Available online at www.sciencedirect.com
ScienceDirect
Optik 121 (2010) 188–194
Optik Optics
www.elsevier.de/ijleo

Another way to shape the comprehensive analytical approach describing electromagnetic energy distribution through four-slab-layer structures

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Received 11 February 2008; accepted 15 June 2008

Abstract

Starting from the generalised four-slab layer in electromagnetic theory and photonics, this paper introduces a convenient method and a new proper change of variables in order to obtain the global analytical expressions of the power flows in such multilayer structures for the TE_m and TM_m optical modes. These proper changes of variables and relevant definitions of apt new parameters (Θ, W, Y and ξ) allow us to derive and shape new general analytical formulations and normalizations in terms of power flows. According to such specific parameters, it can be noted that such a comprehensive result brings in an effective criteria form of the classical results ascribed to three-slab problems. Moreover, we have verified with specific cases regarding three-slab problems the validity of our new global frame for analysing power flows. It clearly appears that classical three-slab-waveguide expressions directly stem from our formulation. Naturally, this global four-slab-waveguides approach can be used directly to the analytical calculus of corresponding ratios of power between the different layers, such as the core compared with buffer layers as upper and lower claddings.

Keywords: Electromagnetic theory; Multilayer structures; Power flow

1. Introduction

Theoretical studies of the field theory of guided waves [1] and integrated optics [2] play a key role in optical telecommunication [3,4] and micro-sensor systems [5,6]. An increasing number of such optical devices is based on multilayer waveguide structures [7] for chemical or biochemical applications. Considering multilayer-slab structures, it is necessary to optimize the opto-geometrical parameters in order to control the localisation of the evanescent wave, to increase the length of such evanescent probes to improve the sensor sensitivity, that is to act on the whole distribution of the energy of the appropriate optical modes regarding a given application.

Intended to unify, this paper introduces first the adequate variables, allowing us to obtain the prevalent analytical expressions of the power flows through any four-slab-multilayer structure for both TE_m and TM_m optical modes. In this formulation, as a main point the definitions of adequate and specific parameters allow us

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doi:10.1016/j.ijleo.2008.06.006

to advance the versatile analytical power-flow expressions that meet the well-known three-slab-waveguide effective criteria form. Moreover, thanks to such a choice of variables, the classical three-slab-waveguide expressions naturally appear to stem from our formulation. Then, such expressions are most suitable to calculate the corresponding ratios of power flows through the core and the upper and lower claddings in any four-slab waveguide.

2. Theoretical approach and analytical expressions of the power flow

2.1. Problem definition: global asymmetric four-slab-waveguide structures and theory

In integrated optics, the resolution and calculation of guided modes in most multilayer slab waveguides directly stem from Maxwell's equations [1,2]. Indeed, the classical propagation wave equation in each dielectric layer of such opto-geometrical structures can be solved by way of the proper and continuous boundary conditions of the electric and magnetic fields at the interfaces: it is actually possible to determine, respectively, the modal or effective propagation constants from the eigenvalue equation for guided modes and the spatial field distributions related to the eigenvectors of such physical systems [8].

Consider the generic four-slab-waveguide structure as shown in Fig. 1; the opto-geometric parameters of such a physical system are the dielectric distribution related to the index profile (n_i, $i = 1$–4), the free propagation constant k_0 (or the wavelength λ_0), and the thicknesses of the core waveguide and the first upper-cladding layer, $2h$ and $2d$. The normalized quantities q, t, p and r hinge on the above parameters together with the modal or effective propagation constant $\beta \equiv \beta_{\mathrm{eff}} = k_0 n_{\mathrm{eff}}$. The guided modes in such an optical multilayer (called TE$_m$ and TM$_m$ polarisations, with the m integer standing for the quantification due to the x-direction's confinement) are assumed to propagate in the z-direction with a phase term given as $\exp[\mathrm{j}(\omega t - \beta z)]$. The design illustrated in Fig. 1 refers to a waveguide structure with $n_3 \leqslant n_4 \leqslant n_2 \leqslant n_1$. Such an adequate approach allows us to include any other distribution regarding the n_i values indices. As shown in Fig. 1, two different families of solutions for guided modes can be defined relative to their m quantification into both light cones: (i) $k_0 n_2 \leqslant \beta \leqslant k_0 n_1$ and (ii) $k_0 n_3 \leqslant \beta \leqslant k_0 n_2$. First inequality (i) deals with the optical family modes involving the upper value of effective propagation constants β (or effective indices n_{eff}) depicting a strong confinement of their spatial field distributions. In the rest of the paper, in order to handle both families of guided modes into a single expression, we will resort to a double notation regarding all following equations and expressions; the upper notations $|^{\bullet}$ account for the first family modes or (i) conditions, whereas the lower notations $|_{\bullet}$ submit to the second family or (ii) prior conditions. Considering such global four-slab waveguides (Fig. 1) the x-spatial distribution of the optical modes (E_y for TE polarisation, and H_y for TM polarisation, Fig. 1) may be expressed as

