transition matrix $Q = (q_{i,j})_{i,j\in S}$ where $S = \{AAA, \ldots, CCC, D\}$ represents the set of credit ratings. Any t-period probability transition matrix is then given by $Q_t = e^{t\Lambda} = \sum_{k\geq 0}(1/k!)(t\Lambda)^k$ where $\Lambda = (\lambda_{i,j})_{i,j}$ is the generator. Following Jarrow *et al.* (1997), no correlation between interest rate risk and credit risk are assumed under the risk neutral probability. In addition, many other assumptions are necessary to ensure correct properties for spreads and transition probabilities. The initial transition matrix, based on historical data, also requires a calibration before risk neutral valuation. Refer to Moraux and Navatte (2001) for a recent exposition of these points.

Let's now assume that the stock price is correctly described under the risk neutral probability by

$$d\ln S = (r - d - \frac{1}{2}\sigma_S^2)\mathrm{d}t + \sigma_S \mathrm{d}W^S \qquad (4)$$

where r is the risk free rate, d the continuous dividend and σ_S the volatility. The value of an American Capped Call options may then be denoted $ACC(S, F, H)$. The corresponding first passage density of the stock price through the barrier H is known to be $h(\tau) = (-u/\sqrt{2\pi\tau^3\sigma_s})e^{-(\mu\tau-u)^2/2\sigma_S^2\tau}$ with $\mu = r - d - (1/2)\sigma_S^2$; $u = \ln(S/H)$. It is important to see that there is no available closed form solution for the pricing of barrier style option when interest rates are stochastic. The term structure of interest rate is therefore assumed to be flat and constant in this optional part.

Under such assumptions, the nowadays price of the option embedded in a CCB is:

$$\begin{aligned} Opt(S_0, H, F) = C(S_0, F) - C_{UI}(S_0, H, F) \\ + H\Psi(S_0, H, F) - \Omega(S_0, H, P) \end{aligned} \qquad (5)$$

with:

$$\Psi(S_0, H, F) = e^{-A+u}N[d(u, -1, T)] + e^{-A-u}N[d(u, 1, T)]$$

$$\Omega(S_0, H, P) = \int_0^T e^{-r(\tau)}P_0(F, c, r, \tau, T; \Theta)h(\tau)\mathrm{d}\tau$$

$u=\ln(S/H), \mu=r-d-(1/2)\sigma_S^2, A^{\pm}=(1/\sigma_S^2)(\mu\pm\sqrt{\mu^2+2r\sigma_S^2})$, $\mathrm{d}(u,\xi,\theta)=(1/\sigma_S\sqrt{\theta})(u+\xi\mu\theta)$. C denotes the price of a standard call option, C_{UI} stands for an *Up&In* barrier option, P_0 is the future investment value viewed from now. Because the credit process is time-homogeneous, the price of a credit risky coupon bond viewed from now is:

$$P_0(F, c, r, \tau, T; \Theta) = \sum_{j=1}^{8} P(F, c, r, \tau, T; \Theta_j(\tau)).Pr[\Theta_j(\tau)/\Theta]$$

$Pr[\Theta_j(\tau)/\Theta_i]$ stands for the risk neutral probability to be rated Θ_j at τ. The first term of Equation 5 represents the voluntary conversion at maturity. The second one warranties that, if the threshold is attained, this conversion at maturity disappears. The third term considers the payment received and the last means that the remaining value of the investment value is surrendered.

V. NUMERICAL RESULTS AND APPROXIMATIONS

This part explores the convertible bond price formula. Parameters for the investment value are fixed such that the interest rate level is 4.5%, and the bond promises 3.5% coupons with a face value of 400. The contractual call price H is 450, the underlying stock pays a 2.5% dividend, its volatility is 20%.

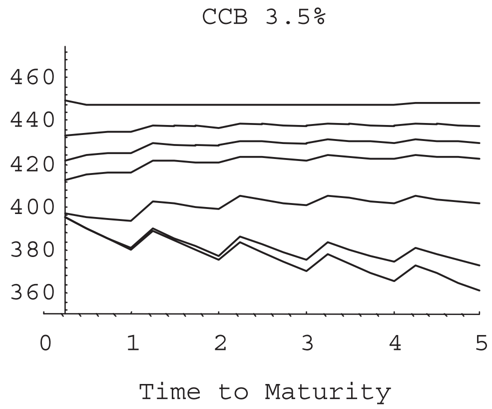
Figure 2 plots the term structure of the callable convertible bond prices for different initial conversion values. It displays the price of certain CCB when one also neglects the interest rate risk. The lowest line belongs to a comparable straight bond. Others stand for CCB price when the conversion value is equal to respectively 250, 350, 400, 415,

[6] Alternative reduced form models are Jarrow and Turnbull (1995) and Duffie and Singleton (1999). The major advantage of the chosen framework is to take explicitly into account any possible future up/downgrading among the default.

