

# Reference Manual

Fortran implementation

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## Introduction

This document describes in details the MUESLI Fortran library routines.

MUESLI is freely available at the following address:

https://perso.univ-rennes1.fr/edouard.canot/muesli

More information can be found in the following documents:

- MUESLI Installation Guide
- MUESLI User's Guide
- MUESLI Inside

The source code of MUESLI comes with many examples which can also give some help.

MUESLI is split in two parts, which correspond to the following Fortran modules:

 Basically, FML (Fortran Muesli Library) contains all necessary materials to numerically work with a dynamic array (dynamic in size, type and structure), called mfArray; all available routines are described in the first section of this guide.

To work with FML, your Fortran source code must include the statement:

```
use fml
```

FGL (Fortran Graphics Library) contains graphic routines which use the mfArray objects; these
routines are described in the second section of this guide.

To work with FGL, your Fortran source code must include the statements:

```
use fml use fgl
```

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#### Typographic convention

In this guide, the colored brackets ('[' and ']') indicate some optional arguments or other things; similarly, the colored pipe ('|') specifies alternates in a choice. These symbols are used only in interface definition of the routines.

Coloring is also used to differentiate source code and execution output. Lines from source code will be always displayed in this color, whereas input/output in a terminal will be always displayed in this color.

For sake of clarity, array constructors use black brackets '[' and ']' (Fortran 2003) instead of '(/' and '/)' (Fortran 95).

#### About the name

Muesli: loose mixture of mainly rolled oats and often also wheat flakes, together with various pieces of dried fruit, nuts, and seeds. There are many varieties, some of which also contain honey powder, spices, or chocolate. (from https://en.wikipedia.org/wiki/Muesli)

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## 1 FML: Numerical Library

FML contains all routines to work with matrices and to do some linear algebra.

Routines beginning with 'mf' are functions, whereas those beginning with 'ms' are subroutines. Functions are used to return only one object (e.g. one mfArray); subroutines can also return many objects, but always via the mfOut facility.

FML also defines some new derived types: mfArray, mfUnit, mf\_Out, mfMatFactor, mfTriConnect, mfTetraConnect, mf\_DE\_Options, mf\_NL\_Options.

Besides, FML defines public constant entities:

– an integer parameter:

```
MF_DOUBLE
```

which defines the working precision in MUESLI. In the current version, it stores the kind of double precision reals. In all this document, when Fortran numerical kind is not specified, real and complex stands for real(kind=MF\_DOUBLE) and complex(kind=MF\_DOUBLE), respectively.

- some mfArray parameters:

```
MF_EMPTY, MF_COLON (and its alias MF_ALL), MF_END, MF_NO_ARG
```

which are, respectively, an empty mfArray, the ':' pseudo-operator (as in Fortran 90 or MATLAB, but specific to FML), an automatic index for pointing to the end of a dimension of an mfArray and a pseudo-keyword to tell that an argument is not present in mfOut.

- some real(kind=MF\_DOUBLE) parameters:

```
MF_PI
MF_EPS
MF_INF
MF_NAN
MF_E
MF_REALMAX
MF_REALMIN
```

and some useful mathematical auxiliairy constants:

```
MF_BESSEL_JO_ROOTS(:)
MF_BESSEL_J1_ROOTS(:)
```

(30 first non-zero roots for both Bessel functions  $J_0$  and  $J_1$ , stored in increasing order)

- one  ${\tt complex(kind=MF\_DOUBLE)}$  parameter:

```
MF_I
```

- and the character parameters:

```
MF_MUESLI_VERSION
MF_COMPILER_VERSION
```

which store, respectively, the MUESLI library version and the compiler vendor and version. The boolean mfIsVersion routine must be used in order to compare each of these parameters with a given version number. Besides, other character functions may be used to retrieve additional information: MF\_LAPACK\_VERSION() (the Blas/Lapack library version) and MF\_COMPILATION\_CONFIG() (the configuration used to compile the whole Muesli library: Debug or Optim).

MF\_INF and MF\_NAN contain special IEEE values (resp. Infinity and Not-a-Number).

FML also defines the following global variables:

- integer variables:

STDERR STDIN STDOUT

which can be modified either directly by the user, or via the msSetStdIO routine.

 logical variables: MF\_NUMERICAL\_CHECK which allows the user to tell Muesli to do additional checks.

The available routines have been arranged into sub-parts:

Core Routines

File Input/Output

Data Analysis Functions

Operators

**Elementary Math Functions** 

Specialized Math Functions

Elementary Matrix Manipulation Functions

Matrix Functions

Polynomial Functions

Optimization and Function Functions

Sparse Matrices.

#### 1.1 Core Routines

mfArray automatic and dynamic array (derived type) mfArray conversion mf mfArray assignment =, msAssign mfArray data modification and extraction msSet, mfGet mfCount true values count msDisplay mfArray pretty print mfDisplayColumns columns used in pretty print mfArray printing format msFormat MUESLI objects deallocation msRelease msAutoRelease mfArray conditional deallocation mfO11t groups output arguments set of output arguments (derived type) mf Out. msPointer, msFreePointer smart pointer between f90 array and mfArray mfNbPointers nb of f90 pointers pointing to an mfArray smart pointer between mfArray and f90 array msEquiv mf\_Int\_List list of integers (derived type) mf\_Real\_List list of reals (derived type) mfIsEmpty mfArray empty test mfArray equality test mfIsEqual, mfIsNotEqual mfIsLogical mfArray boolean test mfArray real test mfIsReal mfIsComplex mfArray complex test mfArray numeric test mfIsNumeric mfArray dense storage test mfIsDense mfIsSparse mfArray sparsity test mfIsScalar, mfIsVector, mfIsMatrix mfArray scalar, vector and matrix test mfArray kind of vector test mfIsRow. mfIsColumn mfIsPerm mfArray permutation vector test All, Any test on boolean mfArray Shape, mfShape shape of an mfArray size of an mfArray Size, mfSize mfInt scalar integer conversion mfDble scalar real conversion mfCmplx scalar complex conversion mfGetMsgLevel, msSetMsgLevel message level tuning mfGetTrbLevel, msSetTrbLevel error traceback tuning msSetColoredMsg colorize Muesli messages on terminal print a colored user message on terminal msPrintColoredMsg msSetTermColor set color for printing on terminal helper for debugging purpose msMuesliTrace usual logical unit inquiry and modification msGetStdIO, msSetStdIO I/O flush msFlush user pause or timing pause msPause msSetTermWidth, mfGetTermWidth set and get terminal character width set and get out-of-range filling by msSet msSetAutoFilling, mfGetAutoFilling msInitArgs, msFreeArgs mfArray arguments' protection and release msSetAsParameter data protection msReturnArray, mfIsTempoArray mark and check temporary mfArray nb of floating-point operations msFlops, mfFlops mfIsFlops0k inquire if flops is available

mfGetAutoComplex, msSetAutoComplex
msEnableFPE, msDisableFPE

 ${\tt mfGetRoundingMode}, \, {\tt msSetRoundingMode}$ 

conversion to complex

run-time floating-point exceptions trapping

floating-point rounding mode

debugging additional checks

MF\_NUMERICAL\_CHECK

mfUnit

msUsePhysUnits msSetPhysDim

msSetPhysUnitAbbrev mfHasNoPhysDim mfHaveSamePhysDim

msPrepHashes msPrintHashes

msPostHashes msPrepProgress msPrintProgress msPostProgress

mfReadLine

msReadHistoryFile msWriteHistoryFile msClearHistory msAddEntryInHistory msRemoveLastEntryInHistory

mfToLower mfToUpper

msFindIOUnit

mfIsVersion msRequMuesliVer

MF\_MUESLI\_VERSION
MF\_COMPILER\_VERSION
MF\_COMPILATION\_CONFIG

physical unit (derived type) Physical units activation set physical unit

set user physical unit abbrevation

dimensionless test dimension equality test

hashes print preparation

hashes print

hashes print conclusion percent progress preparation

percent progress

percent progress conclusion

read a line from terminal with editing and history facilities

read a 'readline' history file write the 'readline' history in a file

clear the 'readline' history

add an entry in the 'readline' history remove last entry in the 'readline' history

lowering string's case uppering string's case

automatically find a free IO unit number

test on version strings

check for a minimum Muesli version

MUESLI version COMPILER version

Configuration used to compile MUESLI

See also:

File Input/Output

Data Analysis Functions

Operators

Elementary Math Functions

Specialized Math Functions

Elementary Matrix Manipulation Functions

Matrix Functions

Polynomial Functions

Optimization and Function Functions

Sparse Matrices

#### mfArray

## automatic and dynamic array (derived type)

Description:

This is the fundamental derived type in MUESLI. Declaration must be made as follows:

type(mfArray) :: A

This derived type may contain real or complex numerical values, using a dense or sparse storage. All numerical values are stored in *double precision*, *i. e.* use 8 bytes of memory.

Special data types are:

- permutation vector (integer vector)
- boolean values (in dense storage only)

All possible data types are sumarized in figure 1.

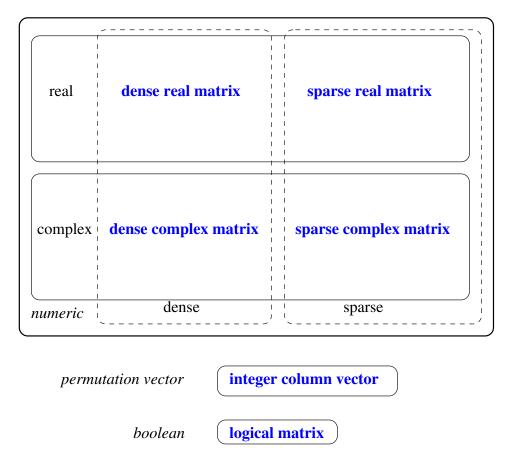


Figure 1: mfArray data types

The data type stored in a mfArray may be retrieved by the following inquiring functions: mfIsReal, mfIsComplex, mfIsDense, mfIsSparse, mfIsNumeric, mfIsPerm, mfIsLogical.

See also: mf, msRelease, msPointer, msEquiv, msSaveAscii

#### mf

## mfArray conversion

#### Description:

This function initializes and returns a temporary mfArray from some common Fortran types: integer, real (single or double), complex (single or double), of rank 0 to 2 (scalar, vector or matrix).

For example:

```
mf([1,3,5,7,11])
mf([1.3,2.05,-5.1e-3])
```

Whatever the argument type of mf (integer, single precision real, double precision real), all numerical numbers are stored in mfArrays as double precision.

By default, this function creates row vectors, so use the following syntax if you want a column vector (avoiding an unnecessary copy):

```
mf([1, 3, 5, 7, 11], transpose=.true.)
```

This latter optional transposition works also for matrices. By default, the transposition is not used (see examples below).

See also: mfArray

```
Example(s):
```

```
real(kind=MF_DOUBLE) :: tab(2,4)
tab(:,:) = reshape( [(1.5d0*i,i=1,8)], [2,4] )
A = mf( tab )
call msDisplay( A, "A" )
At = mf( tab, transpose=.true. )
call msDisplay( At, "At" )
```

# output: A =

```
1.5000 4.5000 7.5000 10.5000
3.0000 6.0000 9.0000 12.0000
At =
```

```
1.5000 3.0000
4.5000 6.0000
7.5000 9.0000
```

12.0000

10.5000

## mfArray assignment

This Fortran operator has been overloaded in MUESLI to make working with mfArray easy.

For example, you can assign any numerical object to A, as simply as:

$$A = [1, 2, 3, 4, 5]$$

The right hand side (RHS) can be of type boolean, integer, real (single or double) or complex (single or double), of rank 0 to 2 (scalar, vector or matrix), or of course mfArray.

As the allocation of data is automatic during assignment, you should free each mfArray as soon as possible (via the msRelease routine), in order to decrease the memory usage.

Sometimes, you could be warned that an assignment is inefficient, when the *RHS* is a temporary mfArray. In such a case, you could use the msAssign routine instead.

Besides, MUESLI allows to assign a dense mfArray A to a boolean, an integer, a real or a complex. So, a statement like:

$$f90_{obj} = A$$

is valid under the following assumptions:

- shapes (of *LHS* and *RHS*) must match;
- types must be consistent (i. e. numeric mfArrays can be assigned to integer or real f90 arrays whereas boolean mfArrays must be assigned to logical f90 arrays mfArrays data types are defined in figure 1).

See also: msPointer, msEquiv

## msAssign

## more efficient mfArray assignment

 ${\it Interface:}$ 

```
subroutine msAssign( dest, source )
  type(mfArray), intent(in) :: source
  type(mfArray) :: dest
```

#### Description:

Ordinarily, you should use the simple = form of assignment. The msAssign routine is intended to make it efficiently, when the RHS is a temporary mfArray.

For example, you should write:

```
call msAssign( C, A + B )
```

which avoids a copy of the data contained in  ${\tt A} + {\tt B}$  (an internal pointer is used instead). In comparison, the statement:

```
C = A + B
```

involves such a copy.

See also: =

#### msSet

#### mfArray data modification

Generic Calling syntax:

```
call msSet( data, x, a1 [, a2] )
```

#### Description:

This routine is used to modify some elements of the mfArray x. The indices of these elements are specified by a1 and a2 for array sections, or only by a1 for scattered elements in the whole array (in this case, a1 is a long-column-index vector).

If a2 is not present, the mfArray x may be a vector, because only one index is given to specify which elements have to be changed. Note that if the index a1 (or a2) is out of range, the shape of x is modified accordingly, i.e. the array is extended (see the remarks at the end of this section).

If a1 (or a2) is a vector, more than one element can be modified; moreover, this vector doesn't need to contain consecutive indices.

If the modification affects more than one element, and if data contains only one value, this latter value is spread across all the target elements.

data may be of type real, complex or mfArray. It doesn't need to be of same data type as x. A conversion is then made, and concerns either data or x, according to the type.

Usually a1 and a2 are integers (scalar or vector), but they may be of type mfArray (in this latter case, they cannot be matrices).

In the special case where a1 (or a2) is MF\_COLON (or its alias MF\_ALL), then all the specified row(s) (or column(s)) are simultaneously modified. If data is empty (i.e. equal to MF\_EMPTY, but only for a dense mfArray x), then the specified row(s) or column(s) are deleted.

a1 and/or a2 may be also special integer sequences, written under the form

```
i1 .to. i2 [.by. i3]
or i1 .by. i3 .to. i2
[ ( i, i = i1, i2 [, i3] ) ]
```

(The operator named '.step.' is an alias for '.by.')

These special integer sequences can be combined as

```
int_seq .and. int_seq
or int_seq .but. i1
```

The step i3 is taken equal to 1 if it is not present. Note that a1, a2, i1 and i2 may also be replaced by an expression using MF\_END (e. g. MF\_END-1) which is an automatic alias for the end value of the corresponding dimension. Be aware that arithmetic involving MF\_END can use only addition or substraction.

Finally, the mfArray x cannot be temporary.

 $\cdots/\dots$ 

#### Remarks:

- to initialize a whole mfArray with 0 or 1, the following routines are available: mfZeros and mfOnes.
- when the mfArray x has a sparse structure, data may be a scalar or a vector (dense or sparse). In such a case, only whole columns of x may be modified. Modifying one element is however allowed, in so far as indices of this element are not out of range (see below, however).
- when the mfArray x has a dense structure, data cannot be sparse.
- not all combinaisons for the types of a1 and a2 are valid. Use preferently the same type for these
  two arguments.
- out-of-range indices are accepted, except when a long-column index is used. For the dense storage, the mfArray is reallocated to the right shape. A message of kind "info" is however emitted (see msSetMsgLevel). During the reallocation, the new elements not targeted are initialized by a predefined value (see below). For the sparse storage, the assignment is done only if the value of nzmax is sufficient (see msSpReAlloc), and, of course, no new elements are added, apart those explicitly targeted by the command.
- the predefined value used to fill the out-of-range indices is by default the NaN (special IEEE number), but this can be changed by use of the msSetAutoFilling routine.

See also:

mfGet

#### mfGet

#### mfArray data extraction

Generic Calling syntax:

```
mfGet( x, a1 [, a2] )
```

#### Description:

This routine extracts some elements (described by the indices a1 and a2, or — when x is a matrix — only by the long-column index a1) of the mfArray x and returns them in an mfArray. Note that if the index a1 (or a2) is out of range, an error results and an empty mfArray is returned.

If a1 (or a2) is a vector, more than one element are extracted; moreover, this vector doesn't need to contain consecutive indices.

Usually a1 and a2 are integers (scalar or vector), but they may be of type real or boolean mfArray.

In the special case where a1 (resp. a2) is MF\_COLON (or its alias MF\_ALL), then all the specified columns (resp. rows) are extracted.

a1 and/or a2 may be special integer sequences, written under the form

```
i1 .to. i2 [.by. i3]
or i1 .by. i3 .to. i2
[ ( i, i = i1, i2 [, i3] ) ]
```

(The operator named '.step.' is an alias for '.by.')

These special integer sequences can be combined as

```
int_seq .and. int_seq
or int_seq .but. i1
```

The step i3 is taken equal to 1 if it is not present. Note that a1, a2, i1 and i2 may also be replaced by an expression using MF\_END (e. g. MF\_END-1) which is an automatic alias for the end value of the corresponding dimension. Be aware that arithmetic involving MF\_END can use only addition or substraction.

Remarks: Sparse mfArrays are partially supported:

- this routine can extract a scalar value, a row or a column, or a submatrix; in this latter case, the submatrix must be contiguous;
- when extracting rows, the indices for these rows must be contiguous (this limitation doesn't hold when extracting columns);
- the above mentioned special integer sequences (built via the .to. and .by. operators) cannot be used (not yet) for indices a1 or a2;
- when extracting a single row or a single column, resulted mfArray has a dense structure.

See also: msSet

mfCount true values count

## ${\it Interface:}$

```
function mfCount( A, dim ) result( out )
  type(mfArray), intent(in) :: A
  integer, intent(in), optional :: dim
  type(mfArray) :: out
```

#### Description:

Returns the number of true values in the columns (or rows) of a boolean mfArray (dense or sparse).

If dim is present, A is always considered as a matrix, and the routine processes each column (if dim = 1) or each row (if dim = 2).

If dim is not present, it returns the number of true values in the whole array.

When out is a vector, it is dense.

The behavior of this routine is similar to the *count* Fortran 90 intrinsic function.

#### Example(s):

```
x = mfMagic(3)
   call msDisplay( x, "x" )
   call msDisplay( mfCount(x>5.0d0), "mfCount(x>5.0d0)")
   call msDisplay( mfCount(x>4.0d0,dim=1), "mfCount(x>4.0d0,dim=1)")
   call msDisplay( mfCount(x>4.0d0,dim=2), "mfCount(x>4.0d0,dim=2)")
output:
 x =
    8
          3
                4
    1
          5
                9
          7
                2
    6
mfCount(x > 5.0d0) =
    4
mfCount( x > 5.0d0, dim=1) =
    2
         1
               1
mfCount(x > 4.0d0, dim=2) =
    1
    2
    2
```

#### msDisplay

#### mfArray pretty print

First calling syntax:

This routine displays an mfArray object (and other object types, as described below) on the screen or terminal, in a pretty form which is column oriented. Not all the decimal digits are printed: their number depends on the format used.

The object x may also be: a Fortran type, integer, real (single or double), complex (single or double), of rank 0 to 2 (scalar, vector or matrix); a MUESLI derived type, mfUnit and mf\_Int\_List.

Only for mfArrays:

- up to four pairs (x, string) can be simultaneously displayed.
- if the optional integer argument head (resp. tail) is present, then only the first (resp. last) rows will be printed. These two arguments can be both present. This convenience is useful to see the begin and the end of a very long column vector. For the case of a long row vector, it will display the head first values, followed by the tail last values, but only if the width of your terminal is sufficient. Internal tests are made about consistency between head, tail and the shape of the array.

For mfArrays (not boolean) and mfUnits, the optional argument unit allows the user to specify another consistent (i. e. of same physical dimension) unit for printing. For example, a length registered in meter (S.I. unit) may be printed in millimeter. The use of the constant parameter SI\_unit enforces the routine to display, if possible, the appropriate S.I. unit.

Only for mf\_Int\_Lists and vectors of mf\_Int\_List:

- only one pair (x, string) can be displayed at a time.
- an additional optional boolean argument (compact) may be used to print the integers rows more compactly.

Second calling syntax:

```
call msDisplay( x, string, legend1 [, legend2, ..., legend9 ] )
```

This latter syntax enables the insertion of legends at the head of each column. Note the maximum number possible, 9, and also that it is restricted to real data in the x mfArray.

N.B.: When displaying an object having a great number of columns, this routine will print first the number of columns adapted to the width of your terminal. If the output is redirected to a file, you may want to change this width: then use the MF\_COLUMNS environment variable (it is the terminal character width, like in the UNIX command resize, and not the matrix column's number).

See also: msFormat, mfDisplayColumns

Example(s): Many outputs from this routine are shown as examples in the MUESLI User Guide.

## mfDisplayColumns

## columns used in pretty print

 ${\it Interface:}$ 

```
function mfDisplayColumns( A, unit ) result( out )
  type(mfArray), intent(in) :: A
  type(mfUnit), intent(in), optional :: unit
  type(mfArray) :: out
```

Description:

mfDisplayColumns returns the number of elements printed by the routine msDisplay on each row.

See also: msDisplay

#### msFormat

#### mfArray printing format

Calling syntax:

```
call msFormat( [ mantissa, exponent ] )
```

The optional argument mantissa may be "short" (5 digits printed), "long" (15 digits printed) or "hex" (hexadecimal output). By default, the short format is used.

The optional argument exponent may be:

- "auto" (automatic format: fixed or exponential format);
- "sci" (scientific format: exponent is always printed);
- "eng" (engineer format: if the mfArray to be displayed is not a scalar, a scale factor is always used and its exponent is moreover a multiple of three).

By default, the automatic format is used.

In the case where this routine is called without any argument, default values for both keywords are used.

 $See \ also: {\tt msDisplay}$ Example(s): x = .t. mfSqrt(mf([2, 3, 5]))call msDisplay( x, "x (short format)" ) call msFormat( "long" ) call msDisplay( x, "x (long format)" ) output: x (short format) = 1.4142 1.7321 2.2361 x (long format) = 1.41421356237310 1.73205080756888 2.23606797749979 x = [1.5, 2.6E1, 3.7E2, 4.8E4]call msFormat( exponent="sci" ) call msDisplay( x, "x (scientific format)" ) call msFormat( exponent="eng" ) call msDisplay( x, "x (engineer format)" ) output: x (scientific format) = 1.5000E+00 2.6000E+01 3.7000E+02 4.8000E+04 x (engineer format) = 1.0E+03 \* 0.0015 0.0260 0.3700 48.0000

#### msRelease

## MUESLI objects deallocation

When you have finished to work with an mfArray, you may deallocate its internal array by using:

```
call msRelease( x1 [, x2, x3, x4, x5, x6, x7 ] )
```

Up to seven mfArrays can be simultaneously released.

Remark: After execution of your program, some compilers or tools may report a warning about some memory leaks. Use of this routine for all of the automatic variables (described below) avoids such warnings.

This routine can be used to free the following derived types: mfArray, mfMatFactor, mfTriConnect, mfTetraConnect, mf\_Int\_List, mf\_NL\_Options and mf\_DE\_Options (only one variable may be freed at a time, except for an mfArray object).

See also: msAutoRelease, msSetAsParameter, msExitFgl

#### msAutoRelease

## mfArray conditional deallocation

This routine is similar to msRelease:

```
call msAutoRelease( x1 [, x2, x3, x4, x5, x6, x7 ] )
```

but the deallocation is done only if the mfArray is temporary.

Is is used only in the context of a user-routine which takes arguments of type mfArray which can be temporary objects.

See also: msRelease

#### mfOut

#### group output arguments

This function is used to group output arguments in calls of some MUESLI subroutines. For example:

```
call msLU( mfOut(L,U,p), A )
```

This facility is only intended to clearly identify the output arguments from the input ones. It is reminiscent of the Matlab syntax:

$$[L,U,p] = lu(A)$$

Arguments of mfOut match always the following MUESLI derived types: mfArray or mfMatFactor.

Case of optional arguments: If some of these arguments are optional, they must be called in order, not by use of keyword as in Fortran 90 syntax. Moreover, for multiple optional arguments, one of them can be omitted by using the special mfArray MF\_NO\_ARG, as in the following example:

Indeed, referring to the msOdeSolve routine, the yp and solve\_log are both optional, and if the user wants to retrieve only the second one, he has to use MF\_NO\_ARG instead of yp.

Remarks: actually, the mfOut function returns an object of type mf\_Out. You don't need to use this latter derived type unless you want to define yourself new routines which use mfOut to group output arguments.

See also: mfArray, mfMatFactor, mf\_Out

## ${\tt mf\_Out}$

# set of output arguments $(derived\ type)$

## Description:

This is a derived type in MUESLI. You need to employ this type only if you wish to define yourself subroutines whose output arguments are grouped with the mfOut function.

See also: mfArray, mfOut

#### msPointer

## smart pointer between f90 array and mfArray

 $Generic\ Interface:$ 

```
subroutine msPointer( A, f90_ptr )

type(mfArray) :: A

real(kind=MF_DOUBLE), pointer :: f90_ptr(:[,:])
or complex(kind=MF_DOUBLE), pointer :: f90_ptr(:[,:])
```

#### Description:

This routine is approximately equivalent to:

```
f90_ptr => A
```

and then, you can access the internal data of A via a classical Fortran array. This association is not always possible, because the types of A and f90\_ptr must match. Ranks however doesn't need to match: you may have a rank-1 array pointing to a rank-2 array; in this latter case, we could see f90\_ptr as a "long column pointer".

A cannot be temporary, because at any time it can be deleted, and then f90\_ptr would become hangling.

Warning:

During all the time when f90\_ptr points to A, some internal properties of A are locked. So, it is strongly recommended to use the routine msFreePointer: (i) to nullify the pointer f90\_ptr and (ii) to unlock properties of A. Be aware that

```
f90_ptr => null()
```

is not sufficient (this way will not unlock the internal mfArray properties of A).

See also: mfNbPointers, msFreePointer, msEquiv

#### msFreePointer

## smart pointer release

 $Generic\ Interface:$ 

```
subroutine msFreePointer( A, f90_ptr )
    type(mfArray) :: A
    real(kind=MF_DOUBLE), pointer :: f90_ptr(:[,:])
or complex(kind=MF_DOUBLE), pointer :: f90_ptr(:[,:])
```

## Description:

This routine releases the link made by msPointer, and unlocks the internal properties of A.

See also: mfNbPointers, msPointer

#### mfNbPointers

## nb of f90 pointers pointing to an mfArray

 $Generic\ Interface:$ 

```
function mfNbPointers( A ) result( n )
  type(mfArray) :: A
  integer :: n
```

Description:

This routine returns the number of Fortran 90 pointers pointing to A.

 $See \ also: {\tt msPointer}, {\tt msFreePointer}$ 

#### msEquiv

## smart pointer between mfArray and f90 array

 $Generic\ Interface:$ 

```
subroutine msEquiv( f90_array, A )
    real(kind=MF_DOUBLE), target :: f90_array(:[,:])
or complex(kind=MF_DOUBLE), target :: f90_array(:[,:])
    type(mfArray) :: A
```

Description:

This routine is approximately equivalent to

```
A => f90_array
```

but the mfArray A becomes restricted, in the sense that you can modify the data but not the shape of your f90\_array, nor its allocation status. As a consequence, you can use most of (but not all) MUESLI routines to work with your f90\_array, via the mfArray A.

When f90\_array is a rank-1 array (here, renamed as f90\_vector), the mfArray A is by default a column vector. Use the optional argument new\_shape to specify the new virtual dimensions of A, as follows:

```
call msEquiv( f90_vector, A, new_shape=[n1,n2] )
```

Of course, you must have consistance, i. e. the product  $n_1 n_2$  must be equal to the number of elements of f90\_vector.

Remark:

To release the link between A and f90\_array, you must use the msRelease routine. Don't write

```
A => null()
```

because usually A is not a pointer, it is just a derived type, *i. e.* a structure which includes the numerical data in a manner that is hidden to the user. Even if A has been declared as a pointer to an mfArray, this is not the good way to release properly A). If you forget to release this link, you could obtain an error when trying an assignment with A.

See also: msPointer

#### mf\_Int\_List

## list of integers (derived type)

Description:

This is a derived type in MUESLI. You may employ it for storing a list of integers of arbitrary length. It's definition is:

```
type :: mf_Int_List
   integer, allocatable :: list(:)
end type mf_Int_List
```

You must allocate yourself the internal component list, which is an ordinary Fortran array of integers. Also, you may declare and use an array of mf\_Int\_List, which is useful to work with a set of varying length lists (see the second example below).

msDisplay and msRelease routines may be used to, respectively, print and free this derived type. These two routines can be also used with an array of mf\_Int\_List.

```
Example(s):
   ! type(mf_Int_List) :: int_list
  allocate( int_list%list(7) )
  int_list%list(:) = [ (i, i = 1, 7) ]
  call msDisplay( int_list, "int_list")
   ! call msRelease( int_list )
output:
int_list =
 1 2 3 4 5 6 7
   ! type(mf_Int_List) :: int_list_vec(5)
  allocate( int_list_vec(1)%list(n) )
  int_list_vec(1)%list(:) = [ (i, i = 1, n) ]
  allocate( int_list_vec(3)%list(n) )
  int_list_vec(3)%list(:) = [ (i, i = 100+1, 100+n) ]
  n = 9
  allocate( int_list_vec(5)%list(n) )
  int_list_vec(5)\%list(:) = [ (i, i = 100000+1, 100000+n) ]
  call msDisplay( int_list_vec, "int_list_vec")
   ! call msRelease( int_list_vec )
output:
 int_list_vec =
  1 2 3 4 5 6
 <EMPTY>
 101 102 103 104 105 106 107 108 109 110 111 112 113 114 115
  <EMPTY>
 100001 100002 100003 100004 100005 100006 100007 100008 100009
```

#### mf\_Real\_List

## list of reals (derived type)

#### Description:

This is a derived type in MUESLI. You may employ it for storing a list of reals of arbitrary length. It's definition is:

```
type :: mf_Real_List
  integer, allocatable :: list(:)
end type mf_Real_List
```

You must allocate yourself the internal component list, which is an ordinary Fortran array of reals. Also, you may declare and use an array of mf\_Real\_List, which is useful to work with a set of varying length lists (see the second example below).

msDisplay and msRelease routines may be used to, respectively, print and free this derived type. These two routines can be also used with an array of mf\_Real\_List.

```
Example(s):
   ! type(mf_Real_List) :: real_list
  allocate( real_list%list(7) )
  real_list%list(:) = [ (i+0.5d0, i = 1, 7) ]
  call msDisplay( real_list, "real_list")
   ! call msRelease( real_list )
output:
 int_list =
 1.500 2.500 3.500 4.500 5.500 6.500 7.500
   ! type(mf_Real_List) :: int_real_vec(5)
  allocate( real_list_vec(1)%list(n) )
  real_list_vec(1)%list(:) = [ (i+0.5d0, i = 1, n) ]
  allocate( real_list_vec(3)%list(n) )
  real_list_vec(3)%list(:) = [(i+0.5d0, i = 10+1, 10+n)]
  n = 9
  allocate( real_list_vec(5)%list(n) )
  real_list_vec(5)%list(:) = [(i+0.5d0, i = 100+1, 100+n)]
  call msDisplay( real_list_vec, "int_list_vec")
   ! call msRelease( real_list_vec )
output:
 int_list_vec =
 1.500 2.500 3.500 4.500 5.500 6.500
 <EMPTY>
 11.500 12.500 13.500 14.500 15.500 16.500 17.500 18.500 19.500 20.500 21.500
  <EMPTY>
```

101.500 102.500 103.500 104.500 105.500 106.500 107.500 108.500 109.500

## mfIsEmpty

## mfArray empty test

 ${\it Interface:}$ 

```
function mfIsEmpty( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

## Description:

Tests if the mfArray A is empty, i. e. has a null size (number of rows multiplied by nupmber of columns).

Note that an sparse matrix (non empty) may have a number of nonzero elements (see  ${\tt mfNnz}$ ) that is zero.

See also: Shape, Size

## mfIsEqual

## mfArray equality test

Interface:

```
function mfIsEqual( A1, A2 ) result( bool )
  type(mfArray), intent(in) :: A1, A2
  logical :: bool
```

#### Description:

Tests if two mfArrays are numerically equal, whatever the structure is (i. e. sparse or dense, real or complex). The shapes can even mismatch, and can take the zero value.

Comparison between boolean mfArray are excluded: All combined with '.eqv.' should be used instead.

Remarks: This function returns a scalar logical, whereas '==' returns a boolean mfArray.

For example:

```
mfIsEqual( A1, A2 )
is equivalent to:
   A11( A1 == A2 )
```

but the former statement is much more efficient for sparse mfArrays.

```
See also: ==, All, mfAll, .eqv.
```

## mfIsNotEqual

## mfArray equality test

 ${\it Interface:}$ 

```
function mfIsNotEqual( A1, A2 ) result( bool )
  type(mfArray), intent(in) :: A1, A2
  logical :: bool
```

Description:

Returns the negation of mfIsEqual(A1,A2).

See also: mfIsEqual

## mfIsLogical

## mfArray boolean test

 ${\it Interface:}$ 

```
function mfIsLogical( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

Description:

Returns '.true.' if the mfArray A is boolean, i.e. if A results from any boolean operation ('==', '>', etc.) on mfArrays.

 $See\ also: ==$ 

#### mfIsReal

mfArray real test

Interface:

```
function mfIsReal( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

Description:

Returns '.true.' if the mfArray A is numeric, but not complex. The internal structure of A may be dense or sparse.

 $See \ also: {\tt mfIsNumeric}, {\tt mfIsComplex}$ 

## mfIsComplex

## mfArray complex test

 ${\it Interface:}$ 

```
function mfIsComplex( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

## Description:

Returns '.true.' if the mfArray A is numeric and complex. The internal structure of A may be dense or sparse.

See also: mfIsNumeric, mfIsReal

#### mfIsNumeric

## mfArray numeric test

 ${\it Interface:}$ 

```
function mfIsNumeric( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

## Description:

Returns '.true.' if the mfArray A is numeric (real or complex). The internal structure of A may be dense or sparse.

 $See \ also: {\tt mfIsReal}, {\tt mfIsComplex}$ 

#### mfIsDense

## mfArray dense storage test

 ${\it Interface:}$ 

```
function mfIsDense( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

Description:

Returns '.true.' if internal structure of A is dense. The data may be boolean, real or complex.

See also: mfIsSparse

# ${\tt mfIsSparse}$

# mfArray sparsity test

 ${\it Interface:}$ 

```
function mfIsSparse( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

Description:

Returns '.true.' if internal structure of A is sparse. The data may be real or complex.

See also: mfIsDense

See also: mfIsVector, mfIsMatrix

#### mfIsScalar

# mfArray scalar test

```
{\it Interface:}
```

```
function mfIsScalar( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool

Description:
Returns '.true.' if A is a scalar.
```

#### mfIsVector

# mfArray vector test

 ${\it Interface:}$ 

```
function mfIsVector( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

Description:

Returns '.true.' if A is a vector.

Remark: A scalar is not a vector, neither a matrix.

 $See \ also: {\tt mfIsScalar}, {\tt mfIsMatrix}, {\tt mfIsRow}, {\tt mfIsColumn}$ 

#### mfIsMatrix

# mfArray vector test

 ${\it Interface:}$ 

```
function mfIsMatrix( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

Description:

Returns '.true.' if A is a matrix.

Remark: A vector is not a matrix.

See also: mfIsScalar, mfIsVector

#### mfIsRow

# mfArray kind of vector test

```
{\it Interface:}
```

```
function mfIsRow( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool

Description:
Returns '.true.' if A is a row vector.
```

See also: mfIsVector, mfIsColumn

See also: mfIsVector, mfIsRow

#### mfIsColumn

# mfArray kind of vector test

```
{\it Interface:}
```

```
function mfIsColumn( A ) result( bool )
    type(mfArray), intent(in) :: A
    logical :: bool

Description:
Returns '.true.' if A is a column vector.
```

#### mfIsPerm

# mfArray permutation vector test

 ${\it Interface:}$ 

```
function mfIsPerm( A ) result( bool )
   type(mfArray), intent(in) :: A
   logical :: bool
```

Description:

Returns '.true.' if A has the type "permutation vector".

To make a real checking for a valid permutation, use  ${\tt mfCheckPerm}$ .

See also: mfIsLogical, mfIsNumeric

### All

# test on boolean mfArray

```
{\it Interface:}
```

```
function All( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

# Description:

Returns '.true.' if all elements of the mfArray A are TRUE (therefore, A must be a boolean mfArray).

See also: mfAll, Any

# Any

# test on boolean mfArray

 ${\it Interface:}$ 

```
function Any( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

Description:

Returns '.true.' if at least one element of the mfArray A is TRUE (therefore, A must be a boolean mfArray).

See also: mfAny, All

### Shape, mfShape

### shape of an mfArray

 ${\it First\ interface:}$ 

```
function Shape( A ) result( out )
  type(mfArray), intent(in) :: A
  integer :: out(2)
```

#### Description:

Returns the shape of the mfArray A (dense or sparse); the returned integer array is always of rank 2, because A is a matrix.

Other interface:

```
function Shape( Qhouse ) result( out )
  type(mfMatFactor), intent(in) :: Qhouse
  integer :: out(2)
```

#### Description:

Returns the shape of the mfMatFactor Qhouse, which comes from the QR decomposition of a sparse matrix; the returned integer array is always of rank 2, because an A is a matrix.

Remark: the mfShape routine is similar, but returns an mfArray instead.

See also: Size, mfMatFactor

#### Size, mfSize

### size of an mfArray

 $First\ interface:$ 

```
function Size( A, idim ) result( out )
  type(mfArray), intent(in) :: A
  integer, intent(in), optional :: idim
  integer :: out
```

#### Description:

If idim is not present, returns the total number of elements of the mfArray A (dense or sparse).

If idim is present (1 or 2), returns the specified dimension.

Other interface:

```
function Size( A, idim ) result( out )
  type(mfMatFactor), intent(in) :: A
  integer, intent(in), optional :: idim
  integer :: out
```

### Description:

If idim is not present, returns the total number of elements of the mfMatFactor A (which comes from some decomposition of a sparse matrix).

If idim is present (1 or 2), returns the specified dimension.

#### Remarks:

- the mfSize routine is similar, but returns an mfArray instead;
- for a sparse matrix, the size returned includes the zero elements: it is the logical size, not the physical size; so, for any storage kind, the size is always the product of the two dimensions returned by shape. Use the mfNnz routine to get the number of non zeros.

See also: Shape

#### mfInt

# scalar integer conversion

Interface:

```
function mfInt( A ) result( out )
    type(mfArray), intent(in) :: A
    integer :: out
```

# Description:

This routine is a facility to convert a scalar mfArray A in an ordinary f90 integer.

A warning is emitted during conversion, in case of loss of precision.

See also: mfDble, mfCmplx

#### mfDble

# scalar real conversion

Interface:

```
function mfDble( A ) result( out )
  type(mfArray), intent(in) :: A
  real(kind=MF_DOUBLE) :: out
```

# Description:

This routine is a facility to convert a scalar mfArray A in an ordinary f90 real.

If A is complex, returns only the real part.

See also: mfInt, mfCmplx

# ${\tt mfCmplx}$

# scalar complex conversion

 ${\it Interface:}$ 

```
function mfCmplx( A ) result( out )
   type(mfArray), intent(in) :: A
   complex(kind=MF_DOUBLE) :: out
```

Description:

This routine is a facility to convert a scalar mfArray A in a complex.

See also: mfInt, mfDble

# ${\tt mfGetMsgLevel}$

# message level tuning

Calling syntax:

level = mfGetMsgLevel( )

returns an integer, ranged from 0 to 3, which is the current message level. See the msSetMsgLevel routine for a description of the message level.

 $See \ also: \ {\tt mfGetTrbLevel}, \ {\tt msSetTrbLevel}, \ {\tt msMuesliTrace}$ 

### msSetMsgLevel

### message level tuning

 $Calling\ syntax:$ 

```
call msSetMsgLevel( level )
```

where level is an integer, ranged from 0 to 3, which specifies the message level, described as follows:

- message level = 3: all messages are printed (verbose mode)
- message level = 2: messages of kind 'ERROR' and 'Warning' are printed [default]
- message level = 1: only messages of kind 'ERROR' are printed
- message level = 0: nothing is printed (quiet mode)

There exists three kinds of messages emitted by the MUESLI library; they are tagged as "info", "Warning" and "ERROR". Only the last kind (the most important) stops temporarily the run-time execution via a "pause" statement, which requires the action of the user to resume the execution. The cause of these errors are generally due to a programming fault; this is the reason why the program stops.

As usual for IEEE-754 arithmetic, floating-point exceptions give Inf and NaN values which quietly propagate. See msEnableFPE to change this default behavior.

*N.B.*: when message level is equal to 0, programming errors (of kind "ERROR") kill the program. This may be useful when the executable must be launched via a batch system.

See also: mfGetMsgLevel, mfGetTrbLevel, msSetTrbLevel, msMuesliTrace

#### mfGetTrbLevel

# error traceback tuning

Calling syntax:

when = mfGetTrbLevel( )

returns a character(len=4), which is the current error traceback policy. See the msSetTrbLevel routine for a description of the error traceback policy.

 $See~also: \verb|mfGetMsgLevel|, \verb|msSetMsgLevel|, \verb|msMuesliTrace||$ 

#### msSetTrbLevel

### error traceback tuning

 $Calling\ syntax:$ 

```
call msSetTrbLevel( when | level )
```

where when is a character(len=4) which specifies the error traceback policy, described as follows:

- when = "all": error traceback is produced for all kind of messages (i. e. ERRORS, Warnings and infos)
- when = "auto": error traceback is produced only for ERRORS [default]
- when = "none": no error traceback

An alternative is to use the integer level argument, which can be equal to 0, 1, 2 or 3. An message level which is greater than or equal to level triggers the traceback. For example, when level is used with the value 2, only messages of kind 'ERROR' and 'Warning' produce a traceback.

N.B.: not all compilers are able to produce an error traceback on demand. See the MUESLI Inside document for more information.

See also: mfGetTrbLevel, mfGetMsgLevel, msSetMsgLevel, msMuesliTrace

# msSetColoredMsg

# colorize messages on terminal

Calling syntax:

```
call msSetColoredMsg( "on" | "off" )
```

Description:

Colorizes all the Muesli messages written on the terminal. Four colours are used:

- red for the ERRORS;
- orange for the Warnings;
- yellow for the infos;
- grey-italic for the traceback.

Disabling the colors are useful before redirecting the program output to a file. Default is "on".

See also: mfGetMsgLevel, msSetMsgLevel, msMuesliTrace

# ${\tt msPrintColoredMsg}$

# print a colored user message on terminal

```
{\it Interface:}
```

```
subroutine msPrintColoredMsg( fmt, msg, color )
    character(len=*), intent(in) :: fmt, msg, color
```

# Description:

Prints the string msg in this color and using format fmt.

Colors are limited to the following ones: "green", "red", "orange", "yellow" and "grey".

See also: msSetTermColor

#### msSetTermColor

# set color for printing on terminal

```
{\it Interface:}
```

```
subroutine msSetTermColor( color )
    character(len=*), intent(in) :: color
```

# Description:

Sets the color for printing user message with msPrintColoredMsg.

Colors are limited to the following ones: "green", "red", "orange", "yellow" and "grey".

To revert to the default color, set color to "normal".

### ${\tt msGetStdIO}$

# usual logical unit inquiry

```
{\it Interface:}
```

```
subroutine msGetStdIO( stdin, stdout, stderr )
  integer, intent(out), optional :: stdin, stdout, stderr

Description:
Gets the Fortran unit for stdin (usually 5), stdout (usually 6) and stderr (usually 0).
```

 $See \ also: {\tt msSetStdIO}$ 

### ${\tt msSetStdIO}$

# usual logical unit modification

 ${\it Interface:}$ 

```
subroutine msSetStdIO( stdin, stdout, stderr )
  integer, intent(in), optional :: stdin, stdout, stderr
```

Description:

Used to change the Fortran units.

 $See \ also: {\tt msGetStdIO}$ 

msFlush I/O flush

```
{\it Interface:}
```

```
subroutine msFlush( unit )
  integer, intent(in) :: unit
```

Description:

Flushes the I/O unit unit.

#### msPause

### user pause or timing pause

Calling syntax:

```
call msPause( [message] [, indent] [, duration] )
```

If the optional argument message is present (character(len=\*)), this routine prints the message and waits for a [RETURN] from the user.

The optional argument indent (a positive integer) allows the user to indent the prompt string

```
[Return] to continue . . .
```

in order to align the beginning of this prompt with other strings printed elsewhere.

If the optional argument duration is present (real), this routine sleeps for the specified number of seconds.

The two arguments indent and duration cannot be used together.

#### msSetTermWidth

### set terminal character width

Syntax:

```
call msSetTermWidth( width | "auto" )
```

### Description:

Set manually the character width of the terminal, for pretty print with the msDisplay routine. The argument width must be an integer greater than 40.

If the character string "auto" is used as argument, then the character width of the terminal will be determined automatically, either from the MF\_COLUMNS environment variable, or from the actual value (using the 'resize' shell command).

 $See \ also: \ {\tt mfGetTermWidth}, \ {\tt msDisplay}, \ {\tt mfDisplayColumns}$ 

#### mfGetTermWidth

### get terminal character width

Interface:

```
function mfGetTermWidth() result ( out )
  integer :: out
```

#### Description:

Get the character width of the terminal. The returned value is actually an internal value used for pretty print in the msDisplay routine. This last value is obtained from:

- first, the value set manually via the msSetTermWidth routine;
- $-\,$  then, from the MF\_COLUMNS environment variable;
- last, via the 'resize' Unix command; if this latter command is not available on your system, an information message is displayed and the default value 80 is used.

See also: msSetTermWidth, msDisplay, mfDisplayColumns

# msSetAutoFilling

# set out-of-range filling by msSet

Syntax:

call msSetAutoFilling( r )

Description:

Set manually the value of the real used to fill the elements of an  $\mathtt{mfArray}$  by  $\mathtt{msSet}$  when the indices are out-of-range.

Default value is the special  $\it IEEE$  number  $\it NaN$ , which means  $\it Not-a-Number$ .

See also: mfGetAutoFilling

# mfGetAutoFilling

# get out-of-range filling by msSet

 ${\it Interface:}$ 

```
function mfGetAutoFilling() result ( r )
    real(kind=MF_DOUBLE) :: r
```

#### Description:

Get the value of the real used to fill the elements of an mfArray by msSet when the indices are out-of-range.

Default value is the special IEEE number NaN, which means Not-a-Number, but this number may be changed by use of msSetAutoFilling.

### msInitArgs

# mfArray arguments' protection

 ${\it Interface:}$ 

# Description:

Inside a user-defined routine, this routine protects some mfArray arguments against a (library) deallocation, in case where they are temporary.

This routine should always be paired with the other routine  ${\tt msFreeArgs}.$ 

# ${\tt msFreeArgs}$

# mfArray arguments' release

 ${\it Interface:}$ 

# Description:

Inside a user-defined routine, this routine ends the argument's protection set by msInitArgs.

This routine should always be paired with the other routine msInitArgs.

#### msSetAsParameter

data protection

### Interface:

```
subroutine msSetAsParameter( x1, x2, x3, x4, x5, x6, x7, param )

type(mfArray) :: x1
type(mfArray), optional :: x2, x3, x4, x5, x6, x7
logical :: param
```

### Description:

When used with "param=.true.", protects the data (and type) inside each mfArrays. Lets the user creating pseudo parameters mfArrays, because the Fortran attribute parameter cannot be used in any circumstances.

Up to seven mfArrays can be simultaneously set.

Before freeing an mfArray via the msRelease routine, the user must apply msSetAsParameter with "param=.false.".

Of course, by default, an mfArray is not data-protected.

# msReturnArray

# temporary mfArray mark

 ${\it Interface:}$ 

```
subroutine msReturnArray( A )
type(mfArray) :: A
```

Description:

At the end of a user-defined function returning an mfArray, marks this object as temporary.

Avoids memory leaks during the execution of your program.

 $See~also: {\tt msInitArgs}, {\tt msFreeArgs}, {\tt mfIsTempoArray}$ 

# ${\tt mfIsTempoArray}$

# temporary mfArray check

 ${\it Interface:}$ 

```
function mfIsTempoArray( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

Description:

Check if the mfArray A is marked as temporary.

See also: msInitArgs, msFreeArgs, msReturnArray

### msFlops

### nb of floating-point operations

 ${\it Interface:}$ 

```
subroutine msFlops( count )
  integer*8, intent(out) :: count
```

Description:

Returns the number of floating-point operations made by the processor, since the last initialization.

Initialization must be made by:

```
call msFlops( init=0 )
The other call:
  call msFlops( count )
```

returns the flops count since the initialization.

count  $\geq 0$ , except if the program has been linked with the dummy version of PAPI (dummy\_papi.o), or if the OS doesn't support hardware counting (e. g. the linux kernel has not been patched for use of the PERFCTR library); in the latter case, msFlops always returns the value -1.

Remarks: requires a specific version of the PAPI library (cf. in papi/version)

 $See \ also: {\tt mfFlops}, {\tt mfIsFlopsOk}$ 

### mfFlops

# nb of floating-point operations

 $Calling\ syntax:$ 

```
count = mfFlops( )
```

Description:

Returns the number of floating-point operations made by the processor, since the last initialization, in the mfArray count.

Initialization must be made by:

```
call msFlops( init=0 )
```

count  $\geq 0$ , except if the program has been linked with the dummy version of PAPI (dummy\_papi.o), or if the OS doesn't support hardware counting (e. g. the linux kernel has not been patched for use of the PERFCTR library); in the latter case, mfflops always returns the value -1.

Remarks: requires a specific version of the PAPI library (cf. in papi/version)

 $See \ also: {\tt msFlops}, {\tt mfIsFlopsOk}$ 

## ${\tt mfIsFlops0k}$

# inquire if flops is available

Calling syntax:

bool = mfIsFlopsOk( )

Description:

Returns a logical revealing the availability of the mfFlops routine.

 $See\ also:\ {\tt mfFlops},\ {\tt msFlops}$ 

## msSetAutoComplex

## set auto conversion to complex

Syntax:

```
call msSetAutoComplex( .true. | .false. )
```

Description:

By default, real mfArrays are automatically converted in complex when operation like  $\sqrt{-1}$  are made.

When calling this routine with argument equal to .false., conversion to complex is not done, then resulting to a special IEEE value (Inf or NaN, according to the operation used).

See also: mfGetAutoComplex

# ${\tt mfGetAutoComplex}$

## get auto conversion to complex

 ${\it Interface:}$ 

```
function mfGetAutoComplex() result( bool )
logical, intent(in) :: bool
```

Description:

Returns a boolean specifying whether MUESLI convert real to complex for some operation (like  $\sqrt{-1}$ ).

 $See \ also: {\tt msSetAutoComplex}$ 

#### msEnableFPE

### run-time floating-point exceptions trapping

#### Interface:

```
subroutine msEnableFPE( exception, full_trapping )
character(len=*), intent(in) :: exception
logical, intent(in), optional :: full_trapping
```

#### Description:

Enables, at run-time, the trapping for a given floating-point exception, i.e. the program stops when the specified exception(s) is (are) encountered.

exception may be one of: "overflow", "zero\_divide", "invalid", "underflow" or "usual\_exceptions".

"usual\_exceptions" is a shortcut for the main three traditional exceptions: "overflow", "zero\_divide", "invalid".

full\_trapping is an optional argument which allows the trapping inside the MUESLI library sources (default is .false.). Note that this features is usually reserved to the MUESLI developpers.

*Remark:* The call may be located anywhere is the program. Usually, it is added at the beginning of the program. However, exceptions' trapping may be added or removed later on.

See also: msDisableFPE

#### msDisableFPE

## run-time floating-point exceptions trapping

### Interface:

```
subroutine msDisableFPE( exception )
character(len=*), intent(in) :: exception
```

#### Description:

Disables, at run-time, the trapping for a given floating-point exception, i.e. the program doesn't stop when the specified exception(s) is (are) encountered, but produces accordingly Infs and NaNs.

exception may be one of: "overflow", "zero\_divide", "invalid", "underflow" or "usual\_exceptions".

"usual\_exceptions" is a shortcut for the main three traditional exceptions: "overflow", "zero\_divide", "invalid".

 $See \ also: {\tt msEnableFPE}$ 

## ${\tt mfGetRoundingMode}$

## floating-point rounding mode

Calling syntax:

```
rounding_mode = mfGetRoundingMode( )
```

### Description:

Returns the floating-point rounding mode in a character string. The  $\it IEEE-754$  rounding mode may be equal to:

- "to\_zero":  $\it IEEE-754$  rounding to zero
- "nearest": IEEE-754 rounding to nearest
- "up": IEEE-754 rounding to pos. infinity
- "down": IEEE-754 rounding to neg. infinity

See also: msSetRoundingMode

#### msSetRoundingMode

### floating-point rounding mode

### ${\it Interface:}$

```
subroutine msSetRoundingMode( rounding_mode )
character(len=*) :: rounding_mode
```

#### Description:

Set the floating-point rounding mode to the specified mode. The  $\it IEEE-754$  rounding mode may be equal to:

```
"to_zero": IEEE-754 rounding to zero
"nearest": IEEE-754 rounding to nearest
"up": IEEE-754 rounding to pos. infinity
"down": IEEE-754 rounding to neg. infinity
```

Warning: The ISO/IEC TR 15580:1998(E) Technical Report, concerning the behavior of Fortran programs, specifies (Section 2.4): "In a procedure, the processor ensures that the flags for rounding have the same values on return as on entry." In the current MUESLI implementation, the rounding mode is set via a C routine, and can violate the preceding requirement; then the use of the msSetRoundingMode routine is left to the programmer responsability.