$$\begin{cases} \dfrac{B\left|\begin{matrix}\cosh\\ \cos\end{matrix}\right.(2\mathrm{d}t+\chi)}{\left|\begin{matrix}\cosh\\ \cos\end{matrix}\right.(\chi)}\exp[r(2d-x)], & x \geqslant 2d \\ \dfrac{B\left|\begin{matrix}\cosh\\ \cos\end{matrix}\right.(tx+\chi)}{\left|\begin{matrix}\cosh\\ \cos\end{matrix}\right.(\chi)}, & 0 \leqslant x \leqslant 2d \\ A\sin(qx) + B\cos(qx), & -2h \leqslant x \leqslant 0 \\ [-A\sin(2hq) + B\cos(2hq)]\exp[p(x+2h)], & x \leqslant -2h \end{cases}, \tag{1}$$

where the quantities $r = (\beta^2 - k_0^2 n_3^2)^{1/2}$, $t = [|\pm(\beta^2 - k_0^2 n_2^2)]^{1/2}$, $q = (k_0^2 n_1^2 - \beta^2)^{1/2}$, $p = (\beta^2 - k_0^2 n_4^2)^{1/2}$ are positive (Fig. 1), and A, B, χ integration constants with respect to the classical resolution of the differential wave equation observed in each layer.

It is clear that such modal expressions E_y or H_y include the continuous boundary conditions between adjacent layers at $x = 2d$, 0 and $-2h$. The usual boundary-value technique can be applied considering the two continuous boundary conditions regarding $x = 0$, $x = 2d$, related to the other components of the fields inferred from the Maxwell's equations $H_z = (\mathrm{j}/\omega\mu_0)\,(\partial E_y/\partial x)$ for TE modes (E_y) and $E_z = (\mathrm{j}/\omega\varepsilon_0 n_i^2)\,(\partial H_y/\partial x)$ for TM modes (H_y); hence, it is possible to define two sets of relations matching both integration constants A and B:

$$\frac{A}{B} = |\pm\eta_{1\to2}\left(\frac{t}{q}\right)\left|\begin{matrix} th(\chi) \\ tg(\chi) \end{matrix}\right. \quad \text{or} \quad \chi = |\pm\left|\begin{matrix} th^{-1} \\ tg^{-1} \end{matrix}\right.\left[\frac{A}{B}\left(\frac{q}{t}\right)\eta_{2\to1}\right], \tag{2}$$

and

$$\left|\begin{matrix} th \\ tg \end{matrix}\right.(2\,\mathrm{d}t+\chi) = |\mp\eta_{2\to3}\left(\frac{r}{t}\right) \quad \text{or} \quad \chi = \left|\begin{matrix} -th^{-1} \\ tg^{-1} \end{matrix}\right.\left(\eta_{2\to3}\frac{r}{t}\right) - 2\,\mathrm{d}t. \tag{3}$$

As a result

$$\frac{A}{B} = |\pm\eta_{1\to2}\left(\frac{t}{q}\right)\left|\begin{matrix} th\left[-th^{-1}\left(\eta_{2\to3}\left(\frac{r}{t}\right)\right) - 2\,\mathrm{d}t\right] \\ tg\left[tg^{-1}\left(\eta_{2\to3}\left(\frac{r}{t}\right)\right) - 2\,\mathrm{d}t\right] \end{matrix}\right.$$

according to the said notation, $\eta_{i\to j} = 1$ for the TE modes, and $(n_i/n_j)^2$ for the TM modes ($i, j = 1$–4).

