Order- Disorder Structural Transitions in Mazes Built by Evaporating Drops

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We show that the evaporation of surfactant solutions confined in quasi-two-dimensional porous media creates micron-sized labyrinthine patterns composing the walls of a centimeter-sized maze. These walls are made of solid deposits formed during drying via a sequence of individual Haines jumps occurring at the pore scale. We rationalize this process driven by simple iterative rules with a cellular automaton that acts as a maze generator. This model well describes the formation dynamics and final structure of an experimental maze as functions of the wettability heterogeneities of a porous medium and its geometry. Also, our findings unveil the crucial role of two geometric dimensionless quantities that control the structural order of a maze.

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Evaporation is central to many industrial and natural processes [1]. Understanding the evaporation of liquid dispersions in porous media is important for various disciplines, e.g., soil physics [2,3] and civil engineering [4], and diverse processes such as the underground sequestration of CO₂ [5]. The key issue in most cases is to comprehend the creation, locus, and morphology of the solid deposits formed during drying. For instance, this topic is crucial in civil engineering, as salt weathering can cause severe damage to the built environment [6] and in material science as evaporation can help the design of novel smart materials [7]. Yet, the development of models for drying in porous media [8] and the proper description of the formation of deposits in such media [9,10] have remained elusive, as these tasks are generally challenging.

Here, we study the evaporation of surfactant solutions in porous media consisting of micron-sized cylindrical posts arranged on square or rectangular lattices. The manufactured evaporation devices are transparent and quasi-two-dimensional (2D). These two essential attributes allow us to precisely determine how and where solid deposits form during drying. As a fluid evaporates, labyrinthine patterns made of solid deposits form in the pores between adjacent posts. These “walls” compose a centimeter-sized structure that resembles a maze when drying ends. Labyrinthine patterns can also emerge in fluid-granular 2D flows [11] and other fluid systems submitted to an external field [12]. Phyllotaxy-inspired drying patterns can even be seen in suitably designed 2D porous media [13]. In our study, we show that the created walls result from the motion of unstable liquid-air menisci between neighboring posts, i.e., Haines jumps. Since these events are sequential and stochastic at the lattice scale and driven by iterative rules at the pore scale, we rationalize our results with a cellular automaton generating mazes. This model, which accounts for pore-scale heterogeneities of wettability known to impact drying in porous media [14,15], well describes the structure and formation dynamics of the observed mazes. For fixed wetting conditions, we show that the structural order of a maze is governed by only two geometric dimensionless quantities built with four key geometric parameters, the post radius, the lattice height, and interpost spacings.

An evaporation device consists of an assembly of posts sealed on top of a smooth solid (see Fig. 1), both surfaces being made of transparent polydimethylsiloxane (PDMS Sylgard 184, Dow Corning) using soft lithography [16]. The posts have a radius \( R = 50 \mu m \) and height \( h = 20–100 \mu m \), the interpost spacings \( \ell_1 \) and \( \ell_2 \) being in the 150–250 \( \mu m \) range (Fig. 1(a)). A drop of a deionized water surfactant (either SDS or Tween 20, concentration 5 cmc, Sigma-Aldrich) mixture is squeezed between textured and smooth surfaces and glue seals the sides of the device. The liquid then pervaporates [17,18] through the ceiling of the device placed in a container with constant temperature and humidity (Figs. 1(a) and 1(b)). For the whole study, experimental and numerical drops begin to evaporate at the origin of time \( t = 0 \).