See also: mfGetRoundingMode

#### MF\_NUMERICAL\_CHECK

### debugging additional checks

### Interface:

logical :: MF\_NUMERICAL\_CHECK

### Description:

This logical global variable is used by the library to do additional checks (validity of certain results, presence of NaN and, if required, precision of the jacobian matrix provided by the user, ...).

When Muesli is compiled in DEBUG mode (usually by the developpers of Muesli), this variable is initially set to TRUE. Otherwise, for ordinary users of the library (which certainly compile it in OPTIM mode), it is initially set to FALSE.

For debugging purpose, you can set its value temporarily to TRUE, especially when using the ODE/DAE solvers (msOdesolve, msDaesolve), or the minimization routines (mf/msLsqNonLin).

 $See \ also: {\tt mf\_DE\_Options}$ 

#### mfUnit

## physical unit (derived type)

#### Description:

The Fortran derived type mfUnit (hidden) contains some usual physical units and constants. It can be used to initialize the physical unit of an mfArray, via the operators '\*' and '='.

No declarations has to be done. Instead, the program may refer to some predefined mfUnits, as e.g.:

$$y = 0.25d0*u_kg$$
  
 $v = 0.10d0*c_speed_of_light$ 

#### Remarks:

- $-\,$  see the  $\it MUESLI$   $\it User$   $\it Guide$  document for a complete list of the available physical units, physical constants and multipliers.
- physical units begin with ' $u_-$ ', multipliers begin with ' $m_-$ ', whereas physical constants begin with ' $c_-$ '.

See also: \*, =, msUsePhysUnits, msSetPhysDim, mfHasNoPhysDim, mfHaveSamePhysDim

## ${\tt msUsePhysUnits}$

# Physical units activation

```
{\it Interface:}
```

```
subroutine msUsePhysUnits( mode )
    character(len=*), intent(in) :: mode
```

Description:

Activates or de-activates the computation of physical units; mode must be "on" or "off".

Default is "off".

 $See \ also: {\tt mfUnit}, {\tt msSetPhysDim}$ 

msSetPhysDim

set physical unit

The first interface:

```
subroutine msSetPhysDim( A,
                                                                       &
                         Mass, Length, Time,
                                                                       &
                         Temp, Electr_Intens, Mole, Lumin_Intens,
                         no_dim )
   type(mfArray), intent(in out) :: A
   real(kind=MF_DOUBLE), intent(in), optional :: Mass,
                                                                       &
                                                  Length,
                                                                       &
                                                  Time,
                                                                       &
                                                  Temp,
                                                  Electr_Intens,
                                                                       &
                                                  Mole,
                                                                       &
                                                  Lumin_Intens
   logical, intent(in), optional :: no_dim
```

is used for setting the physical unit of the mfArray A, from some basic physical dimensions. no\_dim=.true. is used to set A as a non-dimensional quantity.

The second interface:

```
subroutine msSetPhysDim( A, B )

type(mfArray), intent(in out) :: A
type(mfArray), intent(in) :: B
```

allows the user to set the physical unit of the mfArray A, by copying that of the mfArray B.

See the  $MUESLI\ User\ Guide$  document for some examples.

See also: mfUnit, msUsePhysUnits, mfHasNoPhysDim, mfHaveSamePhysDim

## ${\tt msSetPhysUnitAbbrev}$

## set user physical unit abbrevation

```
{\it Interface:}
```

```
subroutine msSetPhysUnitAbbrev( user_unit, abbrev )
    type(mfUnit) :: user_unit
    character(len=*) :: abbrev
is used for setting the abbrevation of a user physical unit.
abbrev is an input string limited to 12 characters.
```

 $See \ also: {\tt mfUnit}, {\tt msUsePhysUnits}$ 

## mfHasNoPhysDim

## dimensionless test

 ${\it Interface:}$ 

```
function mfHasNoPhysDim( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

Description:

Tests if the mfArray A is dimensionless.

 $See~also: {\tt mfUnit, msUsePhysUnits, msSetPhysDim, mfHaveSamePhysDim}$ 

## ${\tt mfHaveSamePhysDim}$

## dimension equality test

 ${\it Interface:}$ 

```
function mfHaveSamePhysDim( A, B ) result( bool )
   type(mfArray), intent(in) :: A, B
   logical :: bool
```

Description:

Tests if the mfArrays A and B have the same physical dimensions.

 $See \ also: \mathtt{mfUnit}, \mathtt{msUsePhysUnits}, \mathtt{msSetPhysDim}, \mathtt{mfHasNoPhysDim}$ 

## msPrepHashes

## hashes print preparation

 $Calling\ syntax:$ 

call msPrepHashes( start, end )

Description:

Prepares data before calls to  ${\tt msPrintHashes}.$ 

start and end are numerical values used to monitor the progress bar. They must be both of integer or real (double) type.

See also: msPrintHashes, msPostHashes

msPrintHashes hashes print

```
Calling\ syntax:
```

```
call msPrintHashes( val )
```

### Description:

Makes on the terminal a progress bar by printing hashes (#).

Usually, the call is located inside a loop; val is the numerical value (either integer or real double) monitoring the progression. It's numerical extremum values have to be passed as arguments of msPrepHashes.

See also: msPostHashes, msPrintProgress

```
Example(s):
```

```
integer :: i_start, i_end, i
   call msPrepHashes( i_start, i_end )
   do i = i_start, i_end
      <some computation>
      call msPrintHashes( i )
   end do
   call msPostHashes( )
or:
   double precision :: time_start, time_end, time
   call msPrepHashes( time_start, time_end )
   time = time_start
   do while( time <= time_end)</pre>
      <some computation>
      call msPrintHashes( time )
      time = <new value of time, increasing but may be not linear with iterations>
   end do
   call msPostHashes( )
```

### msPostHashes

# hashes print conclusion

 $Calling\ syntax:$ 

subroutine msPostHashes()

Description:

Should be added after the loop containing the call to  ${\tt msPrintHashes}.$ 

 $See\ also: \verb|msPrepHashes|, \verb|msPrintHashes||$ 

#### msPrepProgress

#### percent progress preparation

 $Calling\ syntax:$ 

```
call msPrepProgress( start, end [, disp_times, estimator] )
```

Description:

Prepares data before calls to msPrintProgress.

start and end are numerical values used to monitor the progress bar. They must be both of integer or real (double) type.

If the logical optional argument disp\_times is set to .false., time values (left time and estimated remaining time – wall clock time is used, not CPU time) are not displayed (default is TRUE).

The string optional argument estimator prescribes the method to be used to estimate the remaining time. Default is "GLE" (Global Linear Estimator), which corresponds to a fast method but with the assumption of a linear behavior of the cost along all the iterations. The other available method is "LPE" (Local Power Estimator), which involves a Least Square problem, and assumes that the behavior of the cost is under a power form; this latter method is more expensive but should work well with a larger class of problems than "GLE".

See also: msPrintProgress, msPostProgress

#### msPrintProgress

percent progress

 $Calling\ syntax:$ 

```
call msPrintProgress( val )
```

### Description:

Print on STDOUT the percentage of the work already done. According to the setting used during initialization via msPrepProgress, the estimated remaining time may also be displayed.

Usually, the call is located inside a loop (at the end); val is the numerical value (either integer or real double) monitoring the progression. It's numerical extremum values have to be passed as arguments of msPrepProgress.

See also: msPostProgress, msPrintHashes

```
Example(s):
```

```
integer :: i_start, i_end, i
   call msPrepProgress( i_start, i_end )
   do i = i_start, i_end
      . . .
      <some computation>
      call msPrintProgress( i )
   end do
   call msPostProgress( )
or:
   double precision :: time_start, time_end, time
   call msPrepProgress( time_start, time_end )
   time = time_start
   do while( time <= time_end)</pre>
      <some computation>
      call msPrintProgress( time )
      time = <new value of time, increasing but may be not linear with iterations>
   end do
   call msPostProgress( )
```

## ${\tt msPostProgress}$

## percent progress conclusion

 ${\it Interface:}$ 

subroutine msPostProgress()

Description:

Should be added after the loop containing the call to  ${\tt msPrintProgress}.$ 

 $See\ also: \verb|msPrepProgress|, \verb|msPrintProgress|$ 

### mfToLower

# lowering string's case

Calling syntax:

s2 = mfToLower(s1)

Description:

Returns the input strings  ${\tt s1}$  with each letter converted to lower case.

 $See \ also: \ {\tt mfToUpper}$ 

# mfToUpper

# uppering string's case

 $Calling\ syntax:$ 

s2 = mfToUpper(s1)

Description:

Returns the input strings  ${\tt s1}$  with each letter converted to upper case.

 $See \ also: {\tt mfToLower}$ 

## ${\tt msFindIOUnit}$

# automatically find a free IO unit number

 ${\it Interface:}$ 

```
subroutine msFindIOUnit( unit )
  integer, intent(out) :: unit
```

Description:

Returns an integer IO unit which can be used for opening a file.

#### mfIsVersion

## test on version strings

 $Calling\ syntax:$ 

bool = mfIsVersion( $v_1$ , op,  $v_2$ )

### Description:

Compares the two version strings  $v_1$  and  $v_2$  with the operator op ("==", ">", ">=", "<", "<="). The version string is a character string, of size at most 8, matching the rule a.b.c, where a, b and c are integers ranged from 0 to 99.

This function returns a logical and is usually applied to strings as  $MF\_MUESLI\_VERSION$  or  $MF\_LAPACK\_VERSION$ .

 $See \ also: {\tt msRequMuesliVer}$ 

## ${\tt msRequMuesliVer}$

## check for a minimum Muesli version

Interface:

```
subroutine msRequMuesliVer( version )
    character(len=*) :: version
```

#### Description:

Checks that the current Muesli version used at run-time is at least equal to that provided in the "version" argument. This argument must be a string of the form a.b.c, where a, b and c are integers ranged from 0 to 99.

 $See \ also: {\tt MF\_MUESLI\_VERSION}, {\tt mfIsVersion}$ 

### MF\_COMPILER\_VERSION

## compiler vendor and version

 $Calling\ syntax:$ 

string = MF\_COMPILER\_VERSION

returns the compiler vendor and version used during the compilation of the MUESLI library.

 $See \ also: \verb|MF_MUESLI_VERSION|, \verb|MF_COMPILATION_CONFIG|, \verb|MF_LAPACK_VERSION||$ 

### MF\_COMPILATION\_CONFIG

## Configuration used to compile MUESLI

 $Calling\ syntax:$ 

string = MF\_COMPILATION\_CONFIG()

returns either "Debug" or "Optim".

Note the parenthesis used after the name of the variable, because it is implemented as a function.

 $See \ also: \ {\tt MF\_MUESLI\_VERSION}, \ {\tt MF\_COMPILER\_VERSION}, \ {\tt MF\_LAPACK\_VERSION}$ 

### MF\_MUESLI\_VERSION

**MUESLI** version

 $Calling\ syntax:$ 

string = MF\_MUESLI\_VERSION

returns the MUESLI version used during the link of the executable.  $\,$ 

The returned  ${\tt string}$  may be used by the  ${\tt mfIsVersion}$  boolean function.

 $See \ also: \ {\tt MF\_COMPILER\_VERSION}, \ {\tt MF\_COMPILATION\_CONFIG}, \ {\tt MF\_LAPACK\_VERSION}$ 

#### msMuesliTrace

### helper for debugging purpose

Interface:

```
subroutine msMuesliTrace( pause )
    character(len=*), intent(in) :: pause
```

Description: Basically, this subroutine helps the user by printing, if available, a traceback of the program; optionally, it can make a pause (only if the pause argument is equal to "yes").

Contrary to the internal traceback routine, the behavior of this routine depends only on the value of the message level global MUESLI property.

Remark: In order to be useful, the use of this routine requires that the Muesli library has been compiled with debugging turned On. This is the case if the library is used in the DEBUG mode. On the contrary, it suffices that the user adds the appropriate option ( $e.\,g.$  -g for most of compilers) for its own source files

See also: mfGetMsgLevel, msSetMsgLevel

### mfReadLine reads a line from terminal with editing and history facilities

 ${\it Interface:}$ 

```
function mfReadLine( prompt ) result( string )
    character(len=*), intent(in), optional :: prompt
    character(len=1024) :: res
```

Description: This function prompts the string prompt and waits the user to input a character string, using the readline library. As a result, the user can navigate in an history, recall any previous input and modify it.

It returns the string string, which is automatically added in the history only if it differs from the last entry.

The history can be saved with the use of the msWriteHistoryFile routine.

A small example of call can be found in the Muesli User's Guide.

 $See \qquad also: \qquad \texttt{msReadHistoryFile}, \qquad \texttt{msClearHistory}, \qquad \texttt{msAddEntryInHistory}, \\$  msRemoveLastEntryInHistory

## msReadHistoryFile

## read a 'readline' history file

Interface:

```
subroutine msReadHistoryFile( filename )
    character(len=*), intent(in), optional :: filename
```

Description: Reads an 'history' file previously stored by the msWriteHistoryFile routine.

If the optional filename is not present, 'history' is read from the file: /.mfreadline.history

 $See~also: {\tt mfReadLine}, {\tt msClearHistory}, {\tt msAddEntryInHistory}, {\tt msRemoveLastEntryInHistory}$ 

## msWriteHistoryFile

## write a 'readline' history file

Interface:

```
subroutine msWriteHistoryFile( filename )
    character(len=*), intent(in), optional :: filename
```

Description: Writes an 'history' file in filename. This 'history' contains all entries read by the mfReadLine routine.

 $See \qquad also: \qquad \texttt{msReadHistoryFile}, \qquad \texttt{msClearHistory}, \qquad \texttt{msAddEntryInHistory}, \\$  msRemoveLastEntryInHistory

## ${\tt msClearHistory}$

# clear the 'readline' history

 ${\it Interface:}$ 

subroutine msClearHistory( )

Description: Clears all entries read by the mfReadLine routine.

 $See \qquad also: \qquad \texttt{msReadHistoryFile}, \qquad \texttt{msWriteHistoryFile}, \qquad \texttt{msAddEntryInHistory}, \\ \texttt{msRemoveLastEntryInHistory}$ 

## msAddEntryInHistory

add an entry in the 'readline' history

 ${\it Interface:}$ 

```
subroutine msAddEntryInHistory( string )
    character(len=*) :: string
```

Description: Adds the entry specified by the string argument in the 'readline' history.

 $See \quad also: \quad \texttt{mfReadLine}, \quad \texttt{msReadHistoryFile}, \quad \texttt{msWriteHistoryFile}, \quad \texttt{msClearHistory}, \\ \texttt{msRemoveLastEntryInHistory}$ 

msRemoveLastEntryInHistory

remove last entry in the 'readline' history

 ${\it Interface:}$ 

subroutine msRemoveLastEntryInHistory( )

Description: Remove last entry read by the mfReadLine routine.

 $See\ also:$  msReadHistoryFile, msWriteHistoryFile, msClearHistory, msAddEntryInHistory

## 1.2 File Input/Output

msSaveAscii ASCII file saving mfLoadAscii ASCII file loading

msSaveSparse Sparse ASCII file saving mfLoadSparse Sparse ASCII file loading

msLoadSparse Sparse ASCII file (with Right-Hand-Side) loading

msSave Binary file saving mfLoad Binary file loading

mfLoadTriConnect Connectivity Binary file loading

msSaveHDF5 HDF5 file saving mfLoadHDF5 HDF5 file loading

msMedit graphic mfArray editor

#### See also:

Core Routines

**Data Analysis Functions** 

Operators

**Elementary Math Functions** 

Specialized Math Functions

**Elementary Matrix Manipulation Functions** 

Matrix Functions

Polynomial Functions

Optimization and Function Functions

Sparse Matrices

### msSaveAscii

# **ASCII** file saving

Interface:

### Description:

Saves under ASCII form the dense mfArray A in the file named filename.

A is saved via a full 2D format, *i.e.* the number of lines of filename is exactly the number of rows of A. Special IEEE values (NaN and Infs) are written also in ASCII form (using the strings "NaN", "Infinity" and "-Infinity").

If the boolean append is present and TRUE then the data is written at the end of the (previously written) file. The default is to overwrite a pre-existing file.

If the character string format is present, it specifies the *Fortran format* used during the write of individual numbers (for example "ES13.6"). It must be of course a non-empty valid format for writing numerical real values. As usual, if the format is not sufficient to write the required number of digits, stars will be printed out instead (for example, "F3.0" is the shortest format if your are sure that all numbers in the mfArray are integers; and even "F2.0" if they are all positive).

msSaveAscii doesn't store the internal properties of A, so writing a boolean mfArray converts it in real. Another limitation is that only the real part of complex numbers is written; to save the imaginary part, use:

```
call msSaveAscii( filename, mfImag(A) )
```

### Remarks:

- if A contains a permutation, it will be written as a column vector, using integers (the optional argument format will be discarded). However, reading again the same file with mfLoadAscii will convert it into real numbers, as ordinary matrices; msSave is therefore recommended to save a permutation vector.
- to avoid loss of precision during the read of filename, a sufficient number of decimal digits is used to print each real when the optional argument format is not present.
- do not use the optional arguments append and format if you plan to read the file with mfLoadAscii.
- to save sparse matrices, use the msSaveSparse routine.

See also: mfLoadAscii, msSave, msSaveHDF5

### mfLoadAscii

# **ASCII** file loading

 ${\it Interface:}$ 

## Description:

Reads data from an ASCII file named filename and copy it in an mfArray. Inside the file, numerical data are separated by one or more space(s) and tabulation(s). Usually, this file has been created by the msSaveAscii routine but actually, nearly all kinds of ASCII file may be read by mfLoadAscii: see below.

If ieee is present and equal to .true., this routine is able to read special IEEE values, such as NaN and Inf (but this feature makes the read not very efficient for some compilers). The default behavior is to rise an error if any non numeric value is found.

If comment is present, all lines beginning (at any position) by this character are skipped (of course, this character must be different from: a space, a tabulation, a carriage-return, a digit, the signs + and -, the letters I and N which begin Inf and NaN); comments after data are also ignored. Empty lines are treated as commented. The default behavior assumes that the file contains only numerical values and no empty lines.

If rect is present and equal to .false., the file read may contain data which are not under a rectangular layout. In this latter case, the routine returns a column vector mfArray. By default, this routine expects a rectangular layout, *i. e.* the same number of reals stored in each row.

For the standard case only (i. e., rect is equal to .true., or not present), the optional arguments row\_max and col\_max allow the user to read partially the data file. These two arguments are not dependent (and do not need to be both present). Note also that the whole data file may have a non-rectangular layout: it is sufficient that (row\_max, col\_max) covers a rectangular subpart of the data file.

Remark: DOS or UNIX text files are supported.

See also: msSaveAscii, mfLoad, mfLoadHDF5

## msSaveSparse

# Sparse ASCII file saving

 ${\it Interface:}$ 

```
subroutine msSaveSparse( filename, A, format )
    character(len=*), intent(in) :: filename
    type(mfArray), intent(in) :: A
    character(len=3), intent(in), optional :: format
```

### Description:

Saves the sparse matrix mfArray A in the file filename, using the (optionally) specified ASCII format:

"CSC": Compact Sparse Column (default)

The first line contains exactly five items, which are the numbers of rows and columns (nrow and ncol), an integer (1 or 0) specifying whether the entries are row-sorted or not in each column, the tag "CSC" and the type of data ("real" or "complex");

the following line(s) contains the integer vector of the pointers to the columns;

the following line(s) contains the integer vector of the row indices;

the remaining line(s) contains the floating-point values.

- "CSR": Compact Sparse Row

Very close to the previous format, except that row and column role are exchanged.

- "HBO": Harwell-Boeing

A commonly used format for sparse matrices, described at: http://math.nist.gov/MatrixMarket/formats.html#hb

- "MTX": Matrix Market (also called "Coordinates format", or "COO")

A commonly used format for sparse matrices, described at: http://math.nist.gov/MatrixMarket/formats.html#MMformat

If format is not present, this routine uses the "CSC" format.

The routine always overwrites a pre-existing file.

 $See\ also: \verb|mfLoadSparse||$ 

## mfLoadSparse

# Sparse ASCII file loading

 ${\it Interface:}$ 

### Description:

Reads the file filename and copy the sparse matrix it contains in an mfArray.

When reading the file, it uses the (optionally) specified ASCII format:

```
- "CSC": Compact Sparse Column
```

- "CSR": Compact Sparse Row
- "HBO": Harwell-Boeing (without Right Hand Side; on the contrary, use msLoadSparse instead)
- "MTX": Matrix Market (also called "Coordinates format", or "COO")

If format is not present, this routine tries to auto-detect the used format.

The description of these different formats can be found at the msSaveSparse entry.

After reading the matrix:

- eventual duplicated entries are treated according to the optional character argument duplicated\_entries. When this argument is present and equal to "ignored", duplicated entries are ignored; when it is equal to "added" (default value), duplicated entries are added; when it is equal to "replaced", last entries found overwrite previous one.
- entries containing a null value are removed.

See also: msLoadSparse, msSaveSparse, mfSpImport

# ${\tt msLoadSparse}$

# Sparse ASCII file (with Right-Hand-Side) loading

 $Calling\ syntax:$ 

```
call msLoadSparse( mfOut(A[,RHS]), filename [, format, duplicated_entries] )
```

# Description:

Similar to  ${\tt mfLoadSparse}$  but allows the user to retrieve a Right-Hand-Side (HBO format only) together with the sparse matrix.

The input arguments are described in the mfLoadSparse description.

A and RHS are both mfArrays and, after reading the file, contain respectively the sparse matrix and the Right-Hand-Side (dense or sparse, according to the file content).

 $See\ also: {\tt mfLoadSparse}, {\tt msSaveSparse}, {\tt mfSpImport}$ 

### msSave

# Binary file saving

# ${\it Interface:}$

```
subroutine msSave( filename, A, compressed )
    character(len=*), intent(in) :: filename
    type(mfArray), intent(in) :: A
    or type(mfTriConnect), intent(in) :: A
    logical, optional :: compressed
```

### Description:

Saves under a binary form the mfArray (or mfTriConnect) A in the file named filename. The MUESLI Binary Format (MBF) used is specific to the current library. Any filename extension is supported; the ".mbf" used in the MUESLI examples just helps to remind that it is a MUESLI binary file.

If compressed is present and true, or if the specified filename ends with the extension ".gz", then msSave directly stores the file in the 'gzip' compressed format.

### Remarks:

- to read the created file, you must use mfLoad (or mfLoadTriConnect), because a special format
  is used to organize data inside it. MUESLI add a special tag to help with compatibility between
  versions.
- msSave takes care of endianness of your architecture, so read/write this kind of file should be
  possible across big endian and little endian machines.
- msSave stores all the internal properties of A, including its dynamic type, structure and data, hidden matrix properties and physical units; so mfLoad (or mfLoadTriConnect) should restore exactly the same mfArray (but for sparse structure, internal arrays will have the minimal size).
- msSave always overwrites a pre-existing file.

See also: msSaveAscii, msSaveHDF5

mfLoad

# Binary file loading

Interface:

```
function mfLoad( filename ) result( out )
    character(len=*), intent(in) :: filename
    type(mfArray) :: out
```

# Description:

Reads a matrix previously stored in the file named filename (MUESLI Binary Format) by the routine msSave, and copy it in an mfArray.

This routine is able to read directly a compressed gzipped file, as stored by msSave.

See also: mfLoadAscii

## mfLoadTriConnect

# Connectivity Binary file loading

# ${\it Interface:}$

```
function mfLoadTriConnect( filename ) result( out )
    character(len=*), intent(in) :: filename
    type(mfTriConnect) :: out
```

# Description:

Reads a connectivity structure previously stored in the file named filename (MUESLI Binary Format) by the routine msSave, and copy it in an mfArray.

See also: mfLoadAscii

### msSaveHDF5

HDF5 file saving

 ${\it Interface:}$ 

## Description:

By default (or if file\_access is present and equal to "trunc"), this routine creates a new HDF5 file (or overwrites a pre-existing one) and writes the mfArray A (dense or sparse) in it.

If file\_access is present and equal to "append", it tries to add in the existing HDF5 file a new group for A. In the case where a group with the same name already exists, the group will be overwritten only if mfarray\_overwrite is present and equal to true; on the contrary, it will return a non zero value in the status flag (if this later optional argument is present).

The mfArray A is written as one group (named from the optional argument name, or "untitled mfArray" by default), with all its MUESLI properties, stored as group attributes.

The created file can be read in MUESLI by the routine mfLoadHDF5; moreover, it can be browsed by a specific viewer, as 'hdfview' for example.

Remark: No specific extension is required for the "filename", nevertheless ".h5" or ".hdf5" are often used.

See also: msSave, msSaveAscii

### mfLoadHDF5

# HDF5 file loading

## Interface:

# Description:

Reads from an HDF5 file and tries to find an mfArray named from the optional argument name. If this latter in not present, it tries to get the first group which has an attribute named MF\_MUESLI\_VERSION. It doesn't enter in any group: the searched mfArray must be located just under the root structure ("/") of the HDF5 file.

The file filename must have been created by msSaveHDF5.

See also: mfLoad, mfLoadAscii

### msMedit

# graphic mfArray editor

## Interface:

```
subroutine msMedit( A )

type(mfArray), intent(in out) :: A
```

### Description:

This routine allows the user to interactively modify a real mfArray, with a small graphical Matrix-editor.

Actually, it launch another X11 (Qt4-based) application, named 'meditor' (spreadsheet-like), which displays a table containing initially the matrix A. The user can modify the values and extend the shape of the matrix. When the window is closed, new data are automatically saved in the mfArray A.

Special IEEE values can be entered in cells, by typing their ASCII names, NaN and Inf ('meditor' is not case-sensitive). Bad entries are ignored.

To extend the matrix, click anywhere outside the matrix and enter a value (empty cells of the new matrix will be filled with 0).

### Remarks:

- complex numeric matrices, or containing boolean values, are not accepted;
- if A is a sparse matrix, it is first converted to a dense one before display, but the new modified matrix will be converted back to sparse storage;
- the run of the 'meditor' tool blocks the calling program till it's window is closed by the user;
- the size of A is limited to  $200 \times 200$ : it can be changed in the file 'meditor.cpp' if needed (of course, the 'meditor' tool has to be compiled again).

# 1.3 Data Analysis Functions

mfMax, msMaxmfArray maxmfMin, msMinmfArray minmfSumcolumns summfExtremadata extremamfProdcolumns productmfSort, msSortcolumns/rows sortmfIsSortedvector sort test

mfSortRows sort whole rows of a matrix

mfDiff difference and approximate derivative

mfGradient approximate 1D gradient msGradient approximate 2D gradient

mfMeancolumns meanmfMediancolumns medianmfVarcolumns variance

mfStd columns standard deviation

mfRMS root mean square msHist data histogram

mfMoments few first moments of a distribution

mfSmooth smoothing of vector values

mfXCorr autocorrelation of a vector data mfXCorr2 autocorrelation of a matrix data

mfFFT, mfInvFFT discrete Fast Fourier transformation
mfFFT2, mfInvFFT2 2D discrete Fast Fourier transformation
mfFourierCos, mfInvFourierCos
mfFourierSin, mfInvFourierSin discrete Fourier cosine transformation
discrete Fourier sine transformation

 ${\tt mfFourierLeg}, {\tt mfInvFourierLeg} \quad {\tt discrete} \ {\tt Fourier-Legendre} \ {\tt transformation}$ 

## See also:

Core Routines

File Input/Output

Operators

**Elementary Math Functions** 

Specialized Math Functions

Elementary Matrix Manipulation Functions

Matrix Functions

Polynomial Functions

Optimization and Function Functions

Sparse Matrices

mfMax mfArray max

mfMax has two forms. The first one is used to compute the maximum value of the columns (default behavior) or of the rows of an mfArray. The second one compares two matrices.

First interface:

```
function mfMax( A, dim, nanflag ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
character(len=*), intent(in), optional :: nanflag
type(mfArray) :: out
```

If dim is present, A is always considered as a matrix, and the returned vector contains the maximum of each column (if dim = 1) or the maximum of each row (if dim = 2).

If dim is not present: if A is a vector, mfMax returns the largest element; otherwise it returns the max of each columns.

If the string nanflag is present, it must be equal to either "omitNaN", or "includeNaN" (practically, the argument is not case sensitive). By default, NaN values are discarded (see examples below), so "omitNaN" is implied.

For a sparse mfArray A, the result is exactly the same as for the full corresponding matrix. In all cases (sparse or dense), it must contain real values.

Second interface:

```
function mfMax( A, B, nanflag ) result( out )
  type(mfArray), intent(in) :: A
  type(mfArray), intent(in) :: B
  or real(kind=MF_DOUBLE), intent(in) :: B
  character(len=*), intent(in), optional :: nanflag
  type(mfArray) :: out
```

When the two arguments A and B are mfArrays (sparse storage is allowed), they must have the same shape and must contain real values; in this case, the *max* operation is applied element-wise. Otherwise, the second argument B may be a scalar real.

The optional argument nanflag has the same meaning as in the first interface.

See also: mfMin, msMax, mfExtrema

Example(s):
 x = [ 1.0d0, MF\_NAN, 5.0d0, 2.0d0, 4.0d0 ]
 call msDisplay( mfMax(x), "mfMax(x)" )

output:
 mfMax(x) =
 5.0000

 $\cdots / \dots$ 

```
x = [ 1.0d0, MF_NAN, 5.0d0, 2.0d0, 4.0d0 ]
    call msDisplay( mfMax(x,nanflag='includeNaN'), "mfMax(x,nanflag='includeNaN')" )
output:

mfMax(x,nanflag='includeNaN') =

    NaN

x = [ MF_NAN, MF_NAN ]
    call msDisplay( mfMax(x), "mfMax(x)" )
output:

mfMax(x) =

    NaN
```

msMax mfArray max

Calling syntax:

```
call msMax( mfOut(v,i), A [, dim, nanflag] )
```

## Description:

Computes the maximum value of the columns (default behavior) or of the rows of the mfArray A (real values only) and shows where this maximum occurs.

If  $\dim$  is present, A is always considered as a matrix, and the vector v contains the maximum of each column (if  $\dim = 1$ ) or the maximum of each row (if  $\dim = 2$ ). indices of these maxima are returned in the vector i.

If dim is not present: if A is a vector, msMax returns the largest element in v and the corresponding index in i; otherwise it returns the max of each columns in v and the corresponding indices in i.

The optional argument nanflag has the same meaning as in the mfMax routine.

For a sparse mfArray A, the result is exactly the same as for the full corresponding matrix. In all cases (sparse or dense), it must contain real values.

See also: mfOut, msMin, mfExtrema

for index =

2

```
Example(s):
   x = [1.0d0, MF_NAN, 5.0d0, 2.0d0, 4.0d0]
   call msDisplay( x, "x" )
   call msMax( mfOut(a,i), x )
   call msDisplay( a, "max" )
   call msDisplay( i, "for index" )
output:
 x =
    1.0000
                NaN
                        5.0000
                                   2.0000
                                             4.0000
max =
     5
 for index =
     3
   call msMax( mfOut(a,i), x, nanflag='includeNaN' )
   call msDisplay( a, "max" )
   call msDisplay( i, "for index" )
output:
max =
     NaN
```

mfMin mfArray min

mfMin has two forms. The first one is used to compute the minimum value of the columns (default behavior) or of the row of an mfArray. The second one compares two matrices.

First interface:

```
function mfMin( A, dim, nanflag ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
character(len=*), intent(in), optional :: nanflag
type(mfArray) :: out
```

If dim is present, A is always considered as a matrix, and the returned vector contains the minimum of each column (if dim = 1) or the minimum of each row (if dim = 2).

If dim is not present: if A is a vector, mfMin returns the smallest element; otherwise it returns the min of each columns.

If the string nanflag is present, it must be equal to either "omitNaN", or "includeNaN" (practically, the argument is not case sensitive). By default, NaN values are discarded (see examples below), so "omitNaN" is implied.

For a sparse mfArray A, the result is exactly the same as for the full corresponding matrix. In all cases (sparse or dense), it must contain real values.

Second interface:

```
function mfMin( A, B, nanflag ) result( out )
  type(mfArray), intent(in) :: A
  type(mfArray), intent(in) :: B
  or real(kind=MF_DOUBLE), intent(in) :: B
  character(len=*), intent(in), optional :: nanflag
  type(mfArray) :: out
```

When the two arguments A and B are mfArrays (sparse storage is allowed), they must have the same shape and must contain real values; in this case, the *min* operation is applied element-wise. Otherwise, the second argument B may be a scalar real.

The optional argument nanflag has the same meaning as in the first interface.

See also: mfMax, msMin, mfExtrema

Example(s):
 x = [ 1.0d0, MF\_NAN, 5.0d0, 2.0d0, 4.0d0 ]
 call msDisplay( mfMin(x), "mfMin(x)" )

output:
 mfMin(x) =
 1.0000

 $\cdots$ /...

```
x = [ 1.0d0, MF_NAN, 5.0d0, 2.0d0, 4.0d0 ]
    call msDisplay( mfMin(x,nanflag='includeNaN'), "mfMin(x,nanflag='includeNaN')" )
output:

mfMin(x,nanflag='includeNaN') =

    NaN

x = [ MF_NAN, MF_NAN ]
    call msDisplay( mfMin(x), "mfMin(x)" )
output:

mfMin(x) =

    NaN
```

msMin mfArray min

Calling syntax:

```
call msMin( mfOut(v,i), A [, dim, nanflag] )
```

## Description:

Computes the minimum value of the columns (default behavior) or of the rows of the mfArray A (real values only) and shows where this minimum occurs.

If  $\dim$  is present, A is always considered as a matrix, and the vector  $\mathbf{v}$  contains the minimum of each column (if  $\dim = 1$ ) or the minimum of each row (if  $\dim = 2$ ). indices of these minima are returned in the vector  $\mathbf{i}$ .

If dim is not present: if A is a vector, mfMax returns the smallest element in v and the corresponding index in i; otherwise it returns the min of each columns in v and the corresponding indices in i.

The optional argument nanflag has the same meaning as in the mfMin routine.

For a sparse mfArray A, the result is exactly the same as for the full corresponding matrix. In all cases (sparse or dense), it must contain real values.

See also: mfOut, msMax, mfExtrema

for index =

2

```
Example(s):
   x = [1.0d0, MF_NAN, 5.0d0, 2.0d0, 4.0d0]
   call msDisplay( x, "x" )
   call msMin( mfOut(a,i), x )
   call msDisplay( a, "min" )
   call msDisplay( i, "for index" )
output:
 x =
    1.0000
                NaN
                        5.0000
                                   2.0000
                                             4.0000
min =
     1
 for index =
     1
   call msMin( mfOut(a,i), x, nanflag='includeNaN' )
   call msDisplay( a, "min" )
   call msDisplay( i, "for index" )
output:
min =
     NaN
```

data extrema mfExtrema

Interface:

```
function mfExtrema( A, nanflag ) result( out )
   type(mfArray),
                     intent(in)
   character(len=*), intent(in), optional :: nanflag
   type(mfArray)
```

Description:

Returns in an mfArray the two extreme values of an mfArray, in ascent order, i.e. the couple (min,

The optional argument nanflag has the same meaning as in the mfMin and mfMax routines.

See also: msMin, msMax

NaN

NaN

```
Example(s):
   x = [5, 9, 3, 6, 0, 2]
   call msDisplay( x, "x" )
   call msDisplay( mfExtrema(x), "extrema" )
output:
 x =
    5
          9
                3
                      6
                           0
                                  2
 extrema =
    0
          9
   x = [5, 9, MF_NAN, 6, 0, 2]
   call msDisplay( x, "x" )
   call msDisplay( mfExtrema(x,nanflag='includeNaN'), "extrema (include NaN)" )
output:
x =
                           0
     5
          9
                NaN 6
                                  2
 extrema (include NaN) =
```

mfSum columns sum

# Interface:

```
function mfSum( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

# Description:

Computes the sum of the columns (or rows) of an mfArray.

If dim is present, A is always considered as a matrix, and the output contains the sum of each column (if dim = 1) or the sum of each row (if dim = 2).

If  $\dim$  is not present: if A is a vector,  $\operatorname{\mathtt{mfSum}}$  returns the sum of all elements; otherwise it returns the sum of each columns.

See also: mfProd

mfProd columns product

## Interface:

```
function mfProd( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

# Description:

Computes the product of the columns (or rows) of an mfArray. The behavior of this routine is similar to the *product* Fortran 90 intrinsic function.

If dim is present, A is always considered as a matrix, and the output contains the product of each column (if dim = 1) or the product of each row (if dim = 2).

If dim is not present: if A is a vector, mfSum returns the product of all elements; otherwise it returns the product of each columns.

Remarks: this routine cannot be applied to a boolean mfArray.

Warning: this function cannot be applied to sparse matrices.

See also: mfSum

mfSort columns sort

Interface:

```
function mfSort( A, dim, mode ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
character(len=*), intent(in), optional :: mode
type(mfArray) :: out
```

### Description:

If dim is not present, sorts the values of the vector A, and returns the sorted vector in an mfArray.

If dim is present, A may be a matrix. In this case, the sort is applied to the columns if dim is equal to 1, and to the rows if dim is equal to 2.

#### Remarks:

- this routine is appropriate for 2D arrays containing unrelated data in rows. If you want to sort a particular column, keeping the coherence of the lines (i. e. sorting the whole lines by key), you have to write your own code, based on the msSort version (see an example in the Muesli User's Guide); this means also that it is not possible to sort one specific column (or row) while keeping the others unchanged.
- this routine use a quick sort algorithm. By default, or if the optional argument mode is equal to "ascending" values are sorted in increasing order; on the contrary, if the optional argument mode is equal to "descending" values are sorted in decreasing order.
- currently, only rank-1 mfArrays (i. e. vectors) may contain NaN values. These NaNs are moved to the tail of the sorted vector.
- see msSort for an 'in-place' version.

Warning: If the mfArray A is a matrix, it must be real; for a complex vector mfArray, the sort is applied first on the module values, then on the phase-angle.

```
See also: msSort, mfSortRows
```

```
Example(s):
```

```
x = mf([1.0d0, MF_NAN, 5.0d0, MF_NAN, 2.0d0, 4.0d0])
   call msDisplay( x, "x" )
   call msDisplay( mfSort(x), "mfSort(x)" )
   call msDisplay( mfSort(x,'descending'), "mfSort(x,'descending')" )
output:
 x =
    1.0000
                         5.0000
                NaN
                                     NaN
                                              2.0000
                                                        4.0000
mfSort(x) =
              2.0000
                         4.0000
    1,0000
                                   5,0000
                                                \mathtt{NaN}
                                                           NaN
mfSort(x,'descending') =
    5.0000
              4.0000
                         2.0000
                                   1.0000
                                                NaN
                                                           NaN
```

msSort columns sort

```
First calling syntax:
   call msSort ( A [, dim, mode ] )
The 'in-place' version of mfSort.
Second\ calling\ syntax:
   call msSort ( mfOut(v,i), A [, dim, mode ] )
Similar to mfSort but the result is returned in the mfArray v; in addition, indices of the sort are stored
in the vector i.
See also: mfOut, mfSortRows
Example(s):
   x = mfMagic(3)
   call msDisplay( x, "x" )
   call msSort( mfOut(a,i), x, dim=1 )
   call msDisplay( a, "ascending column sort" )
   call msDisplay( i, "with indices" )
   call msSort( mfOut(a,i), x, dim=2, mode="descending" )
   call msDisplay( a, "descending row sort" )
   call msDisplay( i, "with indices" )
output:
 x =
     8
            3
                  4
     1
            5
                  9
            7
                  2
     6
 ascending column sort =
            3
     1
                  2
     6
            5
                  4
     8
            7
                  9
 with indices =
     2
                  3
            1
     3
            2
                  1
     1
            3
 descending row sort =
     8
            4
                  3
     9
            5
                  1
     7
            6
                  2
 with indices =
     1
            3
                  2
     3
            2
                  1
```

mfIsSorted vector sort test

# Interface:

```
bool = mfIsSorted( v, mode )
```

# Description:

Checks that the mfArray vector v is sorted, according the mode, which can be equal to "ascend", "descend" or "either".

If the optional argument mode is not present, then an ascending sort is presumed.

This function returns a logical.

### Remarks:

- currently, only rank-1 mfArrays (i. e. vectors) can be checked.
- the vector  ${\tt v}$  may contain NaN values. The vector is considered as sorted if all NaNs are located at the end.
- for a complex vector mfArray, sorting concerns first the module, then the phase-angle.

See also: mfSort

### mfSortRows

### sort whole rows of a matrix

First interface:

```
function mfSortRows( A, col, mode ) result( out )

type(mfArray), intent(in) :: A
 integer, intent(in), :: col
 character(len=*), intent(in), optional :: mode
 type(mfArray) :: out
```

### Description:

Sorts the rows of the matrix A according to the specified column given by col, and returns the result in an mfArray.

### Remarks:

- this routine use a quick sort algorithm. By default, or if the optional argument mode is equal to "ascending" values are sorted in increasing order; on the contrary, if the optional argument mode is equal to "descending" values are sorted in decreasing order.
- currently, the matrix A should not contain NaN values.

Warning: The mfArray A must be real.

Other interface:

## Description:

Same as before except that the sort of the rows is made by many columns, not necessarily in order, stored in cols (if present). The only constraint is that the size of cols must less or equal the number of columns of A. If cols is not present, then the sort is done by all columns, in order.

In this new interface, the direction of the sort is specified by the sign of the elements of the cols argument: when a column number is positive, the sort is ascending; in the opposite, *i. e.* when it is negative, the sort is descending.

```
See also: mfSort, msSort

Example(s):
x = mfMagic(3)
call msDisplay( x, "x" )
call msDisplay( mfSortRows(x,3), "mfSortRows(x,3)" )
call msDisplay( mfSortRows(x,1,"descending"), "mfSortRows(x,1,'descending')" )
```

```
output:
x =
    8
          3
                4
    1
          5
                9
    6
          7
                2
mfSortRows(x,3) =
          7
    6
                2
    8
          3
                4
          5
                9
     1
mfSortRows(x,1,'descending') =
    6
          7
                2
          5
    1
                9
    8
          3
                4
x = mf([3, 6, 2, 5]).vc.
                                                                    &
   mf([2, 1, 4, 2]).vc.
                                                                    &
   mf([1,0,0,3]).vc.
                                                                    &
   mf([2, 1, 5, 1]).vc.
   mf([2, 3, 3, 1])
call msDisplay(x,"x")
call msDisplay( mfSortRows(x,[3]), "sortrows(x,[3])" )
call msDisplay( mfSortRows(x,[1,-3]), "sortrows(x,[1,-3])" )
output:
x =
    3
                2
          6
                      5
    2
          1
                4
                      2
    1
          0
                0
                      3
    2
          1
                5
                      1
     2
          3
                3
 sortrows(x,[3]) =
          0
                0
    1
                      3
    3
          6
                2
                      5
    2
          3
                3
                      1
     2
          1
                4
                      2
     2
          1
                5
 sortrows(x,[1,-3]) =
    1
          0
                0
                      3
    2
                5
          1
                      1
    2
                4
                      2
          1
    2
          3
                3
                      1
    3
          6
                2
```

### msSortRows

1

### sort whole rows of a matrix

```
First calling syntax:
   call msSortRows ( A, col [, mode ] )
or
   call msSortRows ( A [, cols ] )
The 'in-place' version of mfSortRows.
Second calling syntax:
   call msSortRows ( mfOut(v,i), A, col [, mode ] )
or
   call msSortRows ( mfOut(v,i), A [, cols ] )
Similar to mfSortRows but the result is returned in the mfArray v; in addition, indices of the sort are
stored in the vector i.
See also: mfOut, mfSortRows
Example(s):
x = mf([3, 6, 2, 5]).vc.
                                                                         &
    mf([2, 1, 4, 2]).vc.
                                                                         &
    mf([1, 0, 0, 3]).vc.
                                                                         &
    mf([2, 1, 5, 1]).vc.
                                                                         &
    mf([2, 3, 3, 1])
call msDisplay(x,"x")
call msSortRows( mfOut(a,i), x )
call msDisplay( a,"fully row sorted", i,"with row indices" )
output:
x =
     3
           6
                 2
                        5
     2
                        2
           1
                 4
     1
           0
                  0
                        3
     2
           1
                 5
                        1
     2
           3
                 3
                        1
 fully row sorted =
                  0
     1
           0
                        3
     2
           1
                  4
                        2
     2
           1
                  5
                        1
     2
           3
                  3
                        1
     3
           6
                  2
                        5
 with row indices =
     3
     2
     4
     5
```

### mfDiff

# difference and approximate derivative

## Interface:

```
function mfDiff( A, dim ) result( out )
   type(mfArray), intent(in)
   integer,
                  intent(in), optional :: dim
   type(mfArray)
                                        :: out
```

# Description:

Computes the difference of two consecutive values of the columns (or rows) of an mfArray A.

If dim is present, A is always considered as a matrix, and the routine computes the difference of two consecutive values for each column (if  $\dim = 1$ ) or for each row (if  $\dim = 2$ ).

If dim is not present: if A is a vector, then mfDiff computes the difference of two consecutive values in the natural way; otherwise the same process is applied for each columns.

Remark: this function cannot be applied to sparse matrices.

```
See also: mfGradient, msGradient
```

```
Example(s):
```

```
x = mf([1, 2, 3, 2, 1])
  call msDisplay( x, "x" )
  call msDisplay( mfDiff(x), "mfDiff(x)" )
output:
x =
          2
                3
                      2
                            1
mfDiff(x) =
    1
               -1
                     -1
         1
```

### mfGradient

## approximate 1D gradient

 $Calling\ syntax:$ 

```
D = mfGradient( F, h [, dim, location] )
```

Description:

Computes the gradient D of the rank-1 mfArray F.

If  $\dim$  is present, F is always considered as a matrix, and the routine computes the gradient for each column (if  $\dim = 1$ ) or for each row (if  $\dim = 2$ ).

If dim is not present: if F is a vector, then mfGradient computes the gradient in the natural way; otherwise the same process is applied for each columns.

The data is supposed to be equally spaced with h as increment. h may be a double precision real or a scalar mfArray; it may have negative values.

If the optional argument location (character string) is equal to "centered" then the gradient is computed on an internal centered, staggered grid; otherwise it is computed at the same points as F.

Remark: The gradient is computed using second order formulæ.

See also: msGradient, mfDiff, mfOut

```
Example(s):
```

```
integer :: nx = 5
   real(kind=MF_DOUBLE) :: lx = 1.0d0, hx
   x = mfLinSpace(0.0d0,lx,nx)
  F = x**2
   call msDisplay( F, "F" )
   hx = lx/(nx-1)
   Fx = mfGradient( F, hx )
   call msDisplay( Fx, "Fx = grad_x(F)" )
output:
 F =
    0.0000
              0.0625
                        0.2500
                                   0.5625
                                             1.0000
Fx = grad_x(F) =
    0.0000
              0.5000
                        1.0000
                                   1.5000
                                             2.0000
   fx = mfGradient( F, hx, "centered" )
   call msDisplay( fx, "grad_x(F)" )
output:
 grad_x(z) =
    0.2500
              0.7500
                        1.2500
                                  1.7500
```

### msGradient

## approximate 2D gradient

Calling syntax:

```
call msGradient( mfOut( Fi, Fj ), F, hi, hj [, location] )
```

### Description:

Computes the gradient of the rank-2 mfArray F, along the two dimensions. The data are supposed to be equally spaced in both direction, with hi and hj as increments, which may have negative values.

As a result, Fi is the gradient of F with respect to increasing indices i (i.e. along the rows) and Fj is the gradient of F with respect to increasing indices j (i.e. along the columns).

If the optional argument location (character string) is equal to "centered" then the two components of the gradient are computed on an internal centered, staggered grid; otherwise they are computed at the same points as F.

### Remarks:

- this function cannot be applied to sparse matrices. For all cases and all points, the gradient is computed using second order formulæ.
- if the data matrix F is associated to coordinates X and Y, hi and hj may be chosen to give directly the two components  $F_x$  and  $F_y$  (see the *Muesli User Guide*, section entitled "Data Analysis with mfArrays").

See also: mfGradient, mfDiff, mfOut

```
Example(s):
```

```
integer :: nx = 5, ny = 5
   real(kind=MF_DOUBLE) :: lx = 1.0d0, ly = 2.0d0, hx, hy
   call msMeshGrid( mfOut(x,y), mfLinSpace(0.0d0,lx,nx),
                           .t. mfLinSpace(ly,0.0d0,ny) )
   call msDisplay( x, "x", y, "y" )
   F = (2*x+y)**2
   call msDisplay( F, "F" )
   hx = lx/(nx-1)
  hy = -ly/(ny-1)
   call msGradient( mfOut(Fy,Fx), F, hy, hx )
   call msDisplay( Fx, "Fx = grad_x(F)", Fy, "Fy = grad_y(F)" )
output:
 x =
   0.0000
             0.2500
                       0.5000
                                  0.7500
                                            1.0000
   0.0000
              0.2500
                        0.5000
                                  0.7500
                                            1.0000
   0.0000
              0.2500
                        0.5000
                                  0.7500
                                            1.0000
   0.0000
              0.2500
                        0.5000
                                  0.7500
                                            1.0000
   0.0000
             0.2500
                        0.5000
                                  0.7500
                                            1.0000
```

 $\cdots / \dots$ 

```
у =
                       2.0000
    2.0000
             2.0000
                                 2.0000
                                           2.0000
    1.5000
             1.5000
                       1.5000
                                 1.5000
                                           1.5000
    1.0000
             1.0000
                       1.0000
                                 1.0000
                                           1.0000
    0.5000
              0.5000
                       0.5000
                                 0.5000
                                           0.5000
    0.0000
              0.0000
                       0.0000
                                 0.0000
                                           0.0000
 F =
    4.0000
             6.2500
                       9.0000
                                12.2500
                                          16.0000
    2.2500
             4.0000
                       6.2500
                               9.0000
                                         12.2500
                       4.0000
                                 6.2500
    1.0000
             2.2500
                                          9.0000
    0.2500
              1.0000
                       2.2500
                                 4.0000
                                           6.2500
    0.0000
              0.2500
                     1.0000
                                 2.2500
                                           4.0000
 Fx = grad_x(F) =
     8
         10
               12
                     14
                           16
     6
          8
                10
                     12
                           14
     4
           6
                8
                     10
                           12
     2
                6
                      8
                           10
           4
     0
           2
                            8
                 4
                      6
Fy = grad_y(F) =
     4
           5
                      7
                 6
                            8
     3
           4
                5
                            7
                      6
     2
           3
                 4
                      5
                            6
     1
           2
                 3
                       4
                            5
           1
                 2
     0
                       3
                            4
   call msGradient( mfOut(Fy,Fx), F, hy, hx, "centered" )
   call msDisplay( Fx, "Fx = grad_x(F)", Fy, "Fy = grad_y(F)" )
output:
 Fx = grad_x(F) =
     8
          10
                12
                     14
     6
          8
                10
                     12
     4
                     10
           6
                8
     2
           4
                 6
                      8
Fy = grad_y(F) =
     4
                      7
           5
                 6
     3
           4
                5
                      6
     2
           3
                 4
                      5
     1
           2
                 3
                       4
```

mfMean columns mean

Interface:

```
function mfMean( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

Description:

Computes the mean of the columns (or rows) of an mfArray, as  $\frac{1}{N} \sum_{i=1}^{N} a_i$ .

If dim is present, A is always considered as a matrix, and the output contains the mean of each column (if dim = 1) or the mean of each row (if dim = 2).

If dim is not present: if A is a vector, mfMean returns the mean of all elements; otherwise it returns the mean of each columns.

Warning:

- this function cannot be applied to sparse matrices;
- the user must take care that the mfArray A doesn't contain any NaN values.

See also: mfMedian, mfVar, mfStd, mfRMS, mfMoments

mfMedian columns median

## Interface:

```
function mfMedian( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

# Description:

Computes the median of the columns (or rows) of an mfArray. The median value of a vector is, after sorting this vector, the value located in the middle.

If dim is present, A is always considered as a matrix, and the output contains the median of each column (if dim = 1) or the median of each row (if dim = 2).

If dim is not present: if A is a vector, mfMedian returns the median of all elements; otherwise it returns the median of each columns.

## Warning:

- this function cannot be applied to sparse matrices;
- the user must take care that the mfArray A doesn't contain any NaN values.

See also: mfMean, mfVar, mfStd, mfMoments

mfVar columns variance

Interface:

```
function mfVar( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

Description:

```
Computes the variance of the columns (or rows) of an mfArray, as \frac{1}{N}\sum_{i=1}^{N}(a_i-\bar{a})^2, with \bar{a}=\frac{1}{N}\sum_{i=1}^{N}a_i.
```

If dim is present, A is always considered as a matrix, and the output contains the variance of each column (if dim = 1) or the variance of each row (if dim = 2).

If dim is not present: if A is a vector, mfVar returns the variance of all elements; otherwise it returns the variance of each columns.

## Warning:

- this function cannot be applied to sparse matrices;
- the user must take care that the mfArray A doesn't contain any NaN values.

See also: mfMean, mfMedian, mfStd, mfMoments

### mfStd

## columns standard deviation

## Interface:

```
function mfStd( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

# Description:

Computes the standard deviation of the columns (or rows) of an mfArray. The standard deviation is the square root of the variance.

If dim is present, A is always considered as a matrix, and the output contains the standard deviation of each column (if dim = 1) or the standard deviation of each row (if dim = 2).

If dim is not present: if A is a vector, mfStd returns the standard deviation of all elements; otherwise it returns the standard deviation of each columns.

## Warning:

- this function cannot be applied to sparse matrices;
- the user must take care that the mfArray A doesn't contain any NaN values.

See also: mfMean, mfRMS, mfMedian, mfVar, mfMoments

mfRMS root mean square

 ${\it Interface:}$ 

```
function mfRMS( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

Description:

Computes the Root-Mean-Square of the columns (or rows) of an mfArray.

The RMS is defined by:

$$a_{RMS} = \sqrt{\frac{1}{N} \sum_{i=1}^{N} a_i^2}$$

where N is the number of elements  $x_i$ . The following relation holds:

$$a_{RMS}^2 = \bar{a}^2 + \sigma_a^2$$

so the RMS of a signal is synonym to its standard deviation only when the mean is zero.