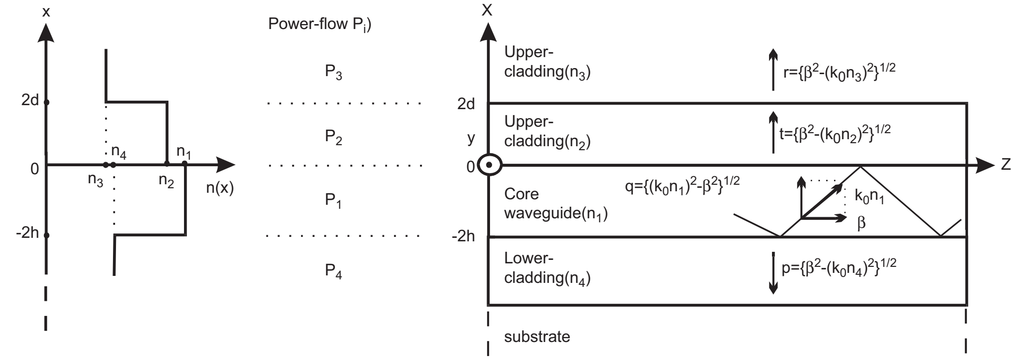

Fig. 1. Schematic diagram of a generic multilayer structure defined as an asymmetric four-slab waveguide. Index profile, normalized power flows (P_i, $i = 1$–4), and the opto-geometric parameters of such a global structure: namely, the respective thicknesses of the core and the buffer layer or upper cladding as $2h$ and $2d$, the indices of the layers n_i ($i = 1$–4), both propagation constants $k_0 = (2\pi/\lambda_0)$ and $\beta \equiv \beta_{\mathrm{eff}} = k_0 n_{\mathrm{eff}}$, together with the defined expressions q, t, p and r functions of the previous parameters are depicted. The geometrical definitions of normalized variables q, t, p and r are inferred from the geometric approach of propagation in such structures (Pythagorean theorem and zig-zag path of ray).

Regarding E_z and H_z fields, consider for $x = -2h$ the third boundary conditions. To this end, Eq. (2) allows us to focus on the eigenvalue equation describing such physical problems (m integer):

$$tg(2hq - m\pi) = \left\{ \frac{\eta_{1\to 4}\left(\frac{p}{q}\right) | \mp \eta_{1\to 2}\left(\frac{t}{q}\right) \left| \begin{matrix} th\left[-th^{-1}\left[\left(\eta_{2\to 3}\frac{r}{t}\right) - 2\,\mathrm{d}t\right]\right] \\ tg\left[tg^{-1}\left[\left(\eta_{2\to 3}\frac{r}{t}\right) - 2\,\mathrm{d}t\right]\right] \end{matrix} \right.}{1 | \pm \eta_{1\to 4}\left(\frac{p}{q}\right) \eta_{1\to 2}\left(\frac{t}{q}\right) \left| \begin{matrix} th\left[-th^{-1}\left[\left(\eta_{2\to 3}\frac{r}{t}\right) - 2\,\mathrm{d}t\right]\right] \\ tg\left[tg^{-1}\left[\left(\eta_{2\to 3}\frac{r}{t}\right) - 2\,\mathrm{d}t\right]\right] \end{matrix} \right.} \right\} \tag{4}$$

Consider now the tg^{-1} of such a $\{(a \mp b)(1 \pm a.b)\}$ -type argument regarding Eq. (4). Then, as the relation is expanded into a sum, the odd parity of th and tg^{-1} functions allows us to rephrase the eigenvalues equation as

$$(2hq) = tg^{-1}\left[\eta_{1\to 4}\left(\frac{p}{q}\right)\right] + tg^{-1}\left[\eta_{1\to 2}\left(\frac{t}{q}\right) \left| \begin{matrix} th\left[th^{-1}\left(\eta_{2\to 3}\frac{r}{t}\right) + 2\,\mathrm{d}t\right] \\ tg\left[tg^{-1}\left(\eta_{2\to 3}\frac{r}{t}\right) - 2\,\mathrm{d}t\right] \end{matrix} \right. \right] + m\pi \tag{5}$$