We use an inverted microscope (IX71, Olympus) and a camera (Eurocam) to record the evolution with time of the surface of an evaporating drop (Fig. 1(b); see Ref. [19] for further details of experimental methods). In an evaporation device, the liquid-air interface consists of circular menisci connecting adjacent posts (see Figs. 1 and 2). As the surface of a drop decreases during drying, all menisci slowly recede with a decreasing radius of curvature \( R_c \) (Fig. 2(a)) until one of them becomes unstable and moves rapidly towards the liquid cell it contacts (see Fig. 2(b) and Video S1 in Ref. [19]); this is a Haines jump. As a result, new menisci or solid films or both are created at the three other edges of this cell [see Figs. 1(c)–2(b) defining the types of posts \( P_k \) and liquid cells \( N_k \) and illustrating the time evolution of these cells]. A wall is built between two neighboring posts when liquid gets trapped between the...
moving unstable meniscus merging with another meniscus located on one edge of the invaded liquid cell [Fig. 2(b)]. The evaporation of this liquid bridge creates a solid wall and possibly the crystallization of the dispersed phase as illustrated by x-ray scattering and scanning electron microscopy when a nanorod-based fluid system is used [20]. Our experiments show that (i) the contact angle \( \theta \) that the liquid makes with PDMS is \( \theta_m \leq \theta \leq \theta_M \) with typically \( \theta_m \approx 25^\circ \) and \( \theta_M \approx 65^\circ \) (see Fig. S1 and related discussion in Ref. [19]), (ii) each meniscus presents an axis of symmetry perpendicular to the line connecting the centers of the two posts it contacts [see Fig. 2(a)], and (iii) \( \theta \) is unchanged during the motion of an unstable meniscus. Also revealed by our observations, the selection of unstable meniscus is stochastic and their motions are sequential; i.e., they move one at a time. Hence, evaporation consists of successive Haines jumps separated by slowly evolving configurations (see Video S1 in Ref. [19]) until the process ends. The interconnected walls then form a centimeter-sized structure resembling a maze [Figs. 1(b) and 2(d)]. Being seen for different solute species (surfactants and colloids), these structures are inherent to the drying process. To study their formation dynamics and topology, we characterize the temporal evolution of \( P_k(t) \) and \( N_k(t) \) by processing images recorded during drying with custom-written C++ software.

An \( \ell_i \) meniscus and an \( \ell_i \) wall \( (i = 1 \text{ or } 2) \) herein denote a meniscus and a wall connecting two adjacent posts with interpost spacing \( \ell_i \), respectively. Using geometry, one easily determines the radius of curvature \( R_i = (\ell_i - 2R_\alpha)/[2 \cos(\theta - \alpha)] \) of an \( \ell_i \) meniscus with \( \alpha \) the angle defining its position on the posts it contacts [see Fig. 2(a)]. The pressure at this meniscus is \( P(\alpha, \theta, \ell_i, h) = P_o + \delta P \) with the ambient pressure \( P_o \) and Laplace pressure \( \delta P = -\gamma[1/\ell_i^2 + 2h^{-1}\cos \theta] \), \( \gamma \) being the air-liquid surface tension [21]. We begin to rationalize our results by considering that \( \theta \) is constant \( (\theta = 45^\circ) \), as this simplified analysis helps us to understand the underlying physics of our experiment. Figure 2(c) shows that \( \delta P \) has a minimum \( \delta P_{\text{min}}^{\alpha} \) for \( \alpha = \alpha_{\text{min}}^{\alpha} \). Also, for any \( h \), we find that \( \delta P_{\text{min}}^{\alpha} \geq \delta P_{\text{min}}^{\alpha} \) when \( \ell_2 \geq \ell_1 \). As shown below in the case \( \ell_2 > \ell_1 \), these basic arguments are sufficient to explain the observed Haines jumps; note that similar arguments can be employed when \( \ell_2 < \ell_1 \).