Table 2. *Relative price of the different component of a corporate callable convertible bond*

Mat	CCB	IV	ACC	Conv	C_{ui}	Ψ	Ω
Conversion value = 400							
0,25	100	95,82	4,18	4.14	3.00	26.27	23.23
0,5	100	94,01	5,99	5.94	5.37	44.04	38.63
0,75	100	92,62	7,38	7.37	7.01	53.69	46.67
1	100	91,43	8,57	8.59	8.35	59.90	51.58
Conversion value = 445							
0.25	100	88.66	11.34	11.20	11.02	92.04	80.89
0.5	100	87.48	12.52	12.53	12.46	94.54	82.08
0.75	100	86.36	13.64	13.71	13.68	95.63	82.03
1	100	85.28	14.72	14.78	14.76	96.29	81.59

IV stands for the Investment value, ACC for the whole optional part, Conv for the voluntary conversion, C_{UI}, Ψ, Ω have been defined in the text.



The lowest price corresponds to the riskless straight coupon bond, the other ones to callable convertible bonds with different initial stock underlying values (respectively 250, 350, 400, 415, 430 and 450). The contractual call price equals to 450 and the share volatility is 20%.

Figure 2. *Callable convertible bond as function of the maturity and conversion values.*

430 and 450. This graph first shows that the term structure may be downward sloping, almost constant and slightly upward sloping. It also appears that the closer the conversion value to the call price $h = 450$, the less the coupon effect is. Finally, when the conversion value reaches the call price, the investor cannot expect more than the call price whatever the maturity of the bonds he holds. As a result, this figure illustrates that the CCB abandons slowly its bond feature as the conversion value rises.

Following Ho and Pfeffer (1996), it is also useful to consider the relative price of the different components of the corporate CCB previously considered in Fig. 2. The conversion value equal to 445 illustrates the case where the CCB is more likely to be called back by the issuer. Table 2 summarizes the results. First it appears that as the probability of early call by the issuer increases, the ACC gets larger. Second, the premium, i.e. the value of the embedded option mainly comes from the difference between the amount received and the surrendered value of the bond rather than the conversion at maturity. In other words, the *Up&In* barrier option compensates the value of a conversion at maturity.

The previous formula may nevertheless appear quite computationally time-consuming. The reason is that the rebate considers any rating scenario at each possible event date, that the bondholder is said to surrender the remainder of the investment value when the convertible bond is called back optimally. One can therefore easily simplify Equation 5 by approximating the surrendered value or the remaining investment value (contemporaneous of the call date). Refer to the Appendix for these operational details.

VI. CONCLUSIONS

This paper considers a reduced approach to price corporate callable convertible bonds. It demonstrates that an adjusted American Capped Call well replicates the optimal policies of both the issuer and investors. It argues that the condition of Brennan and Schwartz (1977, 1980) causing a rational conversion before the dividend date is unrealistic with respect to the usual contractual conversion process of CCBs. Following Ho and Pfeffer (1996) both the corporate underlying bond and stock prices are allowed to be stochastic. The credit process has been considered within a framework *à la* Jarrow *et al.* (1997). Some possible extensions of the proposed setting should include other refinements such as the limited call period.