Such a relation, with no integration's constant, clearly addresses any form of quantification (m integer) of the eigenvalues termed effective propagation constants β (or effective indices n_{eff}) of the guided-modes solutions; the latter depend on the opto-geometric parameters n_i ($i = 1$–4), λ_0, $2h$ and $2d$ as regards the global asymmetric four-slab structures. Considering the definition of the parameter t in Eq. (1) for both light-cones cases together with the properties of the th and tg functions on a given imaginary argument, one can see that the double Eq. (5) with $th \leftrightarrow tg$ is clearly linked to the related change $t \leftrightarrow jt$ ($j^2 = -1$). Moreover, it can be noted that such four-slab-layers eigenvalues equations include the special case involving three-slab layers with no second upper cladding; then Eq. (5) leads to the classical result:

$$(2hq) = tg^{-1}\left[\eta_{1\to 4}\left(\frac{p}{q}\right)\right] + tg^{-1}\left[\eta_{1\to 2}\left(\frac{t}{q}\right)\right] + m\pi.$$

2.2. Proper change of variables and further rewriting of equations

At this stage it is judicious to introduce a set of specific new parameters allowing both simplification and harmonization of the next calculations, leading to the general analytical expressions of the power flows relative to optical guided modes driven through any four-slab-multilayer structure:

$$\Theta = W \left| \begin{matrix} th(\chi) \\ tg(\chi) \end{matrix} \right., \quad W = \eta_{1\to 2} t, \quad \chi = \left| \begin{matrix} -th^{-1} \\ tg^{-1} \end{matrix} \right. \left(\frac{Y}{t}\right) - 2\,\mathrm{d}t,$$
$$Y = \eta_{2\to 3}\, r \quad \text{and} \quad \xi = \eta_{1\to 4}\, p. \tag{6}$$

Specifically, let us underline that we have introduced the prior Θ parameter, related to χ, so as to create a specific variable addressing the four-slab problem. Indeed, the expression of Θ is also the function of all the parameters stemming from the fourth layer, namely the interface parameter $x = 2d$, the normalised functions of the propagation constant t, r, and the index values n_2 and n_3 on each side of such an interface (Fig. 1). In this step, according to considerations regarding calculus, we can stress that the change of variable $t \leftrightarrow jt$, allowing for the passage between the guides modes considering both corresponding light cones (first light cone ↔ second light cone), yields the following changes into new parameters

$W \leftrightarrow jW$, $\chi \leftrightarrow j\chi$ and $\Theta \leftrightarrow -\Theta$ owing to the properties of the functions tg, tg^{-1}, th and th^{-1} dealing with imaginary argument.

Thanks to such specific parameters, the latter relations (2), the eigenvalues Eqs. (4) and (5) can be summed up and significantly simplified as

$$\frac{A}{B} = | \pm \frac{\Theta}{q}, \tag{7}$$

and

$$tg\,(2hq - m\pi) = \frac{q(\xi | \mp \Theta)}{q^2 | \pm \xi\Theta} \quad \text{or}$$

$$(2hq) = tg^{-1}\left[\frac{\xi}{q}\right] + tg^{-1}\left[\frac{\Theta}{q}\right] + m\pi. \tag{8}$$

As a coherent rationale, the introduction of such Θ and ξ parameters allows the eigenvalues Eq. (8) regarding four-slab structures to be refashioned and shaped in a criteria form of effective three-slab problem. But, as mentioned previously, the four-slab-structure criteria are now on one part intrinsic to the Θ parameter, and it is necessary to calculate four global expressions of power flows.

2.3. Derivation of the analytical expressions of the power flow in overall four-slab structures

According to the new parameters, this section together with both appropriate Appendices A and B is devoted to the derivation of the four general analytical expressions of the power flows P_i ($i = 1$–4) within any given four-slab-multilayer structures for both TE and TM optical modes. The time-average power flow P in such a multilayer structure (Fig. 1) can be defined by the integral over the cross-section (x-direction) of the z-component of the Poynting vector $S_z = (1/2)\Re e\,(\vec{E} \wedge \vec{H}^*)|_z$, as the energy flow rate along the direction of propagation [9]. Then, according to each relevant Maxwell's equation describing H_x and E_x field's components asproportional to E_y and H_y for TE modes and TM modes, respectively, such power flows can be written as

$$P^{\mathrm{TE-TM}} = \int_{-\infty}^{+\infty} S_z\,\mathrm{d}x = \begin{cases} \alpha^{\mathrm{TE}}|E_y|^2 \\ \alpha^{\mathrm{TM}}|H_y|^2 \end{cases} \tag{9}$$

with $\alpha_i^{\mathrm{TE-TM}} = (\beta/2\omega\mu_0)$ and $(\beta/2\omega n_i^2\varepsilon_0)$ for both TE and TM modes ($i = 1$–4).