If dim is present, A is always considered as a matrix, and the output contains the mean of each column (if dim = 1) or the RMS of each row (if dim = 2).

If dim is not present: if A is a vector, mfRMS returns the RMS of all elements; otherwise it returns the RMS of each columns.

Warning:

- this function can be applied only to real, dense matrices;
- the user must take care that the mfArray A doesn't contain any NaN values.

See also: mfMean, mfMedian, mfVar, mfStd, mfMoments

msHist data histogram

Calling syntax:

```
call msHist( mfOut( num [, x_bin ] ), x, x_min, x_max, n_bin )
computes a data histogram.
```

The vector mfArray x contains the data. The histogram is computed for data ranged from  $x_min$  to  $x_max$  using a number of bins equal to  $n_bin$ .

The output mfArray num is a vector of the bins values (length equal to n\_bin).

If present, the output  $mfArray x_bin$  is a vector containing the abscissas of the bins (length equal to  $n_bin+1$ ).

Note: to process discrete data, it is recommended to choose  $x\_min$  as the minimum integer value minus 1/2, and  $x\_max$  as the maximum integer value plus 1/2 (see example below).

See also: mf/msBar, mf/msPlotHist, msCumulHist, mfOut

```
Example(s):
```

```
x = .t. mf([ 12, 11, 8, 10, 6, 8, 9, 10, 12, 9, 10, 8, 6, 10 ] )
call msHist( mfOut( y, z ), x, 6.0d0-0.5d0, 12.0d0+0.5d0, 7 )
call msDisplay( y, "y (hist)", z, "z (x_bin)" )
```

#### output:

```
y (hist) =

2
0
3
2
4
1
2
```

```
z (x_bin) =
```

```
5.5000
6.5000
7.5000
8.5000
9.5000
10.5000
11.5000
```

12.5000

#### mfMoments

## few first moments of a distribution

# ${\it Interface:}$

```
function mfMoments( v ) result( out )
  type(mfArray), intent(in) :: v
  type(mfArray) :: out
```

## Description:

Computes some moments of the data distribution provided in the vector mfArray v.

It returns a vector mfArray of length 6, containing in order:

- the mean value of data
- the average deviation
- the standard deviation
- the variance
- the skewness
- the kurtosis

## Warning:

- this function cannot be applied to sparse matrices;
- the user must take care that the mfArray v doesn't contain any NaN values.

See also: mfMean, mfMedian, mfVar, mfStd

#### mfSmooth

# smoothing of vector values

## Interface:

## Description:

Applies the moving average method to the vector mfArray v. It is equivalent to a lowpass filter.

The width of the sliding window is span which must be a non negative integer. The span value should not be greater than the number of elements in v. The default value is 5.

See also: mfSpline

#### mfXCorr

## autocorrelation of a vector data

## Interface:

## Description:

Computes the autocorrelation of the real vector mfArray v.

The output mfArray out is a vector having an odd number of elements, and is symmetric with respect to its center.

Its maximum corresponds to a lag equal to zero. This maximum is equal to 1 when the optional argument scale is set to "normalized".

The maximum lag used can be set by using the optional maxlag argument. By default, its value is equal to N-1, where N is the length of the vector  $\mathbf{v}$ .

See also: mfXCorr2

#### mfXCorr2

## autocorrelation of a matrix data

## Interface:

## Description:

Computes the autocorrelation of the real matrix mfArray A. It is similar to the mfXCorr routine.

The output mfArray out is a matrix having an odd number of rows and columns, and is symmetric with respect to its center.

Its maximum corresponds to a lag equal to zero. This maximum is equal to 1 when the optional argument scale is set to "normalized".

The maximum lag used for the row shift (resp. the column shift) can be set by using the optional maxlag\_r argument (resp. maxlag\_c). By default, their value is equal to the size of the corresponding dimension.

See also: mfXCorr

#### mfFFT

## discrete Fast Fourier transformation

## Interface:

```
function mfFFT( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

## Description:

Applies a Fast Fourier transformation to the columns (or rows) of an mfArray, real or complex. The output mfArray is always complex.

If  $\dim$  is present, A is always considered as a matrix, and the output contains the transformation coefficients of each column (if  $\dim = 1$ ) or the transformation of each row (if  $\dim = 2$ ).

If dim is not present: if A is a vector, mfFFT returns the transformation of all elements; otherwise it returns the transformation of each columns.

Warning: this function cannot be applied to sparse matrices.

Remarks: abscissa are supposed to be equally spaced.

See also: mfInvFFT, mfFFT2, mfFourierCos, mfFourierSin, mfFourierLeg

## mfInvFFT

# inverse Fast Fourier transformation

# ${\it Interface:}$

```
function mfInvFFT( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

# Description:

This is the inverse tranformation of mfFFT. So, for any vector  $\boldsymbol{v},$  we have:

```
v = mfInvFFT(mfFFT(v))
```

See also: mfFFT, mfFFT2, mfFourierCos, mfFourierSin, mfFourierLeg

## mfFFT2

## 2D discrete Fast Fourier transformation

## Interface:

```
function mfFFT2( A ) result( out )
  type(mfArray), intent(in) :: A
  type(mfArray) :: out
```

## Description:

Applies a two dimensional Fast Fourier transformation to the mfArray A, real or complex. The output mfArray is always complex.

Warning: this function cannot be applied to sparse matrices.

 ${\it Remarks:}$  abscissa are supposed to be equally spaced for both dimensions.

See also: mfInvFFT2, mfFFT, mfFourierCos, mfFourierSin, mfFourierLeg

## mfInvFFT2

# inverse 2D Fast Fourier transformation

# ${\it Interface:}$

```
function mfInvFFT2( A ) result( out )
    type(mfArray), intent(in) :: A
    type(mfArray) :: out
```

# Description:

This is the inverse tranformation of mfFFT2. So, for any complex matrix A, we have:

```
A = mfInvFFT(mfFFT(A))
```

See also: mfFFT2, mfFFT, mfFourierCos, mfFourierSin, mfFourierLeg

#### mfFourierCos

## discrete Fourier cosine transformation

## Interface:

```
function mfFourierCos( A, dim ) result( out )

type(mfArray), intent(in) :: A
 integer, intent(in), optional :: dim
 type(mfArray) :: out
```

#### Description:

Applies a discrete Fourier cosine transformation to the columns (or rows) of an mfArray.

If dim is present, A is always considered as a matrix, and the output contains the transformation coefficients of each column (if dim = 1) or the transformation of each row (if dim = 2).

If dim is not present: if A is a vector, mfFourierCos returns the transformation of all elements; otherwise it returns the transformation of each columns.

Warning: this function cannot be applied to sparse matrices. Moreover, the array must contain real values.

#### Remarks:

- abscissa are supposed to be equally spaced;
- actually, data are already considered symmetric on the interval [-n+1, n+1]; therefore, there doesn't exist any constraint on the value of the data vector.

See also: mfInvFourierCos, mfFFT, mfFFT2, mfFourierSin, mfFourierLeg

## mfInvFourierCos

# inverse Fourier cosine transformation

## Interface:

```
function mfInvFourierCos( A, dim ) result( out )

type(mfArray), intent(in) :: A
 integer, intent(in), optional :: dim
 type(mfArray) :: out
```

# Description:

This is the inverse tranformation of mfFourierCos. So, for any vector v, we have:

```
v = mfInvFourierCos(mfFourierCos(v))
```

See also: mfFourierCos, mfFFT, mfFFT2, mfFourierSin, mfFourierLeg

#### mfFourierSin

#### Fourier sine transformation

## Interface:

```
function mfFourierSin( A, dim ) result( out )

type(mfArray), intent(in) :: A
 integer, intent(in), optional :: dim
 type(mfArray) :: out
```

## Description:

Applies a discrete Fourier sine transformation to the columns (or rows) of an mfArray.

If dim is present, A is always considered as a matrix, and the output contains the transformation coefficients of each column (if dim = 1) or the transformation of each row (if dim = 2).

If dim is not present: if A is a vector, mfFourierSin returns the transformation of all elements; otherwise it returns the transformation of each columns.

Warning: this function cannot be applied to sparse matrices. Moreover, the array must contain real values.

#### Remarks:

- abscissa are supposed to be equally spaced;
- actually, data are already considered antisymmetric on the interval [-n+1, n+1]; therefore, there exists some constraints: the conditions y(1) = y(n+1) = 0 are required.

See also: mfInvFourierSin, mfFFT, mfFFT2, mfFourierCos, mfFourierLeg

## mfInvFourierSin

# inverse Fourier sine transformation

## Interface:

```
function mfInvFourierSin( A, dim ) result( out )

type(mfArray), intent(in) :: A
 integer, intent(in), optional :: dim
 type(mfArray) :: out
```

# Description:

This is the inverse tranformation of mfFourierSin. So, for any vector v, we have:

```
v = mfInvFourierSin(mfFourierSin(v))
```

See also: mfFourierSin, mfFFT, mfFFT2, mfFourierCos, mfFourierLeg

## mfFourierLeg

## Fourier-Legendre transform

## ${\it Interface:}$

```
function mfFourierLeg( A, dim ) result( out )

type(mfArray), intent(in) :: A
 integer, intent(in), optional :: dim
 type(mfArray) :: out
```

#### Description:

Applies a discrete Fourier-Legendre transformation to the columns (or rows) of an mfArray.

This transformation is also called "discrete spherical Fourier transform".

If  $\dim$  is present, A is always considered as a matrix, and the output contains the transformation coefficients of each column (if  $\dim = 1$ ) or the transformation of each row (if  $\dim = 2$ ).

If dim is not present: if A is a vector, mfFourierLeg returns the transformation of all elements; otherwise it returns the transformation of each columns.

Warning: this function cannot be applied to sparse matrices. Moreover, the array must contain real values.

#### Remarks:

- abscissa are supposed to be spanned over [-1,1] like a  $\cos(\theta)$  function, with  $\theta$  equally spaced;
- actually, data are already considered symmetric on the interval [-n+1, n+1]; therefore, there doesn't exist any constraint on the value of the data vector.

 $See~also:~{\tt mfInvFourierLeg},~{\tt mfLegendre},~{\tt mfFFT2},~{\tt mfFourierSin},~{\tt mfFourierCos}$ 

## mfInvFourierLeg

# inverse Fourier-Legendre transform

# ${\it Interface:}$

```
function mfInvFourierLeg( A, dim ) result( out )

type(mfArray), intent(in) :: A
 integer, intent(in), optional :: dim
 type(mfArray) :: out
```

# Description:

This is the inverse tranformation of mfFourierLeg. So, for any vector v, we have:

```
v = mfInvFourierLeg(mfFourierLeg(v))
```

 $See \ also: \ {\tt mfFourierLeg}, \ {\tt mfLegendre}, \ {\tt mfFFT2}, \ {\tt mfFourierSin}, \ {\tt mfFourierCos}$ 

# 1.4 Operators

+ - mfMul, .x. mfCross mfKron	mfArray sum mfArray difference mfArray product cross-product of two vectors Kronecker tensor product
* / **	mfArray element-wise product mfArray element-wise quotient mfArray element-wise exponentiation
.t. .h.	mfArray transposition mfArray conjugate transposition
.vchc. msHorizConcat	vertical concatenation horizontal concatenation sparse matrix horizontal concatenation
mfColPerm, msColPerm mfRowPerm, msRowPerm mfInvPerm mfColScale, msColScale mfRowScale, msRowScale msColAutoScale msRowAutoScale	columns permutation rows permutation inverse of a permutation columns scaling rows scaling columns auto scaling rows auto scaling
>=, > <=, < ==, /=	mfArray relational operator mfArray relational operator mfArray relational operator
mfAll, mfAny	mfArray boolean operator
mfColon	mfArray sequence constructor
<pre>.not., .and., .oreqv., .neqv.</pre>	mfArray logical operator mfArray logical operator
mfIsMember mfIntersect mfUnion mfUnique	elements members of a set common elements of two sets all elements of two sets unique elements of a set

 $See\ also:$ 

Core Routines

File Input/Output

Data Analysis Functions

**Elementary Math Functions** 

Specialized Math Functions

Elementary Matrix Manipulation Functions

Matrix Functions

Polynomial Functions

Optimization and Function Functions

Sparse Matrices

+ mfArray sum

 $Calling\ syntax:$ 

C = A + B

Description:

Computes the sum of two  ${\tt mfArrays}$  of same shape.

One of them may be a scalar of type real (single or double) or complex; in such a case, if the mfArray is not a scalar, then the operation is made on all elements.

Remarks: A and B may have a dense or sparse structure.

See also: -

# mfArray difference

 $Calling\ syntax:$ 

$$C = A - B$$

## Description:

Computes the difference of two  ${\tt mfArrays}$  of same shape.

One of them may be a scalar of type real (single or double) or complex; in such a case, if the mfArray is not a scalar, then the operation is made on all elements.

Remarks: A and B may have a dense or sparse structure.

See also: +

## mfMul, .x.

# mfArray product

Interface:

A .x. B is a shortcut for writing: mfMul(A, B)

Description:

Computes the matrix product A \* B (which is not commutative).

When the optional argument transp is present, it computes A' \* B (transp=1) or A \* B' (transp=2). This latter option is valid only for dense mfArrays.

The shape of the two matrices must respect the classical matrix-product rule, according to the value of transp.

Remarks: the element-wise operation is done with the operator '\*'.

This routine can be applied to dense or sparse mfArrays of any type (real or complex).

See also: mfCross

## mfCross

# cross-product of two vectors

Interface:

```
function mfCross( u, v ) result( out )

type(mfArray), intent(in) :: u, v
type(mfArray) :: out
```

Description:

Computes the cross-product of two vectors in  $\mathbb{R}^3$ .

 $See \ also: {\tt mfMul}$ 

## mfKron

# Kronecker tensor product

Interface:

```
function mfKron( A, B ) result( out )

type(mfArray), intent(in) :: A, B
type(mfArray) :: out
```

## Description:

Computes the Kronecker product of A and B. If the shape of A is (m, n) and the shape of B is (p, q), then the shape of the Kronecker product will be (m p, n q).

The  ${\tt mfArrays}\ {\tt A}$  and  ${\tt B}$  may be dense or sparse.

Remark: Currently, only real mfArrays are accepted.

See also: mfMul

# mfArray element-wise product

Calling syntax:

C = A \* B

## Description:

Computes the product (element-wise) of two mfArrays of same shape.

One of them may be a scalar of type real (single or double) or complex; in such a case, if the mfArray is not a scalar, then the operation is made on all elements.

Only for this routine, B may be of type mfUnit. This feature allows the user to change the physical unit of the mfArray A.

Remarks: A and B may have a dense or sparse structure.

See also: mfMul, /

# /

# mfArray element-wise quotient

 $Calling\ syntax:$ 

C = A / B

## Description:

Computes the division (element-wise) of two mfArrays of same shape.

One of them may be a scalar of type real (single or double) or complex; in such a case, if the mfArray is not a scalar, then the operation is made on all elements.

Remarks: A and B may have a dense or sparse structure.

See also: mfLDiv, mfRDiv, \*

# \*\* mfArray element-wise exponentiation

 $Calling\ syntax:$ 

C = A \*\* i

## Description:

Computes the power (element-wise) of the mfArray A to the exponent i, which must be a scalar, of type integer or real.

 ${\it Remarks:}$  A may have a dense or sparse structure.

# .t. mfArray transposition

Calling syntax:

C = .t. A

## Description:

The prefixed operator '.t.' transposes the mfArray A.

A warning is emitted if this routine is applied to a complex mfArray (in this case, the operator '.h.' should be used instead).

Remarks: A may have a dense or sparse structure. This operator can also be applied to integer or real arrays, of rank 1 or 2 (i. e. vector or matrix); in all cases it returns an mfArray.

## .h.

# mfArray conjugate transposition

Calling syntax:

C = .h. A

# Description:

The prefixed operator '.h.' returns the conjugate transpose of the  $mfArray\ A$ .

Remarks: A may have a dense or sparse structure. This operator should be applied to complex arrays, of rank 1 or 2 (i.e. vector or matrix); in all cases it returns an mfArray. If A is a real mfArray, it is converted in a complex mfArray.

See also: .t.

#### .vc.

# vertical concatenation

Calling syntax:

C = A .vc. B

Description:

Vertical concatenation of mfArrays (dense and/or sparse).

 ${\tt A}$  and  ${\tt B}$  must have the same number of columns.

One of the two  $\mathtt{mfArray}s$  can be empty: it enables the construction of a matrix inside a loop, from an empty one.

Remark: This operator also accepts usual Fortran 90 1-rank numeric array (i. e. either real or complex).

See also: .hc.

## .hc.

## horizontal concatenation

 $Calling\ syntax:$ 

C = A .hc. B

## Description:

Horizontal concatenation of mfArrays (dense and/or sparse).

A and B must have the same number of rows.

One of the two mfArrays can be empty: it enables the construction of a matrix inside a loop, from an empty one.

## Remarks:

- when working with sparse mfArrays, the subroutine msHorizConcat is much more efficient.
- this operator also accepts usual Fortran 90 1-rank numeric array (i. e. either real or complex).

See also: msHorizConcat, .vc.

#### msHorizConcat

## sparse matrix horizontal concatenation

## Interface:

```
type(mfArray), intent(in out) :: A
type(mfArray), intent(in) :: data
```

## Description:

This subroutine version of operator(.hc.) is intended to be more efficient, because: (i) modifications of A are made "in place" and (ii) allocation of space is anticipated and over-dimensioned.

A can be empty: it enables the construction of a matrix inside a loop, from an empty one.

The mfArrays A must be sparse; the mfArray data must be sparse if it is a matrix, but can be dense if it is a vector.

Restrictions: the two mfArrays must have the same data type.

Remarks: use operator(.hc.) instead of for dense matrices.

See also: .hc., .vc.

#### mf/msColPerm,

## columns permutation

First calling syntax:

```
B = mfColPerm(A, p)
```

## Description:

Returns the columns permutation of the mfArray A (may be of any type, dense or sparse), using the permutation p: the  $j^{th}$  column of B is the  $p(j)^{th}$  column of A.

In Matlab syntax or Fortran 90 notation, it computes B = A(:, p).

p is either an ordinary vector mfArray, containing the (integer) indices of the permutation, or a true permutation mfArray (see mfPerm).

The second calling syntax:

```
call msColPerm( A, p )
```

applies the same operation, but in-place.

Remark: This routine is much more efficient than computing a column's permutation by calling mfGet(A,MF\_ALL,p).

 $See \ also: \ {\tt mf/msRowPerm}, \ {\tt mf/msColScale}, \ {\tt mf/msRowScale}$ 

## mf/msRowPerm,

## rows permutation

First calling syntax:

```
B = mfRowPerm( A, p )
```

## Description:

Returns the rows permutation of the mfArray A (may be of any type, dense or sparse), using the permutation p: the  $i^{th}$  row of B is the  $p(i)^{th}$  row of A.

In Matlab syntax or Fortran 90 notation, it computes B = A(p, :).

p is either an ordinary vector mfArray, containing the (integer) indices of the permutation, or a true permutation mfArray (see mfPerm).

The second calling syntax:

```
call msRowPerm( A, p )
```

applies the same operation, but in-place.

Remark: This routine is much more efficient than computing a row's permutation by calling mfGet(A,p,MF\_ALL).

 $See \ also: \ {\tt mf/msColPerm}, \ {\tt mf/msColScale}, \ {\tt mf/msRowScale}$ 

# mfInvPerm,

# inverse of a permutation

Calling syntax:

p\_inv = mfInvPerm( p )

Description:

Inverts the permutation  ${\tt p}.$ 

 $See~also: \verb|mf/msColPerm|, \verb|mf/msRowPerm||$ 

# mf/msColScale,

# columns scaling

First calling syntax:

```
B = mfColScale( A, s )
```

Description:

Returns the columns scaling of the mfArray A (must be numeric, dense or sparse), using the dense vector s: the  $j^{th}$  column of B is the  $j^{th}$  column of A multiplied by s(j).

In Matlab syntax, it computes B = A \* diag(s).

Restrictions: the vector **s** must be a real **mfArray**.

The second calling syntax:

```
call msColScale( A, s )
```

applies the same operation, but in-place.

See also: mf/msColPerm, mf/msRowPerm, mf/msRowScale

mf/msRowScale, rows scaling

 $First\ calling\ syntax:$ 

```
B = mfRowScale( A, s )
```

Description:

Returns the rows scaling of the mfArray A (must be numeric, dense or sparse), using the dense vector s: the  $i^{th}$  row of B is the  $i^{th}$  row of A multiplied by s(i).

In Matlab syntax, it computes B = diag(s) \* A.

Restrictions: the vector **s** must be a real **mfArray**.

The second calling syntax:

```
call msRowScale( A, s )
```

applies the same operation, but in-place.

 $See \ also: \ {\tt mf/msColPerm}, \ {\tt mf/msRowPerm}, \ {\tt mf/msColScale}$ 

## msColAutoScale,

# columns auto scaling

 $Calling\ syntax:$ 

call msColAutoScale( mfOut(s), A )

scales the columns of the  $\mathtt{mfArray}\ \mathtt{A}$  (must be numeric, dense or sparse), in such a way that the maximum magnitude of each column is equal to 1.

It ouputs the scaling row vector  $\mathbf{s}$ , which is a real mfArray.

 $See \ also: \verb|msRowAutoScale|, \verb|mf/msColScale|, \verb|mf/msRowScale||$ 

#### msRowAutoScale,

### rows auto scaling

Calling syntax:

call msRowAutoScale( mfOut(s), A )

scales the rows of the mfArray A (must be numeric, dense or sparse), in such a way that the maximum magnitude of each row is equal to 1.

It outust the scaling column vector  $\mathbf{s}$ , which is a real  $\mathtt{mfArray}$ .

 $See \ also: \ {\tt msColAutoScale}, \ {\tt mf/msColScale}, \ {\tt mf/msRowScale}$ 

#### >=

# mfArray relational operator

Calling syntax:

$$C = A >= B$$

#### Description:

Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.

A and B must have the same shape and must be, of course, of type real.

B may be a scalar, real or even integer.

Remarks: a boolean mfArray contains TRUE and FALSE values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '.and.', '.or.', etc.).

See also: >, <=, <, ==, /=

#### >

# mfArray relational operator

 $Calling\ syntax:$ 

$$C = A > B$$

#### Description:

Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.

A and B must have the same shape and must be, of course, of type real.

B may be a scalar, real or even integer.

Remarks: a boolean mfArray contains TRUE and FALSE values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '.and.', '.or.', etc.).

See also: >=, <=, <, ==, /=

#### <=

# mfArray relational operator

 $Calling\ syntax:$ 

$$C = A \le B$$

#### Description:

Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.

A and B must have the same shape and must be, of course, of type real.

B may be a scalar, real or even integer.

Remarks: a boolean mfArray contains TRUE and FALSE values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '.and.', '.or.', etc.).

See also: <, >=, >, ==, /=

#### <

# mfArray relational operator

 $Calling\ syntax:$ 

$$C = A < B$$

#### Description:

Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.

A and B must have the same shape and must be, of course, of type real.

B may be a scalar, real or even integer.

Remarks: a boolean mfArray contains TRUE and FALSE values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '.and.', '.or.', etc.).

See also: <=, >=, >, ==, /=

#### ==

# mfArray relational operator

 $Calling\ syntax:$ 

$$C = A == B$$

#### Description:

Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.

A and B must have the same shape and must be, of course, of type real.

B may be a scalar, real or even integer.

Remarks: a boolean mfArray contains TRUE and FALSE values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '.and.', '.or.', etc.).

See also: /=, mfIsEqual, <=, <, >=, >

### /=

# mfArray relational operator

Calling syntax:

$$C = A /= B$$

#### Description:

Returns a boolean mfArray from two dense mfArrays, by making the comparison element-wise.

A and B must have the same shape and must be, of course, of type real.

B may be a scalar, real or even integer.

Remarks: a boolean mfArray contains TRUE and FALSE values only. It can be displayed as any mfArray and can occur in logical operators ('.not.', '.and.', '.or.', etc.).

See also: ==, mfIsEqual, <=, <, >=, >

#### mfAll

# mfArray boolean operator

#### Interface:

```
function mfAll( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

#### Description:

If A is a vector mfArray, returns a scalar boolean mfArray which is TRUE (i. e. 1) if all elements are TRUE.

If A is a matrix, the routine works down the columns, returning a row vector mfArray.

If dim is present, it indicates on which dimension the routine must be applied.

See also: All, mfAny

#### mfAny

# mfArray boolean operator

#### Interface:

```
function mfAny( A, dim ) result( out )

type(mfArray), intent(in) :: A
integer, intent(in), optional :: dim
type(mfArray) :: out
```

#### Description:

If A is a vector mfArray, returns a scalar boolean mfArray which is TRUE (i. e. 1) if at least one element is TRUE.

If A is a matrix, the routine works down the columns, returning a row vector mfArray.

If dim is present, it indicates on which dimension the routine must be applied.

See also: Any, mfAll

#### mfColon

#### mfArray sequence constructor

 ${\it Interface:}$ 

```
A = mfColon( start, end [, step, tol ] )
```

Description:

Returns an mfArray containing a sequence: a list of reals equally spaced, from start to end, with the specified step.

By default (i. e. when step is not present), increment is unity.

Arguments start, end and step may be of type integer or real (single or double precision), but all of same kind and precision.

Because of the rounding errors, the last number of the sequence seldom matches to the specified value of end, especially when step is not an integer. To avoid this unrequired behavior, the tolerance tol can be used to round the resulting numbers in the sequence to a relative precision equal to tol. Therefore, the tolerance tol should be present only in the case where the first three arguments are reals, all of them having the same precision; moreover, it should be a negative power of ten, e. g. 0.001.

When step is not present, the value of end must be greater than that of start. When step is present, its sign must be the same as that of end-start. step cannot have a zero value.

See also: mfLinSpace, mfLogSpace

```
Example(s):
   call msDisplay( mfColon(1,15,step=3), "mfColon(1,15,step=3)" )
output:
mfColon(1,15,step=3) =
     1
           4
                 7
                      10
                            13
   call msDisplay( mfColon(1.0,3.0,step=0.3333,tol=0.001),
                  "mfColon(1.0,3.0,step=0.3333,tol=0.001)" )
output:
mfColon(1.0,3.0,step=0.3333,tol=0.001) =
    1.0000
              1.3330
                        1.6670
                                   2.0000
                                             2.3330
                                                       2.6670
                                                                  3.0000
```

#### .not.

# mfArray logical operator

 $Calling\ syntax:$ 

C = .not. A

#### Description:

Takes the negation (element-wise) of a boolean mfArray.

Remarks: a boolean mfArray is of type real and contains 1 for emphTRUE and 0 for FALSE. It can be displayed as any mfArray; it is usually built from relational operators ('==', '/=', '>', etc.).

See also: .and., .or., .eqv., .neqv.

#### .and.

# mfArray logical operator

 $Calling\ syntax:$ 

C = A .and. B

Description:

Composes (element-wise) two boolean  ${\tt mfArrays}.$ 

Remarks: a boolean mfArray is of type real and contains 1 for emphTRUE and 0 for FALSE. It can be displayed as any mfArray; it is usually built from relational operators ('==', '/=', '>', etc.).

See also: .not., .or., .eqv., .neqv.

#### .or.

# mfArray logical operator

 $Calling\ syntax:$ 

C = A .or. B

#### Description:

Composes (element-wise) two boolean  ${\tt mfArrays}.$ 

Remarks: a boolean mfArray is of type real and contains 1 for emphTRUE and 0 for FALSE. It can be displayed as any mfArray; it is usually built from relational operators ('==', '/=', '>', etc.).

See also: .not., .and., .eqv., .neqv.

### .eqv.

# mfArray logical operator

 $Calling\ syntax:$ 

$$C = A .eqv. B$$

Description:

Composes (element-wise) two boolean  ${\tt mfArrays}.$ 

Remarks: a boolean mfArray is of type real and contains 1 for emphTRUE and 0 for FALSE. It can be displayed as any mfArray; it is usually built from relational operators ('==', '/=', '>', etc.).

See also: .not., .and., .or., .neqv.

### .neqv.

# mfArray logical operator

Calling syntax:

C = A .neqv. B

Description:

Composes (element-wise) two boolean  ${\tt mfArrays}.$ 

Remarks: a boolean mfArray is of type real and contains 1 for emphTRUE and 0 for FALSE. It can be displayed as any mfArray; it is usually built from relational operators ('==', '/=', '>', etc.).

See also: .not., .and., .or., .eqv.

#### mfIsMember

#### elements members of a set

Calling syntax:

```
E = mfIsMember( A, set [, tol ] )
```

Description:

Returns a boolean mfArray which shows which elements of the mfArray A are member of the mfArray set.

E has the same shape as A.

The tol optional argument is the tolerance used in the comparison test. By default, tolerance is zero, i.e. a strict equality is used to discard elements.

Remark: mfArrays A and set must be real with a dense structure.

See also: mfIntersect, mfUnion, mfUnique

```
Example(s):
```

F

Т

T

F

F

Т

```
a = mf([1, 2, 3]) .vc. mf([4, 5, 6])
   set = [ 0, 12, 8, 6, 2, 10, 4 ]
   call msDisplay( a, "a", set, "set" )
   call msDisplay( mfIsMember(a,set), "mfIsMember(a,set)" )
output:
 a =
     1
           2
                 3
     4
           5
 set =
          12
                             2
     0
                 8
                       6
                                  10
                                         4
mfIsMember(a,set) =
```

#### mfIntersect

#### common elements of two sets

Calling syntax:

```
E = mfIntersect( A, B [, tol ] )
```

#### Description:

Returns an mfArray containing all elements common to the mfArrays A and B. Duplicated elements are discarded; the result is a sorted row vector.

The tol optional argument is the tolerance used in the equality test to discard duplicated elements, if any. By default, tolerance is zero, i. e. a strict equality is used to discard elements.

Remark: mfArrays A and B must be both real with a dense structure.

See also: mfIsMember, mfUnion, mfUnique

```
Example(s):
```

```
A = reshape([9, 7, 20, 4, 1, 5, 9, 7, 21, 1, 5, 25], [3, 4])
B = reshape([25, 15, 8, 8, 6, 5, 5, 4, 20, 10], [5, 2])
call msDisplay( A, "A", B, "B" )
call msDisplay( mfIntersect(A,B), "mfIntersect(A,B)" )
```

```
output:
 A =
     9
            4
                   9
                         1
     7
                   7
            1
                         5
    20
            5
                  21
                         25
 B =
    25
            5
            5
    15
     8
            4
     8
           20
     6
           10
 mfIntersect(A,B) =
```

5

20

25

4

#### mfUnion

#### all elements of two sets

Calling syntax:

```
E = mfUnion( A, B [, tol ] )
```

#### Description:

Returns an mfArray containing all elements of to the mfArrays A and B. Duplicated elements are discarded; the result is a sorted row vector.

The tol optional argument is the tolerance used in the equality test to discard duplicated elements, if any. By default, tolerance is zero, *i. e.* a strict equality is used to discard elements.

Remark: mfArrays A and B must be both real with a dense structure.

See also: mfIsMember, mfIntersect, mfUnique

```
Example(s):
```

```
A = reshape([9, 7, 20, 4, 1, 5, 9, 7, 21, 1, 5, 25], [3, 4])
B = reshape([25, 15, 8, 8, 6, 5, 5, 4, 20, 10], [5, 2])
call msDisplay(A, "A", B, "B")
call msDisplay(mfIntersect(A,B), "mfIntersect(A,B)")
```

# output: A =

```
9 4 9 1
7 1 7 5
```

20 5 21 25

B =

25 5 15 5 8 4

8 20 6 10

mfIntersect(A,B) =

1 4 5 6 7 8 9 10 15 20 21 25

#### mfUnique

#### unique elements of a set

Interface:

```
function mfUnique( A, order, occurrence, tol ) result( out )

type(mfArray) :: A
    character(len=*), intent(in), optional :: order, occurrence
    real(kind=MF_DOUBLE), intent(in), optional :: tol
    type(mfArray) :: out
```

#### Description:

Returns the unique elements of the mfArray A. A must be a vector (either row or column).

By default, the returned elements are sorted in increasing order. However, their layout may be changed by using the following optional parameters:

- the order optional argument (= "asc", "des" or "no") may be used to specify the ordering.
- if order="no", no ordering is applied to the resulting elements but it is required to add the other occurrence optional argument (value may be "first" or "last") to specify if the selected element is the first one or the last one.

The tol optional argument is the tolerance used in the equality test to discard duplicated elements, if any. By default, tolerance is zero, i.e. a strict equality is used to discard elements.

Remark: the mfArray A must be real with a dense structure.

See also: mfIsMember, mfUnion, mfIntersect

```
Example(s):
```

```
a = [2, 4, 6, 8, 9, 8, 7, 1, 5, 3, 6, 2, 4]
   call msDisplay( a, "a" )
   call msDisplay( mfUnique(a), "mfUnique(a)" )
   call msDisplay( mfUnique(a,order="des"), 'mfUnique(a,order="des")' )
   call msDisplay( mfUnique(a,order="no",occurrence="first"),
                  'mfUnique(a,order="no",occurrence="first")' )
   call msDisplay( mfUnique(a,order="no",occurrence="last"),
                                                                           &
                   'mfUnique(a,order="no",occurrence="last")' )
output:
 a =
     2
                 6
                        8
                              9
                                    8
                                          7
                                                 1
                                                       5
                                                             3
                                                                    6
                                                                          2
                                                                                4
mfUnique(a) =
                                          7
     1
           2
                 3
                        4
                              5
                                    6
                                                 8
                                                       9
mfUnique(a,order="des") =
     9
           8
                 7
                        6
                              5
                                    4
                                           3
                                                 2
                                                       1
```

 $\cdots$ /...

mfUnique(a,order="no",occurrence="first") =

2 4 6 8 9 7 1 5 3

mfUnique(a,order="no",occurrence="last") =

9 8 7 1 5 3 6 2 4

#### 1.5 Elementary Math Functions

mfACos inverse cosine

mfACosh inverse hyperbolic cosine

mfACot inverse cotangent

mfACoth inverse hyperbolic cotangent

mfACsc inverse cosecant

mfACsch inverse hyperbolic cosecant

mfASec inverse secant

mfASech inverse hyperbolic secant

mfASin inverse sine

mfASinh inverse hyperbolic sine

mfATan inverse tangent

mfATan2 four quadrant inverse tangent inverse hyperbolic tangent

mfCos cosine

mfCosh hyperbolic cosine

mfCot cotangent

mfCoth hyperbolic cotangent

mfCsc cosecant

mfCsch hyperbolic cosecant

mfSec secant

mfSech hyperbolic secant

mfSin sine

mfSinh hyperbolic sine

mfTan tangent

mfTanh hyperbolic tangent

mfSqrt square root mfExp exponential

mfExpm1 exponential minus one mfLog natural logarithm

mfLog1p natural logarithm of 1 + x

mfLog10 base 10 logarithm
mf/msLog2 base 2 logarithm
mfPow10 base 10 power
mfPow2 base 2 power

mfFun, mfFun2 general function (one or two arguments)

mfGridFun general function (two arguments) applied to coords matrices

mfAbs absolute value mfAngle phase angle

mfComplexcomplex conversionmfConjcomplex conjugatemfImagcomplex imaginary partmfRealcomplex real part

mfHypot hypotenuse

mfRound round towards nearest integer round towards plus infinity

mfFix round towards zero

mfFloor round towards minus infinity
mfMod modulus after division
mfRem remainder after division

mfSign signum function

mfCSign complex signum function

#### See also:

Core Routines

File Input/Output

Data Analysis Functions

Operators

Specialized Math Functions

Elementary Matrix Manipulation Functions

Matrix Functions

Polynomial Functions

Optimization and Function Functions

Sparse Matrices

mfACos inverse cosine

Calling syntax:

C = mfACos(A)

Description:

Returns the inverse cosine (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfCos},~{\tt mfACosh},~{\tt mfCosh}$ 

#### mfACosh

# inverse hyperbolic cosine

 $Calling\ syntax:$ 

C = mfACosh( A )

Description:

Returns the inverse hyperbolic cosine (element-wise) of the  ${\tt mfArray}\ {\tt A}$  (numerical dense only).

 $See~also:~{\tt mfCos},~{\tt mfACos},~{\tt mfCosh}$ 

#### ${\tt mfACot}$

# inverse cotangent

Calling syntax:

C = mfACot(A)

Description:

Returns the inverse cotangent (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfCot},~{\tt mfACoth},~{\tt mfCoth}$ 

#### mfACoth

# $inverse\ hyperbolic\ cotangent$

 $Calling\ syntax:$ 

C = mfACoth( A )

Description:

Returns the inverse hyperbolic cotangent (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See \ also: \ {\tt mfCot}, \ {\tt mfACot}, \ {\tt mfCoth}$ 

mfACsc inverse cosecant

Calling syntax:

C = mfACsc(A)

Description:

Returns the inverse cosecant (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfCsc},~{\tt mfACsch},~{\tt mfCsch}$ 

#### mfACsch

# inverse hyperbolic cosecant

 $Calling\ syntax:$ 

C = mfACsch( A )

Description:

Returns the inverse hyperbolic cosecant (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfCsc},~{\tt mfACsc},~{\tt mfCsch}$ 

mfASec inverse secant

Calling syntax:

C = mfASec(A)

Description:

Returns the inverse secant (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See \ also: \ {\tt mfSec}, \ {\tt mfASech}, \ {\tt mfSech}$ 

### mfASech

# inverse hyperbolic secant

 $Calling\ syntax:$ 

C = mfASech(A)

Description:

Returns the inverse hyperbolic secant (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfSec},~{\tt mfASec},~{\tt mfSech}$ 

mfASin inverse sine

Calling syntax:

C = mfASin( A )

Description:

Returns the inverse sine (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfSin},~{\tt mfASinh},~{\tt mfSinh}$ 

### mfASinh

# inverse hyperbolic sine

 $Calling\ syntax:$ 

C = mfASinh( A )

Description:

Returns the inverse hyperbolic sine (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See \ also: \ {\tt mfSin}, \ {\tt mfASin}, \ {\tt mfSinh}$ 

mfATan inverse tangent

 $Calling\ syntax:$ 

C = mfATan(A)

Description:

Returns the inverse tangent (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfATan2},~{\tt mfTan},~{\tt mfATanh},~{\tt mfTanh}$ 

#### mfATan2

# four quadrant inverse tangent

Calling syntax:

C = mfATan2(Y, X)

Description:

Returns the four quadrant inverse tangent (element-wise) of the quotient Y/X, and the result is in  $(-\pi, \pi]$ .

 ${\tt X}$  and  ${\tt Y}$  must be numerical, dense  ${\tt mfArrays}.$ 

See also: mfATan, mfTan, mfATanh, mfTanh

#### mfATanh

# inverse hyperbolic tangent

 $Calling\ syntax:$ 

C = mfATanh(A)

Description:

Returns the inverse hyperbolic tangent (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfTan},~{\tt mfATan},~{\tt mfTanh}$ 

mfCos

Calling syntax:

C = mfCos( A )

Description:

Returns the cosine (element-wise) of the  ${\tt mfArray}\ {\tt A}$  (numerical dense only).

 $See \ also: \ {\tt mfACosh}, \ {\tt mfCosh}$ 

#### mfCosh

## hyperbolic cosine

Calling syntax:

C = mfCosh( A )

Description:

Returns the hyperbolic cosine (element-wise) of the  ${\tt mfArray}\ {\tt A}$  (numerical dense only).

 $See~also:~{\tt mfCos},~{\tt mfACos},~{\tt mfACosh}$ 

mfCot cotangent

Calling syntax:

C = mfCot(A)

Description:

Returns the cotangent (element-wise) of the  ${\tt mfArray}\ {\tt A}$  (numerical dense only).

 $See \ also: \ {\tt mfACoth}, \ {\tt mfCoth}$ 

#### mfCoth

## hyperbolic cotangent

Calling syntax:

C = mfCoth(A)

Description:

Returns the hyperbolic cotangent (element-wise) of the mfArray A (numerical dense only).

 $See~also:~{\tt mfCot},~{\tt mfACot},~{\tt mfACoth}$ 

mfCsc cosecant

Calling syntax:

C = mfCsc(A)

Description:

Returns the cosecant (element-wise) of the  ${\tt mfArray}\ {\tt A}$  (numerical dense only).

 $See \ also: \ {\tt mfACsc}, \ {\tt mfACsch}, \ {\tt mfCsch}$ 

#### mfCsch

## hyperbolic cosecant

Calling syntax:

C = mfCsch( A )

Description:

Returns the hyperbolic cosecant (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfCsc},~{\tt mfACsc},~{\tt mfACsch}$ 

mfSec secant

Calling syntax:

C = mfSec(A)

Description:

Returns the secant (element-wise) of the  ${\tt mfArray}\ {\tt A}$  (numerical dense only).

 $See \ also: \ {\tt mfASec}, \ {\tt mfASech}, \ {\tt mfSech}$ 

#### mfSech

## hyperbolic secant

Calling syntax:

C = mfSech( A )

Description:

Returns the hyperbolic secant (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfASech},~{\tt mfASec},~{\tt mfSec}$ 

mfSin

Calling syntax:

C = mfSin(A)

Description:

Returns the sine (element-wise) of the  ${\tt mfArray}\ {\tt A}$  (numerical dense only).

 $See \ also: \ {\tt mfASin}, \ {\tt mfASinh}, \ {\tt mfSinh}$ 

## mfSinh hyperbolic sine

Calling syntax:

C = mfSinh( A )

Description:

Returns the hyperbolic sine (element-wise) of the  ${\tt mfArray}\ {\tt A}$  (numerical dense only).

 $See~also:~{\tt mfASin},~{\tt mfASinh},~{\tt mfSin}$ 

mfTan tangent

Calling syntax:

C = mfTan(A)

Description:

Returns the tangent (element-wise) of the  ${\tt mfArray}\ {\tt A}$  (numerical dense only).

 $See \ also: \ {\tt mfATanh}, \ {\tt mfATanh}, \ {\tt mfTanh}$ 

#### mfTanh

## hyperbolic tangent

Calling syntax:

C = mfTanh(A)

Description:

Returns the hyperbolic tangent (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also:~{\tt mfATan},~{\tt mfATanh},~{\tt mfTan}$ 

mfSqrt square root

 $Calling\ syntax:$ 

Description:

Returns the square root (element-wise) of the  ${\tt mfArray}$  A (dense or sparse, real or complex).

The returned mfArray may be of type complex if any element has a negative value.

See also: mfSqrtm

mfExp exponential

Calling syntax:

C = mfExp(A)

Description:

Returns the exponential (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See \ also: \ {\tt mfLog}, \ {\tt mfLog1p}, \ {\tt mfExpm1}, \ {\tt mfExpm}$ 

### mfExpm1

### exponential minus one

 $Calling\ syntax:$ 

C = mfExpm1(A)

Description:

Returns the value of  $\exp(x)-1$ , for each x element of the mfArray A. This operation is made element-wise.

 $See~also:~{\tt mfExp},~{\tt mfLog},~{\tt mfLog1p},~{\tt mfExpm}$ 

### mfLog

# $natural\ logarithm$

Calling syntax:

C = mfLog(A)

Description:

Returns the natural logarithm (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also: {\tt mfLog1p}, {\tt mfExp}, {\tt mfExpm1}, {\tt mfLogm}$ 

### mfLog1p

### natural logarithm of 1 + x

 $Calling\ syntax:$ 

C = mfLog1p(A)

#### Description:

Returns the value of  $\log(1+x)$ , for each x element of the mfArray A (numerical dense only). This operation is made element-wise.

 $See~also:~{\tt mfLog},~{\tt mfExp},~{\tt mfExpm1},~{\tt mfExpm}$ 

 ${\tt mfLog10}$ 

## base 10 logarithm

Calling syntax:

C = mfLog10(A)

Description:

Returns the base 10 logarithm (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also: {\tt mfLog}, {\tt mf/msLog2}$ 

mf/msLog2 base 2 logarithm

Calling syntax:

$$C = mfLog2(A)$$

Description:

Returns the base 2 logarithm (element-wise) of the mfArray A (numerical dense only).

The subroutine version returns two  ${\tt mfArrays}$ :

```
call msLog2( mfOut(M,E), A )
```

The mantissa  ${\tt M}$  has its element ranged in [0.5,1[, whereas elements of  ${\tt E}$  are integer exponents.

Each element of A verifies:  $a=m2^e$ 

See also: mfPow2, mfLog, mfLog10, mfOut

mfPow10 base 10 power

Calling syntax:

C = mfPow10(A)

Description:

Returns the base 10 power (element-wise) of the  ${\tt mfArray}$  A (numerical dense only).

 $See~also: {\tt mfLog10}$ 

mfPow2 base 2 power

First usage:

```
C = mfPow2(A)
```

Description:

Returns the base 2 power (element-wise) of the mfArray A (numerical dense only).

 $Second\ usage:$ 

```
C = mfPow2(M, E)
```

Description:

Returns numbers which write:  $m2^e$ , for each (real) m of M and each (integer) e of E. M and E are two mfArrays.

See also: mfLog2

#### mfFun

#### general function (one arg.)

```
Usage:
    C = mfFun( A, fun )

Description:
Apply the user-defined function (element-wise) fun to the mfArray A (numerical dense only).
fun is a real or complex function having the following interface:
    real(kind=MF_DOUBLE) function fun( x1 )
        real(kind=MF_DOUBLE), intent(in) :: x1
    end function

or
    complex(kind=MF_DOUBLE) function fun( x1 )
        complex(kind=MF_DOUBLE), intent(in) :: x1
    end function
```

See also: mfFunm, mfFun2

#### mfFun2

### general function (two args)

```
Usage:
```

```
C = mfFun2( A, B, fun )
```

#### Description:

Apply the user-defined function (element-wise) fun to the mfArrays A and B (numerical dense only).

fun is a real or complex function having the following interface:

```
real(kind=MF_DOUBLE) function fun( x1, x2 )
    real(kind=MF_DOUBLE), intent(in) :: x1, x2
end function
or
    complex(kind=MF_DOUBLE) function fun( x1, x2 )
        complex(kind=MF_DOUBLE), intent(in) :: x1, x2
end function
```

See also: mfFun, mfGridFun

#### mfGridFun

#### general function (two args) applied to coords matrices

Usage:

```
Z = mfGridFun(v_x, v_y, fun)
```

applies the 2-argument function fun to the couple of coordinate matrices (X, Y) generated by the vectors  $v_x$  and  $v_y$ , without building X and Y (see msMeshGrid about contraints concerning  $v_x$  and  $v_y$ ). It aims at economize memory for large grids.

fun is either a character string giving the name of a 2-arg function (as *complex*, *hypot*, *atan2*, ...), or the name of a user defined function. In this latter case, the user function must have the following interface:

```
real(kind=MF_DOUBLE) function fun( x1, x2 )
    real(kind=MF_DOUBLE), intent(in) :: x1, x2
end function
```

See also: mfFun2

mfAbs absolute value

Calling syntax:

C = mfAbs(A)

Description:

Returns the absolute value (or the module for complex numbers) of the  $mfArray\ A$  (dense or sparse). This operation is made element-wise.

See also: mfAngle

mfAngle phase angle

Calling syntax:

C = mfAngle( A )

#### Description:

Returns the phase angle (element-wise), in radians of the complex mfArray A (dense or sparse).

For real values, this function returns 0 or  $\pi$ . Moreover, it returns 0 when argument is null (real or complex), so maintains the sparsity of the mfArray.

See also: mfAbs

#### mfComplex

#### complex conversion

 $Calling\ syntax:$ 

```
C = mfComplex( A [, B] )
```

Description:

Converts a real mfArray A (dense or sparse) in a complex one.

If the optional argument mfArray B is present, builds a complex mfArray whose real part is A and the imaginary part is B. In this latter case, A and B must be both real (dense or sparse).

See also: mfReal, mfImag

### ${\tt mfConj}$

# ${\bf complex}\ {\bf conjugate}$

 $Calling\ syntax:$ 

C = mfConj(A)

Description:

Returns the conjugate (element-wise) of the complex  ${\tt mfArray}\ {\tt A}$  (dense or sparse).

 ${\tt A}$  may be real, but the returned  ${\tt mfArray}$  is complex.

See also: .h.

### ${\tt mfImag}$

## complex imaginary part

Calling syntax:

C = mfImag( A )

Description:

Returns the imaginary part (element-wise) of the complex  ${\tt mfArray}$  A (dense or sparse).

See also: mfReal

mfReal

## complex real part

Calling syntax:

C = mfReal(A)

Description:

Returns the real part (element-wise) of the complex  ${\tt mfArray}$  A (dense or sparse).

 $See \ also: {\tt mfImag}$ 

mfHypot hypotenuse

 $Calling\ syntax:$ 

C = mfHypot( A, B )

#### Description:

Returns the hypotenuse of a right rectangle whose sides  $(mfArrays\ A\ and\ B,$  in dense format) are provided. This operation is made element-wise.

 ${\it Remarks:}$  the computation is done without overflow or underflow.

#### mfRound

### round towards nearest integer

 $Calling\ syntax:$ 

C = mfRound( A )

#### Description:

Rounds towards nearest integer each element of the  $\mathtt{mfArray}$  A (dense or sparse). The behavior of this routine is similar to the anint Fortran 90 intrinsic function.

See also: mfCeil, mfFix, mfFloor

#### mfCeil

## round towards plus infinity

 $Calling\ syntax:$ 

C = mfCeil(A)

Description:

Rounds towards plus infinity each element of the  ${\tt mfArray}$  A (dense or sparse).

 $See \ also: \ {\tt mfFix}, \ {\tt mfFloor}, \ {\tt mfRound}$ 

#### mfFix

#### round towards zero

Calling syntax:

C = mfFix(A)

Description:

Rounds towards zero each element of the  ${\tt mfArray}$  A (dense or sparse).

 $See \ also: \ {\tt mfCeil}, \ {\tt mfFloor}, \ {\tt mfRound}$ 

#### mfFloor

## round towards minus infinity

 $Calling\ syntax:$ 

C = mfFloor( A )

Description:

Rounds towards minus infinity each element of the  ${\tt mfArray}\ {\tt A}$  (dense or sparse).

 $See \ also: \ {\tt mfCeil}, \ {\tt mfFix}, \ {\tt mfRound}$ 

#### mfMod

#### modulus after division

Calling syntax:

C = mfMod(A, B)

Description:

Returns the modulus after division of each element of the  ${\tt mfArray}$  A (numerical dense only).

N.B.: this is the Fortran 90 modulo function

See also: mfRem

mfRem

## remainder after division

 $Calling\ syntax:$ 

C = mfRem( A, B )

Description:

Returns the modulus after division of each element of the  ${\tt mfArray}$  A (numerical dense only).

N.B.: this is the Fortran 90 mod function

See also: mfMod

mfSign signum function

 $Calling\ syntax:$ 

C = mfSign( A )

Description:

Applies the signum function to each element of the mfArray A (numerical dense only). It returns only 1 or -1.

 $\it N.B.:$  It differs from the Fortran 90 sign intrinsic function.

 $See \ also: {\tt mfCSign}$ 

# ${\tt mfCSign}$

# complex signum function

Calling syntax:

C = mfCSign( A )

Description:

Applies the complex signum function to each element of the  ${\tt mfArray}$  A (numerical dense only).

 $See \ also: {\tt mfSign}$ 

## 1.6 Specialized Math Functions

mfErf Error function

mfErfInv inverse Error function

mfErfC Complementary Error function

mfErfCInv inverse Complementary Error function mfErfCScaled Scaled Complementary Error function

mfExpInt Exponential Integral function

mfGamma Gamma function

mfGammaLn Logarithm of the Gamma function

mfBesselJ Bessel functions of the first kind
mfBesselY Bessel functions of the second kind

mfBesselI modified Bessel functions of the first kind mfBesselK modified Bessel functions of the second kind

mfAiry Airy function

msEllipKE complete elliptic integrals of first and second kinds

mfIsPrime prime number test

mfFactor prime numbers factorization
msCleanPrimeNumbers prime numbers storage cleaning

msRat rational approximation

#### See also:

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mfErf Error function

Calling syntax:

C = mfErf( A )

Description:

Returns the Error function (element-wise) of the  ${\tt mfArray}$  A (dense only, real or complex).

 $See \ also: \ \mathtt{mfErfC}, \ \mathtt{mfErfInv}, \ \mathtt{mfErfCInv}, \ \mathtt{mfErfCScaled}$ 

#### mfErfInv

## inverse Error function

 $Calling\ syntax:$ 

C = mfErfInv( A )

Description:

Returns the inverse Error function (element-wise) of the  ${\tt mfArray}$  A (dense only).

Restriction: A must be of type real.

 $See\ also:$  mfErf, mfErfC, mfErfCInv, mfErfCScaled

## ${\tt mfErfC}$

# Complementary Error function

 $Calling\ syntax:$ 

C = mfErfC( A )

Description:

Returns the Complementary Error function (element-wise) of the mfArray A (dense only, real or complex).

 $See \ also: \ \mathtt{mfErfInv}, \ \mathtt{mfErfCInv}, \ \mathtt{mfErfCScaled}$ 

## mfErfCInv

# inverse Complementary Error function

 $Calling\ syntax:$ 

C = mfErfCInv( A )

Description:

Returns the inverse Complementary Error function (element-wise) of the mfArray A (dense only).

Restriction: A must be of type real.

See also: mfErf, mfErfInv, mfErfC, mfErfCScaled

#### mfErfCScaled

# **Scaled Complementary Error function**

Calling syntax:

C = mfErfCScaled( A )

Description:

Returns the Scaled Complementary Error function (element-wise) of the mfArray A (dense only), defined by mfErfCScaled(x) = exp(x\*\*2) \* mfErfC(x)

Restriction: A must be of type real.

See also: mfErf, mfErfInv, mfErfC, mfErfCInv

## mfExpInt

## **Exponential Integral function**

Calling syntax:

Description:

Returns the exponential integral function (element-wise) of the mfArray A (dense only).

This function is mathematically defined as:

$$E_1(x) = \int_x^\infty \frac{\exp(-t)}{t} dt$$

For positive value of x, it has the following properties:

$$E_1(0) = +\infty, \quad E_1(+\infty) = 0$$

Restriction: A must be of type real and must have a positive value. On the contrary, a warning is emitted.

mfGamma Gamma function

 $Calling\ syntax:$ 

C = mfGamma(A)

Description:

Returns the gamma function (element-wise) of the  ${\tt mfArray}\ {\tt A}$  (dense only).

Restriction: A must be of type real.

See also: mfGammaLn

#### mfGammaLn

# Logarithm of the Gamma function

 $Calling\ syntax:$ 

C = mfGammaLn( A )

Description:

Returns the natural logarithm of the gamma function (element-wise) of the mfArray A (dense only).

Restriction: A must contain positive real values.

See also: mfGamma

#### mfBesselJ

## Bessel functions of the first kind

Calling syntax:

C = mfBesselJ( alpha, A )

## Description:

Returns the Bessel functions of the first kind  $J_{\alpha}(x)$  (element-wise) of the mfArray A (dense only).

The real alpha order doesn't need to be an integer: both positive and negative fractional orders are possible.

Restriction: A must be of type real but, according to the values of  $\alpha$  and x, a complex mfArray may be returned.

Remark: Some roots of  $J_0(x)$  and  $J_1(x)$  are stored respectively in the arrays: MF\_BESSEL\_J0\_ROOTS and MF\_BESSEL\_J1\_ROOTS.

See also: mfBesselY, mfBesselI, mfBesselK, Mf\_BESSEL\_J0\_ROOTS, Mf\_BESSEL\_J1\_ROOTS

#### mfBesselY

## Bessel functions of the second kind

Calling syntax:

C = mfBesselY( alpha, A )

## Description:

Returns the Bessel functions of the second kind  $Y_{\alpha}(x)$  (element-wise) of the mfArray A (dense only).

The real alpha order doesn't need to be an integer: both positive and negative fractional orders are possible.

Restriction: A must be of type real but, according to the values of  $\alpha$  and x, a complex mfArray may be returned.

See also: mfBesselJ, mfBesselK

#### mfBesselI

## modified Bessel functions of the first kind

 $Calling\ syntax:$ 

C = mfBesselI( alpha, A )

Description:

Returns the modified Bessel functions of the first kind  $I_{\alpha}(x)$  (element-wise) of the mfArray A (dense only).

The real alpha order doesn't need to be an integer: both positive and negative fractional orders are possible.

Restriction: A must be of type real but, according to the values of  $\alpha$  and x, a complex mfArray may be returned.

 $See\ also:$  mfBesselJ, mfBesselY, mfBesselK

#### mfBesselK

## modified Bessel functions of the second kind

 $Calling\ syntax:$ 

C = mfBesselK( alpha, A )

## Description:

Returns the modified Bessel functions of the second kind  $K_{\alpha}(x)$  (element-wise) of the mfArray A (dense only).

The real alpha order doesn't need to be an integer: both positive and negative fractional orders are possible.

Restriction: A must be of type real but, according to the values of  $\alpha$  and x, a complex mfArray may be returned.

 $See \ also: {\tt mfBesselI}, {\tt mfBesselI}, {\tt mfBesselI}$ 

mfAiry Airy function

 $Calling\ syntax:$ 

Description:

Returns the Airy function (element-wise) of the mfArray A (dense only).

Restriction: A must be of type real and a real  ${\tt mfArray}$  is returned. Complex function is not yet implemented.

See also: mfBesselJ

## msEllipKE

# complete elliptic integrals of first and second kinds

 $Calling\ syntax:$ 

call msEllipKE( mfOut(K,E), A )

Description:

Computes the complete elliptic integrals of first and second kind K(m) and E(m) (element-wise) of the mfArray A (dense only), which are stored in the two mfArray K and E.

Each element of the array A is taken to be the parameter  $m=k^2$ , where k is the modulus.

The parameter m must be ranged in [0,1] else NaN values are returned for both integrals.

See also: mfOut

#### mfIsPrime

## prime number test

Calling syntax:

B = mfIsPrime( A )

## Description:

Tests if numbers in the mfArray A are prime numbers. This function returns a boolean mfArray of same shape as A. The argument A may be scalar, vector or matrix, and is supposed to contain only integer numbers.

Remark: Integers involved in this routine must be less than 2,147,000,000.

 $See \ also: {\tt mfFactor}, {\tt msCleanPrimeNumbers}$ 

#### mfFactor

## prime numbers factorization

 $Calling\ syntax:$ 

```
B = mfFactor( A )
```

Description:

Returns the prime factors of the scalar mfArray A. The argument A is supposed to contain an integer number.

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Remark: Integers involved in this routine must be less than 2,147,000,000.

See also: mfIsPrime, msCleanPrimeNumbers

```
Example(s):
```

2

2 2 2 3 3 5 5

#### msCleanPrimeNumbers

# prime numbers storage cleaning

 $Calling\ syntax:$ 

call msCleanPrimeNumbers( )

## Description:

Cleans the temporary storage of prime numbers, required for mfIsPrime and mfFactor routines. According to the magnitude of the involved integers, up to 450 MB of memory can be freed.

#### msRat

## rational approximation

 $Calling\ syntax:$ 

```
call msRat( mfOut(N,D), A [, tol] )
```

#### Description:

Computes the rational approximation (element-wise) of the mfArray A (real dense only), and returns the matrix of numerators in the mfArray N and the matrix of denominators in the mfArray D.

Tolerance cannot be smaller than  $10^{-12}$  (default value is  $10^{-6}$ ).

For example, with the default tolerance,  $\pi \approx 355/113$ . Any numerical value has a rational approximation, even Inf = 1/0 and NaN = 0/0.