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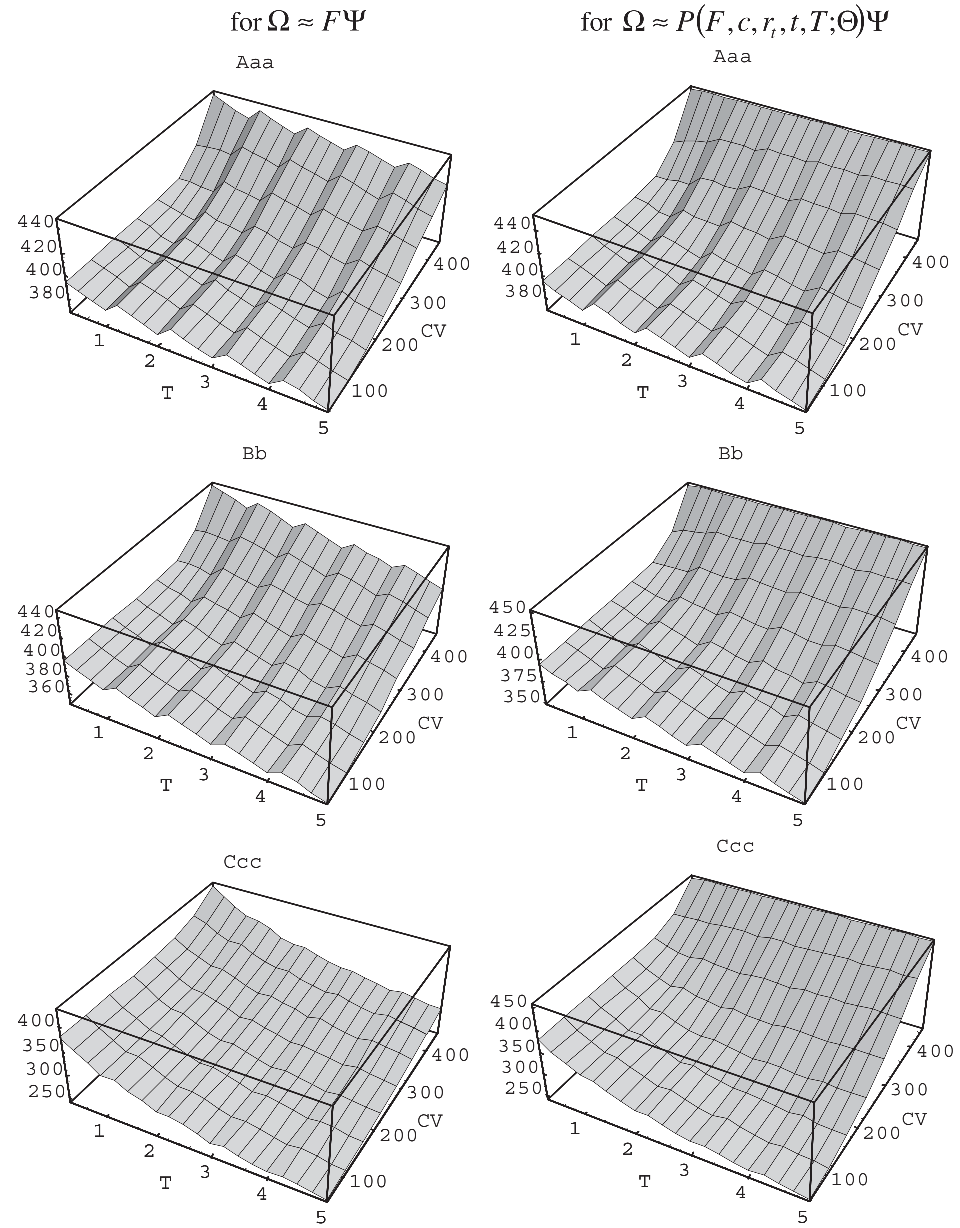
APPENDIX

The previous formula may be simplified by approximating the elements of Equation 5. First, the investor may neglect the remaining coupons, the surrendered value then becomes $\Omega \approx F \int_0^T e^{-r\tau} P(r, \tau, T; \Theta) h(\tau) d\tau$. Second, he may neglect both the remaining coupons and the interest rates and credit risks: $\Omega \approx F e^{-rT} \int_0^T h(\tau) d\tau$. Finally, one may assume that the bondholder gives up only the non-discounted nominal of the investment value, $\Omega \approx F\Psi$. Figure 3 provides approximated prices of corporate CCB for different levels of the underlying conversion value. The investment values are respectively AAA-rated and BB-rated with a recovery rate of 40.32% (according to Standard & Poor, it implies that the convertible debt is a subordinated one). The computed surfaces look like those of Tsiveriotis and Fernandes (1998). They suggest, as already pointed out above, that as the underlying conversion value grows, the convertible bond is less sensible to the coupon payments.



The term structure of interest rates is governed by a single state variable correctly described by a 'square root' process *à la* Cox–Ingersoll–Ross [1985]. The interest rate level $r = 4.5\%$ equals its long-mean run. The coefficient speed of the mean-reversion is unity and its volatility is 20%. The promised face value of the bond is 400 and its annual coupon 3.5%. The contractual call price of the callable convertible bond is 450 and the conversion ratio is unity. The underlying stock volatility is 25%. Recovery = 40.32%.

Figure 3. *Approximated prices of convertible bond for different maturity and conversion value.*