Consider the expression of each field into its respective layer described with system (1) together with the definition of the specific parameters Θ, W, Y and ξ; it is now possible to determine rigorously the analytical expressions of the power flow in each region (Eq. (9)) for the twin-family guided modes (see (i) and (ii) relevant to the conditions in Section 2.1). Then, the general expressions of P_i into each layer (core, lowe and upper claddings) making up the structure (Fig. 1) considering both family modes ascribed to the two light cones can be formulated as

$$\begin{cases} P_3^{\mathrm{TE-TM}} = \dfrac{\alpha_3^{\mathrm{TE-TM}}\,B^2 \left| \begin{array}{c} \cosh^2 \\ \cos^2 \end{array} \right. (2\,\mathrm{d}t + \chi)}{\left| \begin{array}{c} \cosh^2 \\ \cos^2 \end{array} \right. (\chi)} \int_{2d}^{+\infty} \exp[2r(2d - x)]\mathrm{d}x \\ P_2^{\mathrm{TE-TM}} = \dfrac{\alpha_2^{\mathrm{TE-TM}}\,B^2}{\left| \begin{array}{c} \cosh^2 \\ \cos^2 \end{array} \right. (\chi)} \int_0^{2\mathrm{d}} \left| \begin{array}{c} \cosh^2 \\ \cos^2 \end{array} \right. (tx + \chi)\mathrm{d}x \\ P_1^{\mathrm{TE-TM}} = \alpha_1^{\mathrm{TE-TM}} \int_{-2\mathrm{h}}^{0} [A\,\sin\,(qx) + B\cos\,(qx)]^2\mathrm{d}x \\ P_4^{\mathrm{TE-TM}} = \alpha_4^{\mathrm{TE-TM}}[-A\,\sin\,(2hq) + B\cos\,(2hq)]^2 \int_{-\infty}^{-2h} \exp[2p(x + 2h)]\mathrm{d}x \end{cases}, \tag{10}$$

with the quantities $\alpha_i^{\mathrm{TE-TM}}$ defined as previously for both TE and TM modes.

The comprehensive analytical procedure for each calculus yielding P_1, P_4 and P_2, P_3 is described in details in both Appendices A and B. The proper definitions relevant to the new parameters Θ, W, Y and ξ allow us to derive and shape suitably the general analytical formulations and normalizations in terms of power flows. Consequently, Table 1 sums up such general and articulate expressions.

It can be noted that such a comprehensive analytical frame meets the same effective criteria form regarding the classical results related to the three-slab problem [2]. Moreover, it is easy to verify, with the three-slab problem as a limiting case, the validity of the herein general approach. It can be pointed out that both couple of power flows (P_1, P_4) and (P_2, P_3) present a similitude property since they are, respectively, described by the same set of parameters (q, ξ, Θ) and (Y, W, Θ). Indeed, each sum of couples can be included

Table 1. Comprehensive analytical formulations of power flows as functions of specific Θ, W, Y and ξ opto-geometric parameters

Integral domain (x-direction)	Power flows
$[2d, +\infty]$	$P_3^{\mathrm{TE-TM}} = \alpha_3^{\mathrm{TE-TM}} \left(\frac{B^2}{2r}\right)\left(\frac{W^2 \vert \mp \Theta^2}{W^2 \vert \mp (\eta_{1\to 2}^{\mathrm{TE-TM}})^2 Y^2}\right)$
$[0, 2d]$	$P_2^{\mathrm{TE-TM}} = \alpha_2^{\mathrm{TE-TM}} \left(\frac{B^2}{2t}\right)\left(\frac{W^2 \vert \mp \Theta^2}{W^2}\right)\left(2\,\mathrm{d}t \vert \mp \frac{Yt}{t^2 \vert \mp Y^2} - \frac{W\Theta}{W^2 \vert \mp \Theta^2}\right)$
$[-2h, 0]$	$P_1^{\mathrm{TE-TM}} = \alpha_1^{\mathrm{TE-TM}} \left(\frac{B^2}{2}\right)\left(\frac{q^2+\Theta^2}{q^2}\right)\left(2h + \frac{\xi}{(q^2+\xi^2)^2} \vert \mp \frac{\Theta}{(q^2+\Theta^2)^2}\right)$
$[-\infty, -2h]$	$P_4^{\mathrm{TE-TM}} = \alpha_4^{\mathrm{TE-TM}} \left(\frac{B^2}{2p}\right)\left(\frac{q^2+\Theta^2}{q^2+\xi^2}\right)$