```
See also: mfOut
Example(s):
   x = MF_PI
   call msDisplay( x, "x = MF_PI" )
   call msRat( mfOut(y,z), x )
   call msDisplay( y, "numerator", z, "denominator" )
output:
 x = MF_PI =
    3.1416
 numerator =
   355
 denominator =
   113
   x = mfHilb(4)
   call msDisplay( x, "x = mfHilb(4)" )
   call msRat( mfOut(y,z), x )
   call msDisplay( y, "numerator", z, "denominator" )
output:
 x = mfHilb(4)
    1.0000
              0.5000
                      0.3333
                                  0.2500
    0.5000
             0.3333 0.2500
                                  0.2000
    0.3333
             0.2500 0.2000
                                  0.1667
    0.2500
              0.2000
                        0.1667
                                  0.1429
```

num	erator	=

1	1	1	1
1	1	1	1
1	1	1	1
1	1	1	1

#### denominator =

1	2	3	4
2	3	4	5
3	4	5	6
4	5	6	7

# 1.7 Elementary Matrix Manipulation Functions

mfEyeidentity matrixmfLinSpacelinear spaced vectormfLogSpacelog spaced vectormfMagicmagic square

msMeshGrid arrays for interpolation and 3-D plots

mfZeros matrix of zeros mtOnes matrix of ones

mfRanduniformly distributed random numbersmsRandset the random number generator seedmfRandNnormally distributed random numbersmfRandPoissPoisson distributed random numbers

mfRepMat replicate and tile an array mfMerge merge of two arrays

mfPack, mfUnpack packing and unpacking arrays

mfCshift, mfEoshift array shifting

mf/msDiag diagonal matrices and diagonals of a matrix

mfBlkDiag block diagonal concatenation
mf/msFind find indices of nonzero elements

mfNonZeros non-zero elements shape modification mf/msReshape mfTriL lower triangular part upper triangular part mfTriU matrix flip Left-Right mfFlipLR mfFlipUD matrix flip Up-Down mfRot90 matrix rotation Hilbert matrix mfHilb inverse Hilbert matrix mfInvHilb

mfVander Vandermonde matrix
mfCompan Companion matrix
mfHankel Hankel matrix
mfToeplitz Toeplitz matrix

mfPerm permutation vector creation

mfRandPerm random permutation vector creation

mfCheckPerm permutation vector checking

isfinite, mfIsFinite finite test
isinf, mfIsInf infinity test
isnan, mfIsNaN Not-a-Number test

mfIsDiag diagonal pattern test
mfIsTril lower triangular pattern

mfIsTril lower triangular pattern test mfIsTriu upper triangular pattern test

See also:

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mfEye identity matrix

Calling syntax:

$$A = mfEye(n1[, n2])$$

Description:

Returns an mfArray containing the identity matrix.

If it is called with one integer argument, the identity matrix is square, of shape (n1,n1).

If the optional argument n2 is present, the routine builds an identity matrix of shape (n1,n2).

See also: mfOnes, mfZeros

#### mfLinSpace

## linear spaced vector, given nb of values

First calling syntax:

```
A = mfLinSpace( start, end [, nval] )
```

#### Description:

Returns an mfArray (row vector) which contains a list of reals equally spaced, from start to end, with the specified number of values nval. start and end can take any arbitrary finite values, even equal.

By default (i. e. when the integer nval is not present), the list contains nval = 100 reals. If nval is present, it must be greater than or equal to 2.

Second calling syntax:

```
A = mfLinSpace([ start | end ], nval, step )
```

#### Description:

In this second syntax, only one argument among start and end can be present. The additional step argument specifies the step between two consecutive values; it cannot be equal to zero.

Remarks: start, end and step must be of type real (single or double precision).

See also: mfColon, mfLogSpace

```
Example(s):
```

```
call msDisplay( mfLinspace(-MF_PI, MF_PI, 5),
                                                                        &
                  "mfLinspace(-MF_PI, MF_PI, 5)" )
output:
mfLinspace(-MF_PI, MF_PI, 5) =
   -3.1416 -1.5708
                        0.0000
                                  1.5708
                                            3.1416
call msDisplay( mfLinSpace(end=1.0d0, nval=5, step=0.1d0),
                                                                        &
               "mfLinSpace(end=1.0d0, nval=5, step=0.1d0)" )
output:
mfLinSpace(end=1.0d0, nval=5, step=0.1d0) =
  0.6000
            0.7000
                      0.8000
                                 0.9000
                                        1.0000
```

#### mfLogSpace

## log spaced vector, given nb of values

First calling syntax:

```
A = mfLogSpace( start, end [, nval] )
```

#### Description:

Returns an mfArray (row vector) which contains a list of reals logarithmic spaced, from  $10^{start}$  to  $10^{end}$ , with the specified number of values nval.

By default (i. e. when the integer nval is not present), the list contains nval = 20 reals. If nval is present, it must be greater than or equal to 2.

Second calling syntax:

```
A = mfLogSpace( [ start | end ], nval, step )
```

#### Description:

In this second syntaxe, only one argument among start and end can be present. The additional step argument specifies the step between the logarithm of two consecutive values; it cannot be equal to zero.

Remarks: start, end and step must be of type real (single or double precision).

See also: mfColon, mfLinSpace

#### Example(s):

```
call msDisplay( mfLogspace(0.0d0, 5.0d0, 6),
                                                                        &
                  "mfLogspace(0.0d0, 5.0d0, 6)" )
output:
mfLogspace(0.0d0, 5.0d0, 6) =
                      10
                                 100
                                            1000
                                                       10000
                                                                  100000
call msDisplay( mfLogSpace(start=0.0d0, nval=6, step=0.5d0),
                                                                          &
               "mfLogSpace(start=0.0d0, nval=6, step=0.5d0)" )
output:
mfLogSpace(start=0.0d0, nval=6, step=0.5d0) =
   1.0000
            3.1623 10.0000 31.6228 100.0000 316.2278
```

magic square mfMagic

```
Calling syntax:
```

```
A = mfMagic(n)
```

## Description:

Returns an mfArray containing a magic square, from any positive integer  $n \ge 3$ .

## Example(s):

4

9 5

```
call msDisplay( mfMagic(3), "mfMagic(3)" )
call msDisplay( mfMagic(4), "mfMagic(4)" )
```

```
output:
mfMagic(3) =
       3
5
   8
            4
   1
            9
       7
            2
   6
mfMagic(4) =
     12
           8 13
   1
      6
   15
            10
                 3
   14
        7 11
                 2
```

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#### msMeshGrid

#### grid coordinates generation

 $Calling\ syntax:$ 

```
call msMeshGrid( mfOut(X,Y), v_x, v_y )
```

#### Description:

Builds two mfArrays containing the (x, y) coordinates of the nodes of a rectangular, structured grid, generated from the two real vectors  $\mathbf{v}_{-}\mathbf{x}$  and  $\mathbf{v}_{-}\mathbf{y}$ .

Among the two input vectors (called generator vectors), one must be a row vector, and the other a column vector. Then the row vector is vertically replicated as many times as the size of the other to form one of the coordinate matrices, and conversely for the other.

The coordinate matrices X and Y are typically used to visualize a data matrix with an orientation different from that used by default. See more explanation in section "Changing Orientation/Transposition of Matrix Data" in the Muesli User Guide. They can also be used in 2D interpolation (see mfInterp2).

Remark: Note that most of the time, you don't need to compute explicitly the coordinate matrices (X,Y) since you are able to pass the generator vectors directly to graphic routines (msPColor, msContour, etc.), and also to the numerical routine mfGridFun.

```
See also: mfOut
```

```
Example(s):
```

```
v_x = mfLinSpace( -1.0d0, 1.0d0, 5 )
v_y = .t. mfLinSpace( -1.5d0, 1.5d0, 7 )
call msMeshgrid( mfOut(X,Y), v_x, v_y )
call msDisplay( X, "X", Y, "Y" )
```

## output:

```
X =
```

```
-1.0000
         -0.5000
                     0.0000
                               0.5000
                                          1.0000
-1.0000
         -0.5000
                     0.0000
                               0.5000
                                          1.0000
-1.0000
          -0.5000
                     0.0000
                               0.5000
                                          1.0000
-1.0000
          -0.5000
                     0.0000
                               0.5000
                                          1.0000
-1.0000
          -0.5000
                     0.0000
                               0.5000
                                          1.0000
-1.0000
          -0.5000
                     0.0000
                               0.5000
                                          1.0000
-1.0000
          -0.5000
                     0.0000
                               0.5000
                                          1.0000
```

Y =

```
-1.5000
        -1.5000
                  -1.5000
                           -1.5000
                                     -1.5000
-1.0000
        -1.0000
                  -1.0000
                           -1.0000
                                     -1.0000
-0.5000
         -0.5000
                  -0.5000
                            -0.5000
                                      -0.5000
          0.0000
                   0.0000
                             0.0000
0.0000
                                      0.0000
         0.5000
                   0.5000
                             0.5000
                                      0.5000
0.5000
1.0000
        1.0000 1.0000
                             1.0000
                                      1.0000
1.5000
         1.5000 1.5000
                           1.5000
                                      1.5000
```

mfZeros matrix of zeros

 $Calling\ syntax:$ 

```
A = mfZeros(n1[, n2])
```

Description:

Returns an  ${\tt mfArray}$  containing zeros.

If it is called with one integer argument, the output matrix is square, of shape  $(\tt n1,\tt n1)$ .

If the optional argument n2 is present, the routine builds a matrix of shape (n1,n2).

See also: mfEye, mfOnes

mfOnes matrix of ones

 $Calling\ syntax:$ 

```
A = mfOnes(n1[, n2])
```

Description:

Returns an mfArray containing ones.

If it is called with one integer argument, the output matrix is square, of shape  $(\tt n1,\tt n1)$ .

If the optional argument n2 is present, the routine builds a matrix of shape (n1,n2).

See also: mfEye, mfZeros

#### mfRand

## uniformly distributed random numbers

 $Calling\ syntax:$ 

```
A = mfRand(n1[, n2])
```

Description:

Returns an mfArray containing pseudo-random real values, with a uniform distribution in [0, 1].

If it is called with one integer argument, the output matrix is square, of shape (n1,n1).

If the optional argument n2 is present, the routine builds a matrix of shape (n1,n2).

Other syntax:

```
A = mfRand( string )
```

When called with the string "seed", the seed (six long integers stored as double reals) of the random number generator is returned. This value can be later used to reset the seed to a previous state with the msRand routine.

Remark: Generated numbers doesn't depend on the compiler used, because the random\_number Fortran 90 intrinsic routine is not use. The RngStreams package (included in the MUESLI library) is used instead.

See also: mfRandN, mfRandPoiss

#### msRand

## set the random number generator seed

 $Calling\ syntax:$ 

```
call msRand( [ seed ] )
```

Description:

Set the random number generator seed either from given values, or from the clock.

Without argument, the seed is set from the clock.

If the optional argument seed is present, then this argument must be an mfArray whose values are used to reset the seed of the RngStreams package, included in the MUESLI library. This seed must be an array of six integer numbers (stored as double precision reals) satisfying the following constraints:

- the first three values must be less than 4294967087 (and not all zero);
- the last three values must be less than 4294944443 (and not all zero).

Note that a check is done in the current routine.

See also: mfRand, mfRandN, mfRandPoiss

#### mfRandN

# normally distributed random numbers

Calling syntax:

```
A = mfRandN(n1[, n2])
```

## Description:

Returns an mfArray containing pseudo-random real values, with a normal distribution (with zero mean and unit variance).

If it is called with one integer argument, the output matrix is square, of shape (n1,n1).

If the optional argument n2 is present, the routine builds a matrix of shape (n1,n2).

See also: mfRand, mfRandPoiss, msRand

#### mfRandPoiss

## Poisson distributed random numbers

Calling syntax:

A = mfRandPoiss( mu, n1 [, n2] )

## Description:

Returns an mfArray containing pseudo-random integer values, with a Poisson distribution, with mean (and variance) equal to mu.

The argument mu is a real number.

If it is called with only one size, the output matrix is square, of shape (n1,n1).

If the optional argument n2 is present, the routine builds a matrix of shape (n1,n2).

See also: mfRandN, mfRand, msRand

### mfRepMat

### replicate and tile an array

Calling syntax:

```
C = mfRepMat(A, m, n)
```

### Description:

This routine replicates the tile mfArray A, to build a bigger mfArray. The returned mfArray can be viewed as a m-by-n tiling matrix.

The behavior of this routine is similar to the *spread* Fortran 90 intrinsic function.

### Example(s):

```
x = reshape([(i, i = 1, 6)], [2, 3])
  call msDisplay( x, "x", mfRepmat(x,3,2), "mfRepmat(x,3,2)")
output:
x =
   1
        3
             5
   2
        4
             6
mfRepmat(x,3,2) =
        3 5 1 3
4 6 2 4
   1
                           5
   2
                           6
       3 5 1 3
   1
                          5
   2
       4 6 2 4
                          6
       3 5 1 3
4 6 2 4
   1
                           5
    2
                           6
```

### mfMerge

### merge of two arrays

 $Calling\ syntax:$ 

```
C = mfMerge( A, B, mask )
```

## Description:

The three arguments are mfArrays having the same shape. This function merge the two numeric arrays A and B under the control of the boolean mask. Its behavior is like that of the *merge* Fortran 90 intrinsic function.

Sparse arrays are not handled.

```
Example(s):
```

```
A = 3.0d0*mf0nes(3,2)
  B = -1.0d0*mf0nes(3,2)
  mask = mfEye(3,2) > 0.5d0
   call msDisplay( A,"A", B,"B", mask,"mask" )
   call msDisplay( mfMerge(A,B,mask), "mfMerge(A,B,mask)" )
output:
 A =
     3
           3
     3
           3
     3
           3
 B =
    -1
          -1
    -1
          -1
    -1
          -1
mask =
     Т
           F
     F
           Т
     F
           F
mfMerge(A,B,mask) =
     3
          -1
    -1
          3
    -1
          -1
```

mfPack

### packing an array

```
Calling\ syntax:
```

```
C = mfPack( A, mask )
```

### Description:

Packs the elements of the numeric mfArray A in a long column vector, under the control of the boolean mask. The behavior of this routine is similar to the pack Fortran 90 intrinsic function.

Sparse arrays are not handled.

```
See also: mfUnpack
Example(s):
   A = mfMagic(3)
   mask = A >= 5.0d0
   call msDisplay( A,"A", mask,"mask" )
   Ap = mfPack( A, mask )
   call msDisplay( Ap, "A packed" )
output:
 A =
     8
           3
                 4
     1
           5
                 9
     6
           7
                 2
mask =
     Т
           F
                 F
     F
           T
                 T
     Т
           Т
                 F
 A packed =
     8
           6
                 5
                       7
                              9
   R = mfOnes(3)*MF_NAN
   call msDisplay( mfUnpack(Ap,mask,field=R), "A unpacked" )
output:
 A unpacked =
    8.0000
               NaN
                           NaN
      NaN
              5.0000
                         9.0000
    6.0000
              7.0000
                           NaN
```

### mfUnpack

### unpacking an array

 $Calling\ syntax:$ 

```
C = mfUnpack( A, mask, field )
```

### Description:

Unpacks the elements of the numeric vector mfArray A under the control of the boolean mask. The returned mfArray is completed from values of field. A should have been previously processed by the mfPack routine.

The behavior of this routine is similar to the unpack Fortran 90 intrinsic function.

Sparse arrays are not handled.

```
See also: mfPack
Example(s):
   A = mfMagic(3)
   mask = A >= 5.0d0
   call msDisplay( A,"A", mask,"mask" )
   Ap = mfPack( A, mask )
   call msDisplay( Ap, "A packed" )
output:
 A =
     8
           3
                  4
     1
           5
     6
           7
                  2
mask =
     Т
           F
                  F
     F
           Т
                  Т
           T
     Т
                  F
A packed =
                        7
                  5
   R = mfOnes(3)*MF_NAN
   call msDisplay( mfUnpack(Ap,mask,field=R), "A unpacked" )
output:
 A unpacked =
    8.0000
                \mathtt{NaN}
                           NaN
              5.0000
      \mathtt{NaN}
                         9.0000
    6.0000
              7.0000
                           NaN
```

mfCshift array shifting

Calling syntax:

```
C = mfCshift( A, shift )
```

### Description:

Performs a circular shift on the vector mfArray A. shift is an integer. The behavior of this routine is similar to the cshift Fortran 90 intrinsic function.

Sparse arrays are not handled.

See also: mfEoshift

```
Example(s):
```

```
A = [ 1, 2, 3, 4, 5, 6 ]
call msDisplay( A, "A" )
call msDisplay( mfCshift(A,shift=1), "mfCshift(A,shift=1)" )
```

output:

A =

1 2 3 4 5 6

mfCshift(A,shift=1) =

2 3 4 5 6 1

mfEoshift array shifting

```
Calling syntax:
```

```
C = mfEoshift( A, shift )
```

### Description:

Performs an end-off shift on the vector mfArray A. shift is an integer. The behavior of this routine is similar to the *eoshift* Fortran 90 intrinsic function.

Sparse arrays are not handled.

See also: mfCshift

```
Example(s):
```

output:

```
A =
    1    2    3    4    5    6
mfEoshift(A,shift=1,boundary=b) =
    2    3    4    5    6    99
```

#### mf/msDiag

#### diagonal matrices and diagonals of a matrix

The first form is:

```
C = mfDiag( A [, d] )
```

If A is a matrix-like mfArray (dense or sparse), this function extracts the main (or the d-) diagonal (the output is dense, whatever the storage type of A is).

If A is a vector-like mfArray (dense only), this function builds a dense square matrix which has A as main (or as d-) diagonal. For building a sparse matrix, use mfSpDiags.

The subroutine form:

```
call msDiag( A, v [, d] )
```

modifies the main (or the d-) diagonal of the mfArray A (dense or sparse) by copying the elements of the vector-like mfArray v. The data type of A may be changed because A and v can have real or complex values. The vector v is not modified.

Remarks: When d is equal to zero, it points to the main diagonal; when positive, the upper part of the matrix is concerned. If the arg. d is not present, the main diagonal is the default.

```
See also: mfBlkDiag, mfEye, mfSpDiags
Example(s):
   v = [1, 2, 3]
   C = mfDiag(v, 2)
   call msDisplay( C, "C = mfDiag(v,2)" )
output:
            2
                  3
     1
C = mfDiag(v, 2) =
     0
            0
                         0
                                0
                   1
     0
            0
                   0
                         2
                                0
     0
            0
                  0
                         0
                                3
     0
            0
                   0
                         0
                                0
                   0
     0
            0
                         0
                                0
   v = [4, 5, 6]
   call msDisplay( v, "v" )
   call msDiag( C, v, d=-2 )
   call msDisplay( C, "C" )
output:
     4
            5
                   6
 C =
     0
            0
                         0
                                0
                   1
     0
            0
                         2
                                0
                   0
     4
            0
                  0
                         0
                                3
     0
            5
                  0
                         0
                                0
```

#### mfBlkDiag

### block diagonal concatenation

The first calling syntax:

```
C = mfBlkDiag( A, B )
```

gives the concatenation of the two (square) blocks diagonal A and B.

The other calling syntax:

```
C = mfBlkDiag( A, n )
```

concatenates n times (n being non negative) the square matrix A on the diagonal.

Remarks: A (and B, for the first form), may be dense or sparse, and of any type accepted by the .vc. or .hc. operators.

```
See also: mf/msDiag
Example(s):
   A = mfRot90(mfEye(3,3))
   call msDisplay(A,"A")
   call msDisplay( mfBlkDiag(A,3), "blkdiag(A,3)" )
output:
 A =
     0
           0
                 1
     0
           1
                 0
     1
           0
                 0
blkdiag(A,3) =
     0
           0
                       0
                              0
                                    0
                                                       0
                 1
                                          0
                                                0
     0
           1
                 0
                       0
                              0
                                    0
                                          0
                                                0
                                                       0
     1
           0
                 0
                       0
                              0
                                    0
                                          0
                                                0
                                                       0
     0
           0
                 0
                       0
                              0
                                          0
                                                0
                                                       0
                                    1
     0
           0
                 0
                       0
                                    0
                                          0
                                                0
                                                       0
                              1
     0
                                                0
                                                       0
           0
                 0
                       1
                              0
                                    0
                                          0
     0
           0
                 0
                              0
                                    0
                                                       1
                                                       0
     0
           0
                 0
                       0
                              0
                                    0
                                          0
                                                1
```

#### mf/msFind

#### find indices of nonzero elements

The first form:

```
C = mfFind( A )
```

finds  $non\ zero$  (resp. TRUE) elements of the numerical (resp. boolean) mfArray A and returns a long column of their indices (called also linear indices).

Usually, this routine is applied to a boolean mfArray, automatically created in comparison operators; for example, mfFind( A > 1.0d0 ) returns indices of elements which verify the specified property.

The subroutine form:

```
call msFind( mfOut(i,j[,v]), A )
```

allows the user to retrieve the matrix-indices. The two vectors returned (i and j) contain the row-index and column-index for non-zero elements.

If the optional argument v is present, it contains the values of corresponding non-zero elements.

Warning: Contrary to MATLAB, mf/msFind always return row index vectors, instead of column vectors.

See also: mfNonZeros, mfOut

```
Example(s):
```

```
A = mf([9, 0, 0]).vc.
      mf([0, 0, 7]).vc.
      mf([0, 11, 0])
   call msDisplay( A, "A" )
   call msFind( mfOut(i,j,v), x )
   call msDisplay( i,"row indices", j,"col indices", v,"values" )
output:
 A =
    9
          0
                0
     0
          0
                7
          11
 row indices =
     1
          3
                 2
 col indices =
          2
                 3
     1
 values =
     9
         11
                7
```

mfNonZeros non-zero elements

 $Calling\ syntax:$ 

C = mfNonZeros( A )

Description:

Returns all non-zero elements of the  ${\tt mfArray}\ {\tt A}$  in a vector.

Works as  ${\tt msFind}$ , except that indices (i,j) are not returned.

See also: ms/mfFind

#### mf/msReshape

#### shape modification

Generic interface:

```
function mfReshape( A, m, n ) result( out )
  integer, intent(in) :: A(:)
or real(kind=MF_DOUBLE), intent(in) :: A(:)
or type(mfArray) :: A
  integer, intent(in) :: m, n
  type(mfArray) :: out
```

#### Description:

This function extends the Fortran intrinsic reshape; it can be applied to a dense or sparse mfArray.

Remark: in the special call: mfReshape(A, MF\_EMPTY, n) with A being an mfArray only, the first dimension, if not specified, is deduced from the total number of elements in A (the second dimension n can also be replaced by MF\_EMPTY). Same behavior using MF\_NO\_ARG instead of MF\_EMPTY.

Another possible form is:

```
call msReshape( A, m, n )
```

which does the same job with the mfArray A, except that the shape modification is made "in place". Here, m or n cannot be replaced by MF\_EMPTY or MF\_NO\_ARG. This second routine is more efficient than the first one, especially for sparse matrices.

```
See also: mfRepMat
```

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```
Example(s):
```

```
x = [(i, i = 1, 12)]
   call msDisplay( x, "x" )
   call msDisplay( mfReshape(x, 4, 3 ), "mfReshape(x, 4, 3 )" )
   call msDisplay( mfReshape(x, 2, MF_EMPTY ), "mfReshape(x, 2, MF_EMPTY )" )
output:
x =
          2
     1
                3
                      4
                           5 6
                                       7
                                             8
                                                    9
                                                         10
                                                                11
                                                                      12
mfReshape(x, 4, 3) =
          5
     1
                 9
     2
          6
                10
     3
          7
                11
     4
          8
                12
mfReshape(x, 2, MF_EMPTY ) =
          3
                 5
                      7
                            9
     1
                                 11
```

#### mfTriL

## lower triangular part

Calling syntax:

## Description:

This function extracts the lower triangular part of an mfArray, including the main diagonal.

If k is present, it returns the lower part of A up to the k-th diagonal (included). k must be integer; it can be negative.

See also: mfTriU

#### mfTriU

## upper triangular part

 $Calling\ syntax:$ 

## Description:

This function extracts the upper triangular part of an mfArray, including the main diagonal.

If k is present, it returns the upper part of A down to the k-th diagonal (included). k must be integer; it can be negative.

See also: mfTriL

## mfFlipLR

## matrix flip Left-Right

```
Calling\ syntax:
```

```
C = mfFlipLR( A )
```

### Description:

This function flips the  ${\tt mfArray}$  A in the left-right direction.

```
See also: mfFlipUD, mfRot90
```

### Example(s):

```
A = reshape([(i,i=1,15)], [3,5])
call msDisplay(A, "A")
call msDisplay(mfFlipLR(A), "mfFlipLR(A)")
```

#### output:

```
A =
```

```
    1
    4
    7
    10
    13

    2
    5
    8
    11
    14

    3
    6
    9
    12
    15
```

### mfFlipLR(A) =

13	10	7	4	1
14	11	8	5	2
15	12	9	6	3

## mfFlipUD

## matrix flip Up-Down

```
Calling syntax:
```

```
C = mfFlipUD( A )
```

### Description:

This function flips the  ${\tt mfArray}$  A in the up-down direction.

```
See also: mfFlipLR, mfRot90
```

### Example(s):

```
A = reshape( [ (i,i=1,15) ], [3,5] )
call msDisplay( A, "A" )
call msDisplay( mfFlipUD(A), "mfFlipUD(A)" )
```

#### output:

```
A =
```

```
    1
    4
    7
    10
    13

    2
    5
    8
    11
    14

    3
    6
    9
    12
    15
```

```
mfFlipUD(A) =
```

3	6	9	12	15
2	5	8	11	14
1	4	7	10	13

mfRot90 matrix rotation

```
Calling syntax:
```

```
C = mfRot90(A[, k])
```

### Description:

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5

4

This function rotates the mfArray A in the trigonometric direction, by the angle k\*90 in degrees. (i. e. for 90°, last column becomes first row).

k may be any integer (including zero and negative value). It is an optional argument (default value is 1).

```
See \ also: \mathtt{mfFlipLR}, \mathtt{mfFlipUD}
Example(s):
   A = reshape([(i,i=1,6)], [3,2])
   call msDisplay( A, "A" )
   call msDisplay( mfRot90(A,1), "mfRot90(A,1) [90° anticlockwise]" )
   call msDisplay( mfRot90(A,-1), "mfRot90(A,-1) [90° clockwise]" )
output:
 A =
     1
           4
     2
           5
     3
           6
mfRot90(A,1) [90° anticlockwise] =
     4
           5
                  6
     1
           2
                  3
 mfRot90(A,-1) [90° clockwise] =
           2
     3
                  1
```

mfHilb Hilbert matrix

```
Calling\ syntax:
```

```
C = mfHilb(n)
```

## Description:

Builds a square mfArray which contains an Hilbert matrix. It is a well known example of badly conditioned matrix.

```
See also: mfInvHilb, mfCond
```

### Example(s):

```
A = mfHilb(5)
call msDisplay( A, "A = mfHilb(5)" )
call msDisplay( mfCond(A), "cond(A)" )
```

#### output:

### cond(A) =

4.7661E+05

#### mfInvHilb

# inverse Hilbert matrix

Calling syntax:

```
C = mfInvHilb( n )
```

### Description:

Returns the inverse of an Hilbert matrix; therefore the matrix product of mfHilb(n) by mfInvHilb(n) is the identity matrix.

Remarks: this inverse is not computed by some linear algebra algorithm but by an exact formula.

```
See also: mfHilb
```

## Example(s):

```
A = mfHilb(5)
B = mfInvHilb(5)
call msDisplay( B, "B = mfInvHilb(5)" )
call msDisplay( mfNorm(mfEye(5) - mfMul(A,B)), "| I - A*B |" )
```

## output:

```
B = mfInvHilb(5) =
```

25	-300	1050	-1400	630
-300	4800	-18900	26880	-12600
1050	-18900	79380	-117600	56700
-1400	26880	-117600	179200	-88200
630	-12600	56700	-88200	44100

```
| I - A*B | =
```

5.7926E-12

#### mfVander

#### Vandermonde matrix

Calling syntax:

```
A = mfVander( v [, n])
```

### Description:

Returns a Vandermonde matrix. The input vector  $\mathbf{mfArray}$   $\mathbf{v}$  may be either a row or a column vector.

If the optional argument  $\mathbf{n}$  is present, the returned matrix will have n columns, each one constituted by a power of the vector  $\mathbf{v}$ ; else the returned matrix will be square, i.e. will have as much columns as the size of  $\mathbf{v}$ .

```
Example(s):
```

```
x = [1, 2, 3, 4, 5]
   call msDisplay( x, "x" )
   call msDisplay( mfVander(x,3), "A = mfVander(x,3)" )
output:
x =
           2
                 3
     1
                       4
                              5
 A = mfVander(x,3) =
     1
           1
                 1
     4
           2
                 1
     9
           3
                 1
    16
           4
                 1
    25
           5
                 1
```

### mfCompan

### Companion matrix

 $Calling\ syntax:$ 

```
A = mfCompan( v )
```

## Description:

Returns the Companion matrix of a polynomial, represented by its coefficients provided in the input vector mfArray v (this latter array may be either a row or a column vector).

The eigenvalues of the matrix A are the roots of the input polynomial.

```
See also: mfRoots, mfEig
```

```
Example(s):
```

```
! polynom: (x-1)*(x-2)*(x-3)*(x-4) = x^4 - 10*x^3 + 35*x^2 - 50*x + 24,
! so the roots are: {1, 2, 3, 4}
v = [ 1.0d0, -10.0d0, 35.0d0, -50.0d0, 24.0d0 ]
call msDisplay( v, "v" )
A = mfCompan(v)
call msDisplay( A, "A = mfCompan(v)" )
call msDisplay( mfEig(A), "Eigenvalues of A" )
```

output:

```
v =
```

```
1 -10 35 -50 24
```

```
A = mfCompan(v) =
```

```
    10
    -35
    50
    -24

    1
    0
    0
    0

    0
    1
    0
    0

    0
    0
    1
    0
```

### Eigenvalues of A =

```
4.0000 + 0.0000i
3.0000 + 0.0000i
2.0000 + 0.0000i
1.0000 + 0.0000i
```

## mfHankel matrix

Calling syntax:

```
A = mfHankel( C [, R])
```

### Description:

Returns the Hankel matrix of the input vectors mfArray C and R (these latter arrays may be either a row or a column vector).

A Hankel matrix has constant anti-diagonal values.

Hankel matrices are real (because only real input vectors are supported) and symmetric (if square).

See also: mfToeplitz

```
Example(s):
```

```
C = [ 1.0d0, 2.0d0, 3.0d0, 4.0d0 ]
R = [ 4.0d0, 5.0d0, 6.0d0 ]
call msDisplay( C, "C", R, "R" )
call msDisplay( mfHankel(C,R), "mfHankel(C,R)" )
```

## output:

```
C =

1 2 3 4

R =

4 5 6
```

mfHankel(C,R) =

```
1 2 3
2 3 4
3 4 5
4 5 6
```

### mfToeplitz

## Toeplitz matrix

Calling syntax:

```
A = mfToeplitz( C [, R])
```

### Description:

Returns the Toeplitz matrix of the input vectors mfArray C and R (these latter arrays may be either a row or a column vector).

A Toeplitz matrix has constant diagonal values and is symmetric (if square).

Complex input vectors are supported. In such a case, the complex matrix is hermitian if the first element of R is real.

See also: mfHankel

```
Example(s):
```

```
C = [1.0d0, 2.0d0, 3.0d0, 4.0d0]
R = [1.0d0, 5.0d0, 6.0d0]
call msDisplay( mfToeplitz(C,R), "mfToeplitz(C,R)" )
```

output:

1

```
C =
    1
          2
                3
                       4
R =
```

5 A = mfToeplitz(C,R) =

6

```
1
      5
             6
2
      1
             5
3
      2
             1
      3
             2
```

#### mfPerm

## permutation vector creation

Calling syntax:

### Description:

Returns a permutation vector mf Array from the vector of indices  ${\tt v}.$ 

v may be either a Fortran 90 integer vector, or a vector-like mfArray. In this latter case, the mfArray should contains integer values; on the contrary, a warning is emitted by the routine.

See also: mfIsPerm, mfRandPerm, mfCheckPerm, mfColPerm, mfRowPerm, msSaveAscii

#### mfRandPerm

## random permutation vector creation

Calling syntax:

```
p = mfRandPerm( n [, k ] )
```

### Description:

Returns a random permutation vector mfArray. Elements of p are unique integers in the range [1, n].

If the second argument k is present, returns k unique integers in the range [1, n]. In this latter case, p is not a true permutation vector if  $k \neq n$ .

See also: mfPerm, mfIsPerm, mfCheckPerm

#### mfCheckPerm

## permutation vector checking

Interface:

```
function mfCheckPerm( A ) result( bool )
  type(mfArray), intent(in) :: A
  logical :: bool
```

### Description:

Returns '.true.' if A is a valid permutation vector, *i.e.* if the integers are consecutive numbers, not necessarily ordered and appearing only once in the list.

See also: mfPerm, mfIsPerm

isfinite finite test

### Interface:

```
function isfinite( x ) result( out )
  double precision, intent(in) :: x
  logical :: out
```

## Description:

This function returns a logical value, according to the value of x, which must be only of type double precision.

Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.

See also: isinf, isnan, mfIsFinite

isinf infinity test

 ${\it Interface:}$ 

```
function isinf( x ) result( out )
  double precision, intent(in) :: x
  logical :: out
```

Description:

This function returns a logical value, according to the value of x, which must be only of type double precision.

Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.

See also: isfinite, isnan, mfIsInf

isnan

Not-a-Number test

Interface:

```
function isnan( x ) result( out )
  double precision, intent(in) :: x
  logical :: out
```

Description:

This function returns a logical value, according to the value of x, which must be only of type double precision.

Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.

 $See \ also: \mathtt{isfinite}, \mathtt{isinf}, \mathtt{mfIsNaN}$ 

mfIsFinite finite test

 $Calling\ syntax:$ 

C = mfIsFinite( A )

Description:

This elemental function returns a boolean  ${\tt mfArray}.$ 

Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.

See also: isfinite, mfIsInf, mfIsNaN

mfIsInf infinity test

 $Calling\ syntax:$ 

C = mfIsInf( A )

Description:

This elemental function returns a boolean  ${\tt mfArray}.$ 

Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.

See also: isinf, mfIsFinite, mfIsNaN

mfIsNaN

Not-a-Number test

 $Calling\ syntax:$ 

C = mfIsNaN( A )

Description:

This elemental function returns a boolean mfArray.

Remarks: in the IEEE-754 standard, any real value is either finite, infinite, or NaN.

See also: isnan, mfIsFinite, mfIsInf

## ${\tt mfIsDiag}$

# diagonal pattern test

 $Calling\ syntax:$ 

bool = mfIsDiag( A )

returns a  $\mathit{TRUE}$  boolean value if the  $\mathsf{mfArray}$  A (sparse or dense) is a diagonal matrix.

 $See \ also: \ {\tt mfIsTril}, \ {\tt mfIsTriu}$ 

## mfIsTril

# lower triangular pattern test

Calling syntax:

bool = mfIsTril( A )

returns a TRUE boolean value if the mfArray A (sparse or dense) is a lower triangular matrix.

 $See \ also: {\tt mfIsDiag}, {\tt mfIsTriu}$ 

## ${\tt mfIsTriu}$

# upper triangular pattern test

 $Calling\ syntax:$ 

bool = mfIsTriu( A )

returns a TRUE boolean value if the mfArray A (sparse or dense) is a upper triangular matrix.

 $See \ also: {\tt mfIsDiag}, {\tt mfIsTril}$ 

#### 1.8 Matrix Functions

mfDet determinant

mfTrace sum of diagonal elements mfNorm matrix or vector norm

mfRank matrix rank

mfCond matrix condition number

mfRCond reciprocal condition number estimation

 $\begin{array}{ll} \textbf{msLU} & L\,U \text{ factorization} \\ \textbf{msLDLT} & L\,D\,L^t \text{ factorization} \\ \textbf{mf/msChol} & \text{Cholesky factorization} \end{array}$ 

msCholSpSymb Symbolic Cholesky factorization (sparse only)
msCholSpNum Numerical Cholesky factorization (sparse only)

mf/msBalance scaling to improve eigenvalue accuracy

mf/msSVD singular value decomposition

mfInv, .i. matrix inverse mfPseudoInv matrix pseudo-inverse

mfQR, msQR orthogonal-triangular factorization

mfQleft, mfQright left or right multiplication by a Q factor (sparse only)

mf/msEig eigenvalues and eigenvectors

mf/msHess Hessenberg form
mf/msSchur Schur decomposition
mfLDiv, .ix. left matrix divide
mfRDiv, .xi. right matrix divide
msRref reduced row echelon form

mfNull null space

mfOrth matrix orthogonalization

mfExpmmatrix exponentialmfLogmmatrix logarithmmfSqrtmmatrix square rootmfPowmmatrix power

mf/msFunm general matrix function

mf/msEigs few eigenvalues and eigenvectors
mf/msSVDS few singular values and vectors

mfNormEst, mfNormEst1 norm estimations

mfCondEst matrix condition number estimation

mfIsSymmmatrix symmetry inquirymfTolForSymmtolerance for symmetrymfIsPosDefdefinite positiveness inquiry

mfIsDiagDomCol diagonally dominant by cols inquiry

mfIsStrictDiagDomCol strictly diagonally dominant by cols inquiry

mfIsFullRank full rank inquiry

MF\_LAPACK\_VERSIONLAPACK versionmsGetBlasLibGet BLAS library typemsGetLapackLibGet LAPACK library typemsGetArpackInfoGet ARPACK library informationmsGetSuiteSparseLibGet SuiteSparse library version

See also:

Core Routines

File Input/Output

Data Analysis Functions

Operators

Elementary Math Functions

Specialized Math Functions

Elementary Matrix Manipulation Functions

Polynomial Functions

Optimization and Function Functions

Sparse Matrices

mfDet determinant

Calling syntax:

C = mfDet(A)

Description:

Returns the determinant of the mfArray A.

A must be square, and cannot be sparse.

Remarks: using the determinant of a matrix to detect whether it is singular is not a good idea: use mfCond or mfRCond instead.

See also: mfRank

### mfTrace

# sum of diagonal elements

Calling syntax:

C = mfTrace( A )

Description:

Returns the sum of all diagonal element of the  ${\tt mfArray}$  A (dense or sparse, real or complex).

Remarks: the trace of a square matrix is also the sum of its eigenvalues.

See also: mfEig

#### mfNorm

#### matrix or vector norm

Calling syntax:

```
C = mfNorm( A [, norm | p ] )
```

### Description:

Returns the norm of the mfArray A, which can be a vector or a matrix (sparse or dense).

The two optional arguments (norm is a character string whereas p is an integer) cannot be both present.

- For a matrix, only 1-, 2-, "inf"- or "fro"-norms are supported.

By default, returns the 2-norm (which is also the largest singular value of A). If p is present, it returns the p-norm of A (therefore, p can take only the values 1 or 2). If norm is present, it must be inf (infinite-norm) or fro (frobenius-norm).

 $-\,$  For a vector, only  ${\tt p-}>0$  and "inf"- norms are supported.

By default, returns the 2-norm. If p is present, it returns the p-norm of A. If norm is present, it must be inf (infinite-norm).

Note: For sparse matrices, use mfNormEst or mfNormEst1 instead, which are more efficient.

See also: mfCond, mfSVD

mfRank matrix rank

 $Calling\ syntax:$ 

```
C = mfRank( A [, tol ] )
```

### Description:

Computes the rank of a matrix, i.e. the number of linearly independent rows or columns.

If to1, of type real, is present then it returns the number of singular values greater or equal to this optional tolerance. Default tolerance is chosen as  $\max(m,n)||A||_2 \epsilon$ , where m and n are respectively the number of rows and number of columns of the mfArray A.

#### Remarks:

- it is internally based on the Singular Value Decomposition;
- for sparse matrices, the user is invited to use mfSVDS, and examine himself few singular values.

 $See\ also:$  mfSVD, mfSVDS, mfIsFullRank

#### mfCond

### matrix condition number

 $Calling\ syntax:$ 

C = mfCond( A )

### Description:

Returns the condition number of the mfArray A. It is the ratio of the largest singular value to the smallest one.

Remarks: The condition number is always greater than 1. A value equal to infinity (resp. close to the inverse of the  $\epsilon$  machine) indicates that the matrix A is singular (resp. nearly singular).

 $See\ also:$  mfCondEst, mfRCond, mfNorm, mfSVD, mfDet

#### mfRCond

# reciprocal condition number estimation

Calling syntax:

C = mfRCond( A )

Description:

Returns an estimation of the inverse of the condition number (which is called the reciprocal condition number).

Remarks: The reciprocal condition number is always between 0 and 1. A value equal to zero (resp. close to the  $\epsilon$  machine) indicates a singular matrix (resp. nearly singular).

 $See \ also: {\tt mfCond}, {\tt mfCondEst}, {\tt mfNorm}, {\tt mfSVD}, {\tt mfDet}$ 

 ${\tt msLU}$ 

Description:

Computes numerically the LU decomposition of an mfArray.

First calling syntax:

For dense mfArrays, returns the L (lower triangular, with a unit diagonal) and U (upper triangular) factors, and, optionally, the row permutation vector p, such that:

$$LU = A(p,:)$$

If the optional argument p is not present, the routine actually returns a row permuted L, such that:

$$LU = A$$

In this latter case, be aware that you will not be able to solve a linear system using only the factors L and U, because you must know the permutation p to be applied to the right hand side vector. See the  $Muesli\ User$ 's Guide to an example using the LU decomposition.

Second calling syntax:

For sparse mfArrays, the routine msLU behaves in a slightly different manner:

p and q are respectively row and column permutations vectors while r is a vector containing the row scaling of A. All these sparse matrices should verify the following (theoretical) relationship:

$$L U = P R A Q$$

where the matrices P, Q, R are associated to the vectors p, q, r. Practically, the RHS of the previous identity should be computed using mfRowScale, mfColPerm and mfRowPerm.

Moreover, the L factor contains a pointer to an internal structure containing all the five output matrices, so that a call to the mfLDiv routine is simplified.

Third calling syntax: In all cases (sparse or dense matrix A), the following syntax:

```
call msLU( mfOut(factors), A )
```

provides the possibility to factorize only, without copying factors in explicit mfArrays. In this case, the factors (accompanied by the permutations and the scaling) are internally stored (i. e. not available for the end-user) and can be referenced only via the mfMatFactor factors.

The mfMatFactor obtained is intended to be used only in mfLDiv.

Remark: If the matrix A is symmetric and positive definite, the current routine may emit a warning: indeed, a Cholesky factorization (via the mf/msChol routine) should be more efficient and uses less memory.

See also: msQR, msRref, mfOut

## msLDLT $LDL^t$ factorization

 $Calling\ syntax:$ 

```
call msLDLT( mfOut(L,D,P), A )
```

### Description:

Computes numerically the  $LDL^t$  decomposition of a real and symmetric (indefinite) mfArray A, dense or sparse. Returns a lower triangular L, a matrix D (block diagonal 1x1 or 2x2) and a permutation matrix P such that: LDL' = P'AP.

Remark: if A is moreover positive definite, then the Cholesky factorization should be more appropriate!

N.B.: For sparse matrices:

- the matrix D is strictly diagonal;
- be aware that the  $LDL^t$  factorization can fail, especially if the original matrix has some zeros on the main diagonal (the fact that A is non-singular is necessary but not sufficient); in this case, use the LU factorization instead which makes pivoting.

See also: msLU, mf/msChol, mfOut

#### mf/msChol

### Cholesky factorization

Description:

Computes numerically the Cholesky factorization of the mfArray A, which must be symmetric and positive definite; if this latter property doesn't hold, MUESLI emits an error and returns an empty mfArray.

First calling syntax:

```
U = mfChol( A )
```

For dense matrices, returns an upper triangular  $\operatorname{mfArray}$  U, such that: U'U = A

Second calling syntax: For sparse matrices, the calling syntax is slightly different, because a permutation is first applied to the matrix A in order to reduce the fill-in:

```
call msChol( mfOut(L,p), A )
```

returns a lower triangular mfArray L and a permutation p such that: LL' = P'AP. The permutation p is always returned as a special mfArray (see mfPerm), thus the routines mf/msColPerm and mf/msRowPerm must be used to applied this permutation to the matrix A.

Third calling syntax: For sparse matrices only, there is also the possibility to factorize only, without copying the factors (L and P) in explicit mfArrays. In this case, the factors are internally stored (i. e. not available for the end-user) and can be referenced only via the mfMatFactor factors:

```
call msChol( mfOut(factors), A )
```

The mfMatFactor obtained is intended to be used only in mfLDiv.

See also: msLU, msLDLT, mfOut, msCholSpSymb, msCholSpNum

### msCholSpSymb

# Symbolic Cholesky factorization (sparse only)

### Description:

Computes the symbolic Cholesky factorization of the sparse mfArray A, which must be symmetric; if this latter property doesn't hold, MUESLI emits an error and returns an empty mfArray.

#### Interface:

```
subroutine msCholSpSymb( factor, A )
   type(mfArray),   intent(in) :: A
   type(mfMatFactor), intent(out) :: factor
end subroutine
```

After using this routine, the usual step is to call (once or more) the msCholSpNum routine, before solving a linear system with mfLDiv.

See also: mf/msChol

### msCholSpNum

# Numerical Cholesky factorization (sparse only)

#### Description:

After having called the symbolic factorisation (via the msCholSpSymb routine), computes the numerical Cholesky factorization of the sparse mfArray A, which must be positive definite; if this latter property doesn't hold, MUESLI emits an error and returns an empty mfArray.

#### Interface:

```
subroutine msCholSpNum( factor, A )
  type(mfArray),   intent(in)    :: A
  type(mfMatFactor), intent(in out) :: factor
end subroutine
```

After using this routine, the usual step should be to solve a linear system with mfLDiv.

See also: mf/msChol

#### mf/msBalance

### scaling to improve eigenvalue accuracy

The first form:

```
B = mfBalance( A )
```

returns an mfArray which is balanced, i. e. applies similarity transformations to A in order to make the rows and columns as close in norm as possible.

The second form:

```
call msBalance( mfOut(T,B), A )
```

also returns the (non singular) diagonal transformation matrix T, such that:  $B = T^{-1}AT$ 

Remark: there is no permutation applied to the matrix A. Moreover, the matrix A must be square.

See also: mfNorm, mfEig, mfOut

#### mf/msSVD

# singular value decomposition

Calling syntax:

```
S = mfSVD(A)
```

The function form returns only the singular values of the **mfArray** A (size m by n), in a column vector that is sorted from the greatest to the smallest; the size of this vector is  $\min(m, n)$  and its type is always real, whatever the type of A.

The subroutine form allows the user to access to the full decomposition:

```
call msSVD( mfOut(U,S,V), A [, economy_size ] )
```

which returns three mfArrays such that A = USV'. U and V are both unitary matrices whose columns contain, respectively, the left- and right-singular vectors of A. In this case, S is returned as a rank-2 array.

If the logical optional argument economy\_size is present and equal to TRUE, then U contains only the  $\min(m,n)$  first columns of the matrix U, and V contains only the  $\min(m,n)$  first columns of the matrix V. S is always a square matrix of order  $\min(m,n)$ .

Remarks: sparse matrices are not handled; please use mf/msSVDS instead.

See also: mfNorm, mfRank, mfDet, mfOut

mfInv, .i. matrix inverse

Calling syntax:

C = mfInv(A)

.i. A  $(A^{-1})$  is a shortcut for writing: mfInv( A )

Description:

Returns the inverse of the square mfArray A, mathematically noted  $A^{-1}$ .

If A is singular (or close to, ought to the machine  $\epsilon$ ), it returns an infinite mfArray of same shape.

For singular of non square matrices, the Moore-Penrose pseudo-inverse routine mfPseudoInv could also be used.

Remarks: sparse matrices are not handled; it is very unusual to have to explicitly compute the inverse of a matrix. If necessary, solving the system  $Ax = e_i$  for each basis vector  $e_i$  gives each column of  $A^{-1}$ ; but be aware that you will obtain a (nearly) dense matrix.