Between two successive Haines jumps, \( \delta P \) is uniform so that \( \alpha_1 \) and \( \alpha_2 \) are small and different [see Fig. 2(c)]. As water evaporates, these two angles increase as \( \delta P \) decreases until it reaches the value \( \delta P_{\text{min}}^{\alpha} \) [Fig. 2(c)]. Then, if a fluctuation causes an \( \ell_i \) meniscus to move farther than the others, the pressure at this meniscus either decreases \( (i = 1) \) or increases \( (i = 2) \) [Fig. 2(c)]. Hence, the pressure gradient between this local pressure and that of other liquid cells induces a flow that either stabilizes \( (i = 1) \) or destabilizes \( (i = 2) \) the moving front. All \( \ell_2 \) menisci are then potentially unstable. By contrast, all menisci are stable...
when $\delta P > \delta P_{2 \text{min}}$ and $\alpha_i < \alpha_{i \text{min}}$ since $\delta P$ decreases with $\alpha_i$ for both $i = 1$ and 2. Thus, all menisci should recede slowly at a speed set by the evaporation rate when $\delta P > \delta P_{2 \text{min}}$ until a Haines jump occurs for $\delta P = \delta P_{2 \text{min}}$. The motion of an unstable $\ell_2$ meniscus induces a rapid transfer of liquid in the other cells which makes all other menisci advance. So, once a Haines jump is completed, the pressure $\delta P > \delta P_{2 \text{min}}$ is uniform and all menisci are stable again. They recede slowly until another Haines jump occurs when $\delta P = \delta P_{2 \text{min}}$.

Our analysis explains the observed evaporation characterized by a succession of individual Haines jumps. These stochastic events are periodic, their period $\tau$ being the time needed for a liquid cell to evaporate [see Fig. 2(b)]. As depicted in Fig. 2(b), a Haines jump yields the creation of new menisci or walls or both positioned along the edges of the invaded liquid cell according to simple rules. A wall is formed only when the unstable meniscus merges with another meniscus initially present on an edge of this liquid cell; a new meniscus is created otherwise. The final structure shown in Fig. 2(d) concurs with this prediction that should neither depend on $\theta$ nor the lattice anisotropy $\Delta \ell = \ell_2 - \ell_1$ when $\Delta \ell \neq 0$. For square lattices ($\Delta \ell = 0$), the whole picture is not altered. However, all $\ell_j$ menisci are potentially unstable in this case so that the fraction of $\ell_j$ walls should be equal to that of $\ell_2$ walls. This is qualitatively confirmed by the final structure of the maze shown in Fig. 1(b).

To validate our predictions quantitatively, we perform experiments in which we vary $\Delta \ell$ and $h$ for a fixed $\ell_1$. As predicted, the same number of walls is found in the two directions of the lattice for any $h$ when $\Delta \ell = 0$, and the fraction of $\ell_2$ walls is close to 0 when $\Delta \ell > 20 \mu m$ [see Fig. 3(a)]. Unexpectedly, however, a large ($\approx 0.1$–0.4) fraction of $\ell_2$ walls is found experimentally for intermediate $\Delta \ell$ at any $h$ [Fig. 3(a)]; this fraction decreases with increasing $\Delta \ell$ (at fixed $h$) and $h$ (for a given $\Delta \ell$).

Although our simple analysis based on a unique contact angle helps us to understand some results, it fails to explain the existence of unstable $\ell_1$ menisci yielding the finite fraction of $\ell_2$ walls seen for intermediate $\Delta \ell$. To that end, the contact angle is $\theta_m \leq \theta \leq \theta_M$ for the remainder of the study. For this problem driven by the iteration of simple rules, we develop a cellular automaton that acts as a maze generator to rationalize our experiments. This numerical model proceeds as follows. Each pore of a lattice is filled with either liquid or air and menisci connect the two types of cells. At each time step of the algorithm, we count the number $M_i$ of $\ell_i$ menisci, we index their positions for both $i = 1$ or 2 with an integer $1 \leq j_i \leq M_i$, and we randomly select a contact angle $\theta_m \leq \theta_j \leq \theta_M$ for each of them [22]. At each $\ell_i$ meniscus, the excess pressure in the liquid is now $\delta P(\theta_j, \alpha_i, \ell_i, h)$. Similar to our previous analysis assuming a constant angle [see Fig. 2(b)], as menisci move slowly between two Haines jumps, $\delta P$ is uniform and decreases until it reaches the minimum value $\delta P_{\text{min}}(\theta_j, \ell_i, h) = \min[\delta P(\theta_j, \alpha_i, \ell_i, h)]$ of the pressure profile at one meniscus. This minimum being the largest of all pressure profiles, the identified meniscus is unstable, and