with $\Theta = W\left|\begin{matrix} th\,(\chi) \\ tg\,(\chi) \end{matrix}\right.,\ W = \eta_{1\to 2}^{\mathrm{TE-TM}}\, t,\ \chi = \left|\begin{matrix} -th^{-1} \\ tg^{-1} \end{matrix}\right. \left(\frac{Y}{t}\right) - 2\,\mathrm{d}t,\ Y = \eta_{2\to 3}^{\mathrm{TE-TM}} \cdot r,\ \xi = \eta_{1\to 4}^{\mathrm{TE-TM}} . p$

$r = (\beta^2 - k_0^2 n_3^2)^{1/2},\ t = [| \pm (\beta^2 - k_0^2 n_2^2)]^{1/2}, q = (k_0^2 n_1^2 - \beta^2)^{1/2},\ p = (\beta^2 - k_0^2 n_4^2)^{1/2}, \beta = k_0 n_{eff}$

$\alpha_i^{\mathrm{TE-TM}} \equiv \left(\alpha^{\mathrm{TE}} = \frac{\beta}{2\omega\mu_0} \text{ or } \alpha_i^{\mathrm{TM}} = \frac{\beta}{2\omega\eta_i^2 \varepsilon_0}\right), \eta_{i\to j}^{\mathrm{TE-TM}} \equiv \left(\eta^{\mathrm{TE}} = 1 \text{ or } \eta_{i\to j}^{\mathrm{TM}} = \left(\frac{n_i}{n_j}\right)^2\right)$

B integration's constant $\Leftrightarrow \sum_{i=1}^{4} P_i = P_{\mathrm{total}}$ the whole power flow rate.

These expressions are verified and prevalent for both families of guided modes as defined in Section 2.1.

in only one global expression. Thus, writing each quantity $\alpha_j^{\mathrm{TE-TM}}$ ($\forall j = 3$ and 4) regarding P_3 and P_4 power flows as $\alpha_{|_3^4}^{\mathrm{TE-TM}} = \alpha_{|_2^1}^{\mathrm{TE-TM}} \eta_{|_{2\to 3}^{1\to 4}}^{\mathrm{TE-TM}}$, and then factoring the given sums of couple before multiplying judiciously relevant terms by $\eta_{i\to j}$ allowing for Eq. (6), yields both factored expressions:

$$\begin{cases} P_1^{\mathrm{TE-TM}} + P_4^{\mathrm{TE-TM}} = \alpha_1^{\mathrm{TE-TM}} \left(\frac{B^2}{2}\right)\left(\frac{q^2+\Theta^2}{q^2}\right)\left(2h + \frac{\xi^2 + \eta_{1\to 4}^2 q^2}{\xi(q^2+\xi^2)} | \mp \frac{\Theta}{(q^2+\Theta^2)}\right) \\ P_2^{\mathrm{TE-TM}} + P_3^{\mathrm{TE-TM}} = \alpha_2^{\mathrm{TE-TM}} \left(\frac{B^2}{2}\right)\left(\frac{W^2 | \mp \Theta^2}{W^2}\right)\left(2d - \frac{W\Theta}{W^2 | \mp \Theta^2} | \mp \frac{\eta_{2\to 3}^2 W^2 + \eta_{1\to 2}^2 Y^2}{Y(W^2 - Y^2)}\right) \end{cases} \tag{11}$$

cones $k_0 n_2 \leqslant \beta \leqslant k_0 n_1$ and $k_0 n_3 \leqslant \beta \leqslant k_0 n_2$. Moreover, such global expressions are suitable for any TE$_m$- and TM$_m$-guided mode (m quantification integer) according to their dependence on the eigenvalue $\beta = k_0 n_{\mathrm{eff}}$ that can be deduced from Eq. (8).