See also: mfPseudoInv, mfLDiv, mf/msSVD, mfRank

#### mfPseudoInv

### matrix pseudo-inverse

Calling syntax:

```
C = mfPseudoInv( A [, tol ] )
```

Description:

Returns the Moore-Penrose pseudo-inverse of the  ${\tt mfArray}\ {\tt A}.$ 

A may be non square or square singular.  ${\tt mfPseudoInv}$  returns a matrix, based on the SVD of A and any singular values less than the tolerance tol are treated as zero. The default tolerance is  ${\tt max}(m,n)\,||A||_2\,\epsilon$ , where m and n are respectively the number of rows and number of columns of the  ${\tt mfArray}\,$  A.

See also: mfInv, mf/msSVD, mfRank

### mfQR

### orthogonal-triangular decomposition

Calling syntax:

$$R = mfQR(A)$$

performs a QR factorization of the matrix A  $(m \times n$ , dense or sparse mfArray), but returns only the R factor (upper triangular matrix of size  $n \times n$ ) verifying R'R = A'A. This is the Q-less decomposition.

Remark for sparse matrices: The use of this routine should be restricted to small or moderate size sparse matrices because no ordering is applied to prevent the fill-in. For large sparse matrices, use the  ${\tt msQR}$  routine instead, which has also more options.

Note: The complex type is not yet supported for sparse matrices.

See also: msQR, msLU

#### msQR

### orthogonal-triangular decomposition

Calling syntax:

```
call msQR( mfOut(Q,R[,p,RANK]), A [, tol] )
```

applies the orthogonal-triangular decomposition to the mfArray A, dense or sparse, of size  $m \times n$ . It returns the two mfArrays Q and R such that A = QR.

Usually, Q is an orthogonal matrix (size  $m \times m$ ) and R is a upper-triangular matrix (size  $m \times n$ ).

If m > n, the economy size is always used: Q contains only the n first columns of the orthogonal full matrix Q (therefore of size  $m \times n$ ), and R is the square upper part (i. e. the non-zero part) of the full R (therefore of size  $n \times n$ ). This latter case is used for least-square problems.

Optionally, a column permutation vector p may be applied to the matrix A, such that QR = A(:,p):

- for the dense case, the use of this option leads to a factor R having the module of its diagonal terms in decreasing order;
- for the sparse case, the permutation is chosen to obtain a better sparsity for both Q and R factors.

If the mfArray RANK is present, it contains on return the numerical rank of the matrix A:

- for the dense case, a different method is used in order to compute safely the rank: it is the rank-revealing QR;
- for the sparse case, an estimation of the rank is returned.

If the argument tol (real double) is present, it specifies to which accuracy the rank is computed. Default value is the machine precision, MF\_EPS.

Other syntax: for sparse matrices only, the additional syntax can be used:

```
call msQR( mfOut(Qhouse,R[,p,RANK]), A[, tol] )
```

which returns the Householder vectors of the Q matrix in the mfMatFactor Qhouse variable, and the R triangular factor in the mfArray R. Although Qhouse doesn't contain explicitly the matrix Q, the Shape routine applied to it will returned the shape of Q, i.e. (m,m). The shape of R is (m,n).

This second form should be used for large size matrices, Qhouse being much more sparse than Q. The user is invited to use one of the routines mfQleft or mfQright in order to perform the multiplication of Q with another vector or matrix.

The other parameters have the same meaning as in the first calling syntax.

Note: The complex type is not yet supported for sparse matrices.

See also: mfQR, msLU, mfOut

### mfQleft

# left multiplication by a sparse Q factor

Calling syntax:

B = mfQleft( Qhouse, A )

returns the product Q'A.

The mfMatFactor Qhouse holds a sparse Q factor (obtained from a QR decomposition of a sparse matrix); the mfArray A may be a vector (dense) or a matrix (dense or sparse). B has the same structure (dense or sparse) than A.

By performing the operation:  $Q = .t. \ mfQleft(Qhouse, mfSpEye(m,m))$  the standard form of the Q factor can be obtained (but be aware that Qhouse is often much more sparse than Q).

Note: The complex case is not yet implemented.

See also: msQR, mfQright

### mfQright

# right multiplication by a sparse Q factor

 $Calling\ syntax:$ 

B = mfQright( A, Qhouse )

returns the product AQ.

The mfMatFactor Qhouse holds a sparse Q factor (obtained from a QR decomposition of a sparse matrix); the mfArray A may be a vector (dense) or a matrix (dense or sparse). B has the same structure (dense or sparse) than A.

By performing the operation: Q = mfQright( mfSpEye(m,m), Qhouse ) the standard form of the Q factor can be obtained (but be aware that Qhouse is often much more sparse than Q).

Note: The complex case is not yet implemented.

See also: msQR, mfQleft

# mf/msEig

### eigenvalues and eigenvectors

The first form:

$$E = mfEig(A)$$

returns the eigenvalues of the mfArray A in the vector mfArray E.

The subroutine form:

```
call msEig( mfOut(V,D), A )
```

returns two mfArrays. D is a diagonal matrix containing the eigenvalues and V is the matrix whose columns are the corresponding eigenvectors so that: AV = VD (or, equivalently, A = VDV').

#### Remarks:

- Sparse matrices are not handled; please use mf/msEigs instead.
- Eigenvalues are returned in ascending order, according to the magnitude for complex ones.

See also: mf/msEigs, mf/msSchur, mfOut

#### mf/msHess

### Hessenberg form

The first form:

```
H = mfHess(A)
```

computes the Hessenberg form of the mfArray A. The returned mfArray has the same eigenvalues as A but contains zeros below the first subdiagonal.

The subroutine form:

```
call msHess ( mfOut(P,H), A )
```

returns two mfArrays. H is the Hessenberg form and P is a unitary matrix so that: A = PHP'.

Remarks: sparse matrices are not (yet) handled.

See also: mfOut

#### mf/msSchur

#### Schur decomposition

Calling syntax:

```
call msSchur ( mfOut(U,T), A [, form ] )
```

produces a upper triangular Schur matrix T and a unitary matrix U so that: A = UTU'. A must be square.

For a real, dense mfArray A the Schur matrix T is generally quasi-triangular (real form by default, see below).

The optional character argument form may be equal to "real" or "complex".

When processing a complex, dense mfArray A, or when the optional form string is equal to "complex", the matrix T is complex, (strictly) upper triangular.

Other syntax:

just returns the T factor.

Remarks:

in the real form, the matrix T contains 1-by-1 and 2-by-2 diagonal blocks: the 2-by-2 blocks correspond to complex conjugate pairs of eigenvalues of A.
 Each 2-by-2 block is in the form:

$$\begin{pmatrix} a & b \\ c & a \end{pmatrix}$$

where b c < 0. The eigenvalues of such a block are  $a \pm \sqrt{b c}$ .

- the Schur decomposition is intended to be used for non-symmetric eigenvalue problems; if the mfArray A is symmetric/hermitian, it is preferable to compute eigenvalues/vectors via the mf/msEig routines (in such a case, V and D mfArrays returned by mf/msEig correspond exactly to U and T). A warning is therefore emitted if A is symmetric or hermitian.
- sparse matrices are not (yet) handled.

See also: mfOut

mfLDiv, .ix.

left matrix divide

 $Calling\ syntax:$ 

```
x = mfLDiv(A, b)
```

A .ix. b  $(A^{-1}b)$  is a shortcut for writing: mfLDiv(A, b)

Description: solves a linear system of the form Ax = b, i. e. returns  $x = A^{-1}b$  without computing  $A^{-1}$ .

The mfArray A (of size  $m \times n$ ) may be real or complex, sparse or dense, and doesn't need to be square; it can be even rank deficient (tests are made internally to use the most appropriate method):

- if m > n the mean-square solution is returned;
- if m < n the minimum 2-norm solution is returned.

This routine automatically detects if A is triangular and chooses the appropriate method.

Multiple right hand sides may be simultaneously solved, by concatenating all the RHS vectors in one rank-2 mfArray b. This feature is not available for sparse matrices A (but see below the way of solving simultaneously equations by using intermediate factors).

*Note:* For the sparse complex case, only square matrices A are supported.

Using this routine is always more efficient than computing the inverse of the matrix A first followed by the product with b.

Remarks:

- After doing an LU factorization of the (dense) matrix A, this routine can also be called to solve the linear system Ax = PLUx = b, by using the following syntax:

```
x = mfLDiv( L, U, P, b[, option="transp" ] )
```

The optional last argument allows the user to solve the system A'x = b (transpose in the real case, conjugate transpose in the complex case).

- After doing a Cholesky factorization of the (dense) matrix A via mfChol, this routine can also be called to solve the linear system Ax = U'Ux = b, by using the following syntax:

- Similarly,

$$x = mfLDiv(L, U, b)$$

is used for a sparse matrix. As noted in the  ${\tt msLU}$  page, the  ${\tt mfArray}$  L factor contains a pointer to an internal structure containing all necessary data concerning the LU factorization. Note that the RHS b cannot have here multiple columns.

- For using with an mfMatFactor factors (from, e.g., msLU or mf/msChol), the routine must be called as:

```
x = mfLDiv( factors, b[, option="transp" ] )
```

This latter interface allows the user to simultaneously solve multiple RHS. Moreover, for this case only, the RHS b may be dense or sparse. The optional last argument allows the user to solve the system A'x = b.

- When a (near) singular matrix is encountered, a *Warning* message may be emitted, according to the message level parameter (see msSetMsgLevel to change this level).

See also: mfRDiv

mfRDiv, .xi.

#### right matrix divide

 $Calling\ syntax:$ 

```
x = mfRDiv(b, A)
```

b .xi. A  $(bA^{-1})$  is a shortcut for writing: mfRDiv(b, A)

Description: solves linear system of the form x A = b, i. e. returns  $x = b A^{-1}$  without computing  $A^{-1}$ .

The mfArray A (of size  $m \times n$ ) may be real or complex, sparse or dense, and doesn't need to be square; it can be even rank deficient (tests are made internally to use the most appropriate method):

- if n > m the least-square solution is returned;
- if n < m the minimum 2-norm solution is returned.

This routine automatically detects if A is triangular and chooses the appropriate method.

Multiple right hand sides may be simultaneously solved, by concatenating all the RHS vectors in one rank-2 mfArray b. This feature is not available for sparse matrices A (you should use mfLDiv instead, by using transposition).

*Note:* For the sparse complex case, only square matrices A are supported.

Using this routine is always more efficient than computing the inverse of the matrix A first followed by the product with b.

#### Remarks:

- After doing an LU factorization of the (dense) matrix A, this routine can also be called to solve the linear system xA = xPLUx = b, by using the following syntax:

```
x = mfRDiv(b, L, U, P)
```

- After doing a Cholesky factorization of the (dense) matrix A via mf/msChol, this routine can also be called to solve the linear system x A = x U' U = b, by using the following syntax:

```
x = mfRDiv( b, U, form="cholesky" )
```

- Similarly,

$$x = mfRDiv(L, U, b)$$

is used for a sparse matrix. As noted in the  ${\tt msLU}$  page, the  ${\tt mfArray}$  L factor contains a pointer to an internal structure containing all necessary data concerning the LU factorization. Note that the RHS b cannot have here multiple columns.

- When a (near) singular matrix is encountered, a *Warning* message may be emitted, according to the message level parameter (see msSetMsgLevel to change this level).

See also: mfLDiv

#### msRref

### reduced row echelon form

Calling syntax:

```
call msRref ( mfOut(R,jpiv), A [, tol ] )
```

computes the reduced row echelon form of the mfArray A.

The optional real tol is the tolerance (should be positive) used to compute the rank of the matrix. This tolerance cannot be too small. Minimum is the machine precision. When it is known, use the precision of the elements of the matrix A.

Note that the rank provided by this method is only an approximation which may differ from that computed with other methods ( $e.\,g.$ , the Singular Value Decomposition msSVD). However, this approach is pedagogically interesting.

See also: mfRank, mfOrth, mfNull, msQR, mfOut

mfNull null space

Calling syntax:

```
B = mfNull( A [, rational, tol ] )
```

returns an orthonormal basis for the null space of the mfArray A.

The optional logical rational argument specifies whether the rational basis is expected or not.

The optional real tol argument is the tolerance (should be positive) used to compute the rank of the matrix. As mfNull calls msRref, tol cannot be less than the machine  $\epsilon$ .

See also: mfRank, mfOrth, msRref, msQR, msSVD

### mfOrth

# ${\bf matrix} \,\, {\bf orthogonalization} \,\,$

Calling syntax:

B = mfOrth( A )

returns an orthonormal basis for the range of the  ${\tt mfArray}\ {\tt A}.$ 

 $See~also:~{\tt mfRank},~{\tt mfNull},~{\tt msRref},~{\tt msQR},~{\tt msSVD}$ 

# ${\tt mfExpm}$

# matrix exponential

Calling syntax:

B = mfExpm(A)

computes the matrix exponential of the  ${\tt mfArray}\ {\tt A}$  (dense only).

 $See~also:~{\tt mfExp},~{\tt mfLogm},~{\tt mfSqrtm},~{\tt mfPowm},~{\tt mfFunm}$ 

# mfLogm matrix logarithm

Calling syntax:

B = mfLogm(A)

computes the matrix logarithm of the  ${\tt mfArray}\ {\tt A}$  (dense only).

 $See \ also: \ \mathtt{mfLog}, \ \mathtt{mfExpm}, \ \mathtt{mfSqrtm}, \ \mathtt{mfPowm}, \ \mathtt{mfFunm}$ 

# ${\tt mfSqrtm}$

# matrix square root

 $Calling\ syntax:$ 

B = mfSqrtm( A )

computes the matrix square root of the  ${\tt mfArray}\ {\tt A}$  (dense only).

The property BB=A doesn't always hold, because A may not have a square root.

 $See\ also: \mathtt{mfSqrt}, \mathtt{mfExpm}, \mathtt{mfLogm}, \mathtt{mfPowm}, \mathtt{mfFunm}$ 

mfPowm matrix power

Calling syntax:

$$B = mfPowm(A, e)$$

computes the matrix power of the mfArray A, i. e.  $A^e$ .

If e is integer, then A may have a sparse or a dense structure. Any positive integer is accepted.

If e is real, then A must have a dense structure. Any real value for e is accepted.

See also: mfPow10, mfExpm, mfLogm, mfSqrtm, mfFunm

#### mf/msFunm

### general matrix function

Calling syntax:

```
F = mfFunm( A, fun )
```

applies the matrix function fun to the mfArray A (dense only).

fun is either a character string giving the name of a usual function (as sin, cos, asin, ...), or the name of a user function. In this latter case, the user function must have the following interface:

```
function fun(z1) result(z2)
  complex(kind=MF_DOUBLE) :: z1, z2
end function
```

The subroutine form, which has the calling syntax:

```
call msFunm( mfOut(F,e), A, fun )
```

returns also an estimation of the error in the scalar mfArray e. Note that in the case of the exponential function (fun set to "exp"), the mfExpm is directly called, so that the error estimation will be not available (the mfArray e will be empty).

Remark: Use mfFun to apply the function fun in a element-wise manner.

See also: mfExpm, mfLogm, mfSqrtm, mfPowm, mfOut

### mf/msEigs

#### few eigenvalues and eigenvectors

The first form:

```
E = mfEigs( A, k [, which|sigma] [, tol, ncv] )
```

returns few eigenvalues of the matrix A using ARPACK in the vector mfArray E. A may be real/complex, sparse/dense.

The number of requested eignevalues k must be ranged in [2, n-2].

which is a character(len=2) optional argument, can take the values "LM" or "SM", depending on whether the Largest or the Smaller Magnitude eigenvalues/vectors are requested. In addition, which can take the following values:

- for real, symmetric matrices:

"LA" = Largest Algebraic, with an algebraic selection;

"SA" = Smallest Algebraic, with an algebraic selection;

"BE" = Both Ends, with an algebraic selection (half from the largest ones and half from the smallest ones.

- for real, unsymmetric matrices:

"LR" = Largest Real part, with an algebraic selection;

"SR" = Smallest Real part, with an algebraic selection;

"LI" = Largest Imaginary part, with a  $magnitude^1$  selection;

"SI" = Smallest Imaginary part, with a  $magnitude^1$  selection;

for complex matrices:

"LR" = Largest Real part, with an algebraic selection;

"SR" = Smallest Real part, with an algebraic selection;

"LI" = Largest Imaginary part, with a algebraic selection;

"SI" = Smallest Imaginary part, with a algebraic selection;

Eigenvalues are generally returned in a ascending order, excepted for the "LM" and "LA" cases, for which they are returned in a descending order.

sigma is a complex optional argument. If present, mfEigs tries to find the k eigenvalues closest to the complex sigma.

If tol is present, it indicates the tolerance of the computed eigenvalues. By default, the machine epsilon is taken.

If ncv is present, it indicates how many Arnoldi vectors are generated (following the ARPACK documentation, the minimum value for ncv is 2\*k+1 and the maximum value is the order n of the matrix; a warning is emitted if these conditions are not fulfilled).

If ncv is not present, the mfEigs routine tries larger and larger values of ncv, until (hopefully) the convergence of all requested eigenvalues.

In both cases (ncv present or not present), a warning is issued if not all eigenvalues converged. Use the subroutine form, explained below, to obtain a quiet behavior.

.../...

<sup>&</sup>lt;sup>1</sup>in order to keep adjacent the complex conjugate pairs.

The subroutine form:

```
call msEigs( mfOut(V,E,flag), A, k [, which|sigma] [, tol, ncv] )
```

returns the requested eigenvalues in the vector mfArray E, the corresponding eigenvectors in the columns of the mfArray V, and the convergence of the iterative process. If the boolean mfArray flag is TRUE then all the eigenvalues converged; otherwise not all converged. Note that non-converged eigenvalues/eigenvectors are set to NaN.

The remaining arguments are processed as described above.

See also: mf/msEig, mf/msSVDS, mfOut

mf/msSVDS

#### few singular values

The first form:

```
S = mfSVDS( A, k [, which, tol, ncv ] )
```

returns few singular values of the matrix A using ARPACK in the vector mfArray S. A may be real/complex, sparse/dense; it doesn't need to be square.

k, the number of required singular values, cannot be larger than  $\min(m, n) - 2$ . If so, NaN values are used to fill the vector S.

which is a character(len=2) optional argument, which must be equal to "LM", "SM" or "BE". If present, mfSVDS tries to find the k singular values of largest magnitude ("LM"), or smallest magnitude ("SM"). Only for real matrices, "BE" allows to compute the both-end singular values. Default value is "LM".

If tol is present, it indicates the tolerance of the computed singular values. By default, the machine epsilon is taken.

If ncv is present, it indicates how many Arnoldi vectors are generated (following the ARPACK documentation, the minimum value for ncv is 2\*k+1 and the maximum value is  $\min(m,n)$  of the matrix; a warning is emitted if these conditions are not fulfilled).

If ncv is not present, the mfSVDS routine tries larger and larger values of ncv, until (hopefully) the convergence of all requested singular values.

In both cases (ncv present or not present), a warning is issued if not all singular values converged. Use the subroutine form, explained below, to obtain a quiet behavior.

The subroutine form:

```
call msSVDS( mfOut(S,flag), A, k [, which, tol, ncv ] )
```

returns the requested singular values in the vector mfArray S and the convergence of the iterative process. If the boolean mfArray flag is TRUE then all the singular values converged; otherwise not all converged. The remaining arguments are processed as described above.

See also: mf/msSVD, mf/msEigs, mfOut

# mfNormEst, mfNormEst1

### norm estimations

Syntax:

```
C = mfNormEst( A [, tol ] )
```

### Description:

Returns an estimation of the 2-norm of a rank-2  $\mathtt{mfArray}$  (dense or sparse, real or complex, square or non-square).

The optional argument allows the user to choose the tolerance used in the iterative algorithm. By default, tolerance is  $10^{-6}$ .

Syntax:

```
C = mfNormEst1( A )
```

returns an estimation of the 1-norm of a square matrix (sparse or dense).

See also: mfNorm

#### mfCondEst

# matrix condition number estimation

Calling syntax:

C = mfCondEst( A )

Description:

Returns an estimation of the condition number of the rank-2 mfArray A.

Remarks: The condition number is always greater than 1. A value equal to infinity (resp. close to the inverse of the  $\epsilon$  machine) indicates that the matrix A is singular (resp. nearly singular).

See also: mfCond, mfRCond

### mfIsSymm

# matrix symmetry inquiry

Calling syntax:

returns a boolean value according to the symmetry property of the mfArray A. If the matrix is complex, this property means hermitian.

For a sparse mfArray A, if the optional argument option is present and equal to "pattern", then the routine checks only for the symmetry of the sparse structure.

If the optional tolerance tol is present, the symmetry test (for the real case) is made according to the rule:

$$||A - A'||_1 < 2 \operatorname{tol} ||A||_1$$

Default tolerance is MF\_EPS.

See also: mfIsPosDef, mfTolForSymm

# ${\tt mfTolForSymm}$

# tolerance for symmetry

Calling syntax:

tol = mfTolForSymm( A )

returns the tolerance for which the  ${\tt mfArray}\ {\tt A}$  should be symmetric.

 $See \ also: {\tt mfIsSymm}$ 

### mfIsPosDef

# definite positiveness inquiry

Calling syntax:

bool = mfIsPosDef( A )

returns a boolean value according to the definite positiveness property of the  ${\tt mfArray}$  A.

 $See \ also: {\tt mfIsSymm}$ 

# ${\tt mfIsDiagDomCol}$

# diagonally dominant by cols inquiry

 $Calling\ syntax:$ 

bool = mfIsDiagDomCol( A )

returns a boolean value according to the "diagonally dominant by cols" property of the  ${\tt mfArray}$  A.

 $See \ also: \verb|mfIsStrictDiagDomCol||$ 

# mfIsStrictDiagDomCol

# strictly diagonally dominant by cols inquiry

 $Calling\ syntax:$ 

bool = mfIsStrictDiagDomCol( A )

returns a boolean value according to the "strictly diagonally dominant by cols" property of the  ${\tt mfArray}$  A.

 $See \ also: {\tt mfIsDiagDomCol}$ 

#### mfIsFullRank

# full rank inquiry

Calling syntax:

bool = mfIsFullRank( A [, tol ] )

returns '.true.' if the mfArray A is full rank, i. e. if its rank is equal to the minimum of its dimensions.

If to1, of type real, is present then this value is used as tolerance in the rank computation. Default tolerance is chosen as  $\max(m,n)||A||\epsilon$ .

See also: mfRank

#### MF\_LAPACK\_VERSION

# LAPACK version

Calling syntax:

string = MF\_LAPACK\_VERSION()

returns the LAPACK version used during the link of the executable.

Note the parenthesis used after the name of the variable, because it is implemented as a function.

The returned string may be used by the mfIsVersion boolean function.

 $See \quad also: \quad \texttt{MF\_COMPILER\_VERSION}, \quad \texttt{MF\_MUESLI\_VERSION}, \quad \texttt{msGetBlasLib}, \quad \texttt{msGetLapackLib}, \\ \texttt{msGetArpackInfo}, \\ \texttt{msGetSuiteSparseLib}$ 

#### msGetBlasLib

# Get BLAS library type

Calling syntax:

call msGetBlasLib( string )

returns the BLAS library type used during the link of the executable, that is, either Reference for most compilers or a specific package for others ( $e.\,g.\,MKL$  for the INTEL compiler suite; OpenBLAS or ATLAS for the GNU compiler suite).

string is a character string of length at least 128.

The current routine is able to detect the dynamic switch between BLAS implementations like OpenBLAS and ATLAS.

 $See \quad also: \quad {\tt MF\_COMPILER\_VERSION}, \quad {\tt MF\_MUESLI\_VERSION}, \quad {\tt MF\_LAPACK\_VERSION}, \quad {\tt msGetLapackLib}, \\ {\tt msGetArpackInfo}, \\ {\tt msGetSuiteSparseLib}$ 

### msGetLapackLib

# Get LAPACK library type

 $Calling\ syntax:$ 

call msGetLapackLib( string )

returns the LAPACK library type used during the link of the executable, that is, either Reference for most compilers or a specific package for others ( $e.\,g.\,MKL$  for the INTEL compiler suite or ATLAS for the GNU compiler suite).

string is a character string of length at least 128.

The current routine is able to detect the dynamic switch between BLAS implementations like Reference and ATLAS.

 $See \quad also: \quad {\tt MF\_COMPILER\_VERSION}, \quad {\tt MF\_MUESLI\_VERSION}, \quad {\tt MF\_LAPACK\_VERSION}, \quad {\tt msGetBlasLib}, \\ {\tt msGetArpackInfo}, \\ {\tt msGetSuiteSparseLib}$ 

## msGetArpackInfo

# Get ARPACK library information

 $Calling\ syntax:$ 

```
call msGetArpackInfo( version[, info] )
```

version is a character string of length at least 24, containing on return the version of the library. The "NG" letters indicates the *new generation* version, which can be found at: http://forge.scilab.org/index.php/p/arpack-ng/.

info is an optional character string of length at least 128, containing on return additional information, especially various non official fixes applied to the library.

 $See \quad also: \quad {\tt MF\_COMPILER\_VERSION}, \quad {\tt MF\_MUESLI\_VERSION}, \quad {\tt MF\_LAPACK\_VERSION}, \quad {\tt msGetBlasLib}, \\ {\tt msGetLapackLib}, \\ {\tt msGetSuiteSparseLib}$ 

# ${\tt msGetSuiteSparseLib}$

# Get SuiteSparse library version

Calling syntax:

call msGetSuiteSparseLib( version )

version is a character string of length at least 8, containing on return the version of the library.

The returned  ${\tt string}$  may be used by the  ${\tt mfIsVersion}$  boolean function.

 $See \quad also: \quad {\tt MF\_COMPILER\_VERSION}, \quad {\tt MF\_MUESLI\_VERSION}, \quad {\tt MF\_LAPACK\_VERSION}, \quad {\tt msGetBlasLib}, \\ {\tt msGetLapackLib}, \\ {\tt msGetArpackInfo}$ 

## 1.9 Polynomial Functions

mfPolyVal mfPolyFit, msPolyFit mfFunFit, msFunFit

mfRoots mfPoly

mfSpline, msSpline

mfLegendre mfPPVal mfPPDer

mfInterp1 mfInterp2

mfPSLG

msPrintPSLG
mfDelaunay
mfTriConnect
msBuildTriConnect
msUpdateTriConnect
msExtractTriConnect
msCheckDomainConvexity

msPrintTriConnect
msTriNodeNeighbors
mfTriSearch, mfNodeSearch

mfVoronoi
mfVoronoiStruct
msPrintVoronoi
mfGridData

mfDelaunay3D
msEndDelaunay3D
mfTetraConnect
msBuildTetraConnect
msExtractTetraConnect
msPrintTetraConnect
msDel3DNodeNeighbors

msDel3DNodeNeighbors mfTetraSearch, mfNodeSearch3D mfGridData3D

miGridData3D

polynomial evaluation polynomial fitting nonlinear function fitting roots of a polynomial

polynomial with specified roots

cubic spline interpolation or smoothing

Legendre polynomials

piecewise polynomial evaluation piecewise polynomial derivative

1D interpolation

regular 2D interpolation

Planar Straight Line Graph (derived type)
Planar Straight Line Graph display
Delaunay 2D tessellation (triangulation)
2D triangular connectivity (derived type)
2D triangular connectivity initialization
2D triangular connectivity update

get components from a mfTriConnect structure

Check convexity of a 2D triangulation 2D triangular connectivity display 2D triangulation node neighbors

2D triangulation search

2D Voronoi diagram from a set of points 2D Voronoi structure (derived type) 2D Voronoi structure display irregular 2D interpolation

Delaunay 3D tessellation (tetrahedralization) Internal free storage after Delaunay 3D 3D tetrahedral connectivity (derived type) 3D tetrahedral connectivity initialization

get components from a mfTetraConnect structure

3D tetrahedral connectivity display 3D tetrahedralization node neighbors

3D tetrahedralization search irregular 3D interpolation

### See also:

Core Routines

File Input/Output

**Data Analysis Functions** 

Operators

Elementary Math Functions

Specialized Math Functions

Elementary Matrix Manipulation Functions

Matrix Functions

Optimization and Function Functions

Sparse Matrices

# mfPolyVal

# polynomial evaluation

Interface:

```
function mfPolyVal( p, x ) result( out )
  type(mfArray), intent(in) :: p, x
  type(mfArray) :: out
```

Description:

Evaluates polynomial p at points x.

The  $mfArray\ p$  is a vector of length k+1 whose elements are the coefficients of the polynomial in descending powers.

$$out = p_1 x^k + p_2 x^{k-1} + \dots + p_k x + p_{k+1}$$

See also: mfPPVal, mfPolyFit

mfPolyFit

polynomial fitting

Interface:

```
function mfPolyFit( x, y, n ) result( p )
  type(mfArray), intent(in) :: x, y
  integer, intent(in) :: n
  type(mfArray) :: p
```

Description:

Fit polynomial to real data.

The mfArray x and y must be column vectors of same shape. They are coordinates of the data. n is the degree of the polynomial to be obtained, so n + 1 real coefficients are returned in the vector p. These coefficients are sorted for descending powers.

 $See \ also: {\tt msPolyFit}, {\tt mfPolyVal}, {\tt mfFunFit}$ 

# msPolyFit

# polynomial fitting

Calling syntax:

Description:

Similar to mfPolyFit, but also returns (in mfArrays) the norm-2 of the residuals (in normr) and optionally the correlation coefficient  $r_{COTT}^2$  (in r2).

Nota: The correlation coefficient of the approximation is computed as follows:

$$r_{\rm COTT}^2 = 1 - \frac{{\tt normr}^2}{M \; \sigma^2} \label{eq:corr}$$

where  $\sigma^2$  is the standard deviation (as computed by mfVar(y)) and M is the number of data points in x and y.

 $See \ also: {\tt mfPolyVal}, {\tt msFunFit}, {\tt mfOut}$ 

#### mfFunFit

### nonlinear function fitting

 ${\it Interface:}$ 

```
function mfFunFit( x, y, fun, p0, n, tol ) result( out )
   type(mfArray), intent(in) :: x, y, p0
   interface
      function fun( x, p, n ) result( res )
         import :: MF_DOUBLE
         \verb"real(kind=MF_DOUBLE)", intent(in) :: x
         real(kind=MF_DOUBLE), intent(in) :: p(n)
         integer,
                               intent(in) :: n
         real(kind=MF_DOUBLE) :: res
      end function fun
   end interface
                         intent(in) :: n
   integer,
   real(kind=MF_DOUBLE), intent(in), optional :: tol
   type(mfArray) :: out
```

### Description:

Fit arbitrary (nonlinear) function to real data.

Data is stored in the mfArrays x and y, which must be column vectors of same shape. There are m couples (x, y).

fun is the real function which tries to model the data. This function is parameterized by n variables stored in the mfArray p.

At input, p0 contains an initial guess for the values of the n unknown parameters. Final values are stored in the mfArray out.

tol is the tolerance for the convergence. Its default value is  $10^{-6}$ .

#### Remarks:

- the problem is formulated as a non-linear least-square problem, solved by package MINPACK;
- therefore, m (the number of couples (x,y)) must be bigger than the number of parameters n.

See also: msFunFit, mfPolyFit, mfLsqNonLin

#### msFunFit

### nonlinear function fitting

 $Calling\ syntax:$ 

```
call msFunFit( mfOut(p[, r2]), x, y, fun, p0, n [, tol ] )
```

Description:

Similar to mfFunFit, but in addition it returns the correlation coefficient  $r_{corr}^2$  (in the mfArray r2).

For this subroutine, the output of the desired n variables is stored in the mfArray p.

Remark: Please note that the correlation coefficient  $r_{\text{COTT}}^2$  is ranged in the interval [-1, +1]. A value close to +1 indicates a perfect correlation while a null value indicates no correlation. A negative value means that the relationship between original data and fitted data is very bad most of time (practically, this should indicates that your function fun is not adapted to the data).

See also: mfOut

#### mfRoots

# roots of a polynomial

Calling syntax:

r = mfRoots(p)

Description:

Returns the roots of the polynomial whose coefficients are provided in the  $\mathtt{mfArray}$  p (row or column vector).

This polynomial writes:

$$P(x) = p_1 x^{n-1} + p_2 x^{n-2} + \dots + p_{n-1} x + p_n$$

where  $[p_1, p_2, \dots, p_n]$  are the *n* components of the input vector **p**.

Even if the roots are real, the returned mfArray is complex. The roots are not sorted.

Remarks:

- p must be of type real.
- The mfPoly function is the inverse of the mfRoots function.

 $See \ also: \mathtt{mfCompan}, \mathtt{mfEig}, \mathtt{mfFZero}$ 

# mfPoly

# polynomial with specified roots

Calling syntax:

$$p = mfPoly(r)$$

Description:

Returns the coefficients of the polynomial whose roots are provided in the mfArray r (row or column vector).

If n is the size of  ${\tt p}$  then the output polynomial reads:

$$P(x) = p_1 x^{n-1} + p_2 x^{n-2} + \dots + p_{n-1} x + p_n$$

Remarks:

- The specified roots in **r** may be complex.
- $-\,$  The  ${\tt mfRoots}$  function is the inverse of the  ${\tt mfPoly}$  function.

 $See \ also: {\tt mfCompan}, {\tt mfEig}$ 

### mfSpline

### cubic spline interpolation

#### Interface:

```
&
function mfSpline(x, y,
                                                                       &
                   BC_type_1, BC_val_1, BC_type_2, BC_val_2,
                                                                       &
                   periodic )
                                                                       &
result( y_sec )
   type(mfArray),
                         intent(in)
                                               :: x, y
                         intent(in), optional :: xi
   type(mfArray),
                         intent(in), optional :: BC_type_1
   integer,
   real(kind=MF_DOUBLE), intent(in), optional :: BC_val_1
                         intent(in), optional :: BC_type_2
   integer,
   real(kind=MF_DOUBLE), intent(in), optional :: BC_val_2
                         intent(in), optional :: periodic
   logical,
   type(mfArray) :: y_sec
```

#### Description:

If xi is not present, this function returns the coefficients of the spline (piecewise cubic polynomial form) which interpolates the values y at the abscissas x. The returned coefficients in y\_sec (second derivative at abscissas) are intended to be used with the mfPPVal function.

If xi is present, mfSpline returns only the interpolated values at the entries contained in the mfArray xi.

By default, the spline form is natural, which means that the first and the last coefficients are null (i.e. curvature is null at both ends). Optionally, boundary conditions of various (and mixed) types may be specified at the ends of the interval:

```
BC_type_1 = 1 implies that y' = BC_val_1 at first end;

BC_type_1 = 2 implies that y'' = BC_val_1 at first end;

BC_type_2 = 1 implies that y' = BC_val_2 at second end;

BC_type_2 = 2 implies that y'' = BC_val_2 at second end;
```

If periodic is present and equal to TRUE, then the value of y, along with the first and second derivatives are equal at the two ends of the interval. Default is FALSE.

#### Remarks:

- x and y must be vectors having the same shape, but xi may have any shape.
- Abscissas x doesn't need to be equally spaced; x must contain at least 2 points.
- -x, y and xi may be temporary mfArrays.
- interpolated values, at abscissas different than the nodes in x may be obtained by using the mfPPVal function.
- smoothing of data may be obtained by using the  ${\tt msSpline}$  routine.

### msSpline

### cubic spline smoothing

 ${\it Interface:}$ 

#### Description:

Similar to the mfSpline function (in particular the last four optional arguments), but apply a smoothing to the data, instead of a strict interpolation. The new (smoothed) values of y are stored in the y\_smooth mfArray, and the cubic spline coefficients are stored in the y\_sec mfArray.

#### Remarks:

- The weights mfArray contains the weight of the smoothing, to be applied to each value of the y mfArray; it may be a scalar: in such a case, all weights are equal. Weights must be strictly positive reals from 0 to infinity. If some weights are less or equal to zero, this will lead to an error.
- When choosing infinite weights, this routine gives the same results than the mfSpline function,
   i. e. a strict interpolation of the data; on the other end, choosing very small weights leads to the least-square linear regression of all the data points.
- Typically, the values of the weights vary according to the powers of ten. If the abscissas are not equally spaced, you may choose to scale your weights with  $h^3$ , where h is the interval between the x values.

See also: mfOut

# ${\tt mfLegendre}$

# Legendre polynomials

 $Calling\ syntax:$ 

C = mfLegendre( n, A )

### Description:

Returns the evaluation of the n-th Legendre polynomial (degree n) of each element of the mfArray A. Restriction: A must be of type real.

#### mfPPVal

### piecewise polynomial evaluation

### ${\it Interface:}$

## Description:

Computes the interpolated values at the entries xi.

The way this function treats out-of-range input values in **xi** is similar to that of **mfInterp1** (see the remarks below).

p is the vector of Spline coefficients – of the curve (x,y) – returned by mfSpline.

#### Remarks:

- -x, y, p and xi may be temporary mfArrays.
- xi mfArray may have any shape.
- By default, an element of yi is set to MF\_NAN if the corresponding value in xi is outside the range of x. The optional argument extrapol may be used to override this behaviour: a real value allows the use of a constant for out-of-range inputs. More interestingly, a logical value can be used to compute an extrapolated value by using the interpolation law of the nearest valid interval.

See also: mfPolyVal, mfPPDer

#### mfPPDer

# piecewise polynomial derivative

# ${\it Interface:}$

# Description:

Similar to  ${\tt mfPPVal}$  but returns the derivative instead of the function value.

p is the vector of Spline coefficients – of the curve (x,y) – returned by mfSpline.

### mfInterp1

### 1D interpolation

Interface:

#### Description:

Interpolates data y, which are defined along a vector x, at the entry(ies) specified by xi.

x and y must be vectors with the same shape; x must have strictly monotonous values.

xi may be a scalar or a vector; yi, which contains interpolated data, will have the same shape as xi.

#### Remarks:

- By default a linear interpolation (2-point stencil) is used (order=1); set the optional argument order to 2 (resp. 3) in order to use a quadratic (resp. cubic) interpolation using a 3-point (resp. 4-point) stencil. A nearest interpolation (1-point stencil) is also possible: set order to 0.
- By default, an element of yi is set to MF\_NAN if the corresponding value in xi is outside the range of x. The optional argument extrapol may be used to override this behaviour: a real value allows the use of a constant for out-of-range inputs. More interestingly, a logical value can be used to compute an extrapolated value by using the interpolation law of the nearest valid interval.

See also: mfSpline, mfInterp2, mfGridData, msMeshGrid

```
Example(s):
```

```
x = mfLinspace(0.0d0, 1.0d0, 5)
   call msDisplay( x, "x" )
   print *, "y = x**2"
   y = x**2
   xi = mfLinspace(0.05d0, 0.95d0, 4)
   call msDisplay( xi, "xi" )
   yi = mfInterp1( x, y, xi, order=2 )
   call msDisplay( yi, "interpolated data at many points" )
output:
x =
    0.0000
              0.2500
                        0.5000
                                  0.7500
                                            1.0000
 y = x**2
 xi =
    0.0500
              0.3500
                        0.6500
                                  0.9500
 interpolated data at many points =
   0.0025
              0.1225
                        0.4225
                                  0.9025
```

 $\cdots$ /...

```
xi = mfLinspace(-0.15d0, 1.15d0, 7)
   call msDisplay( xi, "xi" )
  yi = mfInterp1( x, y, xi, order=2 )
   call msDisplay( yi, "interpolated data at many points (2 NaNs expected)" )
output:
xi =
   -0.1500
             0.0667
                       0.2833
                                 0.5000
                                           0.7167
                                                     0.9333
                                                            1.1500
 interpolated data at many points (2 NaNs expected) =
             0.0044
                       0.0803
                                 0.2500
     NaN
                                           0.5136
                                                     0.8711
                                                                NaN
   yi = mfInterp1( x, y, xi, order=2, extrapol=MF_INF )
   call msDisplay( yi, "interpolated data at many points (2 Infs expected)" )
output:
 interpolated data at many points (2 Infs expected) =
             0.0044
                       0.0803
                                 0.2500
                                           0.5136
                                                     0.8711
                                                                Inf
   yi = mfInterp1( x, y, xi, order=2, extrapol=.true. )
   call msDisplay( yi, "interpolated data at many points (using extrapolation)" )
output:
 interpolated data at many points (using extrapolation) =
   0.0225
           0.0044 0.0803 0.2500 0.5136 0.8711
                                                              1.3225
```

### mfInterp2

### regular 2D interpolation

### ${\it Interface:}$

#### Description:

Interpolates data z, which are defined on a rectangular, horizontal grid (x,y), at the entry(ies) specified by xi and yi.

x, y and z must be matrices with the same shape; x, y must have strictly monotonous values. Usually, x and y are built with the routine msMeshGrid.

(xi,yi) also must have the same shape (scalars, vectors, or matrices); zi, which contains interpolated data, will have the same shape as xi and yi.

#### Remarks:

- By default a bilinear interpolation (4-point stencil) is used (order=1); set the optional argument order to 2 (resp. 3) in order to use a biquadratic (resp. bicubic) interpolation using a 9-point (resp. 16-point) stencil. A nearest interpolation (1-point stencil) is also possible: set order to 0.
- Note that the method used is valid only for rectangular, horizontal grid. For a general, irregular grid, see mfGridData.
- By default, an element of zi is set to MF\_NAN if the corresponding value in (xi,yi) is outside the range of x and y. The optional argument extrapol may be used to override this behaviour: a real value allows the use of a constant for out-of-range inputs. More interestingly, a logical value can be used to compute an extrapolated value by using the interpolation law of the nearest valid interval.

See also: mfInterp1, msMeshGrid

#### mfPSLG

## Planar Straight Line Graph (derived type)

Description:

This derived type encapsulates internal arrays describing a 2D domain definition, using polygonal shapes.

Declaration is made as follows:

```
type(mfPSLG) :: PSLG_domain
```

Contrary to other Muesli derived types, all internal fields are public, letting the user to allocate/deallocate and set the data himself. The complete definition of this kind of object is:

```
type :: mfPSLG
    real(kind=MF_DOUBLE), allocatable :: n_xy(:,:), holes_xy(:,:)
    integer, allocatable :: edge_n(:,:)
end type
```

n\_xy contains the coordinates of the nodes; must be of size (nn, 2), with nn > 3.

edge\_n contains the definition of each edge segment, by its two endpoints; ; must be of size (ne, 2), with  $ne \ge 0$ .

holes\_xy contains the definition of holes; they are points having two coordinates; must be of size (nh, 2), with  $nh \geq 0$ . A hole should be, ideally, located inside a closed boundary of edges or, alternatively, outside the domain but inside the convex hull (in this later case, its role is to allow concavities). The hole points must not be located exactly on any edge.

#### Remarks:

- This structure can be used directly by msDelaunay to triangulate the corresponding domain.
- $-\,$  See examples of PSLG domains in the Muesli User's Guide.
- A variable of this type may be freed, at the end of its use, by the msRelease routine.

 $See \ also: {\tt msPrintPSLG}, {\tt msPlotPSLG}$ 

#### msPrintPSLG

# Display of a Planar Straight Line Graph

Calling syntax:

```
call msPrintPSLG( PSLG_domain [, short_info ] )
```

### Description:

For a small number of nodes (typically less than few dozens), this routine print on the screen the definition of a Planar Straight Line Graph, which is used to represent a domain geometry.

The input argument PSLG\_domain must be of type mfPSLG.

When the boolean optional argument  $short_info$  is present and equal to TRUE then the routine just print the number of items, i.e. nodes, edges and holes. Therefore, it is the only way to inspect the structure when it is big.

See also: msPlotPSLG

### mfDelaunay

## Delaunay 2D tessellation (triangulation)

First calling syntax:

```
tri = mfDelaunay( x_in, y_in )
```

### Description:

Builds a Delaunay 2D triangulation from a list of nodes whose coordinates are given in x\_in and y\_in (these two mfArrays must be vectors having the same shape).

The returned mfArray contains indices for the triangles: each row of tri provides three integers which describes a triangle (direct orientation, when travelling from first node to the second and the third ones).

Second calling syntax:

#### Description:

Builds a constrained conforming Delaunay 2D triangulation (as previously, from a given list of nodes whose coordinates are stored in  $x_i$  and  $y_i$ ), giving a quality mesh by adding new nodes where it is required. Two different criteria are used:

- A minimal angle theta\_min, to avoid triangles with too small angles. This angle, in degree, must be less than 28.6 (otherwise the algorithm may fail to converge – anyway, the routine will emit an error). The recommanded value ranges from 20 to 25.
- A maximum area area\_max, to avoid big triangles.

These two optional arguments are real. At least one is required.

Third calling syntax:

#### Description:

Same as the above interface, but take a PSLG domain on input (see mfPSLG), instead of a list of nodes. PSLG means *Planar Straight Line Graph* and, very briefly, can be viewed as a polygonal description of the boundaries of a domain; it is intended to store the description of a plane domain to be meshed.

See also: msBuildTriConnect, mfNodeSearch, mfTriSearch, mfDelaunay3D

#### mfTriConnect

# 2D triangular connectivity (derived type)

## Description:

This derived type encapsulates internal arrays describing the connectivity of a 2D triangulation. It must be initialized with the msBuildTriConnect routine.

Declaration is made as follows:

```
type(mfTriConnect) :: tri_connect
```

Note that this structure embeds not only the connectivity tables, but also all nodes coordinates.

See msExtractTriConnect for more information about the content of this structure.

A variable of this type must be freed, at the end of its use, by the msRelease routine.

See also: msPrintTriConnect, msTriMesh

#### msBuildTriConnect

### 2D triangular connectivity initialization

### Interface:

```
subroutine msBuildTriConnect( x, y, tri, tri_connect,
                                                                            &
                              check_tri_orient,
                                                                            &
                              tri_renum, equil_face_orient )
   type(mfArray),
                       intent(in)
                                             :: x, y
   type(mfArray),
                       intent(in out)
                                             :: tri
   type(mfTriConnect), intent(out)
                                             :: tri_connect
   logical,
                       intent(in), optional :: check_tri_orient
   logical,
                       intent(in), optional :: tri_renum, equil_face_orient
```

#### Description:

This routine initializes the internal structures of the 2D triangulation connectivity tri\_connect. See mfTriConnect.

Arguments are mfArrays describing (i) the coordinates of all nodes (in x and y) and (ii) the definition of the triangles in terms of nodes (in tri).

The indices in the mfArray tri describe the 2D triangulation; they may come from the mfDelaunay routine applied to the coordinates x and y.

The optional logical argument <code>check\_tri\_orient</code> is used to check that all triangles, described in the <code>mfArray tri</code>, have a direct orientation; this is required if you plan to apply, later on, the <code>msCheckDomainConvexity</code> routine. Default is to apply such a check. As this check is costly, you should omit it only if you are sure on the constraint about triangles' orientation, for example when <code>tri</code> results from the use of the <code>mfDelaunay</code> routine. When used, this check may change some indices in the <code>tri mfArray</code>.

The optional logical argument tri\_renum is used to apply a renumbering of the triangles, in such a way that they are, wherever possible, geometrically contiguous. In this case, the table of triangles tri can be modified (row ordering). Default is to not apply such a renumbering.

The optional logical argument equil\_face\_orient is used to modify the orientation of triangles' faces, in such a way that they are, wherever possible, equilibrated at each node. Default is to not modify the faces' orientation.

#### Remarks:

- After use, the tri\_connect object should be freed via a call to the routine msRelease.
- The nodes coordinates are embedded in the tri\_connect structure. Therefore, you should NOT change the nodes coordinates after calling the current routine: indeed, while the connectivity remains the same, the convexity (which is a geometrical property embedded in the structure, see msExtractTriConnect) may be affected. Otherwise, especially if you DO modify the coordinates of the boundary nodes, you will have to compute again the connectivity.

See also: mfTriSearch, mfNodeSearch, msPrintTriConnect

# ${\tt msUpdateTriConnect}$

# 2D triangular connectivity update

# ${\it Interface:}$

# Description:

Update the nodes' position in an  ${\tt mfTriConnect}$  structure.

Arguments x and y are mfArrays containing the new (x, y) position of the nodes in the 2D triangulation.

See also: msBuildTriConnect

#### msExtractTriConnect

### get components from a mfTriConnect structure

### Interface:

```
subroutine msExtractTriConnect( tri_connect, tri_f, face_n,
                                                                                &
                                face_tri, convexity,
                                                                                &
                                boundary_nodes, boundary_faces )
   type(mfTriConnect),
                                   intent(in)
                                                          :: tri_connect
                                   intent(out), optional :: tri_f(:,:),
   integer, allocatable,
                                                                                &
                                                             face_n(:,:),
                                                             face_tri(:,:)
                                   intent(out), optional :: convexity
   integer,
   type(mf_Int_List), allocatable, intent(out), optional :: boundary_nodes(:)
   type(mf_Int_List), allocatable, intent(out), optional :: boundary_faces(:)
```

#### Description:

Usually, the 2D mesh connectivity is embedded (with hidden components) in a mfTriConnect structure. If the user want to know/use the detailed links between nodes, triangles and faces, he has to extract himself the corresponding connectivity tables.

The input 2D connectivity tri\_connect must have been initialized by the msBuildTriConnect routine.

According to the optional argument presence, this routine returns the following integer tables, containing positive integers (when nothing else is specified):

- tri\_f, of dimension (nt, 3), which contains the three faces of a given triangle; nt is the total number of triangles.
- face\_n, of dimension (nf, 2), which contains the two nodes which define a given face; nf is the total number of faces.
- face\_tri, of dimension (nf, 2), which contains the two triangles separated by a given face; nf is the total number of faces. In the case where the given face is on the boundary domain, only the first triangle number is valid; the second one is set to a value which is negative or zero (see below).
- convexity, an integer variable whose value may be -1 (UNKNOWN), 0 (FALSE) or 1 (TRUE). It is recommended to ask for its value when interpreting the value of the second column of the face\_tri table. When the domain convexity is UNKNOWN or TRUE, the second element of the face\_tri table is always zero for a boundary face. When the domain convexity is FALSE, the value zero for a boundary face indicates the convex exterior part, the value -1 indicates the non-convex part of the exterior boundary, whereas other negative values indicates an interior bondary (i. e. a hole; add one to this number to obtain the numbering of the hole).
  - Note that the order of the interior boundaries' numbering is determined by the smallest value of the nodes for each of these internal boundaries.
  - Note also that the convexity can be explicitly determined by use of the msCheckDomainConvexity routine.
- boundary\_nodes, a structure of type mf\_Int\_List, containing the indices of the boundary nodes, stored in several lists, if appropriate (i. e., if internal boundaries exist, in addition to the external one). This implies computing the convexity of the domain (see above).
- boundary\_faces, a structure of type mf\_Int\_List, containing the indices of the boundary nodes, stored in several lists, if appropriate (*i. e.*, if internal boundaries exist, in addition to the external one). This implies computing the convexity of the domain (see above).

#### Remarks:

- the connectivity doesn't depend on the exact location of the nodes, but the convexity does.
- the number of faces (nf) can be retrieve by asking the shape of the face\_n array.
- the allocatable arrays (tri\_f, face\_n and others) are allocated by the routine itself; in the case where one of them is already allocated, it will be deallocated and reallocated if needed.

 $See \ also: {\tt mfTriConnect}, {\tt msBuildTriConnect}$ 

## ${\tt msCheckDomainConvexity}$

# Check convexity of a 2D triangulation

Calling syntax:

call msCheckDomainConvexity( tri\_connect )

Description:

Computes the convexity of a 2D triangulation, given as argument in the tri\_connect connectivity.

After use of this routine, the convexity component of the connectivity may be only TRUE or FALSE, not UNKNOWN (see msExtractTriConnect).

See also: mfTriConnect, msBuildTriConnect, msPrintTriConnect

#### msPrintTriConnect

## Display of a 2D triangular connectivity

 $Calling\ syntax:$ 

```
call msPrintTriConnect( tri_connect [, short_info ] )
```

## Description:

For a small number of nodes (typically less than few dozens), this routine print on the screen the connectivity tables of a triangular mesh (the connectivity could be computed by the msBuildTriConnect routine, or by other ways).

The input argument tri\_connect must be of type mfTriConnect.

When the boolean optional argument short\_info is present and equal to TRUE then the routine just print the number of items, i.e. nodes, triangles and faces. Therefore, it is a simple way to inspect the structure when it is big. Another way is to extract some components of tri\_connect by using the msExtractTriConnect routine.

See also: msTriMesh

## msTriNodeNeighbors

## 2D triangulation node neighbors

## Interface:

## Description:

This routine computes the list of the neighbors (either nodes, stored in connected\_nodes, or faces, stored in connected\_faces) for each node belonging to the triangulation given in tri\_connect.

The ouput lists, connected\_nodes and connected\_faces are both vectors of type mf\_Int\_List, a special structure which can contain a variable number of elements. Be aware that the allocation is done by the current routine, not by the user. The length of these vectors is, of course, equal to the number of nodes of the triangulation.

The input mesh connectivity tri\_connect must have been built before via a call to msBuildTriConnect.

#### mfTriSearch

## 2D triangulation search (for triangle)

Generic Interface:

```
function mfTriSearch( tri_connect, x, y, strict ) result( num )
    type(mfTriConnect), intent(in) :: tri_connect
    type(mfArray), intent(in) :: x, y
or real(kind=MF_DOUBLE), intent(in) :: x, y
    logical, intent(in), optional :: strict
    integer :: num
```

### Description:

Searches in a Delaunay triangulation the triangle which encloses the point (x,y).

The input mesh connectivity tri\_connect must have been built before via a call to msBuildTriConnect.

For a general search (i. e. when the optional argument strict is FALSE) the routine always returns a valid triangle index which is the enclosing triangle to the targeted point, or the nearest triangle if the targeted point is outside the meshed domain.

On the contrary, for a *strict* search (*i. e.* when the optional argument **strict** is TRUE, which is the default), if the search successes, then **num** contains the (non zero) index of the triangle; else it contains a negative or null value, which is related to concavities in the meshed domain: 0 means a targeted point outside the domain, whereas -k means a point located in the hole number k (a warning is emitted).

*Remark:* The belonging of a point to a triangle is not strict: this means that the same point can belong to several adjacent triangles. In particular, a node of the triangulation belongs to all triangles who share it.

