# A guided mode in the range of the radiation modes for a rib waveguide
# Exemple d'un mode guidé plongé parmi les modes de radiation pour un guide ruban

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## Abstract

The stratified cladding of integrated optical waveguides is dispersive, which induces quite original phenomena. We present an example of a dielectric waveguide for which there exists a guided mode which propagates faster than some waves in the stratified cladding. Such a guide presents a specific symmetry and the key point is a separate study of the symmetrical and antisymmetrical modes.

### Résumé

Le caractère dispersif du milieu stratifié de référence des microguides réalisés en optique intégrée induit des phénomènes originaux. Nous présentons un exemple de guide pour lequel nous montrons qu'il existe un mode guidé qui se propage plus vite que certaines ondes dans la gaine stratifiée. Ce guide présente un symétrie particulière et le résultat est obtenu de l'étude séparée des modes symétriques et antisymétriques.


In integrated optics the cross-section of a rib microguide is a bounded perturbation; the core of the guide; a stratified medium; the cladding. The stratified nature of the cladding induces theoretical difficulties and leads to original results when we study the guided modes. As a matter of fact the perfectly stratified medium is dispersive: light waves propagate with different speeds.

Are there guided modes which propagate faster than some waves in the stratified medium? The propagation constant of a such guided mode would be also the one of a radiation mode.

In the simpler case where the medium is homogeneous, with index $n_\infty$, all the waves propagate at the same speed $c/n_\infty$ ($c$ is the light speed in the vacuum). Accordingly, in a guide with an homogeneous cladding a guided mode cannot propagate faster than $c/n_\infty$ [1]. A similar result is proved when the cladding is a diopter [2,3].

In the case of a stratified medium, we can construct a waveguide whose cladding is a slab guide (with 3 layers) and which presents a guided mode inside the range of the radiation modes. We give sufficient conditions for the existence of such a guided mode. The guide extends to infinity in all directions and is invariant in the propagation direction $(Oz)$. It is defined by its refractive index profile $n(x,y)$. We note $\bar{n}(y)$ the refractive index profile of the perfectly stratified medium.

In the weak guidance approximation, looking for guided modes reduces to solving the following scalar problem:

$$-\triangle u - k^2 n^2 = -\beta^2\, u, \quad \text{in } \mathbb{R}^2, \tag{1}$$

where $k$ is the wave number, $\beta$ the propagation constant of the mode, with $u \neq 0$ and $\int_{\mathbb{R}^2} |u|^2\, dxdy < +\infty$. Such a mode propagates at the speed $kc/\beta$ in the $Oz$-direction.

We follow a method already used in the work of Witsch[4]. We consider the index profile $n(x,y)$ defined by:

$$\begin{aligned} n(x,y) \quad &= n_+ \text{ if } (x,y) \in (\mathbb{R}\times]-h,h[) \\ &\qquad \cup (]-d,d[\times]-H,H[), \\ &= n_\infty \text{ otherwise,} \end{aligned} \tag{2}$$

with $n_+ > n_\infty$, see Figure 1. The associated slab

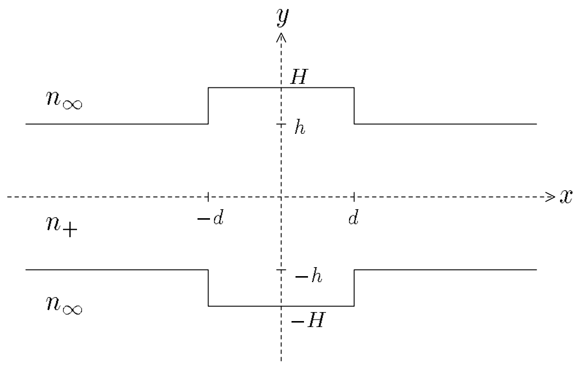


Figure 1: Index profile of the guide.

waveguide is defined by:

$$\begin{aligned} \bar{n}(y) &= n_+ \quad && \text{if } y \in ]-h, h[, \\ &= n_\infty && \text{otherwise.} \end{aligned} \tag{3}$$

Since the guide is perfectly symmetrical with respect to $(Ox)$, there is no connection between the symmetrical waves and the antisymmetrical waves. For the antisymmetrical waves the minimal speed in the external stratified medium is given by the speed $s_a$ of the first antisymmetrical mode of the associated slab waveguide. This one propagates faster than the fundamental mode (symmetrical) of the slab waveguide, whose speed is denoted $s_s$. We can check that some antisymmetrical guided modes propagate with a speed lying between $s_s$ and $s_a$.

Indeed, the solutions of (1) are either symmetrical or antisymmetrical with respect to $(Ox)$. Then we solve (1) by solving two problems in $\mathbb{R}\times]0, +\infty[$ which differ by the condition on $(Ox)$: $\frac{\partial u}{\partial \nu} = 0$ or $u = 0$, where $\nu$ is the unit external normal vector. In order to derive the announced result, we need to study two slab waveguides. The modes of the slab waveguide defined by (3) are given[5] by the solutions of:

$$-\frac{d^2\varphi}{dy^2} - k^2\bar{n}^2\varphi = -b^2\varphi \ \text{ on } \mathbb{R}, \tag{4}$$

with $\varphi \neq 0$ and $\int_{\mathbb{R}} |\varphi|^2 \, dy < +\infty$. We note $b_s$ (respectively $b_a$) the propagation constant of the first symmetrical (respectively antisymmetrical) guided mode. Then $b_s$ is given by the smallest solution $\kappa$ of the dispersive equation:

$$\tan(\kappa h) = \frac{\delta}{\kappa}$$

and $b_a$ by the solution of:

$$\tan(\kappa h) = -\frac{\kappa}{\delta},$$

with $\kappa^2 = k^2n_+^2 - b^2$ and $\delta^2 = b^2 - k^2n_\infty^2 = k^2(n_+^2 - n_\infty^2) - \kappa^2$. So, the solutions are given by the intersections of the curve of $\kappa \longmapsto \tan(\kappa h)$ with curves of two functions of $\kappa$.