3. Discussion and conclusion

According to the specific double notation $\left|\begin{matrix} \bullet \\ o \end{matrix}\right.$ defined in Section 2.1, the analytical expressions summed up in Table 1 allow a cogent formulation of the power-flow rates for both different families of guided modes into a global four-slab structure (Fig. 1) considering both light

It can be noticed that each power flow formulation P_i ($i = 1$–4) stands as a function involving the opto-geometric parameters regarding each ith layer plus the parameters of the other adjacent layers; such a property stems from the continuity of relevant physics at the interfaces on various steps of this rationale. As an example, P_1 hinges on the opto-geometric parameters ascribed to the core (layer $i = 1$) as q, $2h$, $\alpha_1^{\mathrm{TE-TM}}$, plus the other parameters regarding the two adjacent layers, as ξ (function of n_4 and p) for the lower cladding ($i = 4$) and Θ (function of W and χ that depends on n_2, $2d$ and t) for the upper cladding ($i = 2$) (Fig. 1). We proved that proper changes of variable and the determination of apt new parameters (Θ, W, Y and ξ) allowed us to come up with a general analytical formulation and normalization in terms of power flows. The comprehensive laws summarized in Table 1 show off an effective criterion form of the classical results in a three-slab problem [2].

As the integration constant B (defined as $\sum_{i=1}^{4} P_i = P_{\text{total}}$) is the same in the four expressions P_i of the power flows (Table 1), such formulations are proper and commendable to assess directly the distribution of energy as relevant ratios of power between the different involved layers P_i/P_j (with $i \neq j$); it is then possible to compare the localisation of energy in versatile four-slab structures, regarding the corresponding ratios of power into the core $[P_1| + (P_2)]/P_{\text{total}}$ and in the cladding $[P_2 + P_3 + P_4| - (P_2)]/P_{\text{total}}$ for both families' guided modes.

Such a paper presents another way of shaping a convenient analytical method and a new proper change of variables so as to obtain the global expressions of the power flows in such multilayer structures, respectively, for the TE_m and TM_m optical modes.

Appendix A

According to Eq. (10), the calculus of the power flow $P_1^{\mathrm{TE-TM}}$ is deduced from the term $(\alpha_1^{\mathrm{TE-TM}} I_1)$, with $I_1 = \int_{-2h}^{0} [A\sin(qx) + B\cos(qx)]^2 \mathrm{d}x$. Then, by using Eq. (7), the integration of I_1 yields:

$$I_1 = \left(\frac{B^2}{2}\right)\left[2h\left(\frac{q^2+\Theta^2}{q^2}\right)\right] + T, \tag{A.1}$$

with

$$T = |\mp\left(\frac{B^2}{2}\right)\left(\frac{\Theta}{q^2}\right) + \sin(4qh)\left(\frac{B^2}{4q} - \frac{\Theta^2}{4q^3}B^2\right) \\ |\pm\cos(4hq)\left(\frac{\Theta B^2}{2q^2}\right).$$

Then, consider the trigonometric relations between $\sin(4hq)$ and $\cos(4hq)$ with the quotient formula of functions $tg(2hq)$ and $tg2(2hq)$, respectively, plus the first Eq. (8) called eigenvalues equation, we can write:

$$\sin(4hq) = \frac{2q(\xi \mp \Theta)(q^2 \pm \Theta\xi)}{D} \quad \text{and} \\ \cos(4hq) = \frac{(q^2 \pm \Theta\xi)^2 - q^2(\xi \mp \Theta)^2}{D}, \tag{A.2}$$

with $D = (q^2 \pm \Theta\xi)^2 + q^2(\xi \mp \Theta)$ that reduces to $D = (q^2 + \xi^2)(q^2 + \Theta^2)$.

Substituting for (A.2) into the expression of T (A.1), expanding and simplifying the numerator of each term of the sum, re-arranging and judiciously factoring them allow us to reduce T to

$$T = \left(\frac{B^2}{2}\right)\left[\Big|\mp\frac{\Theta}{q^2} + \frac{\xi(q^2+\Theta^2)^2}{q^2(q^2+\xi^2)(q^2+\Theta^2)}\right] \\ = \left(\frac{B^2}{2}\right)\left[\left(\frac{q^2+\Theta^2}{q^2}\right)\left(\frac{\xi}{(q^2+\xi^2)}| \mp \frac{\Theta}{(q^2+\Theta^2)}\right)\right]. \tag{A.3}$$