See also: mfNodeSearch

#### mfNodeSearch

## 2D triangulation search (for node)

Interface:

```
function mfNodeSearch( tri_connect, x, y, strict ) result( num )
    type(mfTriConnect), intent(in) :: tri_connect
    type(mfArray), intent(in) :: x, y
or real(kind=MF_DOUBLE), intent(in) :: x, y
    logical, intent(in), optional :: strict
    integer :: num
```

#### Description:

Searches in a Delaunay triangulation the nearest node to the point (x,y).

The input mesh connectivity tri\_connect must have been built before via a call to msBuildTriConnect.

For a general search (i. e. when the optional argument strict is FALSE) the routine always returns a valid node number which is the nearest node to the targeted point, even if this latter point is outside the meshed domain.

On the contrary, for a *strict* search (*i. e.* when the optional argument **strict** is TRUE, which is the default), if the search successes, then **num** contains the (non zero) index of the node; else it contains a negative or null value, which is related to concavities in the meshed domain: 0 means a targeted point outside the domain, whereas -k means a point located in the hole number k (a warning is emitted).

See also: mfTriSearch

#### mfVoronoi

## 2D Voronoi diagram from a set of points

First calling syntax:

```
voronoi = mfVoronoi( x_in, y_in, what )
```

## Description:

Builds a 2D Voronoi diagram from a list of nodes whose coordinates are given in x\_in and y\_in (these two mfArrays must be vectors having the same shape).

The returned structure voronoi (mfVoronoiStruct derived type) contains optionally the vertices and the neighbors.

More specifically, the returned structure have a content which depends on the value argument what (character string):

- if what is equal to "vertices", then voronoi contains only the coordinates of the vertices defining each Voronoi cell, and the list of corresponding indices of these vertices for each input points;
- if what is equal to "neighbors", then voronoi contains only the list of neighbors for each input points.
- if what is equal to "both", the two previous components are stored in the voronoi structure.

Remark: Note that the returned structure contains also the input points, making it an autonomous structure.

See also: msPrintVoronoi, mfPlotVoronoi, mfDelaunay

#### mfVoronoiStruct

# 2D Voronoi structure emph(derived type)

#### Description:

This derived type encapsulates internal arrays describing a 2D Voronoi diagram. It is created by the mfVoronoi routine.

Declaration is made as follows:

```
type(mfVoronoiStruct) :: voronoi
```

The content of this structure, useful to the user, is:

```
type :: mfVoronoiStruct
    real(kind=MF_DOUBLE), pointer :: n_xy(:,:)
    real(kind=MF_DOUBLE), pointer :: v_xy(:,:)
    type(mf_Int_List), pointer :: vertices(:)
    type(mf_Int_List), pointer :: neighbors(:)
end type
```

 $n_xy$  contains the coordinates of the input points; its size is (nn, 2), where nn is the number of points.  $v_xy$  contains the position of the vertices; its size is (nv, 2), where nv is the number of (unique) vertices.  $v_xy$  contains the position of the vertices; its size is (nv, 2), where nv is the number of (unique) vertices.  $v_xy$  contains the position of the vertices, its size is (nv, 2), where nv is the number of (unique) vertices.

**neighbors** is a vector of lists (size nn); each list contains integer indices of the neighbors, for each Voronoi cell.

## Remarks:

- It is of course not recommended to set or change yourself the components of this structure.
- A variable of this type may be freed, at the end of its use, by the msRelease routine.

See also: msPrintVoronoi, mfPlotVoronoi, mf\_Int\_List

### msPrintVoronoi

# 2D Voronoi structure display

Calling syntax:

call msPrintVoronoi( voronoi )

Description:

For a small number of nodes (typically less than few dozens), this routine print on the screen a Voronoi diagram (this structure must be computed by the  ${\tt mfVoronoi}$  routine).

The input argument voronoi must be of type mfVoronoiStruct.

 $See \ also: {\tt mfVoronoiStruct}, {\tt mfPlotVoronoi}$ 

#### mfGridData

## irregular 2D interpolation

### Interface:

```
function mfGridData( x, y, f, xi, yi ) result( fi )
  type(mfArray), intent(in) :: x, y, f, xi, yi
  type(mfArray) :: fi
```

### Description:

Interpolates data f, which are defined on an irregular grid (x,y), at the entries specified by xi and yi.

x, y and f must be vectors with the same shape.

(xi,yi) also must have the same shape (scalars, vectors, or matrices); fi, which contains interpolated data, will have the same shape as them.

fi is set to MF\_NAN if (xi,yi) is outside the convex hull of x and y.

Remarks: a linear interpolation is used.

See also: mfInterp2, mfGridData3D

### mfDelaunay3D

# Delaunay 3D tessellation (tetrahedralization)

## ${\it Interface:}$

```
function mfDelaunay3D( x, y, z ) result( tetra )
    type(mfArray), intent(in) :: x, y, z
    type(mfArray) :: tetra

or

function mfDelaunay3D( coords ) result( tetra )
    type(mfArray), intent(in) :: coords
    type(mfArray) :: tetra
```

### Description:

Builds a Delaunay 3D tetrahedralization from a list of nodes whose coordinates are given in x, y and z (these three mfArrays must be vectors having the same shape). In the second possible interface, coords must have three columns.

The returned mfArray contains indices for the tetrahedra: each row of tetra provides four integers which describes a tetrahedron.

All tetrahedra have a direct orientation: nodes 1, 2 and 3 is a direct triangle if node 4 is above them.

See also: msBuildTetraConnect, mfDelaunay

## msEndDelaunay3D

## Internal free storage after Delaunay 3D

 $Calling\ syntax:$ 

call msEndDelaunay3D( )

## Description:

After using the  ${\tt mfDelaunay3D}$  (and perhaps also  ${\tt msBuildTetraConnect}$ ) routine(s), it is recommended to call  ${\tt msEndDelaunay3D}$  in order to clean the auxiliary memory used to build the tetrahedralization.

Currently, the Muesli library is not designed to handle the cleaning of intricated 3D connectivities creation. In other words, you should follow each mfDelaunay3D call by (if nedeed) a msBuildTetraConnect call, and last by a mfDelaunay3D call, otherwise you will be faced with some memory leaks.

### mfTetraConnect

# 3D tetrahedral connectivity (derived type)

## Description:

This derived type encapsulates internal arrays describing the connectivity of a 3D tetrahedralization. It must be initialized with the  ${\tt msBuildTetraConnect}$  routine.

Declaration is made as follows:

```
type(mfTetraConnect) :: tetra_connect
```

A variable of this type must be freed, at the end of its use, by the msRelease routine.

 $See \ also: {\tt msPrintTetraConnect}, {\tt mfDelaunay3D}$ 

#### msBuildTetraConnect

## 3D tetrahedral connectivity initialization

## Interface:

```
subroutine msBuildTetraConnect( x, y, z, tetra, tetra_connect )
    type(mfArray), intent(in) :: x, y, z, tetra
    type(mfTetraConnect), intent(out) :: tetra_connect

or
subroutine msBuildTetraConnect( coords, tetra, tetra_connect )
    type(mfArray), intent(in) :: coords, tetra
    type(mfTetraConnect), intent(out) :: tetra_connect
```

#### Description:

This routine initializes the internal structures of a 3D tetrahedralization connectivity.

The indices in the mfArray tetra describe the 3D tetrahedralization; they MUST come from the mfDelaunay3D routine applied to the coordinates x, y and z.

Arguments are mfArrays describing (i) the coordinates of all nodes (in x, y and z) and (ii) the definition of the triangles in terms of nodes (in tetra).

The second possible interface allows the user to group all the columns vector coordinates in one matrix having a coherent shape.

After use, the tetra\_connect object should be freed via a call to the routine msRelease.

Remarks: This routine may lead to an error if internal information have been erased by using msEndDelaunay3D. As a consequence, you should call this latter cleaning routine after getting the mfTetraConnect connectivity.

See also: mfTetraSearch, mfNodeSearch3D

## msExtractTetraConnect get components from a mfTetraConnect structure

### Interface:

#### Description:

Usually, the 3D mesh connectivity is embedded (with hidden components) in a mfTetraConnect structure. If the user want to know/use the detailed links between nodes, tetrahedra and faces, he has to extract himself the corresponding connectivity tables.

The input 3D connectivity tetra\_connect must have been initialized by the msBuildTetraConnect routine.

According to the optional argument presence, this routine returns the following integer tables, containing positive integers (when nothing else is specified):

- tetra\_f, of dimension (nt, 4), which contains the four faces of a given tetrahedra; nt is the total number of tetrahedra.
- face\_n, of dimension (nf, 3), which contains the three nodes which define a given face; nf is the total number of faces.
- face\_tetra, of dimension (nf, 2), which contains the two tetrahedra separated by a given face; nf is the total number of faces. In the case where the given face is on the boundary domain, only the first triangle number is valid; the second one is set to a value which is zero.

#### Remarks:

- the number of faces (nf) can be retrieve by asking the shape of the face\_n array.
- the allocatable arrays (tretra\_f, face\_n and face\_tetra) are allocated by the routine itself; in the case where one of them is already allocated, it will be deallocated and reallocated if needed.

See also: mfTetraConnect, msBuildTetraConnect

#### msPrintTetraConnect

## Display of a 3D tetrahedral connectivity

 $Calling\ syntax:$ 

```
call msPrintTetraConnect( tetra_connect [, short_info ] )
```

## Description:

For a few numbers of nodes (typically less than few dozens), this routine print on the screen the connectivity tables of a tetrahedral mesh (the connectivity could be computed by the msBuildTetraConnect routine, or by other ways).

The input argument tetra\_connect must be of type mfTetraConnect.

When the boolean optional argument  $short\_info$  is present and equal to TRUE then the routine just print the number of items, i.e. nodes, tetrahedra and faces. Therefore, it is a simple way to inspect the structure when it is big. Another way is to extract some components of  $tetra\_connect$  by using the msExtractTetraConnect routine.

## msDel3DNodeNeighbors

# 3D tetrahedralization node neighbors

## Interface:

```
type(mfArray), intent(in) :: tetra
type(mf_Int_List), intent(out) :: node_neighbors(:)
```

### Description:

This routine computes the list of the neighbors of all nodes in the allocatable array node\_neighbors (of type mf\_Int\_List). Be aware that the allocation is done by the current routine, not by the user.

The indices in the mfArray tetra describe the 3D tetrahedralization; they may come from the mfDelaunay3D routine applied to the coordinates (x, y, z) of a set of nodes.

 $See \ also: {\tt msBuildTetraConnect}$ 

#### mfTetraSearch

## 3D tetrahedralization search (for tetrahedron)

## Generic Interface:

```
function mfTetraSearch( tetra_connect, x, y, z ) result( num )
    type(mfTetraConnect), intent(in) :: tetra_connect
    type(mfArray), intent(in) :: x, y, z
or real(kind=MF_DOUBLE), intent(in) :: x, y, z
    integer :: num
```

#### Description:

Searches in a Delaunay tetrahedralization the tetrahedron which encloses the point (x,y,z).

tetra\_connect must be built before via a call to msBuildTetraConnect.

If the search successes, then num contains the (non zero) index of the tetrahedron, else it contains 0.

*Remark:* The belonging of a point to a tetrahedron is not strict: this means that the same point can belong to several adjacent tetrahedra. In particular, a node belongs to all tetrahedra who share it.

See also: mfNodeSearch3D, msBuildTetraConnect

#### mfNodeSearch3D

## 3D tetrahedralization search (for node)

### Interface:

```
function mfNodeSearch3D( tetra_connect, x, y, z ) result( num )
    type(mfTetraConnect), intent(in) :: tetra_connect
    type(mfArray), intent(in) :: x, y, z
or real(kind=MF_DOUBLE), intent(in) :: x, y, z
    integer :: num
```

### Description:

Searches in a Delaunay tetrahedralization the nearest node to the point (x,y,z).

tetra\_connect must be built before via a call to msBuildTetraConnect.

If the search successes, then num contains the (non zero) index of the node, else it contains 0. If num is 0, this means that the point is outside the convex hull (a Warning is emitted).

See also: mfTetraSearch, msBuildTetraConnect

#### mfGridData3D

## irregular 3D interpolation

Interface:

```
function mfGridData3D( x, y, z, f, xi, yi, zi ) result( fi )
  type(mfArray), intent(in) :: x, y, z, f, xi, yi, zi
  type(mfArray) :: fi
```

### Description:

Interpolates data f, which are defined on an irregular 3D grid (x,y,z), at the entries specified by xi, yi and zi.

x, y, z and f must be vectors with the same shape.

(xi,yi,zi) also must have the same shape (scalars, vectors, or matrices); fi, which contains interpolated data, will have the same shape as them.

fi is set to MF\_NAN if (xi,yi,zi) is outside the convex hull of x, y and z.

Remarks: a linear interpolation is used.

See also: mfGridData

#### 1.10 **Optimization and Function Functions**

single-variable non-linear zero finding mf/msFZero mf/msFSolve multiple-variable non-linear zero finding non-linear least-square minimization mf/msLsqNonLin

mfTrapz numerical evaluation of integrals by trapezoidal method

cumulative integrals by trapezoidal method mfCumTrapz

mfSimpson numerical evaluation of integrals by Simpson method numerical evaluation of integrals (adaptive method) mfQuad, msQuad

mfDblQuad, msDblQuad numerical evaluation of double integrals (adaptive method)

mfOdeSolve, msOdeSolve integrator of ODE (explicit) systems mfDaeSolve, msDaeSolve integrator of DAE (implicit) systems

options for differential solvers (derived type) mf\_DE\_Options mf\_NL\_Options options for non-linear solvers (derived type)

See also:

Core Routines

File Input/Output

**Data Analysis Functions** 

Operators

Elementary Math Functions

Specialized Math Functions

Elementary Matrix Manipulation Functions

**Matrix Functions** 

Polynomial Functions

Sparse Matrices

#### mf/msFZero

### single-variable non-linear zero finding

#### Interface:

```
function mfFZero( fun, x0, tol ) result( out )
  interface
    real(kind=MF_DOUBLE) function fun( x )
        real(kind=MF_DOUBLE), intent(in) :: x
    end function
  end interface

  type(mfArray), intent(in) :: x0
  real(kind=MF_DOUBLE), optional :: tol

  type(mfArray) :: out
```

#### Description:

Search for a solution of the non-linear equation fun(x) = 0, via the Dekker algorithm (combination of bisection, linear and quadratic interpolation).

fun is a user-supplied function which has the prescribed interface above.

x0 must be a real vector mfArray containing exactly two values; it defines the interval for searching x. These two values must be set such that the sign of fun(x) differs.

The optional argument tol specifies the relative tolerance for the stopping criterium. It should be a non negative and not too small real value; default value is  $2 \epsilon$ .

#### Remarks:

- the convergence is usually fast and the Dekker method theoretically never fail, providing the initial interval given in x0 is correct;
- fun has an imposed list of arguments; the user may use module's data in order to exchange other information between his program and this function. Therefore, except for very simple cases, this user's routine must be located inside a module, USEd also by the user's program.

The subroutine form:

```
call msFZero( mfOut(x,fval[,status]), fun, x0 [, tol ] )
```

allows the user to access to other information: fval is the function value associated to the zero x, and a status (optional) which has the following values:

status	explanation
0	Normal termination of the algorithm.
-1	Bad initialization: fun, applied to the two elements of x0, must have opposite
	sign.
-2	Bad argument value: x0 has not enough components.

Remark: Adding the optional argument status allows the user to launch its program in a batch mode: indeed, errors in the current routine will not stop the program (it is the responsability of the programmer to test the value of the status variable before using the result). Otherwise (i. e. when the status optional argument is not used), an error will stop the program and, in debug mode, a traceback of this error will be displayed.

See also: mfRoots, mfFSolve, mfOut

#### mfFSolve

## multiple-variable non-linear zero finding

Interface:

```
function mfFSolve( fcn, n, x0, options, jac, sparse ) result( out )
   interface
      subroutine fcn( n, x, fvec, flag )
         integer,
                               intent(in) :: n
         real(kind=MF_DOUBLE), intent(in) :: x(n)
         real(kind=MF_DOUBLE)
                                          :: fvec(n)
         integer
                                          :: flag
      end subroutine
   end interface
   type(mfArray), intent(in)
                                 :: x0
   type(mf_NL_Options), intent(in), optional :: options
   external,
                                    optional :: jac
   logical,
                       intent(in), optional :: sparse
   type(mfArray) :: out
```

#### Description:

Search for a solution of n non-linear equations with n unknowns:

$$F_i(x_1, x_2, \dots, x_n) = 0, \quad i = 1, \dots, n$$

described in the subroutine fcn. This user-supplied subroutine has the interface prescribed above and computes fvec(i) =  $F_i(x_1, x_2, ..., x_n)$ , i = 1, ..., n.

flag is actually an in-out argument of fcn: on entry, its value is always zero. The user can stop the iteration process, by setting it to any negative integer.

x0 is an initial guess for the solution x.

The mfArray options%tol contains the tolerance(s) for the convergence:

- when it is a scalar, both the function tolerance FTOL and the parameter tolerance XTOL are set to the corresponding value;
- when it is a vector of two elements, the values are used to set the two tolerances: FTOL and XTOL, respectively.

When the mfArray options%tol is empty (i. e. not set by the user), FTOL and XTOL are both set to  $10^{-9}$ .

See mf\_NL\_Options for other options.

Optionally, the user may provide the computation of the jacobian via the subroutine jac. Its interface must be as follows:

 $\cdots / \dots$ 

The jacobian matrix is defined as  $J = \frac{\partial F}{\partial x}$ , therefore, the rank-2 array jacobian must be filled as follows: jacobian(i,j) =  $J_{i,j} = \frac{\partial F_i}{\partial x_i}$ . Only non-zero values of J need to be defined.

For the sparse case, the optional argument sparse must be set to .true. and the jac routine must be defined as:

The pd,ipd,jpd f90 arrays describe the CSC (Compact Sparse Column) representation of the sparse jacobian matrix, as follows:

- pd(1:nnz) contains the non-zero matrix entries;
- ipd(1:nnz) contains the row indices;
- ipd(1:ncol+1) is the pointer to the beginning of the columns, in arrays (pd,ipd).

pd,ipd must contain all diagonal terms, even if they are null. Moreover, row indices must be sorted in ascending order. Lastly, this routine must include a mechanism such that only the value of nnz is computed when job=0. You must also use the convention: jpd(ncol+1)=nnz+1.

#### Remarks:

- the problem is solved by the package MINPACK;
- the algorithm may fail (it is not unusual to have no solution for the non-linear problem the returned value will be therefore a vector of NaNs); in such a case, using the msFSolve subroutine helps in giving some information about the reason of the failure.
- fcn and jac have imposed lists of arguments; as for mfLsqNonLin, mfOdeSolve and mfDaeSolve the user may use module's data in order to exchange other information between his program and these routines (especially for checking the number of equations). Therefore, except for very simple cases, these two user's routines must be located inside a module, USEd also by the user's program.

See also: mfFZero, mfLsqNonLin

#### msFSolve

## multiple-variable non-linear zero finding

Calling syntax:

```
call msFSolve( mfOut( x[, fvec, status ] ), fcn, n, x0 [, options, jac] )
```

## Description:

Similar to mfFSolve, but also returns the vector function (in the mfArray fvec) and, optionally, a status which has the following values:

status	explanation
1	Normal termination of the algorithm. The stopping criterion about xtol has been reached first.
2	Normal termination of the algorithm. The stopping criterion about ftol has been reached first.
-1	Bad initialization for x0 (size or type).
-2	Bad argument value: tol is not strictly positive.
-3	Bad termination: number of calls of fcn has reached or exceeded max_iter*(n+1) (if jac is not present) or max_iter (if jac is present).
-4	Bad termination: tol is too small. No further improvement in the approximate solution x is possible.
-5	Bad termination: iteration is not making good progress.
-10	The user has stopped the algorithm via the argument flag of the fcn subroutine.
-14	The user-supplied fcn routine returned invalid entries (NaN value).

For this subroutine, the solution is stored in the mfArray x.

If the status value is not positive (i. e. when algorithm fails), both x and fvec are vectors of NaNs.

Remark: Adding the optional argument status allows the user to launch his program in a batch mode: indeed, errors in the current routine will not stop the program (It is the responsability of the programmer to test the value of the status variable before using the result). Otherwise (i. e. when the status optional argument is not used), an error will stop the program and, in debug mode, a traceback of this error will be displayed.

See also: mfOut

### mfLsqNonLin

### nonlinear least-square minimization

Interface:

```
function mfLsqNonLin( m, fcn, p0, n, options, jac ) result( out )
  integer,
                 intent(in) :: m, n
  type(mfArray), intent(in) :: p0
  interface
     subroutine fcn( m, n, p, fvec, flag )
                             intent(in) :: m, n
        real(kind=MF_DOUBLE), intent(in) :: p(n)
        real(kind=MF_DOUBLE)
                               :: fvec(m)
        integer
                                        :: flag
     end subroutine fcn
  end interface
  type(mf_NL_Options), optional :: options
  external,
                      optional :: jac
  type(mfArray) :: out
```

#### Description:

Minimize the sum of the squares of m nonlinear functions  $F_i$  depending on n parameters.

fcn is the user-supplied subroutine which calculates the functions; it has the interface prescribed above and computes fvec(i) =  $F_i(p)$ , i = 1, ..., m, where p represents the vector of the n parameters.

flag is actually an in-out argument of fcn: on entry, its value is always zero. The user can stop the iteration process, by setting it to any negative integer. See just below for constrained minimization.

Constraints of type BOX are possible, for each parameter (see mf\_NL\_Options).

At input, p0 contains an initial guess for the values of the n unknown parameters. Final values are stored in the mfArray out.

All the following options are stored in the structure options (see mf\_NL\_Options).

The mfArray options%tol contains the tolerance(s) for the convergence:

- when it is a scalar, both the function tolerance FTOL and the parameter tolerance XTOL are set to the corresponding value, while the orthogonality tolerance GTOL (it measures the orthogonality between the function vector and the columns of the jacobian) is set to zero;
- when it is a vector of three elements, the values are used to set the three tolerances: FTOL, XTOL and GTOL, respectively.

When the mfArray options%tol is empty (i. e. not set by the user), FTOL and XTOL are both set to  $10^{-6}$  while GTOL is set to zero.

options%max\_iter is the maximum number of iterations. Its default value is 50.

.../...

options%epsfcn is the approximation error of the functions  $F_i$ . Its default value is zero. It is used only to compute the step for the finite differences computation of the Jacobian; this argument will not be referenced if jac is present. In the case of an approximate computation in the fcn routine (e. g. an integration or whatelse), the value of epsfcn may be critical to obtain good convergence; it must be chosen carefully; see mf\_NL\_Options for additional information.

The logical options%print can be used to print some information during the iterative process. Default is FALSE.

Optionally, the user may provide the computation of the jacobian via the subroutine jac. Its interface must be as follows:

The jacobian matrix is defined as  $J=\frac{\partial F}{\partial p}$ , and the rank-2 array jacobian must be filled as follows: jacobian(i,j) =  $J_{i,j}=\frac{\partial F_i}{\partial p_j}$ . Only non-zero values of J need to be defined.

The integer options%check\_jac allows the user to numerically check (via finite differences) the components of the jacobian each time it is called. A quick global check is done by setting this argument to 1 while a complete (but more expensive) check is done by setting it to 2. Default is 0, *i. e.* no check.

Lastly, the integer print\_check\_jac allows the printing (either on the screen if the value is 1, or in files if the value is 2) of the results of the previous jacobian check. Default is 0, *i. e.* no check.

#### Remarks:

- the problem is solved by the LMA method (Levenberg-Marquardt Algorithm) of the MINPACK package;
- m must be bigger than the number of parameters n.
- fcn and jac have imposed lists of arguments; as for mfFSolve, mfOdeSolve and mfDaeSolve the user may use module's data in order to exchange other information between his program and these routines (especially for checking the number of equations). Therefore, except for very simple cases, these two user's routines must be located inside a module, USEd also by the user's program.
- the initial step bound for the parameters' vector is based on a factor whose default value is 100. Sometimes, we may want to change this factor to a smaller value, especially if you use the log() function to ensure the positiveness of your unknowns; in such a case, factor = log(100) is more appropriate, by setting options%init\_step\_bound\_factor to this value.

See also: mfFunFit, msLsqNonLin

## msLsqNonLin

## nonlinear least-square minimization

Calling syntax:

#### Description:

Similar to mfLsqNonLin, but also returns the 2-norm residue (in the mfArray resnorm) and, optionally, a status which has the following values:

status	explanation
1	Normal termination of the algorithm. Both actual and predicted relative reductions in the sum of squares of the functions fvec are at most ftol.
2	Normal termination of the algorithm. Relative error between two consecutive iterates of the parameters' vector is at most xtol.
3	Normal termination of the algorithm. Conditions for status = 1 and status = 2 both hold.
-1	Improper input parameters.
-4	fvec is orthogonal to the columns of the jacobian. This means that the gradient is approximatively null (with respect to gtol): it may correspond to a minimum, or a maximum, or a saddle point, so the algorithm cannot continue.
-5	number of calls of fcn has reached max_iter*(n+1) (if jac is not present) or max_iter (if jac is present).
-6	tol is too small. No further reduction in the sum of squares is possible.
-7	tol is too small. No further improvement in the approximate solution <b>p</b> is possible.
-8	fvec is orthogonal to the columns of the jacobian to machine precision. This means that the gradient is approximatively null: it may correspond to a minimum, or a maximum, or a saddle point, so the algorithm cannot continue. Incidently, gtol is too small.
-9	The algorithm has been stopped because a constraints' interval for one of the unknown parameters is too small, which makes the finite-differences approximation of the jacobian impossible.
-10	The algorithm has been stopped via the argument flag of the fcn subroutine.
-14	The user-supplied fcn routine returned invalid entries (NaN value).

For this subroutine, the output of the desired n variables is stored in the mfArray p.

res\_log and p\_log are mfArrays which contain the history of the norm of the residue and the parameters, along the iterations. Each row of these matrices corresponds to an iteration.

ident is an mfArray which contains the identifiability of each parameter. This vector has values ranged between 0 and 1, and computed internally by the ratio  $|R_i|/|R_1|$ , where  $R_i, i=1\ldots n$ , are the diagonal values of the R matrix obtained after a QR decomposition of the jacobian (sensitivity matrix). The elements of ident have been reordered to match the original order of the parameters. A value equal or close to 0 means that the corresponding parameter is not identifiable. Actually, the user should compare the values of ident to the numerical precision of the fvec functions.

 $\cdots$ /...

Remark: Adding the optional argument status allows the user to launch his program in a batch mode: indeed, errors in the current routine will not stop the program (It is the responsability of the programmer to test the value of the status variable before using the result). Otherwise (i. e. when the status optional argument is not used), an error will stop the program and, in debug mode, a traceback of this error will be displayed.

See also: mfOut

### mfTrapz

## integration by trapezoidal method

First calling syntax:

```
res = mfTrapz( y )
```

## Description:

Performs the numerical integration of the data y, assuming constant unit spacing for the abscissas, using the trapezoidal rule. If the spacing is different from unity, the result res must be multiplied by the effective spacing.

 $Second\ calling\ syntax:$ 

```
res = mfTrapz( x, y )
```

#### Description:

Performs the numerical integration of the data y, spread against the abscissas x.

#### Remark:

- x and y must be real vector mfArrays.
- the method used is of order 2, *i. e.* the resulting error (when the exact result is known or can be estimated) should be divided by 4 when the number of values in **x** and **y** is doubled. If this behavior is not reached, this may indicate that the integral is not convergent.

 $See \ also: \mathtt{mfSimpson}, \mathtt{mfQuad}, \mathtt{mfCumTrapz}$ 

### mfCumTrapz

## cumulative integration by trapezoidal method

First calling syntax:

```
res = mfCumTrapz( y )
```

## Description:

Performs the numerical cumulative integration of the data y, assuming constant unit spacing for the abscissas, using the trapezoidal rule. If the spacing is different from unity, the result res must be multiplied by the effective spacing.

 $Second\ calling\ syntax:$ 

```
res = mfCumTrapz( x, y )
```

#### Description:

Performs the numerical cumulative integration of the data y, spread against the abscissas x.

#### Remark:

- x and y must be real vector mfArrays.
- the method used is of order 2, *i. e.* the resulting error (when the exact result is known or can be estimated) should be divided by 4 when the number of values in **x** and **y** is doubled. If this behavior is not reached, this may indicate that the integral is not convergent, at least at some abscissas.

See also: mfTrapz

### mfSimpson

## integration by Simpson method

 $Calling\ syntax:$ 

```
res = mfSimpson( y )
```

## Description:

Performs the numerical integration of the data y, assuming constant unit spacing for the abscissas, using the Simpson rule. If the spacing is different from unity, the result res must be multiplied by the effective spacing.

 $Second\ calling\ syntax:$ 

```
res = mfSimpson( x, y )
```

#### Description:

Performs the numerical integration of the data y, spread against the abscissas x.

#### Remarks:

- x and y must be real vector mfArrays.
- the number of points must be odd.
- the method used is of order 4, *i. e.* the resulting error (when the exact result is known or can be estimated) should be divided by 16 when the number of values in x and y is doubled. If this behavior is not reached, this may indicate that the integral is not convergent.

See also: mfTrapz, mfQuad

#### mfQuad

## numerical evaluation of integrals

## Interface:

#### Description:

Performs the numerical computation of the integral of the function fun(x) over the interval [a, b].

The optional arguments abs\_tol and rel\_tol allows the user to give the absolute and/or the relative precision of the result. Default tolerance is  $1 \times 10^{-12}$ .

The second optional argument sing should be used when the function fun(x) has a singularity at x = a or/and at x = b. 'sing=.true.' leads to use another quadrature method, which usually involves a fewer number of function evaluation. This optional argument cannot be used when the interval range is infinite.

#### Remarks:

- out is set to MF\_NAN if required precision cannot be achieved; an error is emitted by the library.
   Call the subroutine version if you don't want such a behavior.
- a and/or b may be infinite values. In such a case, an appropriate routine is called; a correct result
  is expected as long as the integral is convergent.

See also: msQuad, mfTrapz, mfDblQuad, msDblQuad

### msQuad

### numerical evaluation of integrals

Calling syntax:

### Description:

The behavior of this routine is similar to those of mfQuad.

The subroutine call allows to get more information returned by the integrator:

- q contains the numerical result
- abserr contains an estimation of the absolute error
- status (optional) contains the returned error code:

status = 0: normal and reliable termination of the routine. It is assumed that the requested accuracy has been achieved.

status < 0: abnormal termination of the routine. The estimates for q and abserr are less reliable. It is assumed that the requested accuracy has not been achieved. Only a warning is emitted by the library. (Exact meaning of the error can be found below)

- neval (optional) contains the effective number of function evaluation

All these variable must be of type(mfArray).

Remark: Adding the optional argument status allows the user to launch its program in a batch mode: indeed, errors in the current routine will not stop the program (It is the responsability of the programmer to test the value of the status variable before using the result). Otherwise (i. e. when the status optional argument is not used), an error will stop the program and, in debug mode, a traceback of this error will be displayed.

See also: mfQuad, mfOut, mfDblQuad, msDblQuad

status	explanation
-1	Maximum number of subdivisions allowed has been achieved.
-2	The occurrence of roundoff errors is detected, which prevents the requested tolerance from being achieved. The error may be under-estimated.
-3	Extremely bad integrand behaviour occurs at some points of the integration interval.
-4	The algorithm does not converge. Roundoff error is detected in the extrapolation table. It is presumed that the requested tolerance cannot be achieved, and that the returned result is the best which can be obtained.
-5	The integral is probably divergent, or slowly convergent. It must be noted that divergence can occur with any other value of status.
-6	Invalid input (this error should have been trapped by the routine msQuad itself).

#### mfDblQuad

## numerical evaluation of double integrals

#### Interface:

```
function mfDblQuad( fun, xa, xb, ya, yb, abs_tol, rel_tol ) result( out )
    interface
        real(kind=MF_DOUBLE) function fun( x, y )
            real(kind=MF_DOUBLE), intent(in) :: x, y
        end function
    end interface

    real(kind=MF_DOUBLE), intent(in) :: xa, xb, ya, yb
    real(kind=MF_DOUBLE), intent(in), optional :: abs_tol, rel_tol
        type(mfArray) :: out

Other calling syntaxes:
    out = mfDblQuad( fun, fun_xa, fun_xb, ya, yb, abs_tol, rel_tol )

or
    out = mfDblQuad( fun, xa, xb, fun_ya, fun_yb, fun_y=.true., abs_tol, rel_tol )
```

where fun\_xa, fun\_xb, fun\_ya and fun\_ya are all double precision real function of one real argument. By using these latter syntaxes, the user is able to integrate over non rectangular domains. Note that in the second case, the additional argument fun\_y is mandatory (no other way to disambiguate a unique interface for differente routines).

## Description:

Performs the numerical computation of the double integral of the function fun(x, y) over the interval [xa, xb]×[ya, yb].

The optional arguments abs\_tol and rel\_tol allows the user to give the absolute and/or the relative precision of the result. Default tolerance is  $1 \times 10^{-12}$ .

Remarks: out is set to MF\_NAN if required precision cannot be achieved; an error is emitted by the library. Call the subroutine version if you don't want such a behavior.

See also: mfQuad, msQuad

#### msDblQuad

## numerical evaluation of double integrals

Calling syntax:

### Description:

The behavior of this routine is similar to those of mfDblQuad (including the fact to do integration on non rectangular domains).

The subroutine call allows to get more information returned by the integrator:

- q contains the numerical result
- abserr contains an estimation of the absolute error
- status (optional) contains the returned error code:

status = 0: normal and reliable termination of the routine. It is assumed that the requested accuracy has been achieved.

status < 0: abnormal termination of the routine. The estimates for q and abserr are less reliable. It is assumed that the requested accuracy has not been achieved. Only a warning is emitted by the library. (Exact meaning of the error can be found below)

- neval (optional) contains the effective number of function evaluation

All these variable must be of type(mfArray).

Remark: Adding the optional argument status allows the user to launch its program in a batch mode: indeed, errors in the current routine will not stop the program (It is the responsability of the programmer to test the value of the status variable before using the result). Otherwise (i. e. when the status optional argument is not used), an error will stop the program and, in debug mode, a traceback of this error will be displayed.

See also: mfDblQuad, mfOut

status	explanation
-1	Maximum number of subdivisions allowed has been achieved.
-2	The occurrence of roundoff errors is detected, which prevents the requested tolerance from being achieved. The error may be under-estimated.
-3	Extremely bad integrand behaviour occurs at some points of the integration interval.
-4	The algorithm does not converge. Roundoff error is detected in the extrapolation table. It is presumed that the requested tolerance cannot be achieved, and that the returned result is the best which can be obtained.
-5	The integral is probably divergent, or slowly convergent. It must be noted that divergence can occur with any other value of status.
-6	Invalid input (this error should have been trapped by the routine msDblQuad itself).

#### mfOdeSolve

## integrator of ODE (explicit) systems

Main interface:

```
function mfOdeSolve( deriv, t_span, y_0, options, jac, sparse ) result( y )
   interface
      subroutine deriv( t, y, yprime, flag )
         real(kind=MF_DOUBLE), intent(in)
                                               :: t, y(*)
         real(kind=MF_DOUBLE), intent(out)
                                              :: yprime(*)
         integer,
                               intent(in out) :: flag
      end subroutine
   end interface
   type(mfArray),
                        intent(in)
                                              :: t_span, y_0
   type(mf_DE_Options), intent(in), optional :: options
   external,
                                    optional :: jac
   logical,
                        intent(in), optional :: sparse
   type(mfArray) :: y
```

#### Description:

Performs the numerical integration of a system of (explicit<sup>1</sup>) ordinary differential equations (ODEs) of first order, which is written under the form:

$$y' = F(t, y)$$

described via the subroutine deriv (the variables t, y and yprime represent respectively t, y and y'). Usually, the integration is done between time  $t_0$  and  $t_{end}$  (extremal values of t\_span, which is a column vector mfArray of at least two components) or, in the continuation mode<sup>2</sup>, up to the new values specified in t\_span (in this case, t\_span may contain only one value).

The output of mfOdeSolve is the row vector y which represents the solution y at time  $t_{end}$ . If the vector  $t\_span$  contains more than two values (or more than one value in the continuation mode), the routine returns in y the intermediate values of y, as specified by all time values present in  $t\_span$ ; in this case, y is a rank-2 mfArray with as many rows as components in  $t\_span$ .

The integration uses the prescribed initial condition stored in the row vector y\_0. Note that this mfArray is not referenced in the continuation mode.

The last argument flag of deriv is always set to zero (internally by the solver) on input, and should be altered (inside the routine deriv) only in some special situations:

- set flag to -1 for an *Illegal Condition*; the value of y' at this time may not be available. The routine will try to continue the integration (reducing the time step) as long as possible without getting the illegal condition. [You can put this assignment anywhere in the **deriv** routine.]
- set flag to -2 for an *Emergency Exit*; the value of y' at this time may not be available. The routine will return control immediately to the calling program. [You can put this assignment anywhere in the deriv routine.]
- set flag to 3 for an  $End\ Condition$ ; the value of y' at this time must be always available. The routine will adapt its time step to finish exactly at the current time given by the argument t of deriv. [WARNING: you must put this assignment at the end of the deriv routine, to insure that all the derivatives have been computed.]

 $\cdots$ /...

<sup>&</sup>lt;sup>1</sup>For implicit systems, the mfDaeSolve routine should be used instead.

 $<sup>^2</sup>$ Currently, only the BDF integrator has this feature.

The previous three situations are called *stopping conditions*. Note that when a *stopping condition* is found in the user subroutine resid, then the vector  $t_span$  (which contains the value of  $t_{end}$ ) is modified. Note also that the three *stopping conditions* doesn't have the same numerical cost; their cost is mentioned in the following table:

Flag to used in resid	Stopping condition	Numerical cost
-1	Illegal Condition	expensive
-2	Emergency Exit	very cheap
3	End Condition	cheap

All following options are stored in the structure options (see mf\_DE\_Options).

The boolean mfArray options%continuation allows the user to continue the integration to a new value of  $t_{end}$ , without specify a new  $t_0$ . Thanks to specific calls to mf/msOdeSolve, the user is even able to save internal data of the integrator, to stop the program, and to launch again the integrator after the reload of saved data. See the next section *Other calling syntaxes*.

The mfArray options%tol allows the user to give the precision of the result, and must contain strictly positive values. Default values are  $10^{-3}$  for the relative tolerance and  $10^{-6}$  for the absolute tolerance; this mfArray may be of rank 0, 1 or 2:

- if it is a scalar, it specifies the relative tolerance for all the components of the solution; the absolute tolerance is fixed to  $10^{-6}$ .
- if it is a vector, it must have two elements which specifies the relative and the absolute tolerance, respectively.
- if it is a matrix, it must have two columns (corresponding to the relative and absolute tolerance)
   and a number of rows equal to the number of equations.

The mfArray options%y\_ind\_out allows the user to get only a subpart of the mfArray y; if present, it must contain integer indices of the wanted components of the y vector.

The string options%method specifies the method to be used. Possible choices are "RKF" (Runge-Kutta Fehlberg), "ABM" (Adams-Bashforth-Moulton), or "BDF" (Backward Differentiation Formula). By default, the Runge-Kutta Fehlberg method is selected.

When the "BDF" method is used, the user may provide the jac routine to compute the jacobian matrix (otherwise, it will be numerically evaluated by finite differences). Moreover, this jacobian matrix may be defined in dense, band or sparse format. For the first two formats, the interface of the jac routine is:

```
interface
   subroutine jac( t, y, jacobian, ldjac )
     real(kind=MF_DOUBLE), intent(in) :: t, y(*)
     integer, intent(in) :: ldjac
     real(kind=MF_DOUBLE), intent(out) :: jacobian(ldjac,*)
   end subroutine
end interface
```

The jacobian matrix is defined as  $J=\frac{\partial F}{\partial y}$ , therefore, the rank-2 array jacobian must be filled as follows: jacobian(i,j) =  $J_{i,j}=\frac{\partial F_i}{\partial y_i}$ .

Please note that, for the dense format, only non-zero values need to be defined.

If the jacobian is symmetric and positive definite (abbreviated as SPD), you should set the jac\_symm\_pos\_def option (see mf\_DE\_Options) in order to economize storage and obtain a better performance for its factorization; in such a case, you must give only the upper part of the jacobian.

 $\cdots / \dots$ 

Furthermore, when the jacobian J is banded, the mfArray options%band may be used to economize both memory and CPU time. It must be a vector of exactly two elements: ML and MU which are the widths of the lower and upper parts of the band, respectively, with the main diagonal being excluded (e. g. for a tridiagonal system, both elements are equal to 1).

In this case, routine jac must store the elements  $J_{i,j}$  in the matrix jacobian as follows:

```
irow = i - j + ML + MU + 1
jacobian(irow,j) = J_{i,j}
```

Actually, ldjac is equal to 2\*ML+MU+1: indeed, an extra storage is needed for the matrix factorisation. For an SPD matrix, simply take ML equal to zero in the previous formulae.

For the sparse case, the optional argument sparse must be set to .true. and the jac routine must be defined as:

The pd,ipd,jpd f90 arrays describe the CSC (Compact Sparse Column) representation of the sparse jacobian matrix, as follows:

- pd(1:nnz) contains the non-zero matrix entries;
- ipd(1:nnz) contains the row indices;
- jpd(1:ncol+1) is the pointer to the beginning of the columns, in arrays (pd,ipd).

pd,ipd must contain all diagonal terms, even if they are null. Moreover, row indices must be sorted in ascending order. Lastly, this routine must include a mechanism such that only the value of nnz is computed when job=0. You must also use the convention: jpd(ncol+1)=nnz+1.

Be aware that the sparse structure (that is to say, the indices' vectors ipd and jpd) must remain the same during all the integration process. Otherwise, use the option spjac\_const\_struct of the mf\_DE\_Options derived type. Moreover, you can even force to reuse the same sparse structure if you make many calls to mf/msOdeSolve during, e.g., an iterative procedure: see the option reuse\_spjac\_struct of the mf\_DE\_Options derived type.

### Remarks:

- on output, y may contain *NaN* values for some reasons. In such a case, use the msOdeSolve routine to further investigate the problem.
- deriv and jac have imposed lists of arguments; the user may use module's data in order to exchange other information between his program and these routines (especially for checking the number of equations). Therefore, except for very simple cases, these two user's routines must be located inside a module, USEd also by the user's program.
- for debugging purposes, the user may inspect the values of y and y' during the integration process. See explanations about the fields monitor\_y\_ind, monitor\_yp\_ind and monitor\_pause of the mf\_DE\_Options derived type.
- the *stopping conditions* are not (yet) available for the "ABM" method.

 $\cdots$ /...

Other calling syntaxes

```
call msOdeSolve( A, "save" [, jac_sparse] )
```

By using this call, the user is able to save all internal data of the integrator in the mfArray A, and then continue the integration later on. The optional argument jac\_sparse, which is the name of the routine computing the sparse Jacobian matrix, is required when the Jacobian has a sparse structure.

```
call msOdeSolve( A, "restore" [, jac_sparse] )
```

This call is required to restore all internal data of the integrator, before a new integration sequence. The optional argument <code>jac\_sparse</code>, which is the name of the routine computing the sparse Jacobian matrix, is required when the Jacobian has a sparse structure.

```
call msOdeSolve( "finalyze" )
```

This call is useful to avoid memory leaks at the end of the integration process.

See also: mfDaeSolve

#### msOdeSolve

## integrator of ODE (explicit) systems

Calling syntax:

### Description:

The behavior of this routine is similar to those of mfOdeSolve, but it allows to get more information returned by the integrator (via mfArrays which must be enclosed to the mfOut function):

- y contains the numerical result.
- status contains a returned error code:

status > 0: normal and reliable termination of the routine. It is assumed that the requested accuracy has been achieved. (Exact meaning of the status can be found below)

status < 0: abnormal termination of the routine. The estimates for y is less reliable. It is assumed that the requested accuracy has not been achieved. Moreover, in some circumstances, a message is printed; in such cases, the user cannot use numerical values in y and yp and therefore an appropriate decision must be taken. (Exact meaning of the error can be found below)

- tolout is different from tol only if status = -2: in such a case, it contains an appropriate value for continuing the integration.
- yp contains an estimation of the derivative at time  $t_{end}$  (or at all intermediate values defined in t\_span); moreover, the role of the option options $y_{ind}$ out applies also to yp.
- solve\_log contains additional information about the integration process. It is a vector of 4 or 6 elements, according to the method used. These elements are:

nb\_step is the total number of steps used

dt\_min the smallest step used

dt\_max the largest step used

nb\_deriv is the number of calls of the deriv routine

nb\_jac is the number of evaluation of the jacobian matrix (only for the "BDF" method)

nb\_solve is the number of times the linear system is solved using the jacobian matrix (only for the "BDF" method)

- t\_log contains the whole history of the times used during the integration. This can be useful to detect where a critical behavior is located.
- order\_log contains the whole history of the order used during the integration. The first value, not defined, is set to NaN (Not-a-Number special IEEE floating-point value); as a consequence, this mfArray has the same size as t\_log.

Among these returned variables, only the first two are required, while the others are optional. Moreover, it is forbidden to use the same mfArray in place of two or more actual arguments, even empty.

### Remarks:

The presence of the argument status allows the user to launch its program in a batch mode: indeed, errors in the current routine will not stop the program (it is the responsability of the programmer to test the value of the status variable before using the result). If the mfOdeSolve routine was used instead, an error would stop the program and, in debug mode, a traceback of this error would be displayed.

.../...

- The special calling syntaxes used to save/restore the internal data (in case of continuation in overlay situations), or to finalize the integrator, have been presented at the end of the previous entry.

status	explanation	
0	Normal termination of the solver.	
4	Normal termination of the solver, but a large amount of work has been expended.	
5	Normal termination of the solver, but the $RKF$ integrator has been used very inefficiently because the natural step size has been restricted by too frequent output.	
12	Normal termination of the solver, but an <i>End Condition</i> was found (flag, last argument in deriv, was equal to 3 and the solver has been called to finish at the new current point).	
-2	The error tolerances are too stringent.	
-3	The local error test cannot be satisfied because you specified a zero value for the absolute error in tol and the corresponding computed solution component is zero. Thus, a pure relative error test is impossible for this component.	
-4	The problem appears to be stiff $(RKF \text{ and } ABM \text{ integrators only}).$	
-6	The $BDF$ integrator had repeated convergence test failures on the last attempted step.	
-7	The $BDF$ integrator had repeated error test failures on the last attempted step.	
-10	Abnormal termination of the solver, due to an <i>Illegal Condition</i> (flag, last argument in deriv, was equal to $-1$ and the solver did its best to not obtain $-1$ ).	
-11	Abnormal termination of the solver, due to an <i>Emergency exit</i> (flag, last argument in deriv, was equal to $-2$ and control has been returned to the calling program).	
-14	The user-supplied deriv routine returned invalid entries (NaN value).	
-20	Allocation error because the maximum of available memory has been reached. An explanation is printed.	
-33	Invalid argument.	
-34	Invalid implementation of the user-defined sparse jacobian.	
-40	On calling, the number of output argument(s) is incorrect.	
-100	The routine wasn't able to start, nor choose an initial timestep (flag, last argument in deriv, was different from 0).	

 $See~also: {\tt mfOdeSolve, msDaeSolve, mfOut, mf\_DE\_Options}$ 

#### mfDaeSolve

## integrator of DAE (implicit) systems

 ${\it Interface:}$ 

```
function mfDaeSolve( resid, t_span, y_0, yp_0, options, jac, sparse ) result( y )
   interface
      subroutine resid( t, y, yprime, delta, flag )
                                           :: t, y(*), yprime(*)
         real(kind=MF_DOUBLE), intent(in)
         real(kind=MF_DOUBLE), intent(out)
                                              :: delta(*)
         integer,
                               intent(in out) :: flag
      end subroutine
   end interface
   type(mfArray),
                        intent(in out)
                                                 :: t_span
   type(mfArray),
                       intent(in)
                                                 :: y_0, yp_0
   type(mf_DE_options), intent(in),
                                        optional :: options
   external,
                                        optional :: jac
   logical,
                       intent(in),
                                        optional :: sparse
   type(mfArray) :: y
```

#### Description:

Performs the numerical integration of a system of (fully implicit <sup>3</sup>) differential algebraic equations (DAEs) of first order (and differential index 1 only<sup>4</sup>), which can be written in the general form:

$$R(t, y, y') = 0$$

described via the user subroutine resid, which must define the components  $R_i(t, y, y')$  and store the values in the delta array (the variables t, y and yprime represent respectively t, y and y'). Usually, the integration is done between time  $t_0$  and  $t_{end}$  (extremal values of t\_span, which is a column vector mfArray of at least two components) or, in the continuation mode, up to the new values specified in t\_span (in this case, t\_span may contain only one value).

The output of mfDaeSolve is the row vector y which represents the solution y at time  $t_{end}$ . If the vector  $t\_span$  contains more than two values, the routine returns in y the intermediate values of y, as specified by all time values present in  $t\_span$ ; in this case, y is a rank-2 mfArray with as many rows as components in  $t\_span$ .

The integration uses the prescribed initial conditions stored in the row vectors y\_0 and yp\_0. By default, it is assumed that the data y\_0 and yp\_0 are consistent (note that a consistency test is performed; if it fails, an error is returned), i. e. verify the equation

$$R(t_0, y_0, y_0') = 0;$$

otherwise, the user must set the options%IC\_known argument to .false., so that the initial conditions will be computed by the solver, taking y\_0 and yp\_0 as initial guesses; yp\_0 may be empty but y\_0 must always be allocated and must have its size equal to the number of equations (see mf\_DE\_Options for more details; in particular, the user must indicate which components of these two vectors should be held fixed).

• • • / . . .

<sup>&</sup>lt;sup>3</sup>Not just linearly implicit, which would have a mass matrix. For explicit systems, the mfOdeSolve routine should be used instead.

<sup>&</sup>lt;sup>4</sup>For higher-index problems, the current algorithm may or may not work!

The last argument flag of resid is always set to zero (internally by the solver) on input, and should be altered (inside the routine resid) only in some special situations:

- set flag to -1 for an *Illegal Condition*; the value of y' at this time may not be available. The routine will try to continue the integration (reducing the time step) as long as possible without getting the illegal condition. [You can put this assignment anywhere in the **resid** routine.]
- set flag to -2 for an *Emergency Exit*; the value of y' at this time may not be available. The routine will return control immediately to the calling program. [You can put this assignment anywhere in the resid routine.]
- set flag to 3 for an End Condition; the value of y' at this time must be always available. The routine will adapt its time step to finish exactly at the current time given by the argument t of resid. [WARNING: you must put this assignment at the end of the resid routine, to insure that all the residues delta have been computed.]

The previous three situations are called *stopping conditions*. Note that when a *stopping condition* is found in the user subroutine resid, then the vector  $t_span$  (which contains the value of  $t_{end}$ ) is modified. Note also that the three *stopping conditions* doesn't have the same numerical cost; their cost is mentioned in the following table:

Flag to used in resid	Stopping condition	Numerical cost
-1	Illegal Condition	expensive
-2	Emergency Exit	very cheap
3	End Condition	cheap

All following options are stored in the structure options (see mf\_DE\_Options).

The boolean mfArray options%continuation allows the user to continue the integration to a new value of  $t_{end}$ , without specify a new  $t_0$ . Thanks to specific calls to mf/msDaeSolve, the user is even able to save internal data of the integrator, to stop the program, and to launch again the integrator after the reload of saved data. See the next section *Other calling syntaxes*.

The mfArray options%tol allows the user to give the precision of the result, and must contain non negative values. Default values are  $10^{-3}$  for the relative tolerance and  $10^{-6}$  for the absolute tolerance; this mfArray may be of rank 0, 1 or 2:

- if it is a scalar, it specifies the relative tolerance for all the component of the solution; the absolute tolerance is fixed to  $10^{-6}$ .
- if it is a vector, it must have two elements which specifies the relative and the absolute tolerance, respectively.
- if it is a matrix, it must have two columns (corresponding to the relative and absolute tolerance)
   and a number of rows equal to the number of equations.

The mfArray options%non\_neg allows the user to apply non-negativity constraints to a subset (or all the set) of the components of the solution y. It must be a vector describing the indices of such constraints.

The mfArray options%y\_ind\_out allows the user to get only a subpart of the mfArray y; if present, it must contain integer indices of the wanted components of the y vector.

The user may provide the jac routine to compute the jacobian matrix (otherwise, it will be numerically evaluated by finite differences). This matrix may be defined in dense, band or sparse format. For the first two formats, the interface of the jac routine is:

 $\cdots / \dots$ 

The generalized jacobian<sup>5</sup>  $\widetilde{J}$  must be defined as:

$$\widetilde{J} = \frac{\partial R}{\partial y} + c_J \frac{\partial R}{\partial y'}$$

where  $c_J$  is a scalar computed by the solver; therefore the numerical values of  $\widetilde{J}$  must be stored in the jacobian rank-2 array as follows: jacobian(i,j) =  $\widetilde{J}_{i,j} = \frac{\partial R_i}{\partial y_i} + c_J \frac{\partial R_i}{\partial y_i'}$ .

Please note that, for the dense format, only non-zero values of  $\widetilde{J}$  need to be defined.

If the jacobian is symmetric and positive definite (abbreviated as SPD), you should set the jac\_symm\_pos\_def option (see mf\_DE\_Options) in order to economize storage and obtain a better performance for its factorization; in such a case, you must give only the upper part of the jacobian.

Furthermore, the jacobian  $\widetilde{J}$  may be banded, in order to economize both memory and CPU time. In such a case, the mfArray options%band must be set; it must contain a vector of exactly two elements: ML and MU which are the widths of the lower and upper parts of the band, respectively, with the main diagonal being excluded (e. g. for a tridiagonal system, both elements are equal to 1). In this case, routine jac must store the elements  $\widetilde{J}_{i,j}$  in the matrix jacobian as follows:

```
\label{eq:invariant} \begin{split} \text{irow = i - j + ML + MU + 1} \\ \text{jacobian(irow,j) = } & \widetilde{J}_{i,j} \end{split}
```

Actually, <code>ldjac</code> is equal to <code>2\*ML+MU+1</code>: indeed, an extra storage is needed for the matrix factorisation. For an SPD matrix, simply take <code>ML</code> equal to zero in the previous formulae.