FIG. 3. (a) Experimental fraction of $\ell_2$ walls vs $\ell_2$ for $\ell_1 = 175 \mu m$ and four post heights $h$: 20 $\mu m$ ( ), 25 $\mu m$ ( ), 50 $\mu m$ ( ), and 80 $\mu m$ ( ). The lines correspond to our model with $\theta_m = 25^\circ$ and $\theta_M = 65^\circ$. As indicated by the arrow, the shades of gray of the lines stand for different $h$ varying from the lightest gray for the smallest post (10 $\mu m$) to black for the tallest one (200 $\mu m$) with intermediate values 20, 50, 80 and 100 $\mu m$. (b) Data shown in Fig. 3(a) vs $h \Delta \ell$ vs $\ell_2$ ( ) and $\ell_1$ ( ). (c) Close-up views of a few tens of pores depicting the final numerical structures for different $h \Delta \ell$ vs $\ell_1$ ( ) vs $\ell_2$ ( ). (d) Fractions of $\ell_2$ walls vs $\ell_2$ vs $h = 50$ $\mu m$ and different $\ell_1$: 175 $\mu m$ ( ), 200 $\mu m$ ( ), and 225 $\mu m$ ( ). The three lines are the outcomes of the cellular automaton for the water-SDS mixture with $\theta_m$ and $\theta_M$ as in Fig. 3(a). The open squares denote experiments conducted with the water-Tween 20 mixture.
onto a master curve when plotted as a function of \( \theta \). In addition, our model predicts well the variations of the fraction of \( \text{h} \) (see Fig. 4), and our experiments seem to be independent of the nature of the surfactant [see Fig. 3(d)]. In our study with four geometric parameters (\( h, \theta_1, \theta_2, R \)) at play, Fig. 3(b) reveals the governing dimensionless quantity \( h \Delta l / (\ell_1 \ell_2) \).

To conclude, studying the evaporation of surfactant solutions in transparent 2D porous media, we unravel the mechanisms governing the formation of solid deposits in the micron-sized pores, these deposits composing the walls of a centimeter-sized maze. Interestingly, two posts of a maze are connected by one path only; these mazes are called “perfect,” and they are equivalent to a tree in graph theory. We model our findings with a cellular automaton accounting for wettability heterogeneities and acting as a maze generator that captures the structure of a created maze and its formation dynamics. Also, our findings unveil two geometrical dimensionless quantities that allow for a fine-tuning of the structural order of a maze. These results could help the design of multiscale materials using evaporation as a route for controlled self-assembly. In particular, the design rules described here could find applications in flexible electronics as they offer possibilities for tailoring microwire networks.

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\[ \text{FIG. 4. Final fractions of } P_k \text{ vs } h \Delta l / (\ell_1 \ell_2) \text{ for } \ell_1 = 175 \mu m / (5 \ell_1 / R = 3.5) \text{ and four post heights. The symbols are identical to those of Fig. 3(a). The lines are the outcomes of the cellular automaton with } \theta_0 = 25^\circ \text{ and } \theta_M = 65^\circ; \text{ the shades of color of the lines stand for the four heights varying from the lightest shade for the smallest post to the darkest shade for the tallest post.} \]

For a review on models of drying in porous media, see, for instance, M. Prat, Recent advances in pore-scale models for drying of porous media, Chem. Eng. J. (London) 86, 153 (2002).


See Supplemental Material at http://link.aps.org/supplemental/10.1103/PhysRevLett.121.078002 for details of experimental methods, typical distributions of contact angles, a brief discussion on governing parameters, two figures comparing experimental, and numerical formation dynamics for rectangular and square lattices and three videos. Video S1 illustrates experimental Haines jumps and the resulting formation of walls. Videos S2 and S3 show that the experimental structures of a maze compare well to numerical predictions for both kinds of lattices.


The surface tensions determined by pendant drop tensiometry are 35 and 36 mN m$^{-1}$ for the SDS-based system and the Tween 20-based one, respectively.

We consider a typical length scale of wettability heterogeneities given by that of a meniscus and a rectangular distribution of contact angles $\theta$ for the sake of simplicity.