If the width of the slab guide is $2H$ ($H > h$), instead of $2h$, the equations are identical, replacing $h$ by $H$. For this guide, we note $\tilde{b}_a$ the propagation constant of the first antisymmetrical mode, see the figure 2 for the curves. Notice

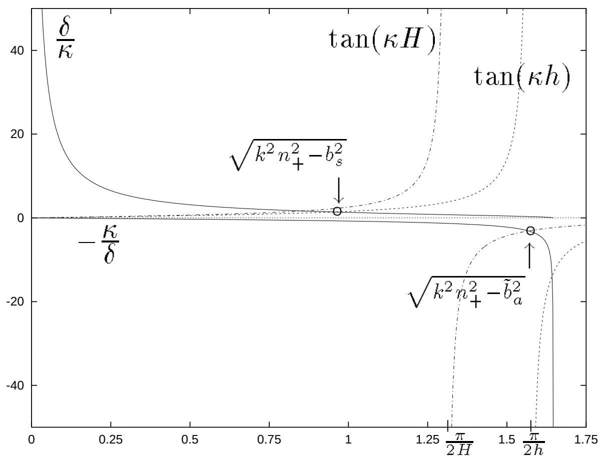


Figure 2: Graphic definition of $b_s$ and $\tilde{b}_a$.

that there exists an antisymmetrical mode for the slab guide with index $\bar{n}$ if and only if the pole of $\kappa \longmapsto -\kappa/\delta$ is greater than $\pi/(2h)$, see Figure 2:

$$kh > \frac{\pi}{2\sqrt{n_+^2 - n_\infty^2}}. \tag{5}$$

Moreover we have $-b_s^2 < -\tilde{b}_a^2$ if $\pi/(2H) > \sqrt{k^2n_+^2 - b_s^2}$, see Figure 2:

$$kH < \frac{\pi}{2\sqrt{n_+^2 - n_s^2}}, \tag{6}$$

with $n_s = b_s/k$ the effective index of the first mode.

We can now use the conditions (5) and (6) linking the dimensions of the guide (except the width $d$) and the wave number $k$.

With the min-max principle we prove[6] the existence of a symmetrical mode of the rib waveguide with a propagation constant $\beta_s$ such that

$\beta_s^2 > b_s^2$. Under the assumption (5), there exists an antisymmetrical mode with a propagation constant $\beta_a$[6] such that $\beta_a^2 > b_a^2$.

The effective index of the symmetrical mode is between the effective index of symmetrical modes of the two slab guides with width $h$ and $H$. The effective index of the antisymmetrical mode satisfies a similar comparison: $b_a^2 < \beta_a^2 \leq \tilde{b}_a^2$.

Under hypothesis (6) we have therefore:

$$b_a^2 < \beta_a^2 < b_s^2.$$

Moreover $\beta$ is the propagation constant of a symmetrical radiation mode if and only if $\beta^2 \leq b_s^2$. Then $\beta_a$ is a propagation constant characterizing an antisymmetrical guided mode and a symmetrical radiation mode. To illustrate this result we present a numerical example (finite element method). If we choose $n_\infty = 3.34$, $n_+ = 3.44$ and $h = 1\mu m$, (5) and (6) are satisfied with $k = 4.053668\mu m^{-1}$ (wavelength: $\lambda = 1.55\mu m$) and $H = 1.2\mu m$. The width $d$ is arbitrarily fixed to $1\mu m$. With a finite elements method we compute[6,7] 2 guided modes. The second is in the range of the radiation modes, see Table 1.

| | |
|---|---|
| $kn_\infty$ | 13.53925 |
| $b_a$ | 13.74391 |
| $\beta_a$ | 13.76629 |
| $b_s$ | 13.89269 |
| $\beta_s$ | 13.89502 |
| $kn_+$ | 13.94462 |

Table 1: Values of propagation constants.

See Figure 3 for the isovalues of the normalized modes in the transverse section. The minimum is zero for the external isovalue and 1 or $-1$ in the center. To conclude we notice that the assumptions (5) and (6) are very restrictive and non necessary. We have expressed these conditions in the simple case defined by (2) but the method is valid for more complex devices if they are symmetrical with respect to $(Ox)$.

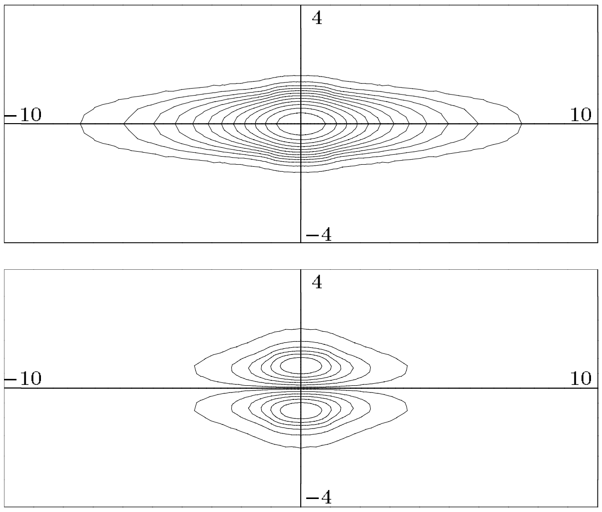


Figure 3: Isovalues of the first and second modes.

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