For the sparse case, the optional argument sparse must be set to .true. and the jac routine must be defined as:

```
interface
   subroutine jac(t, y, yprime, cj, nrow, job, pd, ipd, jpd, nnz)
      real(kind=MF_DOUBLE), intent(in)
                                           :: t, y(*), yprime(*), cj
                            intent(in)
      integer,
                                           :: nrow, job
      real(kind=MF_DOUBLE), intent(out)
                                           :: pd(*)
                                         :: pu(.,
:: ipd(*), jpd(*)
      integer,
                            intent(out)
      integer,
                            intent(in out) :: nnz
   end subroutine
end interface
```

 $c_J$  having the same meaning as before.

 $\cdots$ /...

<sup>&</sup>lt;sup>5</sup>It is so called because it is the linear combination of two jacobian. Actually, it is close to the iteration matrix of the non-linear solver. See also this remark.

The pd,ipd,jpd f90 arrays describe the CSC (Compact Sparse Column) representation of the sparse jacobian matrix, as follows:

- pd(1:nnz) contains the non-zero matrix entries;
- ipd(1:nnz) contains the row indices;
- jpd(1:ncol+1) is the pointer to the beginning of the columns, in arrays (pd,ipd) (actually, a jacobian matrix is always square, hence ncol=nrow).

Row indices must be sorted in ascending order. Moreover, this routine must include a mechanism such that only the value of nnz is computed when job=0; therefore the argument nnz has generally an intent(y) for the first call and an intent(in) for subsequent calls.

Be aware that the sparse structure (that is to say, the indices' vectors ipd and jpd) must remain the same during all the integration process. Otherwise, use the option spjac\_const\_struct of the mf\_DE\_Options derived type. Moreover, you can even force to reuse the same sparse structure if you make many calls to mf/msOdeSolve during, e.g., an iterative procedure: see the option reuse\_spjac\_struct of the mf\_DE\_Options derived type.

#### Remarks:

- on output, y may contain *NaN* values for some reasons. In such a case, use the msDaeSolve routine to further investigate the problem;
- resid and jac have imposed lists of arguments; the user may use module's data in order to exchange other information between his program and these routines (especially for checking the number of equations). Therefore, except for very simple cases, these two user's routines must be located inside a module, USEd also by the user's program.
- for debugging purposes, the user may inspect the values of y and y' during the integration process. See explanations about the fields monitor\_y\_ind, monitor\_yp\_ind and monitor\_pause of the mf\_DE\_Options derived type.

Other calling syntaxes

```
call msDaeSolve( A, "save" [, jac_sparse] )
```

By using this call, the user is able to save all internal data of the integrator in the mfArray A, and then continue the integration later on. The optional argument jac\_sparse, which is the name of the routine computing the sparse Jacobian matrix, is required when the Jacobian has a sparse structure.

```
call msDaeSolve( A, "restore" [, jac_sparse] )
```

This call is required to restore all internal data of the integrator, before a new integration sequence. The optional argument <code>jac\_sparse</code>, which is the name of the routine computing the sparse Jacobian matrix, is required when the Jacobian has a sparse structure.

```
call msDaeSolve( "finalyze" )
```

This call is useful to avoid memory leaks at the end of the integration process.

See also: mfOdeSolve

#### msDaeSolve

## integrator of DAE (implicit) systems

Calling syntax:

#### Description:

The behavior of this routine is similar to those of mfDaeSolve, but it allows to get more information returned by the integrator (via mfArrays which must be enclosed by the mfOut function):

- y contains the numerical result
- status contains a returned error code:

status > 0: normal and reliable termination of the routine. It is assumed that the requested accuracy has been achieved. (Exact meaning of the status can be found below)

status < 0: abnormal termination of the routine. The estimates for y is less reliable. It is assumed that the requested accuracy has not been achieved. Moreover, in some circumstances, a message is printed; in such cases, the user cannot use numerical values in y and yp and therefore an appropriate decision must be taken. (Exact meaning of the error can be found below)

- tolout is different from tol in some few cases. First, if status = -2, it contains an appropriate value for continuing the integration. Second, if status = -101, it contains the absolute value of the residual vector  $R(t_0, y_0, y'_0)$  (initial data consistency).
- yp contains an estimation of the derivative at time  $t_{end}$  (or at all intermediate values defined in t\_span); moreover, the role of the optional argument y\_ind\_out applies to yp.
- solve\_log contains additional information about the integration process. It is a vector of 6 elements, which are:

nb\_step is the total number of steps used;

dt\_min the smallest step used;

dt\_max the largest step used;

nb\_resid is the number of calls of the resid routine;

nb\_jac is the number of evaluation of the jacobian matrix;

nb\_solve is the number of solving the linear system using the jacobian matrix.

 init\_log contains specific information about the computation of consistent initial conditions (when this has been required). It is a vector of 4 elements, which are:

nb\_resid\_0 is the number of calls of the resid routine;

nb\_jac\_0 is the number of evaluation of the jacobian matrix;

nb\_solve\_0 is the number of solving the linear system using the jacobian matrix;

cpu\_time\_0 is the CPU time spent in this initialization part.

- t\_log contains the whole history of the times used during the integration. This can be useful to detect where a critical behavior is located.
- order\_log contains the whole history of the order used during the integration. The first value, not defined, is set to NaN (Not-a-Number special IEEE floating-point value); as a consequence, this mfArray has the same size as t\_log.

 $\cdots$ /...

Among these returned variables, only the first two are required, while the others are optional. Moreover, it is forbidden to use the same mfArray in place of two or more actual arguments, even empty.

Remark: The presence of the argument status allows the user to launch its program in a batch mode: indeed, errors in the current routine will not stop the program (it is the responsability of the programmer to test the value of the status variable before using the result). If the mfDaeSolve routine was used instead, an error would stop the program and, in debug mode, a traceback of this error would be displayed.

status	explanation
0	Normal termination of the solver.
4	Normal termination of the solver, but a large amount of work has been expended.
12	Normal termination of the solver, but an <i>End Condition</i> was found (flag, last argument in resid, was equal to 3 and the solver has been called to finish at the new current point).
-2	The error tolerances are too stringent.
-3	The local error test cannot be satisfied because you specified a zero value for the absolute error in tol and the corresponding computed solution component is zero. Thus, a pure relative error test is impossible for this component.
-6	The method used had repeated error test failures on the last attempted step.
-7	The corrector could not converge.
-8	The matrix of partial derivatives is singular.
-9	The corrector could not converge: there were repeated error test failures in this step.
-10	Abnormal termination of the solver, due to an <i>Illegal Condition</i> (flag, last argument in resid, was equal to $-1$ and the solver did its best to not obtain $-1$ ).
-11	Abnormal termination of the solver, due to an <i>Emergency Exit</i> (flag, last argument in resid, was equal to $-2$ and control has been returned to the calling program).
-13	Some indices for the non-negativity constraints stored in the non-neg array of the mf_DE_Options are not in the range [1,Neq] (where Neq is the number of equations).
-14	The user-supplied resid routine returned invalid entries (NaN value).
-15	The user-supplied $\mathtt{jac}$ routine (either dense or sparse) returned invalid entries (NaN value).
-20	Allocation error because the maximum of available memory has been reached. An explanation is printed.
-33	Invalid argument.
-34	Invalid implementation of the user-defined sparse jacobian.
-40	On calling, the number of output argument(s) is incorrect.
-100	The routine wasn't able to start, neither choose an initial timestep, nor check initial conditions for consistency (flag, last argument in resid, was different from 0).
-101	Initial conditions y_0 and yp_0 are not consistent.
-102	msDaeSolve failed to compute the initial vector yp_0.

 $See \ also: {\tt msOdeSolve}, {\tt mf\_DE\_Options}$ 

### mf\_NL\_Options

# options for non-linear solvers (derived type)

#### Description:

This derived type gathers options used by the two routines mfFSolve and mfLsqNonLin, and is defined as follows:

```
type :: mf_NL_Options
```

```
type(mfArray)
                         :: tol
                                                    = 50
   integer
                         :: max_iter
   type(mfArray)
                         :: epsfcn
   real(kind=MF_DOUBLE) :: init_step_bound_factor = 1.0d2
   logical
                         :: print
                                                    = .false.,
                                                                             &
                                                    = .false.,
                                                                             &
                            print_using_transf
                            reuse_spjac_struct
                                                    = .false.
   type(func_ptr), allocatable :: f_inv_transf(:)
   integer
                                                    = 0,
                                                                             &
                         :: check_jac
                            print_check_jac
                                                    = 0
   logical
                         :: box_constrained
                                                    = .false.
   type(mfArray)
                         :: lower_bounds,
                            upper_bounds
   real(kind=MF_DOUBLE) :: sing_jac_tol
                                                    = epsilon(1.0d0)
   logical
                         :: print_sing_val
                                                    = .false.
end type mf_NL_Options
```

The following fields may be used in both two routines:

- tol specifies the tolerance to be used. See the appropriate routine to know the usage of this option.

- max\_iter specifies the approximated maximum number of iterations.
- epsfcn specifies the optimum small increment (in a relative sense) used for computing the derivatives (the elements of the jacobian matrix) by finite differences. Therefore, its value is not used when the jacobian matrix is provided by the user. It may be a scalar or a vector; in the latter case (but only for LsqNonLin), each components of epsfcn corresponds to an unknown. The value(s) of epsfcn must be carefully chosen; there doesn't exist a universal constant because the optimum value depends on the algorithm in the user-supplied fcn routine. Its role is basically to equalize the numerical error which comes from the scheme and the roundoff error inherent to the floating-point model. If fcn is coded as a mathematical function, it can be set to zero (because internally, the finite-difference routine will choose  $\sqrt{\epsilon}$ , where  $\epsilon$  is the machine precision; this is the default). If fcn uses an interpolation over a given interval  $\Delta x$  (e. g. from a table lookup) then the epsfcn should be related to some power of this  $\Delta x$  value, according the accuracy of the interpolation scheme.
- init\_step\_bound\_factor specifies the initial step bound factor. The default value of 100 is recommended in most cases. However it can be changed for another value as low as 0.1. For example, if you use the log() function to ensure the positiveness of your unknowns, you should choose log(100) instead.
- print is used to print on screen the following information during the iterative process: the iteration number, the value of each unknown parameter and the norm of the residue. If some transformation is used between the physical value of the parameters and the actual value in the algorithm, it may be convenient to print the physical value of each parameter; to do that, the user must set the print\_using\_transf field to .true. and associate the function pointers stored in the field f\_inv\_transf to appropriate user-defined inverse functions. The last pointer array must be allocated by the user. See the Muesli User's Guide for more details.

 $\cdots/\dots$ 

- reuse\_spjac\_struct must be set to .true. only when you are sure that the sparse structure of the jacobian is not modified during many different calls of the FSolve solver: this can occur inside a loop when using the same set of equations. Be aware that this option can only set to .true. after the first call of the solver; on the contrary, an error will arise. Note that this option is used only for the sparse case.
- for debugging purposes (but only when MF\_NUMERICAL\_CHECK is TRUE), check\_jac can be used to make a check of the user jacobian (dense only):
  - 1. when check\_jac is set to 1, a global and quick check is done;
  - 2. when check\_jac is set to 2, a complete (but more expensive) check is done.

In both cases, a message is displayed according to the errors obtained concerning the accuracy of the jacobian. Note that this is an estimation because (i) discrepancies found may be strongly affected by both floating-point roundoff errors and badly scaled variables or equations, and (ii) relative errors are based on a sort of norm of the jacobian; when this latter is large, some terms in the error matrix may remain small.

print\_check\_jac can be used to print the results of the jacobian check:

- 1. when print\_check\_jac is set to 1, information is displayed on the screen;
- 2. when print\_check\_jac is set to 2, information is stored in files (one different file for each jacobian call).

In both cases, (i) the given information depends on the level of the check and (ii) the output is done at each call of the jac routine, regardless of the severity of the discrepancies. See the *Muesli User's Guide* where a detailed example of use can be found.

- (for LsqNonLin only) box\_constrained can be used to constrain the parameters to be inside a box.
   Set it to .true. and specify the box bounds as follows:
  - 1. lower\_bounds is a vector containing the lower bound for each parameter. The length of this mfArray vector must be equal to the number of unknown parameters; use -MF\_INF to release the constraint.
  - 2. upper\_bounds is a vector containing the upper bound for each parameter. The length of this mfArray vector must be equal to the number of unknown parameters; use +MF\_INF to release the constraint.

These constraints of type box are dynamic, which means that during the iterations, the user can change the value of the bounds (e. g. in the fcn routine).

- Only when MF\_NUMERICAL\_CHECK is TRUE, sing\_jac\_tol allows the user to know whether his system of equations is singular or not. It concerns only the LsqNonLin routine. Normally the jacobian is not singular, which means that all the parameters are independent; on the contrary, the routine displays the linear relation between the parameters, by showing the vector of the basis of the nullspace. Note that the nullspace basis is always computed from a local point-of-view (linearized equations); if this basis changes along the iteration, this means that the relationship between the parameters is nonlinear. The default value of this tolerance is the machine  $\epsilon$ . Set sing\_jac\_tol to zero to avoid the computation of the nullspace.
- Only when MF\_NUMERICAL\_CHECK is TRUE, print\_sing\_val allows the user to print the singular values of the jacobian matrix. Default is FALSE.

All fields of this derived type are public, so the user can access each component via the '%' Fortran 90 selector.

Lastly, the user can use the msRelease routine to free all fields of this structure (and to re-initialize the components between two different calls to any of these integrators).

See also: mf\_DE\_Options

### mf\_DE\_Options

## options for differential solvers (derived type)

#### Description:

This derived type gathers options used by the two integrators mfOdeSolve and mfDaeSolve, and is defined as follows:

#### type :: mf\_DE\_Options

```
logical
                   :: continuation = .false.
character(len=3)
                  :: method = ""
type(mfArray)
                  :: tol, y_ind_out, band
type(mfArray)
                  :: non_neg, y0_ind, yp0_ind
                                     = .true.,
logical
                   :: IC_known
                                                                   &
                      print_progress
                                       = .false.,
                                                                   &
                      disp_times
                                         = .false.,
                                                                   &
                      jac_symm_pos_def = .false.,
                                                                   &
                      spjac_const_struct = .true.,
                                                                   &
                      reuse_spjac_struct = .false.,
                                                                   &
                      jac_investig = .false.,
                                                                   &
                      rational_null_basis = .false.,
                                                                   &
                      pseudo_inverse
                                        = .false.
real(kind=MF_DOUBLE) :: jac_rcond_min
                                        = MF_EPS
logical
                   :: save_sing_jac
                                         = .false.
type(mfArray)
                   :: monitor_y_ind,
                                                                   &
                     monitor_yp_ind
logical
                   :: monitor_pause
                                         = .false.
                                         = 0,
integer
                   :: check_jac
                                                                   Хr.
                      print_check_jac
type(mf_DE_Named_Group), allocatable :: named_eqn(:),
                                                                   &
                                     named_var(:)
```

end type

The following fields may be used in both integrators:

- continuation specifies that the integrator is not restarted. Using such a feature to continue the integration leads to much better performance than restarting the integrator.
- tol specifies the tolerance in many ways (relative or absolute, as a whole or by component).
- y\_ind\_out allows the user to get only a subpart of the result.
- band specifies that the jacobian is stored in a band format.
- print\_progress specifies that the percentage of work already done is printed in the terminal. This
  options should not be used when using the main program in a batch mode, redirecting output in
  a file.
- disp\_times specifies that time values will also be displayed wall clock time is used, not CPU time.

(See msPrepProgress and msPrintProgress for other information concerning the last two options.)

.../...

- jac\_symm\_pos\_def must be set to .true. only when you are sure that the jacobian is symmetric and positive definite; in such a case, you must only provide the upper part of the jacobian: this will economize some storage. A Cholesky factorization will be used (instead of a general LU one) and, especially for sparse, large matrices, this will be more efficient.
- spjac\_const\_struct must be set to .false. only when you are sure that the sparse structure of
  the jacobian is modified along all the time integration (this is not common!); note that this option
  is used only for the sparse case.
- reuse\_spjac\_struct must be set to .true. only when you are sure that the sparse structure of the jacobian is not modified during many different calls of the DAE or ODE solvers: this can occur during an iterative procedure using the same mesh. Be aware that this option can only set to .true. after the first call of the solver; on the contrary, an error will arise. Note that this option is used only for the sparse case.
- for debugging the DAE integration (but only if jac\_investig is TRUE), the user may set a threshold value (not greater than unity) in jac\_rcond\_min in order to detect whether the jacobian is singular or not<sup>6</sup>. When (an estimation of) the reciprocal of condition number is less than or equal to the value of jac\_rcond\_min, then the jacobian is considered as singular: a warning message is then emitted. Moreover, for the dense case only:
  - 1. additional information is provided to the user, about the null space of the transpose of the jacobian matrix: this may help the user in understanding why the jacobian is singular.
  - 2. the jacobian and the nullspace of its transpose can be saved on disk according to the value of the boolean save\_sing\_jac (the filenames are respectively 'jacobian.dat' and 'nullspace.dat').

The default value of <code>jac\_rcond\_min</code> is the machine epsilon and, if set by the user, it cannot be smaller than this default value. The nullspace basis may be returned in the form of a rational basis (set <code>rational\_null\_basis</code> to .true.) but this should be useful for small problem only (few number of equations) or for pedagogical reasons. The <code>pseudo\_inverse</code> component is reserved for a future usage.

- for debugging purposes again, the user can tell the appropriate integrator to store all intermediate values (note: at the internal time steps!), by specifying a number of indices in monitor\_y\_ind, and/or indices in monitor\_yp\_ind. In such a case, the selected values of the vector y (resp. y') are copied in the file 'odesolve\_y.out' or 'daesolve\_y.out' (resp. 'odesolve\_yp.out' or 'daesolve\_yp.out'); each line contains the current time (independant variable) in the first column, then the selected values of the dependant variables. The boolean monitor\_pause allows the user to pause after each successful internal step of the ODE/DAE solver (the default behavior is not to make such an interruption). Moreover, the data files are flushed continuously, so you can use an external graphic tool to plot the data which are monitored.
- recall that the use of MF\_NUMERICAL\_CHECK is recommended, because it allows internal routines to check the validity of the equations or of the jacobian elements, especially non finite values (Infinities, NaNs).

· · · / . . .

<sup>&</sup>lt;sup>6</sup>For the ODE solver, the jacobian provided by the user may be singular, it doesn't matter for the integration process. See the remark at the end of this entry.

- for debugging purposes (but only when MF\_NUMERICAL\_CHECK is TRUE), check\_jac can be used to make a check of the user jacobian (dense only):
  - 1. when check\_jac is set to 1, a global and quick check is done;
  - 2. when check\_jac is set to 2, a complete (but more expensive) check is done.

In both cases, a message is displayed according to the errors obtained concerning the accuracy of the jacobian. Note that this is an estimation because (i) discrepancies found may be strongly affected by both floating-point roundoff errors and badly scaled variables or equations, and (ii) relative errors are based on a sort of norm of the jacobian; when this latter is large, some terms in the error matrix may remain small.

print\_check\_jac can be used to print the results of the jacobian check:

- 1. when print\_check\_jac is set to 1, information is displayed on the screen;
- 2. when print\_check\_jac is set to 2, information is stored in an overwritten file (a unique file along all the integration process). The library makes as many pauses as needed to let the user to inspect this file.

In both cases, (i) the given information depends on the level of the check and (ii) the output is displayed at each call of the <code>jac</code> routine, regardless of the severity of the discrepancies. See the <code>Muesli User's Guide</code> where a detailed example of use can be found.

- the named\_eqn and named\_var fields allow the user to name his equations and variables by grouping them under useful, physical names. The derived type mf\_DE\_named\_group is defined as follows:

```
type :: mf_DE_Named_Group
   character(len=132) :: name
   integer :: begin, last
end type
```

The user must allocate himself the arrays named\_eqn(:) and named\_var(:), then correctly initializes the three internal fields: name (the name of the group), begin and last (the indices of equation in deriv or resid which define the beginning and the end of the each named group). See the *Muesli User's Guide* where an example of use can be found.

The field method is specific to the mfOdeSolve integrator. It may contains "RKF" (Runge-Kutta Fehlberg, which is the default method), "ABM" (Adams-Bashforth-Moulton), or "BDF" (Backward Differentiation Formula).

The remaining are specific to the mfDaeSolve integrator:

- non\_neg allows the user to constrain some components of the solution to remain non negative.
- IC\_known indicates whether initial conditions are known (and then supposed to be consistent) or not (see also just below).
- in the case where initial conditions have to be computed (i. e. when IC\_known is equal to .false.) and only when the jacobian matrix is provided, the user may indicate which components of the vectors y\_0 and yp\_0 are prescribed in, respectively, y0\_ind and yp0\_ind (at most N components may be prescribed, where N is the number of the system's equations).

If used, these two vectors must have a length equal to N; free components are selected by putting a zero at appropriate locations, whereas prescribed components may be selected by putting any non-zero value.

The user may also leave the fields empty, which is equivalent to specify that no component at all are prescribed. Values present in the two  $mfArrays\ y_0$  and  $yp_0$  are then considered as initial guesses.

It is worth mention that initialization may fail in the case where the user prescribed too many components. A very common error is to prescribed the N components of  $y_0$  by setting all com-

ponents of  $y0\_ind$  to one, for a DAE systems which have  $N_a$  algebraic equations; in such a case, all components referencing to the algebraic equations should be left free, and the user can only prescribed the  $N-N_a$  components of  $y\_0$  corresponding to the differential equations.

All fields of this derived type are public, so the user can access each component via the '%' Fortran 90 selector.

Lastly, the user can use the msRelease routine to free all fields of this structure (and to re-initialize the components between two different calls to any of these integrators).

Remark: The jacobian matrix of the ODE solver may be singular, because the iteration matrix M writes:

$$M = I + \Delta t J = I + \Delta t \frac{\partial F}{\partial y}$$

It can be seen easily that for a sufficiently small time-step  $\Delta t$ , M is not singular so there is no constraint over J: it could be even the null matrix! Otherwise, the iteration matrix M used by the DAE solver is close to the following generalized jacobian:

$$M \approx \widetilde{J} = \frac{\partial R}{\partial y} + c_J \frac{\partial R}{\partial y'}$$

where  $c_J$  is related to the inverse of the time step  $\Delta t$ . In this latter case,  $\widetilde{J}$  cannot be singular. (The two functions F(t,y) and R(t,y,y') have been defined respectively at the mfOdeSolve and mfDaeSolve entries.)

See also: mf\_NL\_Options

## 1.11 Sparse Matrices

mfSparsesparse matrix conversionmfFulldense matrix conversionmfSpAlloc, msSpReAllocsparse matrix space allocationmfNnznumber of nonzero matrix elements

mfNzMaxsparse matrix internal sizemfNcolMaxsparse matrix internal sizemfSpEyesparse identity matrix

mfSpOnes replace nonzero elements with ones

mfSpRandsparse uniformly distributed random matrixmfSpRandNsparse normally distributed random matrixmfSpDiagssparse matrix formed from diagonals

mfSpCut cut small elements

mfSpImport, msSpExport sparse matrix conversion from/to f90 arrays

mfIsRowSortedrow sorted inquirymsRowSortsparse matrix row sortmsGetAutoRowSortedrow sorting policymsSetAutoRowSortedrow sorting policy

mfMatFactor handle to internal matrix factors (derived type)

msFreeMatFactor deallocation of internal matrix factors

#### See also:

Core Routines

File Input/Output

**Data Analysis Functions** 

Operators

Elementary Math Functions

Specialized Math Functions

Elementary Matrix Manipulation Functions

Matrix Functions

Polynomial Functions

Optimization and Function Functions

# mfSparse

# sparse matrix conversion

Calling syntax:

B = mfSparse( A )

converts the  ${\tt mfArray}$  A (which should be dense) into a sparse  ${\tt mfArray}$ .

To create a sparse matrix from scratch, use the  ${\tt mfSpAlloc}$  routine instead; from existing contents, use  ${\tt mfSpImport}$ .

See also: mfFull, msSpExport

# mfFull

# dense matrix conversion

Calling syntax:

B = mfFull(A)

converts the  ${\tt mfArray}$  A (which should be sparse) into a dense  ${\tt mfArray}$ .

 $See \ also: \ {\tt mfSparse}$ 

## mfSpAlloc

## sparse matrix space allocation

# ${\it Interface:}$

### Description:

This routine is used to create new space for a sparse mfArray of size m by n (logical dimensions) which will contain at most nzmax (if present) non-zero values. If nzmax is not present, it is set to zero.

If ncolmax is present, it reserves space for a greater number of columns. The value of ncolmax cannot be less than n; it defaults to n. This argument is usually employed with the routine msHorizConcat.

kind may be equal to "real" or "complex"; it defaults to "real".

See also: msSpReAlloc, msHorizConcat

### msSpReAlloc

### sparse matrix space re-allocation

# ${\it Interface:}$

#### Description:

This routine changes the space for elements in the sparse mfArray A. At least one argument, among the two optional ones nzmax and ncolmax, must be present.

The argument nzmax corresponds to the maximum number of non zero entries in the sparse structure. It can be increased to any value, but you will get an error if the value of nzmax is too small.

The argument ncolmax corresponds to the maximum number of the columns. Here again, you will get an error if the value of ncolmax is too small. Be careful when changing its value, because the logical size of the matrix is determine by ncolmax.

Usually the two arguments are used to *increase* the corresponding value. It may be dangerous to try to *decrease* their value without knowledge about the internal sparse structure. See below to decrease as much as possible the internal size of the structure automatically, without loss of data.

#### Other calling syntax:

To compact an mfArray to its minimal room space (both in terms of the maximum number of non zero entries nzmax, and maximum number of columns ncolmax), use the following special calling sequence:

```
call msSpReAlloc( A, "minimal" )
```

Note that this will remove also any (unwanted) zeros actually stored.

 $See \ also: \ {\tt mfSpAlloc}$ 

### mfNnz

# number of nonzero matrix elements

 ${\it Interface:}$ 

```
function mfNnz( A ) result( out )
    type(mfArray), intent(in) :: A
    integer :: out
```

Description:

Returns the number of non-zero element of the sparse  ${\tt mfArray}\ {\tt A}.$ 

See also: mfNzMax

### mfNzMax

# sparse matrix internal size

 ${\it Interface:}$ 

```
function mfNzMax( A ) result( out )
  type(mfArray), intent(in) :: A
  integer :: out
```

# Description:

Returns the maximum number of non-zero element of the sparse mfArray A.

See also: mfNnz, mfNcolMax

### mfNcolMax

# sparse matrix internal size

 ${\it Interface:}$ 

```
function mfNcolMax( A ) result( out )
  type(mfArray), intent(in) :: A
  integer :: out
```

# Description:

Returns the maximum number of columns of the sparse  ${\tt mfArray}\ {\tt A}.$ 

See also: mfNnz, mfNzMax

## mfSpEye

# sparse identity matrix

## Interface:

## Description:

This routine creates a sparse mfArray of size m by n, having ones on its main diagonal.

The optional argument kind allows the user to choose a complex array (if kind is equal to "complex"); by default, a real matrix is returned.

See also: mfSpDiags, mfSpOnes

# mfSpOnes

# replace nonzero elements with ones

 $Calling\ syntax:$ 

B = mfSpOnes( A )

## Description:

This routine creates a sparse matrix by replacing all non-zero values of the mfArray A by one.

Be aware that this routine is NOT the sparse equivalent of  ${\tt mfOnes}.$ 

See also: mfSpEye

## mfSpRand

# sparse uniformly distributed random matrix

 $First\ calling\ syntax:$ 

```
B = mfSpRand( A )
```

## Description:

This routine creates a sparse matrix by replacing all non-zero values of the  $mfArray\ A$  (which may be dense or sparse) by a uniformly distributed random value. Therefore, the  $mfArray\ B$  has the same sparsity than A.

 $Second\ calling\ syntax:$ 

```
B = mfSpRand( m, n, density )
```

### Description:

creates a random, m-by-n, sparse matrix with approximately density\*m\*n uniformly distributed nonzero entries (the real density must be in [0,1]).

 $See \ also: \mathtt{mfSpEye}, \mathtt{mfSpOnes}, \mathtt{mfSpRandN}$ 

## mfSpRandN

# sparse normally distributed random matrix

First calling syntax:

```
B = mfSpRandN( A )
```

## Description:

This routine creates a sparse matrix by replacing all non-zero values of the  $mfArray\ A$  (which may be dense or sparse) by a normally distributed random value. Therefore, the  $mfArray\ B$  has the same sparsity than A.

 $Second\ calling\ syntax:$ 

```
B = mfSpRandN( m, n, density )
```

### Description:

creates a random, m-by-n, sparse matrix with approximately density\*m\*n normally distributed nonzero entries (the real density must be in [0,1]).

See also: mfSpEye, mfSpOnes, mfSpRand

### mfSpDiags

### sparse matrix formed from diagonals

 ${\it Interface:}$ 

```
function mfSpDiags( m, n, v, d ) result( out )
  integer, intent(in) :: m, n
  type(mfArray) :: v
  integer, intent(in), optional :: d
  type(mfArray) :: out
```

#### Description:

Creates a sparse matrix (of size m by n) whose d-diagonal is the input vector v (dense).

The vector v may have a length greater than those of the d-diagonal (in such a case, subsequent elements will not be referenced); it can be also a scalar (in such a case, the scalar value is spread over the diagonal).

Remarks: When d is equal to zero, it points to the main diagonal; when positive, the upper part of the matrix is concerned. If the arg. d is not present, the main diagonal is the default.

 $Simplified\ interface:$ 

```
function mfSpDiags( v ) result( out )
  type(mfArray) :: v
  type(mfArray) :: out
```

#### Description:

When only one argument is used (it must be a vector mfArray A), then the routine outputs a diagonal square sparse matrix whose dimensions are deduced from the length of the vector d.

 $Other\ interface:$ 

```
function mfSpDiags( m, n, A, d ) result( out )
  integer, intent(in) :: m, n
  type(mfArray) :: A
  integer, intent(in) :: d(:)
  type(mfArray) :: out
```

#### Description:

In this second interface the input mfArray A may be a rank-2 array. So, each column of A is used to fill a diagonal of the returned sparse matrix, whose index is specified in the vector d.

If A is a row vector then each constructed diagonal will have the same elements.

 $See \ also: {\tt mfSpEye}, {\tt mf/msDiag}$ 

mfSpCut cut small elements

Calling syntax:

B = mfSpCut( A, threshold )

## Description:

This routine removes all elements of the mfArray A (sparse, real or complex) whose magnitude is less or equal than the prescribed threshold (positive real arg.).

### mfSpImport

### sparse matrix conversion from f90 arrays

Calling syntax:

```
A = mfSpImport( i, j, d[, m, n, nzmax, format, duplicated_entries, pattern ] )
```

creates the sparse mfArray A from a sparse structure stored in classical f90 arrays; i and j are integer f90 vectors whereas d is a real or a complex f90 vector.

The vector  $\mathbf{d}$  may have a null size, in which case the allocation of data space is made by this routine but filled with NaN value (initialization may occur later on).

If present, the character string format specifies the sparse format; it may be equal to "COO" (COOrdinates), "CSC" (Compact Sparse Columns) or "CSR" (Compact Sparse Rows). By default, format="COO" is assumed.

If present, m and n define the (logical) shape of the sparse mfArray; these shape must be large enough to contain all the input indices specified in i and j. By default, the logical shape is deduced as follows:

- if the format is "C00", then the matrix shape is deduced from the maximum value of arrays i and
   j. It follows than these two arrays are inspected.
- if the format is "CSC" (resp. "CSR"), the number of columns (resp. rows) comes from the size of the j array (resp. of the i array). Then the other dimension is deduced by inspecting the appropriate part of the other integer array; this is because the long index array may be overdimensioned.

If present, nzmax allows to reserve more place than the actual size.

During importing f90 arrays:

- eventual duplicated entries are treated according to the optional character argument duplicated\_entries. When this argument is present and equal to "ignored", duplicated entries are ignored; when it is equal to "added" (default value), duplicated entries are added; when it is equal to "replaced", last entries found overwrite previous ones.
- entries containing a null value are removed.

If present, the character string pattern indicates that the input matrix has a specific pattern, and consequently that only a part of the values may be present in the vector d:

- "symm": the input matrix is symmetric (real or complex) and, possibly, only the lower or the upper part is provided by the user.
- "skew": the input matrix is skew-symmetric (real or complex) and, possibly, only the lower or the upper part is provided by the user. No diagonal element can be found in the entries.
- "herm": the input matrix is hermitian (complex) and, possibly, only the lower or the upper part is provided by the user. For a real matrix, it is equivalent to "symm".

Note that a check about the pattern is done after importing the input values; in other words, giving a random matrix will raise an error.

See also: msSpExport, mfSparse, mfLoadSparse, msSaveSparse

## msSpExport

# sparse matrix conversion to f90 arrays

 $Calling\ syntax:$ 

```
call msSpExport( A, i, j, d [, format ] )
```

copies the sparse mfArray A to a sparse structure using classical f90 arrays.

 $\mathtt{i}$  and  $\mathtt{j}$  are integer f90 vectors.  $\mathtt{d}$  is a real or a complex f90 vector.

If present, the character string format specifies the sparse format; it may be equal to "COO" (COOrdinates), "CSC" (Compact Sparse Columns) or "CSR" (Compact Sparse Rows). By default, format="COO" is assumed.

 $See \ also: \mathtt{mfSpImport}, \mathtt{mfSparse}, \mathtt{mfLoadSparse}, \mathtt{msSaveSparse}$ 

### mfIsRowSorted

# row sorted inquiry

 ${\it Interface:}$ 

```
function mfIsRowSorted( A ) result( bool )
  type(mfArray), intent(in out) :: A
  logical :: bool
```

Description:

Returns '.true.' if the columns of the sparse mfArray A are "row sorted" and contain no duplicated entries.

 $See~also: {\tt msRowSort}, {\tt msGetAutoRowSorted}, {\tt msSetAutoRowSorted}$ 

### msRowSort

# sparse matrix row sort

 $Calling\ syntax:$ 

```
call msRowSort( A [, struct_only ] )
```

## Description:

Sorts all columns of the sparse mfArray A in increasing order, and remove all duplicated entries.

If the logical optional argument struct\_only is present and equal to '.true.' then the data array is discarded.

 $See~also:~{\tt mfIsRowSorted},~{\tt msGetAutoRowSorted},~{\tt msSetAutoRowSorted}$ 

#### ${\tt msGetAutoRowSorted}$

# row sorting policy

 ${\it Interface:}$ 

```
subroutine msGetAutoRowSorted( auto_row_sorted )
    logical, intent(out) :: auto_row_sorted
```

### Description:

If the returned logical  $auto\_row\_sorted$  is TRUE, then all sparse matrices are row sorted when needed.

 $See~also: {\tt msRowSort}, {\tt mfIsRowSorted}, {\tt msSetAutoRowSorted}$ 

#### ${\tt msSetAutoRowSorted}$

# row sorting policy

Interface:

```
subroutine msSetAutoRowSorted( auto_row_sorted )
    logical, intent(in) :: auto_row_sorted
```

#### Description:

Only if  $auto\_row\_sorted$  is TRUE, then all sparse matrices will be row sorted when needed. Therefore, this routine can change this row sorting policy.

By default, auto\_row\_sorted is TRUE.

See also: msRowSort, mfIsRowSorted, msGetAutoRowSorted

#### mfMatFactor

# handle to internal matrix factors (derived type)

# Description:

This derived type is a handle to (sparse or dense) factors computed by some factorization routine (e. g. msLU, msQR or msChol). Declaration is made as follows:

#### type(mfMatFactor) :: factor

The user cannot manipulates the contents of this derived type: it is indeed intended to be used only by other MUESLI routines.

The user may release this structure by use of the msFreeMatFactor routine (but use of msRelease is equivalent).

### ${\tt msFreeMatFactor}$

# deallocation of internal matrix factors

 $Calling\ syntax:$ 

call msFreeMatFactor( factor )

Description:

Deallocate the mfMatFactor factor.

Remark: The routine msRelease may also be used in an equivalent way.

# 2 FGL: Graphical Library

FGL contains all routines to make figures and plots (only in 2D).

Routines beginning with 'ms' are subroutines, whereas those beginning with 'mf' are functions. Usually, the two forms of the same routine do the same job, but the latter returns an identification (actually, a handle to the graphic object).

The available routines have been grouped into sub-parts:

Global graphic settings

Window's and figure's management

Figure properties

Figure annotation – Low level graphic object's manipulation

High level plotting routines

Interactive routines

# 2.1 Global graphic settings

msSetBackgroundColor
msSetColorOverflowPolicy
mfGetColorOverflowPolicy
msExitFgl

mfGetX11Device, msSetX11Device
mfGetX11ColorDepth

mfGetDefaultCapStyle, msSetDefaultCapStyle
mfGetDefaultJoinStyle, msSetDefaultJoinStyle

mfGetCharEncoding, msSetCharEncoding

background color setting set color overflow policy get color overflow policy FGL closing

X11 device status get X11 screen color depth

default cap style for lines default join style for lines

character encoding

#### See also:

Window's and figure's management

Figure properties

Figure annotation – Low level graphic object's manipulation

High level plotting routines

Interactive routines

### ${\tt msSetBackgroundColor}$

### background color setting

 ${\it Interface:}$ 

```
subroutine msSetBackgroundColor( color_string )
    character(len=*), intent(in) :: color_string
```

#### Description:

Set the background color. The only two possibilities are "white" (the default) and "black".

This routine must be used at the beginning of the graphic part. The background color must be the same for all the opened windows; it cannot be changed during the execution, until FGL graphic part is closed (msExitFg1) and automatically reinitialized by opening any figure.

*Remark:* When the background has been selected as black, printing leads to different behavior for EPS and PDF formats (see msPrint).

### mfGetColorOverflowPolicy

# get color overflow policy

Interface:

```
function mfGetColorOverflowPolicy() result( policy )
      character(len=10) :: policy(2)
```

#### Description:

Returns two strings. The first one (resp. second one) refers to color overflow in the low part (resp. high part) of the colormap.

The color overflow policy may be "signaled" or "truncated". When it is "signaled", a contrasted color is used to signal the colormap overflow. When it is "truncated", the lowest color (or highest color) is used, giving a smooth aspect to the colors.

Default behavior is "signaled" for both ends.

 $See \ also: {\tt msSetColorOverflowPolicy}$ 

### msSetColorOverflowPolicy

# set color overflow policy

### ${\it Interface:}$

```
subroutine msSetColorOverflowPolicy( low, high )
    character(len=*), optional :: low, high
```

#### Description:

This routine is used to monitor the colormap overflow. low and high are two strings which must contain "signaled" or "truncated".

When it is "signaled", a contrasted color is used to signal the colormap overflow. When it is "truncated", the lowest color (or highest color) is used, giving a smooth aspect to the colors.

Default behavior is "signaled" for both ends.

*Remark:* if none of the two arguments is present, then nothing is changed.

See also: mfGetColorOverflowPolicy

### mfGetX11Device

Get X11 device status

 ${\it Interface:}$ 

status = mfGetX11Device( )

Description:

Returns the status of the X11 device, which can be "on" (the default) or "off".

 $See \ also: {\tt msSetX11Device}$ 

#### msSetX11Device

#### Set X11 device status

### Interface:

```
subroutine msSetX11Device( flag )
    character(len=*), intent(in) :: flag
```

#### Description:

During execution, as MUESLI is able to produce printed images (EPS or PDF) in an independant way, the X11 device can be de-activated, by setting the flag parameter to "off". It can be useful for batch executions.

### Remarks:

- the X11 device is "on" by default;
- it can be enabled/disabled by the MFPLOT\_X11\_DEVICE environment variable, which can take the value 0 or 1.

 $See \ also: {\tt mfGetX11Device}$ 

# ${\tt mfGetX11ColorDepth}$

get color depth

 ${\it Interface:}$ 

```
function mfGetX11ColorDepth() result( depth )
  integer :: depth
```

Description:

Returns the color depth currently used by the X11 server. A figure must have been previously opened.

See also: mf/msColormapSize, mf/msColormap, msColorbar

msExitFgl FGL closing

Calling syntax:

```
call msExitFgl()
```

This routine is intended to be used at the end of your graphical program. It leaves a prompt to the user and waits for an answer ('y' or 'Y'), after warning the user by a text like:

\*\*\*

End of MUESLI plotting:

- all figures are going to be closed.
- graphic memory will be properly cleaned.

Do you really close graphics ? [y|Y]

It is a useful command to avoid that all graphic windows disappear; as mentioned, it also cleans the FGL internal memory (all graphic objects); moreover, X11 windows position and size are saved in a small database.

Remarks: If you want that your program terminates without a user action, you can redirect the standard input (via '<' in a shell command) and read a file containing the 'y' symbol. Another possibility is to use the 'yes' unix command which outputs repeatedly the character 'y' to your program via a unix pipe, e. g.:

\$ yes | a.out

See also: msRelease

# ${\tt mfGetDefaultCapStyle}$

# Get default cap style for lines

 $Calling\ syntax:$ 

```
cap_style = mfGetDefaultCapStyle( )
```

Description:

Returns the current default cap style, which may be 0 ("CapButt"), 1 ("CapRound") or 2 ("CapProjecting").

The signification of these three attributes may be found easily on the web.

 $See~also:~{\tt msSetGrObj},~{\tt msSetDefaultCapStyle},~{\tt msSetDefaultJoinStyle}$ 

# ${\tt msSetDefaultCapStyle}$

# Set default cap style for lines

Interface:

```
subroutine msSetDefaultCapStyle( cap_style )
    character(len=*), intent(in) :: cap_style
```

#### Description:

Change the global value for the default cap style. The argument  ${\tt cap\_style}$  may be set to "CapButt", "CapRound" or "CapProjecting".

The signification of these three attributes may be found easily on the web.

Note also that this graphic attribute may be change for a particular object, via the msSetGrObj routine.

 $See\ also:$  mfGetDefaultCapStyle, mfGetDefaultJoinStyle

### ${\tt mfGetDefaultJoinStyle}$

# Get default join style for lines

Calling syntax:

```
join_style = mfGetDefaultJoinStyle( )
```

### Description:

Returns the current default join style, which may be 0 ("JoinMiter"), 1 ("JoinRound") or 2 ("JoinBevel").

The signification of these three attributes may be found easily on the web.

 $See~also:~{\tt msSetGrObj},~{\tt msSetDefaultJoinStyle},~{\tt msSetDefaultCapStyle}$ 

### ${\tt msSetDefaultJoinStyle}$

# Set default join style for lines

Interface:

```
subroutine msSetDefaultJoinStyle( join_style )
    character(len=*), intent(in) :: join_style
```

#### Description:

Change the global value for the default join style. The argument join\_style may be set to "JoinMiter", "JoinRound" or "JoinBevel".

The signification of these three attributes may be found easily on the web.

Note also that this graphic attribute may be change for a particular object, via the msSetGrObj routine.

 $See\ also:$  mfGetDefaultJoinStyle, mfGetDefaultCapStyle

# mfGetCharEncoding

# Get character encoding

 ${\it Interface:}$ 

encoding = mfGetCharEncoding( )

Description:

Returns the current character encoding, which is "Latin" (the default) or "UTF-8".

This is used in the following graphic routines, which display character strings in a figure: mf/msText, msTitle, msXLabel, msYLabel, mf/msFigure and msLegend.

 $See \ also: {\tt msSetCharEncoding}$ 

### msSetCharEncoding

### Set character encoding

 ${\it Interface:}$ 

```
subroutine msSetCharEncoding( encoding )
    character(len=*), intent(in) :: encoding
```

#### Description:

When processing character's strings (e.g. in the msTitle routine), MUESLI accepts two different character encodings. The charset may be "Latin" (default one) or "UTF-8".

So, the encoding argument can take only the following values:

```
- "Latin" (or "ISO-8859-1" which is an alias) - "UTF-8"
```

Warning: Not all the characters defined in the "UTF-8" encoding are valid. Actually, only a subpart of the 256 first characters (i.e. mostly of the "Latin" encoded characters, but not all) are correctly processed.

See also: mfGetCharEncoding

# 2.2 Window's and figure's management

mf/msFigure figure creation
msResizeWindow window resizing

mfGetWinId current figure identification

msClose figure deletion

 ${\tt mfGetColorScheme}, \, {\tt msSetColorScheme} \quad {\tt color} \, \, {\tt scheme}$ 

mfGetColorInd, msSetColorInd index in color scheme

msPrint file printing (EPS and PDF)

msSetPdfOC PDF Optional Contents management

#### $See\ also:$

Global graphic settings

Figure properties

Figure annotation – Low level graphic object's manipulation

High level plotting routines

Interactive routines

mf/msFigure figure creation

Calling syntax:

```
call msFigure( [win_id] [, position] [, size] [, title] )
```

where arguments are all optional.

This routine opens a new window or selects (and therefore makes active) a previously opened one.

The win\_id argument (integer) indicates the identification of the window; if not present, a sequential integer from 1 is used.

The position argument is used to impose the position of the window on the screen. It must be a Fortran array of two integer values specifying the position of the newly created window; the position in pixels of the top-left corner of the window is taken from the top-left corner of the whole screen (virtual screen in case of many monitors), counting positively downward for the y value. Default position is approximately (0,0) for the first window, and shifted both in x- and y-position, in order to be not completely overlapped (for the windows below others, only their title is visible).

The size argument is used to impose the size of the window on the screen. It must be a Fortran array of two integer values specifying the size of the newly created window; the two values indicates the width and height in pixels of the window. Default size is 800 by 600.

The title argument (character string, of maximum length 80) specifies the title of the figure. This title is visible in the X11 window title; it applies also for the printed EPS and PDF files.

#### Remarks:

- the default size of X11 windows can be modified via the environment variables MF-PLOT\_X11\_WIDTH and MFPLOT\_X11\_HEIGHT.
- the window title always contains win\_id and the name of the executable program.
- ordinarily, if you have no figure opened, most of graphical routines automatically open a new one.
- maximum number of concurrent figures is 64. However, win\_id values don't need to be contiguous, and may be even greater than 64.

The other form:

```
win_id = mfFigure()
```

creates a new figure and returns its (integer) window identification.

Remarks: use mfGetWinId to get the window identification of the current active figure.

See also: msClose, msClf

#### msResizeWindow

# window resizing

 $Calling\ syntax:$ 

```
call msResizeWindow( [width, height] )
```

### Description:

If the optional arguments width and height are both present, then the window is resized by the library.

On the contrary, i.e. when the optional arguments are not present, this routine becomes interactive, prints a prompt on the terminal, and wait the resizing of the window by the mouse.

See also: msFigure, msClf

# ${\tt mfGetWinId}$

# current figure handle

 $Calling\ syntax:$ 

win\_id = mfGetWinId( )

Description:

Returns the window identification (integer) of the current active figure.

 $See \ also: {\tt msFigure}$ 

msClose figure deletion

 $Calling\ syntax:$ 

```
call msClose( [win_id] )
```

### Description:

Closes the specified window, or the current window if the optional argument win\_id is not present.

Note that after closing a window, the current selected window is undefined, even if other figures yet exist. It is required to select another existing window if you wish to plot inside, else a new figure will be automatically created.

See also: msFigure, msClf

#### mfGetColorScheme

Get color scheme

 $Calling\ syntax:$ 

color\_scheme = mfGetColorScheme( )

### Description:

Returns the color scheme attached to the selected window, which is an integer ranged in [1-4]. Default is 3.

There are four different color schemes, used for recycling colors when you plot several lines on the same figure. They are described in the *Muesli User's Guide* (cf. § 5.2 Color management).

 $See \ also: {\tt msSetColorScheme}$ 

#### msSetColorScheme

Set color scheme

Interface:

```
subroutine msSetColorScheme( color_scheme )
integer, intent(in) :: color_scheme
```

#### Description:

Set the color scheme for the selected window, via an integer argument ranged in [1-4].

There are four different color schemes, used for recycling colors when you plot several lines on the same figure. They are described in the *Muesli User's Guide* (cf. § 5.2 Color management).

The default color scheme is the third one, i. e. the new colors in Matlab (from release R2014b).

See also: mfGetColorScheme

#### mfGetColorInd

# Get color scheme index

Calling syntax:

ind = mfGetColorInd( )

Description:

Returns the color scheme index attached to the selected window.

This is the index of the next color to be used.

See also: msSetColorInd

#### msSetColorInd

### Set color scheme index

Interface:

```
subroutine msSetColorInd( ind )
  integer, intent(in) :: ind
```

#### Description:

Set the color scheme index for the selected window, and for the current color scheme.

Usually, when plotting many curves on the same figure, colors are cycled inside the colortable of the selected color scheme. This routine allows to select a specific index, for example to avoid the cycling in the colors.

See also: mfGetColorInd

msPrint file printing

 $Calling\ syntax:$ 

```
call msPrint( filename [, file_format ] )
```

#### Description:

Prints the current figure in a file named filename.

Currently, file\_format may be only "EPS" or "PDF". If this optional argument is not present, MUESLI tries to deduce the device type from the extension of the provided filename. By default, an EPS file is created.

For EPS (resp. PDF) files, and for debugging purpose, comments can be inserted in the generated file by setting the environment variable MFPLOT\_EPS\_COMMENTS (resp. MFPLOT\_PDF\_COMMENTS) to 1 (default is no comments). Be aware that this will increase the size of the file.

Note, if you have selected a black background (see msSetBackgroundColor), that the EPS only will have also a black background. For the PDF files have always a white background.

#### Remarks:

- EPS and PDF files intensively make use of language-based shading. This means that all coloured gradients (from msPatch or msPColor for example) are treated by using appropriate PostScript or PDF commands, and leads to smaller files.
- Of course, printed files may be generated without any X11 capability. This is useful under Windows (where there is no X11 access) or for batch processes.
- For PDF files only, optional contents (also known as layers) may be inserted. See msSetPdfOC.

#### msSetPdf0C

#### PDF Optional Contents management

The first interface is:

It is used to tag a graphic object of the current window as *Optional Content* in the PDF. Doing so, and once the *Layers* of your PDF viewer is opened, you should be able to choose to display or not this graphic object (which appears under the name) on the screen.

The graphic object, referenced by its handle, may be visible or not. The initial state of visibility in the PDF layers will be consistent with the visibility in the X11 window at the time of printing, except when the persistent argument is used: in such a case, only the graphic object tagged as *persistent* will be initially visible in the *mutex* group.

If name is not present, then it will be assigned the corresponding graphic object number (as "handle i").

The mutex optional argument (MUTually EXclusive) is used to group some optional contents in a Radio-Button group: at most one of these optional contents may be visible in the PDF (but that does not prevent the existence of a state where none of these are visible — see below the section concerning persistent if you want that exactly one content must be visible at a time). The mutex name is the name which will appear in the "layers" panel of the PDF viewer, which enclosed all different names. Same thing for the super\_group optional argument, except that the groups inside are not in mutually exclusion.

When using the mutex feature, replacing 'name=name' by 'persistent=.true.' allows that one object is always visible, and it is the concerned object by default. A typical application is to group text annotations in the figure in mutually exclusive groups, one for a language; each group may have its own name, for example, equal to Français, Deutsch, etc. The mutex name could be then, Other languages (default: English). See the Labels\_and\_Languages\_test.f90 program under src/test/fgl.

The other interface:

concerns a vector of handle, and can be used to the following aims:

- when the two arguments merge and name are both present, it merges a group of graphic objects under the same name as optional contents; as an example of use, see the PlotCubicBezier\_test program under src/test/fgl.
- when the two arguments merge and name are both not present, it gets a different name for each graphic objects of the group, from the legend of the figure. Of course, such a legend should exist; on the contrary, a generic name (as "handle i") will be used instead. As an example of use, see the figure 4 of the Legend\_test program under src/test/fgl.

The mutex optional argument has the same meaning as in the first interface. Its use implies the merge of all optional contents specified by the array of handles. Same thing for super\_group.

•••/...

Constraints about the presence of optional arguments:

- mutex and super\_group arguments cannot be both present.
- persistent and name arguments cannot be both present.
- in a given group of optional contents, the persistent argument can be used only one time.
- currently (but this may change in a future version), there are only one Radio-Button group and only one Super-group for each figure.

#### Remarks:

- this routine must be called before creating the PDF file via msPrint.
- you are allowed to first call the second interface, using an array of handle, getting the names from the legend and then, call the first interface to modify the name of one specific graphic object.
- the PDF Optional Contents will be ignored if you print in the EPS format.

### 2.3 Figure properties

mf/msAxisfigure axes settingmsAxisFontSizeaxis font size settingmsAxisLineWidthaxis line width settingmsCharInPixelscharacter size policy

mf/msCAxisget or set color axismsClfclear current figuremsColorbarcolor bar settingmf/msColormapget or set colormapmf/msColormapSizeget or set colormap size

msDrawBox redraw the box in current figure

msDrawGrid if needed, redraw the grid in current figure

msRedrawFigure redraw the current figure
msGetX11Pixmap X11 pixmap saving

msGrid grid setting

msHold plot command superposition
msRemoveClipBox remove a clipping box inside axes
define a clipping box inside axes

msShading color shading mode

mfGetXAxisTicksNb get ticks number of the X-axis

msSetXAxisUserLabels set customized ticks labels of the X-axis

mf/msTitle title setting

msTitleFontSize title font size setting
mf/msXLabel x-label setting
mf/msYLabel y-label setting

msLabelFontSize descriptive labels font size setting

msAxisLabelFormat kind of labelling for Axis msSetWinProp window properties setting

msLegend add legends to curves, in one frame

mfLegend add legends to curves, possibly in many frames

#### See also:

Global graphic settings

Window's and figure's management

Figure annotation – Low level graphic object's manipulation

High level plotting routines

Interactive routines

mf/msAxis

#### figure axes setting

The function form

returns the current axis values of the selected figure, always in the order: *left*, *right*, *bottom*, *top*, whatever the mode ("xy" or "ij" – see below) is. This array is a vector of length 4 or 2, containing extremal values of the x- and/or y-axis, according to the presence of the optional argument axis which must be equal to the single character "x" or "y".