Then (A.3) and (A.1) enable us to conclude that (Table 1)

$$I_1 = \left(\frac{B^2}{2}\right)\left(\frac{q^2+\Theta^2}{q^2}\right)\left(2h + \frac{\xi}{(q^2+\xi^2)^2}| \mp \frac{\Theta}{(q^2+\Theta^2)^2}\right). \tag{A.4}$$

By doing so, the calculus of the power flow $P_4^{\mathrm{TE-TM}}$ (Eq. (10)) is defined as $(\alpha_4^{\mathrm{TE-TM}} I_4)$, giving after integration:

$$I_4 = \frac{K^2}{2p},$$

with

$$K = [-A\sin(2hq) + B\cos(2hq)] \\ = B\cos(2hq)\left[1 - \frac{A}{B}tg(2hq)\right]. \tag{A.5}$$

Regarding the calculus of K^2, consider the trigonometric relation that transforms $\cos^2(2hq)$ to $tg^2(2hq)$; thanks to Eqs. (7) and (8), after convenient simplifications, it can be part in the form

$$K^2 = B^2\frac{[(q^2| \pm \xi\Theta)| \mp \Theta(\xi| \mp \Theta)]^2}{[(q^2| \pm \xi\Theta)^2 + q^2(\xi| \mp \Theta)^2]}. \tag{A.6}$$

A whole expansion of both the numerator and the denominator allows significant respective simplification as $(q^2+\Theta^2)^2$ and $(q^2+\xi^2)(q^2+\Theta^2)$, yielding (Table 1)

$$I_4 = \left(\frac{B^2}{2p}\right)\left(\frac{q^2+\Theta^2}{q^2+\xi^2}\right). \tag{A.7}$$

Appendix B

According to Eq. (10), the power flow $P_3^{\mathrm{TE-TM}}$ can be found after integration as the following form $(\alpha_3^{\mathrm{TE-TM}}(B^2/2r)H)$, with

$$H = \frac{\left|\begin{array}{l}\cosh^2\\ \cos^2\end{array}\right.(2\,\mathrm{d}t + \chi)}{\left|\begin{array}{l}\cosh^2\\ \cos^2\end{array}\right.(\chi)}. \tag{B.1}$$

Then it is necessary to explain H as a function of the new parameters defined in Eq. (6). By way of classical trigonometric relations between the respective functions $\cos^2$ and ch^2 with regard to both functions tg^2 and th^2, and considering the set of Eq. (6), we can write

$$\left|\begin{matrix}\cosh^2\\ \cos^2\end{matrix}\right.(\chi) = \frac{W^2}{W^2 \mp \Theta^2} \text{ and}$$

$$\left|\begin{matrix}\cosh^2\\ \cos^2\end{matrix}\right.(2\,\mathrm{d}t + \chi) = \frac{W^2}{W^2 \mp (\eta_{1\to 2}^{\mathrm{TE-TM}})^2 Y^2}. \tag{B.2}$$

Thus, both (B.1) and (B.2) yield the expression of P_3 as given in Table 1.

Regarding the power flow $P_2^{\mathrm{TE-TM}} = \alpha_2^{\mathrm{TE-TM}} \cdot I_2$ (Eq. (10)), the trigonometric relations between both functions $\cos^2$ and ch^2 linked to the functions cos and ch are applied before integration, so as to come up with

$$I_2 = \frac{B^2}{2t\left|\begin{matrix}\cosh^2\\ \cos^2\end{matrix}\right.(\chi)} \cdot \times \left[2\,\mathrm{d}t + \frac{\left|\begin{matrix}\sinh\\ \sin\end{matrix}\right.(2(2\,\mathrm{d}t+\chi))}{2} - \frac{\left|\begin{matrix}\sinh\\ \sin\end{matrix}\right.(2\chi)}{2}\right]. \tag{B.3}$$

Then, the first Eq. (B.2) is substituted in the first term of (B.3), while considering the trigonometric relations that link both functions sinh and sin to the respective quotients of both functions *th* and *tg* is taken into account with the proper Eq. (6) so as to give (Table 1):

$$I_2 = \left(\frac{B^2}{2t}\right)\left(\frac{W^2| \mp \Theta^2}{W^2}\right)\left(2\,\mathrm{d}t| \mp \frac{Yt}{t^2| \mp Y^2} - \frac{W\Theta}{W^2| \mp \Theta^2}\right). \tag{B.4}$$

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