The *subroutine form* has the following calling syntax:

```
call msAxis( mode | dp_range | mf_range [, axis ] )
```

If the character string mode is present, it may have the following values:

- "on" / "off": axes are drawn (default) or hidden (and so for X label and Y label).
- "auto": axes' ranges are computed from the actual numerical values of data to be plotted, giving smart extremal values; this is the default. Note that graphic data involved do not include arrows or texts, which are considered as annotations.
- "tight": axes' ranges are computed from actual values of data (see just above what is considered as "data"), strictly keeping their extremal values.
- "manual": axes' ranges are not automatic; the current axis values are frozen, whatever the following plots. This mode is also set when a range is explicitly used (see below). Only another call using the "auto" or "tight" argument can unlock it.
- "equal": x- and y- axis are drawn with the same scale; the default behaviour is "unequal". Note that the equal mode is possible only when axes have the same scaling (both linear or both logarithmic).
- "xy" / "ij": the origin of the x- and y-axis is at the bottom left corner of the figure (default location for most plotting commands) or at the top left corner (when the data is interpreted as a matrix, which is the case for msSpy, msImage and msPColor in the absence of (X,Y) coordinates). In the "xy" mode, the x-axis labels are written at the bottom side of the box, whereas they are written at the top one in the other mode.
- "inverted": in this case, the second optional argument axis must be present and equal to "x" or "y". Corresponding extremal values of this axis are then swapped together. This doesn't imply that the axis becomes "manual". There is no "normal" state, i.e. two successive calls with the "inverted" argument is equivalent to do nothing; in other words, "inverted" is not a property but an action to realize.
- "linlin" / "loglog": x- and y-axis are both linear (default) or logarithmic (base 10).
- "loglin" / "linlog": as specified, one of the two axes is linear whereas the other is logarithmic (base 10); Note that if the *equal* mode is set, it will be disabled.

If the real dp\_range, or mfArray mf\_range, is present (containing 4 values), it is used to initialize the axes' range; the *manual* mode is automatically set. The additional optional argument axis can be used to set only the x- or the y-axis: in this case, the length of the range vector must be equal to 2, instead of 4, and the axis argument set to the single character "x" or "y".

The axis optional argument can also be used when the following modes are set: auto, tight and manual.

Axes may be inverted (e.g., for the x-axis, the maximum value on the left and the minimum value on the right); to do this, simply give a range in decreasing order.

See also: msAxisFontSize, msAxisLineWidth, msLabelFontSize, msGrid

#### msAxisFontSize

# axis font size setting

Interface:

```
subroutine msAxisFontSize( size_factor )
real(kind=MF_DOUBLE), intent(in) :: size_factor
```

#### Description:

This routine is used to change the font size of the axis numbering.

size\_factor must be ranged in [0.4, 2.5]. Default size factor is unity. Actual font size also depends on the char\_height\_pixel mode (see the msCharInPixels routine).

See also: msAxis, msTitleFontSize, msLabelFontSize, msAxisLineWidth

#### msAxisLineWidth

### axis line width setting

Interface:

```
subroutine msAxisLineWidth( width )
real(kind=MF_DOUBLE), intent(in) :: width
```

#### Description:

This routine is used to change the line width of the axis and the associated numbering.

Default width is unity.

Remark: If you increase the value of width, you will remark that the effective line width in pixels is approximately half of this latter value. Actually, the pixel width is a discontinuous function of width, especially at small values. The corresponding behaviour is however continuous for the drawing generated in EPS and PDF files.

See also: msAxis, msAxisFontSize, msLabelFontSize

#### msCharInPixels

# character size policy

Interface:

```
subroutine msCharInPixels( mode )
    character(len=*), intent(in) :: mode
```

#### Description:

If mode is equal to "off", the character size is proportional to the window size (default behavior).

If mode is equal to "on", the character size is absolute. Unity is about 12 pixels.

 $See \ also: {\tt msAxisFontSize}, {\tt msText}$ 

#### mf/msCAxis

get or set color axis

The first form:

```
function mfCAxis() result ( out )
    type(mfArray) :: out
```

returns the color axis of the current figure. It is a 2-value vector which contains the min and max values used in the colorbar. The default value (if not set by the user) is the range [0,1].

The subroutine form:

```
call msCAxis( col_range | mf_range | string )
```

is used to set the color axis. One argument is required, it is either a real 2-value vector, an mfArray or a character string.

When the argument contains numerical values, it specifies the color range. Be aware that the two values must not be too close together: in such a case, the routine enlarges slightly the given interval and emits a warning. Once the color axis has been defined, it will remain the same until the next call.

When the argument is a string, it must be equal to "reset": the color axis takes the initial [0,1] range, whereas it becomes extensible again, when used repeatedly (by successive calls of msContour, for example).

*Remark:* The two input numerical values defining the color axis must be ordered: first is the minimum, second is the maximum.

See also: msColorbar, msColormap, msColormapSize, msPColor, msContour, msPatch

## ${\tt msClf}$

# clear current figure

 $Calling\ syntax:$ 

call msClf()

Description:

Clears the current figure, but don't close the corresponding window.

 $See \ also: \mathtt{msFigure}, \mathtt{msClose}, \mathtt{msCla}$ 

## msColorbar color bar setting

### $Calling\ syntax:$

```
call msColorbar( mode [, position ] [, label ] )
```

### Description:

If mode is equal to "on", adds a colorbar to the current figure.

Its position can be specified by an optional argument, position, which can be "vert" (i. e. right side) or "horiz" (i. e. bottom side); by default the position is automatically chosen.

The optional argument label can be used for the labelling of the color bar.

The three arguments are of type character(len=\*).

#### Remarks:

- In some situations, in particular when axis scaling are equal (see msAxis) and one figure side is much smaller than the other, it is recommended to left the routine choose itself the position; otherwise the colorbar may have a too short length.
- It is recommended to call this routine after drawing all objects.

See also: msCAxis, msColormap, msColormapSize

### mf/msColormap

### get or set colormap

The function form:

```
colormap = mfColormap()
```

returns the current colormap in an mfArray (rank-2 array of 3 columns) which contains the Red-Green-Blue components of each color. These components are reals, ranged from 0.0 to 1.0.

The other function form:

```
bool = mfColormap( "init_status" )
```

returns a logical value according to the initialization status of the colormap.

The subroutine form:

```
call msColormap( name [, "inverted"] )
or
call msColormap( colormap [, int_max] )
```

is used to set the colormap.

The user can choose a predefined colormap, via the character name argument: "rainbow" (or its alias "jet"), "parula", "hot", "bluered", "fusion", "flag" or "grey" (a grey scale). Number of colors is 256, but it can be changed via msColormapSize. You can see their color representation in the *Muesli User's Guide*. If the optional argument "inverted" is added, then the colormap is inverted.

User-defined colormap can also be registered, via the mfArray colormap. This array must have 3 columns and a number of rows up to 4096.

By default, colormap contains real color components, ranged from 0.0 to 1.0. If the optional argument int\_max is present, it indicates that entries are integers whose max value is provided (e. g. 255 for 256 values for each R,G,B component, giving a subset among 16 millions of colors. Integer color components always start from 0).

#### Remarks:

- Before using this routine, a window must be opened via msFigure.
- There is no default colormap. The user must select one of them before using colors.

See also: msCAxis, msColorbar

## mf/msColormapSize

# get or set colormap size

The function form:

```
size = mfColormapSize()
```

returns the size (integer) of the current colormap.

The subroutine forms:

```
call msColormapSize( size )
```

is used to set the size of the colormap. A maximum of 4096 colors is allowed; note however that in most of case, 256 colors are sufficient to eliminate color artifacts on the screen.

```
call msColormapSize( "auto" )
```

sets the size of the color map to its default size,  $i.\,e.\,256.$ 

See also: mf/msColormap, msColorbar

### msDrawBox

# redraw the box in the current figure

Calling syntax:

call msDrawBox()

Redraws the framed box in the current figure.

This routine is useful after drawing some plots which partially erase the ticks and marks of the box. It doesn't redraw the numerical label neither the optional grid (which should remain in the background).

 $See \ also: {\tt msRedrawFigure, msDrawGrid}$ 

### msDrawGrid

# if needed, redraw the grid in the current figure

Calling syntax:

call msDrawGrid()

Redraws the grid in the current figure, if needed.

This routine is useful during animation: if the grid has been set with the msGrid routine, the current routine must be called in the iteration loop, before any plotting with FGL.

See the Muesli User's Guide about a typical sequence of Muesli routines to include in an animation loop.

See also: msRedrawFigure, msDrawBox

# msRedrawFigure

# redraw the current figure

 $Calling\ syntax:$ 

call msRedrawFigure( )

redraws the current figure.

Typically, this routine should be used after any graphic object change, modified by the routine msSetGr0bj, in order to make visible the modification on the screen.

See also: msDrawBox, msDrawGrid, msResizeFigure

## msGetX11Pixmap

# X11 pixmap saving

 $Calling\ syntax:$ 

call msGetX11Pixmap( A1, A2, A3 )

## Description:

This routine returns in three mfArrays, the color components of the X11 pixmap drawn on the screen.

If the background color is black, then it gets the R,G,B planes.

If the background color is white, then it gets the C,M,Y planes.

Remark: the mfArray arguments should not be pointed by another ordinary Fortran pointer (see msPointer); on the contrary, a Warning is emitted.

 $See \ also: {\tt msSetBackgroundColor}$ 

msGrid grid setting

Calling syntax:

```
call msGrid( mode [, minor ] )
```

The string mode may have the following values:

- "on": a background grey dashed grid is drawn for major axis ticks; (default)
- "off": grid is hidden;

The boolean optional minor can be set to .true. to get minor lines (aligned on minor ticks). By default, these minor dotted lines are not drawn.

See also: msAxis

### msHold

# plot command superposition

 $Calling\ syntax:$ 

```
call msHold( "on" | "off" )
```

## Description:

Usually, each new plot command erase the previous one. If you want to keep the previous graphic objects drawn in the figure, you must call this routine with "on".

Use "off" to come back to the default behavior.

Remark: to avoid unwanted behavior, place this command after the first plot (msPlot or whatever).

# ${\tt msRemoveClipBox}$

# remove a clipping box inside axes

 ${\it Interface:}$ 

subroutine msRemoveClipBox( )

Description:

Remove the clipping box set by the msSetClipBox routine.

 $See \ also: \ {\tt msSetClipBox}$ 

# ${\tt msSetClipBox}$

# define a clipping box inside axes

 ${\it Interface:}$ 

```
subroutine msSetClipBox( dp_range )
    real(kind=MF_DOUBLE), intent(in) :: dp_range(:)
```

Description:

Define a clipping box (inside current axes) for subsequent plotting commands.

See also: msRemoveClipBox

### msShading

# color shading mode

 $Calling\ syntax:$ 

```
call msShading( mode )
```

where mode is a character string which can be "flat" or "interp".

#### Description:

Sets the shading mode used when drawing patches:

- "flat" implies that only one color is used;
- "interp" leads to interpolated colors between the vertices of the polygonal shape.

"flat" is the default mode.

Note that the "interp" mode doesn't lead to bigger files when printing in PDF. Only for EPS, the use of transparency leads to the inclusion of bitmap images.

The shading has no effect on the display of the colorbar. Indeed, the colormap may have sharp gradients or even jumps in colors, or may use of a very small number of colors (e. g. 8 or 16): that must appear clearly in the colorbar.

See also: mf/msPatch, mf/msPColor, msColorbar

### mfGetXAxisTicksNb

# get ticks number of the X-axis

Calling syntax:

```
nb = mfGetXAxisTicksNb( "major" | "all" )
```

returns the number of ticks of the X-axis (linear scaling only). This can be the number of major ticks (the default, or if "major" is used as argument) or all ticks (both major and minor) when "all" is used as argument.

Usually, this routine is used in conjunction with a call to msSetXAxisUserLabels.

#### msSetXAxisUserLabels

#### set customized ticks labels of the X-axis

First calling syntax:

```
call msSetXAxisUserLabels( labels )
```

This call must occur only after the function mfGetXAxisTicksNb, which returns the number N of ticks of the (linear) X-axis.

labels must be an array (of size N) of character strings. The ticks labels (major or all, according the previous call) are then replaced by each string in labels. Only the first 6 characters are used for each label (truncation beyond).

Remark: Using customized labels can be useful for a Bar plot (see msBar), either because numerics are not appropriate, or because you want to use a logarithmic scaling when it is not allowed.

Other calling syntax:

```
call msSetXAxisUserLabels( .false. )
```

This later call is used to manually disable the replacement of the labels. Note that this replacement is also automatically disabled when the range of the X-axis is modified, either by the user (explicit call to msAxis) or by a plot of other graphic objects which would modify this range.

mf/msTitle title setting

 ${\it Interface:}$ 

```
subroutine msTitle( title )
    character(len=*), intent(in) :: title
```

Description:

Adds a title to the current figure. Not drawn if axis is off.

Maximum length of the title is 128 characters.

This routine should be called after the other data plotting routines.

Escaped sequences may be use in the string (see msText) to change the font, the style, the position (sub-or superscript), etc. Note also that the global character size may be changed by using msTitleFontSize.

Remark: the function mfTitle has the same argument list; it returns the (integer) handle of the created graphic objet; this is useful to create many strings in different languages, and set them as optional content for PDF creation (see msSetPdfOC).

See also: msXLabel, msYLabel, msAxis

### msTitleFontSize

# title font size setting

Interface:

```
subroutine msTitleFontSize( size_factor )
    real(kind=MF_DOUBLE), intent(in) :: size_factor
```

### Description:

This routine is used to change the font size of the title.

size\_factor must be ranged in [0.5, 3.5]. Default size factor is 2. Actual font size also depends on the char\_height\_pixel mode (see the msCharInPixels routine).

 $See \ also: {\tt msAxis}, {\tt msAxisFontSize}, {\tt msTitle}, {\tt msLabelFontSize}$ 

mf/msXLabel x-label setting

 ${\it Interface:}$ 

```
subroutine msXLabel( xlabel )
    character(len=*), intent(in) :: xlabel
```

#### Description:

Adds a label to the x-axis of the current figure. Not drawn if axis is off.

Maximum length of this label is 96 characters.

This routine should be called after the other data plotting routines.

Escaped sequences may be use in the string (see msText) to change the font, the style, the position (sub-or superscript), etc. Note also that the global character size may be changed by using msLabelFontSize.

Remark: the function mfXLabel has the same argument list; it returns the (integer) handle of the created graphic objet; this is useful to create many strings in different languages, and set them as optional content for PDF creation (see msSetPdfOC).

See also: msTitle, msYLabel, msAxis

mf/msYLabel y-label setting

 ${\it Interface:}$ 

```
subroutine msYLabel( ylabel )
    character(len=*), intent(in) :: ylabel
```

#### Description:

Adds a label to the y-axis of the current figure. Not drawn if axis is off.

Maximum length of this label is 96 characters.

This routine should be called after the other data plotting routines.

Escaped sequences may be use in the string (see msText) to change the font, the style, the position (sub-or superscript), etc. Note also that the global character size may be changed by using msLabelFontSize.

Remark: the function mfYLabel has the same argument list; it returns the (integer) handle of the created graphic objet; this is useful to create many strings in different languages, and set them as optional content for PDF creation (see msSetPdfOC).

See also: msTitle, msXLabel, msAxis

#### msLabelFontSize

# descriptive labels font size setting

# ${\it Interface:}$

```
subroutine msLabelFontSize( size_factor )
    real(kind=MF_DOUBLE), intent(in) :: size_factor
```

### Description:

This routine is used to change the font size of the axis labels.

size\_factor must be ranged in [0.4, 2.5]. Default size factor is unity. Actual font size also depends on the char\_height\_pixel mode (see the msCharInPixels routine).

 $See \ also: \verb|msAxis| | \verb|msAxis| | \verb|msAxis| | \verb|msTitleFontSize| | \verb|msXLabel| | \verb|msYLabel| | \\$ 

#### msAxisLabelFormat

# kind of labelling for Axis

Interface:

```
subroutine msAxisLabelFormat( x_axis_mode, y_axis_mode )
    character(len=*), intent(in), optional :: x_axis_mode, y_axis_mode
```

### Description:

This routine is used to set the format of labelling axis.

Default mode is "std", i. e. numeric numbers without unit. If set to "time" then numbers are supposed to be seconds and are written under the format (DD) HH MM SS.S.

See also: msAxisFontSize

## msSetWinProp

## window properties setting

 ${\it Interface:}$ 

```
call msSetWinProp( property, data )
```

Description: Used to modify a specified property of the current window.

property may take the following values:

Remark: Contrary to the specific following routines: msAxisFontSize, msAxisLineWidth, msLabelFontSize, msTitleFontSize, msXLabel, msYLabel, msTitle, the redraw of the figure is not done; therefore, the user must call the msRedrawFigure to see the change effects.

### msLegend

### add legends to curves, in one frame

First interface:

```
subroutine msLegend( legend_1 [, legend_2, legend_3, ... ]
                                                                               &
                        [, location | position ] )
     character(len=*), intent(in)
                                             :: legend_1
     character(len=*), intent(in), optional :: legend_2, ..., legend_13
      character(len=2), intent(in), optional :: location
     type(mfArray),
                        intent(in), optional :: position
Second interface:
  subroutine msLegend( legend_1, handle_1 [, legend_2, handle_2, ... ]
                        [, location | position ] )
     character(len=*), intent(in)
                                             :: legend_1
                        intent(in)
                                             :: handle_1
     integer,
     character(len=*), intent(in), optional :: legend_2, ..., legend_13
     integer,
                       intent(in), optional :: handle_2, ..., handle_13
     character(len=2), intent(in), optional :: location
     type(mfArray),
                       intent(in), optional :: position
```

#### Description:

Adds legends to the curves plotted in the figure. Up to 13 legends (noted above as legend\_1, legend\_2, ...) in one frame are supported, each containing a string of at most 80 characters.

The handles of the curves may be provided (see the second interface), making possible to legend only a subset of the curves. Of course, all handled must be valid and must be integers returned by *Muesli* graphic functions. Note that the two calling syntaxes above cannot be mixed.

Numbered legends and handles must be used in increasing order, otherwise some legends may not appear.

The optional argument location (not to be used in conjonction with position) can be used to specify in which corner of the figure the legend is displayed. Possible values are "TL" (top-left), "TR" (top-right), "BL" (bottom-left), "BR" (bottom-right) or "outside" (see below). Default is top-left corner.

The optional argument position (not to be used in conjonction with location) specify the position of the legend's frame, *i.e.* the coordinates of the top-left angle of the frame. The user is even allowed to locate the legend "outside" of the axes, by specifying any value of the coordinates. The legend frame may or may not be entirely visible in the X11 window, but it should appear in the EPS and PDF files. This is useful in case of a very high legend. A better way, however, is to use location="outside"; in such a case, the legend will be drawn in a small, additional window (but the legend will appear in the same EPS or PDF after printing).

Legends are grouped in a unique frame which can be moved inside or outside axes via the msMoveLegend routine. See mfLegend to create many frames. A legend frame cannot be removed without knowing its handle, *i. e.* via mfLegend.

#### Remarks:

Usually, the number of string arguments (or pairs of string and associated handle) specifying the legends is equal to the current number of curves. However, it is possible to specify less legends than the actual number of curves: a simple warning will be emitted during the call. Similarly, specifying a number of legends larger than the number of current curves leads to a simple warning, and additional legends will be discarded.

 $\cdots / \dots$ 

For bar plot, the different items in the legend correspond to the data series plotted; this works only
for grouped or stacked bars (see mf/msBar). Currently, the second interface above is not compatible
with bar plots.

# $Third\ interface:$

### Description:

Allows the user to group all the legends (and, optionally, the associated handles) in one array of character strings.

## mfLegend

# add legends to curves, possibly in many frames

### Description:

As opposed to most of graphic routines, the function version of the Legend family behaves differently.

It has the same interfaces as msLegend and, as usual, returns the (integer) handle of the graphic objet; moreover, it can be called many times for the same figure, making it useful to create legends in different languages, and to set them as optional content for PDF creation (see msSetPdfOC).

# 2.4 Figure annotation – Low level graphic object's manipulation

mf/msArrow arrow drawing
mf/msText text display

mfGetAllGrObj get all graphic objects
mfGetTypeGrObj get the type of a graphic object
mfSelectTypeGrObj select graphic objects by type
msSetGrObj graphic object setting
msRemoveGrObj graphic object deletion

See also:

Global graphic settings

Window's and figure's management

Figure properties

High level plotting routines

Interactive routines

mf/msArrow arrow drawing

The first form:

draws an arrow from the position  $(x\_start, y\_start)$  to the position  $(x\_end, y\_end)$ . These coordinates must be of type real.

The optional argument color have the same meaning as in the routine mf/msPlot.

Optional arguments linewidth and headsize (real numbers) can be used to specify the line width and the head size of the arrow. Default values are unity.

If the optional boolean argument clipping is set to FALSE, then the arrow can be displayed out of the viewport (default is TRUE, i. e. the arrow is clipped at the viewport; this default behavior is recommended when it is an annotation at some location inside the viewport and you might want to pan or zoom inside the axes).

The second form:

draws only an arrow head, given a position (x,y) and an angle in radian (all are real variables).

The remaining arguments have the same meanings as above.

#### Remarks:

- the function mfArrow has the same argument list; it returns the (integer) handle of the created graphic object.
- in future, the four first arguments will be able to take also the type mfArray, in order to display many arrow at a time.

See also: mf/msText

mf/msText text display

 ${\it Interface:}$ 

#### Description:

Writes the string text at the position (x, y). These coordinates may be of type real or mfArray. Many escape codes allow the use of different kinds of fonts, the use of subscript or superscript, greek letters, etc. See the Muesli User's Guide at section 5.5 for more information.

If the real argument angle is present then text is written with this angle (in degrees).

The real argument just specifies the justification of the string in comparison with the position (-1 for left (default), 0 for centered, +1 for right — other real values ranged from -1 to +1 are permitted, giving intermediate justifications). Similarly, the real just\_vert argument concerns the vertical justification (default is -1, *i.e.* the baseline of the glyph located at y coordinate, and 0 to obtain the topline of the glyph at y; use -0.5 for a vertically centered glyph). Concerning only the vertical justification, the optional boolean argument pix\_voffset can be used (under X11) to move vertically the string box by a 2 pixel shift — this is especially useful when the bottom or the top of the string box is against a drawn line (default is pix\_voffset=.false.).

If the optional argument color is present, it specifies the color of the text displayed (see msSetGrObj). Default color is black when the figure background is white (see msSetBackgroundColor).

If the optional argument bg is present, it specifies the color of the background (see msSetGrObj). Default is transparent.

The optional real argument height specifies the height of the characters drawn (default is 1). This height may be relative (the default, character size is proportional to the window size) or absolute (character height is in pixels, unity is about 12 pixels), according to the mode defined in the routine msCharInPixels.

If the optional boolean argument clipping is set to FALSE, then text can be displayed out of the viewport (default is TRUE, i. e. the text is clipped at the viewport; this default behavior is recommended when text is an annotation at some location inside the viewport and you might want to pan or zoom inside the axes).

The optional output arguments xbox and ybox allows the user to retrieve the value of the rectangle frame of the string displayed (4 values for both x and y, because the string may be inclined). These arguments must be real arrays of length 4.

### Remarks:

- usually, the current routine is used to annotate a graphic, so it should be employed after any use of mf/msPlot; on the contrary, the msHold routine must be added before the plot command, else all texts from mf/msText will be erased.
- the function mfText has the same argument list; it returns the (integer) handle of the created graphic object.

See also: mf/msArrow, msSetCharEncoding

# mfGetAllGrObj

# get all graphic objects

Calling syntaxe:

hdle\_vec = mfGetAllGrObj( )

Description: Returns a vector of handle for all graphic objects in the current window.

The hdle\_vec variable, declared by the user, must be an integer vector. For an easy programming use, it should be allocatable: due to a feature of modern Fortran, it will be allocate with the appropriate size during the assignment; moreover, there is no need to deallocate it between calls.

See also: mfGetTypeGrObj, msSetGrObj

# mfGetTypeGrObj

## get the type of a graphic object

 $Calling\ syntaxe:$ 

```
type = mfGetTypeGrObj( handle )
```

Description: Returns the type of a graphic object, identified by its handle, as a character string.

The type may be one of the followings:

```
- "line", "point" or "line+point", typically created by mf/msPlot;
- "image", created by mf/msImage;
- "text", created by mf/msText;
- "polygon", created by mf/msPatch;
- "pcolor", created by mf/msPColor;
- "contour", created by mf/msContour;
- "quiver", created by mf/msQuiver;
- "streamline", created by mf/msStreamline or mf/msTriStreamline;
- "arrow" or "arrow_head_only", created by mf/msArrow;
- "quadr_bezier", created by mf/msPlotQuadrBezier;
- "cubic_bezier", created by mf/msPlotCubicBezier or mf/msPlotCubicSpline;
- "histogram", created by mf/msPlotHist;
- "errorbar_x_line",
                        "errorbar_y_line",
                                                 "errorbar_xy_line",
                                                                          \verb"errorbar_x_pt",
  "errorbar_y_pt", "errorbar_xy_pt", "errorbar_x_line+pt", "errorbar_y_line+pt" or
  "errorbar_xy_line+pt", created by mf/msErrorBar;
- "pcolor_spy", "pcolor_spy_sparse", "pcolor_spy_sparse_2", "plot_spy_sparse", created by
  mf/msSpy;
- "legend", created by mf/msLegend;
- "tri_fill", created by mf/msTriFill;
- "trimesh_by_fac", "trimesh_by_tri" or "mesh_boundary_unstruct", created by mf/msTriMesh;
- "tri_pcolor", created by mf/msTriPColor;
- "tri_quiver", created by mf/msTriQuiver;
- "tri_contour", created by mf/msTriContour;
- "xlabel", created by mf/msXLabel;
- "ylabel", created by mf/msYLabel;
- "title", created by mf/msTitle;
- "set_clip_box", created by msSetClipBox;
- "remove_clip_box", created by msRemoveClipBox.
```

See also: mfGetAllGrObj, msSetGrObj

# mfSelectTypeGrObj

## select graphic objects by type

 $Calling\ syntaxe:$ 

```
hdle_vec = mfSelectTypeGrObj( handles, grobj_type )
```

Description: Returns a vector of handles for the graphic objects which match the type specified as argument, among a list of input handles.

handles must be a integer vector, containing a set of valid graphic object handles.

grobj\_type is a character string, containing the type of the graphic object, as returned by
mfGetTypeGrObj.

The hdle\_vec variable, declared by the user, must be an integer vector. For an easy programming use, it should be allocatable: due to a feature of modern Fortran, it will be allocate with the appropriate size during the assignment; moreover, there is no need to deallocate it between calls.

 $See \ also: {\tt mfGetAllGr0bj}, {\tt msSetGr0bj}$ 

### msSetGrObj

### graphic object setting

 $First\ interface:$ 

```
call msSetGrObj( handle, property, data )
```

Description: Changes one property of the graphic object identified by its handle. This handle may be a scalar or vector integer, or even a vector of handles stored in an mfArray.

The type of data depends on the target property, as detailed below.

property may take the following values:

- "visible": data may be any string, but the object will be displayed only if data is "on"; an exception occurs if the object is tagged as *Optional Content* for the PDF driver (see msSetPdfOC).
- "linestyle": data must be a string among "-", "--", "-." (or ".-" as an alias) or ":" (or ".." as an alias), for continuous, dashed, dashed-dotted, and dotted, respectively.
- "linewidth": data must be of type real, positive (default value is 1).
- "cap\_style": data must be a string among "CapButt", "CapRound" or "CapProjecting" (but not case sensitive). Signification of these three keywords may be found easily on the web and concerns all devices (X11, EPS and PDF). Default is "CapRound".
- "join\_style": data must be a string among "JoinMiter", "JoinRound" or "JoinBevel" (but not case sensitive). Signification of these three keywords may be found easily on the web and concerns all devices (X11, EPS and PDF). Default is "JoinRound".
- "marker": data may be a single character among ".", "+", "\*", "o", "x", "s", "^" or "d" (for Dot, Plus, Asterisk, Circle, X, Square, TriangleUp and Diamond, respectively), or an escaped sequence as "\M01" to "\M26" for using the complete set of markers, described in Fig. 27 of the Muesli User's Guide. Negative codes "\M-14" to "\M-21" tell Muesli to draw the 8 filled markers with a small white border around them. For a more comprehensive reference, the markers may be also specified by an escaped text sequence, as "\CircleFilled" (the character case doesn't matter; it can be adapted for a better reading).
- "markersize": data must be of type real, positive (default value is 1).
- "color": data may be a single character, among "w", "k", "r", "g", "b", "c", "m" or "y" (for white, black, red, green, blue, cyan, magenta and yellow, respectively), or an escaped sequence of the form "\Cnn" (e. g. "\C01", "\C02", ...) for using the corresponding color of the current color scheme (see msSetColorScheme).
- "col\_name": data may be any string which represents a valid colorname from the X11 RGB database.
- "col\_rgb": data must be a vector of type real and size 3.
- "text": data may be any string.
- "position": data must be a string among "first", "last", "up" or "down". This affects the position in the stack, *i. e.* the drawing order.
- "opacity": data must be of type real, ranged from 0.0 (full transparency) to 1.0 (full opacity).
- "coordinates": (for text string) data must be of type real and size 2.
- "relat\_coords": (for a legend frame created by mfLegend) data must be of type real and size 2, and is understood as relative coordinates (left and bottom sides correspond to 0, right and top sides correspond to 1).
- "EPS\_user\_comment" (not case sensitive): data can be any character string. This text will be add only in the EPS file in order to find easily the PostScript commands related to the graphic object.

Changing the coordinates of the points in a polyline needs to change both x and y coordinates together by using the  $Other\ interface$ :

```
call msSetGrObj( handle, "x", x, "y", y )
```

### Remarks:

- the user must call the msRedrawFigure to see the change effects.
- properties are not all editable for a given graphic object. The following table describes what can be modified for all types of graphic objects.

 $See \ also: {\tt msSetWinProp}, {\tt msRemoveGrObj}$ 

### msRemoveGrObj

## graphic object deletion

 $Calling\ syntax:$ 

```
call msRemoveGrObj( handle [, redraw ] )
```

### Description:

Removes the graphic object in the current figure, specified by its handle, and redraws the figure.

handle may be a scalar or a vector of handle (always integer); it can be also an mfArray containing integer values of handles.

If the optional argument redraw is used and set to FALSE, the figure is not redrawn. This could be justified when you have to remove a lot of graphic objects. Don't forget to specify redraw = TRUE on the last call, or call the msRedrawFigure to redraw the whole figure.

Remark: After the grobj removing, the handles are set to zero. In the case of an mfArray, all elements are set to zero but the mfArray itself is not released.

See also: msSetWinProp, msSetGrObj

# 2.5 High level plotting routines - 2D

mf/msPlot data plot (using straight segments)

mf/msErrorBar data plot with error bars

mf/msPlotQuadrBezier data plot (using quadratic curved segments)
mf/msPlotCubicSpline data plot (using cubic curved segments)
data plot (using cubic curved segments)

mf/msBar bar data plot

mf/msPColor pseudo-color plot mf/msContour data contouring

mf/msContourF data contouring with filled regions

mf/msPatch graphic patch mf/msPlotHist data histogram

msCumulHist cumulative data histogram msImRead, msImWrite image read and write

mf/msImage image display

mf/msPlotPSLG plot a PSLG domain
mf/msTriMesh plot a triangulation
mf/msPlotVoronoi plot a Voronoi diagram

mf/msTriPColor pseudo-color on a triangulation
mf/msTriFill coloring on a triangulation
mf/msTriContour data contouring on triangles

mf/msTriContourF data contouring on triangles with filled regions

mf/msTriQuiver vector field plot on a triangulation

mf/msTriStreamline vector field streamline on a triangulation

mf/msSpy sparsity pattern visualization

See also:

Global graphic settings

Window's and figure's management

Figure properties

Figure annotation – Low level graphic object's manipulation

Interactive routines

### mf/msPlot

# data plot (using straight segments)

Calling syntax:

### Description:

Plots the vector mfArray y versus the vector mfArray x.

If y is not present then x is plotted against its integer indices. One exception is when x is complex: in such a case, msPlot(x) is equivalent to msPlot(real(x),imag(x)), so the drawing is done in the complex plane.

The optional argument linespec is a string which contains a multiple attribute vector (color, linestyle, marker) as in MATLAB. See msSetGrObj for further information.

The optional argument color may be a character string (one-letter color code, escaped sequence, color-name from the RGB database) or a vector of type real and length 3 containing the RGB components of a color.

The optional argument linewidth must be a positive number of type real (default is 1). Under X11 the unit is pixel, therefore specifying a value less than 1 makes no difference; however, in EPS and PDF, this will lead to very thin lines.

The optional argument markersize must be a positive number of type real; it is used only for marker symbols. If this argument is not present, then the marker size is automatically chosen to be coherent with the line width value.

The optional argument dashes\_inverted must be a boolean; it is used only for dashed lines: it shifts the dashes in such a way that two identical dashed lines using different colors are both visible, when this argument is used for the second curve.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

Log representation of the data is made by using the msAxis routine.

# Remarks:

- the drawing is always clipped at the viewport.
- the function mfPlot has the same argument list; it returns the (integer) handle of the created graphic object.
- when calling many time this routine in the same figure, without providing any color information, there is an automatic color selection, cycling the colors from the current colortable (for more information about colortables or color schemes see the *Muesli User's Guide*).
- by default, the axis range presents its minimum on the left and its maximum on the right. To invert the axis, use the msAxis routine.

See also: mf/msBar, mf/msPlotQuadrBezier, mf/msPlotCubicBezier, mf/msPlotCubicSpline

### mf/msErrorBar

# data plot with error bars

Calling syntax:

Description:

Plots the mfArray y versus the mfArray x, using line specification given in linespec and adding error bars at each point.

The argument x is optional (the x-axis will use a vector of sequential integers from 1). x and y must be vectors (row or col) of same length, not matrices.

The optional argument linespec (character string)) has exactly the same meaning as in the msPlot routines. In particular, when not specified, colors will be recycled from the current colortable (for more information about colortables – or color schemes – see the *Muesli User's Guide*).

The optional arguments linewidth, color and markersize have exactly the same meaning as in the msPlot routines.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

#### Remark:

- the drawing is always clipped at the viewport.
- the function mfErrorBar has the same argument list; it returns the (integer) handle of the created graphic object.
- currently, axis must be both linear.
- due to the simple implementation and the number of optional arguments, it is required, most of times, to use the name of each argument as keywords (on the contrary, errors will occur at compilation and even at run-time). The examples below show typical calls.

See also: mf/msPlot, msAxis

## Example(s):

For adding error bars only along the y-axis, the first call:

will give at run-time:

```
(MUESLI msErrorBar:) ERROR: bad optional arg combinaison (see doc) !
```

because the first actual argument is associated to  ${\tt x}$  and the second one to  ${\tt y}$ .

The second one:

will not compile:

```
Compilation Error: Missing actual argument for argument 'y'

Only this third call is correct:

call msErrorBar( y=mf([2.00d0,1.50d0,3.00d0,2.50d0]),
```

y\_err=mf([0.10d0,0.15d0,0.20d0,0.25d0]) )

## mf/msPlotQuadrBezier

# data plot (using quadratic curved segments)

# ${\it Interface:}$

### Description:

Draws a quadratic Bézier curve from the control points whose coordinates are given in the mfArrays x and y. The drawn object is a quadratic piecewise polynomial parametric curve. The curve is continuous (but doesn't have necessarily continuous derivatives); therefore, the total number of points in the x and y mfArrays must be of the form: 2k + 1, where k is the number of segments.

Optional arguments linespec, color, linewidth and dashes\_inverted have the same meaning as in the routine msPlot.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- the drawing is always clipped at the viewport.
- the marker choice, when specified, is discarded.
- the function mfPlotQuadrBezier has the same argument list; it returns the (integer) handle of the created graphic object.
- when not specified, colors will be recycled from the current colortable (for more information about colortables or color schemes see the *Muesli User's Guide*).

See also: mf/msPlotCubicBezier, mf/msPlotCubicSpline

## mf/msPlotCubicBezier

# data plot (using cubic curved segments)

# ${\it Interface:}$

### Description:

Draws a cubic Bézier curve from the control points whose coordinates are given in the mfArrays x and y. The drawn object is a cubic piecewise polynomial parametric curve. The curve is continuous (but doesn't have necessarily continuous derivatives); therefore, the total number of points in the x and y mfArrays must be of the form: 3k + 1, where k is the number of segments.

Optional arguments linespec, color, linewidth and dashes\_inverted have the same meaning as in the routine msPlot.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- the drawing is always clipped at the viewport.
- the marker choice, when specified, is discarded.
- the function mfPlotCubicBezier has the same argument list; it returns the (integer) handle of the created graphic object.
- when not specified, colors will be recycled from the current colortable (for more information about colortables or color schemes see the *Muesli User's Guide*).

See also: mf/msPlotQuadrBezier, mf/msPlotCubicSpline

# mf/msPlotCubicSpline

# data plot (using cubic curved segments)

# ${\it Interface:}$

### Description:

Draws a cubic Spline curve from given points whose coordinates are given in the mfArrays x and y. The drawn object is a cubic piecewise polynomial curve parametrized by the vector  $abs\_curv$ . It is  $C^2$  continuous at segment boundaries (i. e. both the tangent vector and the curvature are continuous). The two vectors wx and wy must have been computed before the call by use of the routine mfSpline.

Optional arguments linespec, color, linewidth and dashes\_inverted have the same meaning as in the routine msPlot.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

## Remarks:

- the drawing is always clipped at the viewport.
- the marker choice, when specified, is discarded.
- the function mfPlotCubicSpline has the same argument list; it returns the (integer) handle of the created graphic object.
- when not specified, colors will be recycled from the current colortable (for more information about colortables or color schemes see the *Muesli User's Guide*).

See also: mf/msPlotQuadrBezier, mf/msPlotCubicBezier

mf/msBar bar data plot

Calling syntax:

## Description:

Plots the mfArray y versus the mfArray x, using vertical bars. The argument x is optional (the x-axis will use a vector of sequential integers from 1). When y is a vector (row or col) then x must have the same shape as y; however, when y is a matrix (processed by columns), then x must be a column vector of same length as that of y columns.

The optional argument color specifies the color used to fill the bars; the color convention is the same as in the msPlot routine. See also msSetGrObj for further information. It may have many types and shapes:

- it may be a single character string, containing a one-letter color code or a color name from the RGB database; it may be also a vector of N character strings, N being the number of columns of y.
- it may be a vector of type real with 3 RGB components (describing one color); or a matrix of dimension (3, N), N being the number of columns of y.

The optional argument "width" must be a positive number of type real ranged in [0, 1]. It is the relative width of the bars (default is 0.8).

The optional argument "baseline" must be of type real. Its role depends on whether the Y-axis is linear or logarithmic:

- in the case of a linear Y-axis, baseline can take any value (default value is zero). The routine plots a thin line to visualize the baseline.
- in the case of a logarithmic Y-axis, baseline is the minimum value for data (no default value).
   Negative values, or those which are less than baseline are discarded.

The optional argument style works only for matrix data. When it is equal to "grouped" (default value), grouped bars are used; when it is equal to "stacked bars are used.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

#### Remarks:

- the function mfBar has the same argument list; it returns the (integer) handle of the created graphic object.
- only the Y-axis can be of logarithmic type; in this case, of course, all values in y must be positive.
- when not specified, colors will be recycled from the current colortable (for more information about colortables or color schemes see the Muesli User's Guide).

See also: mf/msPlot, msAxis

mf/msPColor

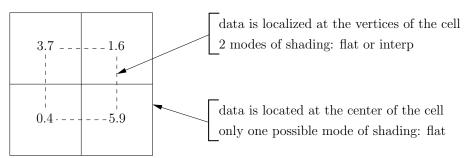
# graphic pseudo-color

Calling syntax:

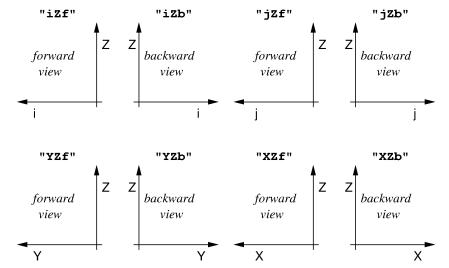
Draws a pseudo color plot of the data contained in the Z mfArray.

The (i, j) indices are mapped to the axis (-Y, X), so data visualisation is coherent with the matrix layout and with other routines working with matrix data (msContour, msQuiver, msStreamline). If this orientation is not appropriate, use the coordinates (mfArrays X and Y), as described below.

The optional data\_centering argument specifies how the data stored in the matrix are graphically displayed; it is a string which must be equal to "vertex" (default value — (i,j) points to the vertex of a cell) or "cell" ((i,j) points to the center of a cell). In the cell centered model, the shading of the color is only flat, whatever the value specified by the use of msShading. This is summarized in the following figure:



The optional view argument is a character string specifying the type of view for displaying the data in the matrix Z: the default view is the usual top view, noted "ji" or "XY", according to the presence of the arguments X, Y. When these latter arguments are not present, view may be equal to "iZf", "iZb", "jZf" or "jZf", specifying a side view as noted by the letters i, j or Z (always the vertical axis); the third letter is a modifier, indicating a forward or a backward view. In the other case, i. e. when the arguments X, Y are presents, the corresponding strings are "YZf", "YZb", "XZf" and "XZf". The different side views are summarized below:



Note that, in projection mode, only the default data centering (i. e. data\_centering="vertex") is accepted.

 $\cdots / \dots$ 

A thin line may be drawn to show the boundary of each quadrilateral cell by using the optional argument grid equal to TRUE. The default is to not show these boundaries. The color of this thin line may be specified in the optional argument grid\_color, with the same convention as in the msPlot, routine; by default, it is the foreground color. The optional argument grid\_step allows to plot only some lines of the grid, indicating the step to be used along the X or the Y coordinate; it must be a vector of two integers. This is particularly useful when the size of the data is large.

The optional argument lighting (boolean variable) changes the use of colors: instead of selecting shades from the Z values, they come from the lighting of the surface. In this case, it is recommended to select the <code>grey</code> colormap from the <code>msColormap</code> routine. Note also that this option is applicable only when X and X coordinates are provided.

When arguments X and Y are also provided, then their values are used as coordinates for the data points in Z:

- usually, X and Y are matrices and their shape must be the same as that of the data matrix Z. Use of these coordinates is required when, e.g., you want to swap the indices i and j, or apply a transformation (linear or not) between the (i,j) indices and the (x,y) coordinates; these coordinate matrices may be created by the msMeshGrid routine.
- to economize memory, it is possible to pass only vectors: in this case, X and Y, provided as rank1 mfArrays, must obey to the same constraint as edicted for the generator vectors (see again
  msMeshGrid).

Be aware that X and Y are *not used* to defined the axes of the plot: to do so, you must use the msAxis routine with appropriate argument(s).

The color axis may be set explicitly by the user via the msCaxis routine; otherwise it will be chosen automatically by the routine itself. In the color axis manual mode, data should be theoretically ranged entirely inside the color axis; on the contrary, the resulting behavior depends on the shading value (cf. msShading). When shading is equal to "flat", a quiet behavior is obtained and the colormap overflow can be monitored by use of the msSetColorOverflowPolicy routine; when shading is equal to "interp", a warning is emitted by the library because strange colors can be drawn. This latter case should be avoided if possible.

Note that a colormap must have been defined before the call, (via msColormap); otherwise, a warning is emitted and strange or unexpected results can occur.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

# Remarks:

- The drawing is always clipped at the viewport.
- The function mfPColor has the same argument list; it returns the (integer) handle of the created graphic object.
- For making a pseudo color plot on a triangle (or a triangular patch), use the msTriPColor routine.
- NaN values are not (yet) allowed: an error is returned by the routine.

See also: msPatch, msContour, msImage, msQuiver

mf/msContour

data contouring

Calling syntax:

### Description:

Draws a contour plot of the data contained in the mfArray Z. The plot is done in the current figure. As opposed as msPColor routine, msContour accepts only vertex-centered data.

It doesn't fill the regions between the contours with colors; for such a feature, see the mf/msContourF routine.

(i, j) indices of Z are mapped to the axis (-Y, X), so data visualisation is coherent with the matrix layout and with other routines working with matrix data (msPColor, msQuiver, msStreamline). If this orientation is not appropriate, use the coordinates (mfArrays X and Y), as described below.

When arguments X and Y are also provided, then their values are used as coordinates for the data points in Z:

- usually, X and Y are matrices and their shape must be the same as that of the data matrix Z. Use of these coordinates is required when, e.g., you want to swap the indices i and j, or apply a transformation (linear or not) between the (i,j) indices and the (x,y) coordinates; these coordinate matrices may be created by the msMeshGrid routine.
- to economize memory, it is possible to pass only vectors: in this case, X and Y, provided as rank1 mfArrays, must obey to the same constraint as edicted for the generator vectors (see again
  msMeshGrid).

If the optional C mfArray is given in front of (X,Y) coordinate matrices, then it will contain on output a two-row matrix storing the contour lines. Each contiguous drawing segment contains the value of the contour, the number of (x,y) drawing pairs, and the pairs themselves. The segment are appended end-to-end as

```
C = [ level_1, x_1, x_2, ..., level_2, x_1, x_2, ...; pairs_1, y_1, y_2, ..., pairs_2, y_1, y_2, ...]
```

Be aware that these point coordinates must be interpreted in different ways, according the value of order (see below).

The optional (integer) argument nb\_levels specifies the number of contours to be drawned; it cannot be used together with levels.

The optional argument levels, which must be a rank-1 mfArray, is used to specify the level values; it cannot be used together with nb\_levels.

By default, the number of level curves is equal to 9, and the level values are equally spaced within the *min* and *max* values of the data, or within thoses of the Color Axis if it has been set before (see msCAxis); this can be changed by using the optional arguments nb\_levels and levels (see above). A warning is emitted if a level is outside the Color Axis, excepted when the color is prescribed in linespec.

By default, the level curves are labelled (use labels=.false. to remove the labels) and the contour curves are colored using the current colormap (use a color inside linespec to specify the color of all curves). Labels are displayed in the foreground color (e. g. black on white) by default; the user may choose another color via the labelscolor optional argument. Moreover the label size may be changed by using the optional labelsize real argument, which lets the user to specify a percentage of the standard size (default is labelsize=1.0d0).

.../...

The optional argument order is an integer (1 or 2) specifying the quality of the approximation of the contour lines. Default is order = 1: in this case, segments are straight lines. If order is set to 2 then a bilinear interpolation is used inside the cells, and segments are constituted by quadratic Bezier arcs. Note that in all cases, the approximation provides only continuity at the cell boundaries, therefore the tangent lines may appear as broken. Note also that the default case (order = 1) is well adapted for most situations; only for situations where you have a small number of cells, or if you want to strongly zoom inside a contour map, you would choose order = 2.

Optional arguments linespec (character string) and linewidth (real value) allow the user to change the appearance of the contour curves. Possible values for these arguments are described in msSetGrObj, under the "color", "linestyle" and "linewidth" entries. If a color has been specified, then all contour lines will be drawn with the same color. Be aware, however, that the choice of markers is disabled.

Note that a colormap (see msColormap) must have been defined before the call if a specific color is not present in linespec; otherwise, a warning is emitted and strange or unexpected results can occur.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

#### Remarks:

- The drawing is always clipped at the viewport.
- The function mfContour has the same argument list; it returns the (integer) handle of the created graphic object.
- For making contours on a triangle (or a triangular patch), use the msTriContour routine.
- NaN values are not (yet) allowed: an error is returned by the routine.

See also: mf/msContourF, mf/msImage, mf/msQuiver

# mf/msContourF

# data contouring with filled regions

Calling syntax:

# Description:

Similar to the  ${\tt msContour}$  routine, except that the regions between the contours are filled with appropriate colors.

The whole region whose values are greater than the level is filled with the corresponding color. As a consequence, the region less than all specified levels is left in white. To avoid white regions, it is sufficient to add a level less than or equal the smaller value in the Z array.

Be aware however that some arguments of msContour are not available here.

See also: mf/msTriContourF

mf/msQuiver

vector field plot

Calling syntax:

Draws the vector field whose 2D components are stored in the u and v mfArrays, located at position (X,Y) or whows location is deduced from the layout of u and v when X and Y are not present. The plot is done in the current figure.

(i, j) indices are mapped to the axis (-Y, X), so data visualisation is coherent with the matrix layout and with other routines working with matrix data (msContour, msPColor, msStreamline). If this orientation is not appropriate, use the coordinates (mfArrays X and Y), as described below.

When arguments X and Y are also provided, then their values are used as coordinates for the data points in u and v:

- usually, X and Y are matrices and their shape must be the same as those of the data matrices u and v. Use of these coordinates is required when, e.g., you want to swap the indices i and j, or apply a transformation (linear or not) between the (i,j) indices and the (x,y) coordinates; these coordinate matrices may be created by the msMeshGrid routine.
- to economize memory, it is possible to pass only vectors: in this case, X and Y, provided as rank1 mfArrays, must obey to the same constraint as edicted for the generator vectors (see again
  msMeshGrid).

Be aware that X and Y are *not used* to defined the axes of the plot: to do so, you must use the msAxis routine with appropriate argument(s).

u and v must be rank-2 (i.e. matrices) mfArrays of same shape. They represent respectively the X and Y components of the vector field to be drawn. If the orientation or the scaling is not appropriate, use the second form of call described below, by adding coordinates.

The optional data\_centering argument specifies how the data are stored in the matrix; it is a string which must be equal to "vertex" (default value) or "cell".

The optional color argument specifies the color used for drawing the arrows: it is a one-letter color symbol or an escaped sequence, as described in the msSetGrObj routine.

The optional arrow\_length argument (real) specifies the maximum length of the arrow field as a multiplicative factor applied to the mesh size; by default its value is 1 so that each arrow is inside the rectangular cell where its center is located. A value much less that 1 is useless, whereas a big value is not recommended.

The optional step argument (integer) the step in the loops over the (i, j) indices to display the arrow; the default value is 1. It is useful to avoid a too big number of arrows for high resolution meshes.

The optional linewidth argument (real) specifies the line width, as a relative factor; by default its value is 1. The line width is constant and is the same for all arrows, whatever their length is.

The optional arrow\_head argument (real) specifies the size of the arrow head, based on the character height; by default its value is 1.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

.../...

## Remarks:

- The drawing is always clipped at the viewport.
- An arrow is not drawn if any component of the corresponding vector (u, v) is equal to NaN.
- The function mfQuiver has the same argument list; it returns the (integer) handle of the created graphic object.
- For drawing a 2D vector field on a triangular mesh, use the msTriQuiver routine.
- When specifying a color, the routine msColormap should be used before for definition of the colormap.

See also: mf/msContour, mf/msPColor

## mf/msStreamline

### vector field streamline

Calling syntax:

Based on a velocity vector field whose x- and y-components are respectively  $\mathbf{u}$  and  $\mathbf{v}$  (mfArrays), draws streamlines in the current figure. There is no restriction about the velocity field: it may have a non-zero divergence, or a non-zero curl.

The streamlines are specified by their starting points (2D-point coordinates stored in the start mfArray, which may have any number of lines, but exactly two columns). The integration always begins at the starting point, in two possible direction: forward and/or backward (see below the explanation about the direction optional argument). Usually, the end-points of a streamline are located at the boundary of the domain, but in some rare cases (velocity field having a non zero divergence) they can end inside the domain, at a stationnary point.

(i, j) indices are mapped to the axis (-Y, X), so data visualisation is coherent with the matrix layout and with other routines working with matrix data (msContour, msPColor, msQuiver). If this orientation is not appropriate, use the coordinates (mfArrays X and Y), as described below.

When arguments X and Y are also provided, then their values are used as coordinates for the data points in u and v:

- usually, X and Y are matrices and their shape must be the same as those of the data matrices u and v. Use of these coordinates is required when, e.g., you want to swap the indices i and j, or apply a transformation (linear or not) between the (i,j) indices and the (x,y) coordinates; these coordinate matrices may be created by the msMeshGrid routine.
- to economize memory, it is possible to pass only vectors: in this case, X and Y, provided as rank1 mfArrays, must obey to the same constraint as edicted for the generator vectors (see again
  msMeshGrid).

Be aware that X and Y are *not used* to defined the axes of the plot: to do so, you must use the msAxis routine with appropriate argument(s).

u and v must be rank-2 (i. e. matrices) mfArrays of same shape. They represent respectively the X and Y components of the vector field to be drawn. If the orientation or the scaling is not appropriate, use the second form of call described below, by adding coordinates.

The optional argument direction (character string) specifies whether the integration must be done following the flow direction ("forward"), against the flow direction ("backward") or in both direction ("both"); this latter case is the default.

The optional n\_arrow argument specifies the number of arrow heads to be drawn at equally curvilinear abscissa along each streamline. It must be an integer greater or equal zero (default value is 3); set it to zero if you want to suppress the arrow heads.

The optional color argument specifies the color used for drawing the streamlines; it has the same meaning as in the routine msPlot.

The optional linestyle argument (character string) specifies the line style, in the same way as msPlot; default line style is *continuous*.

The optional linewidth argument (real) specifies the line width, as a relative factor; by default its value is 1.

The optional arrow head argument (real) specifies the size of the arrow head; by default its value is 1.

 $\cdots / \dots$ 

The optional npt\_max argument is the maximum number of points for each half-streamline (indeed, you can ask for a forward integration, a backward one or both – see above); its default value is 1000.

The optional curv\_tol argument concerns the integration process and more specifically the tolerance for high curvature detection; it is relative to the mesh size; its default value is 0.5.

The optional stop\_zone argument indicates to the integration process that some regions of the (x, y) plane are forbidden. For example, this is useful to avoid entering in a region near a singularity of the velocity vector field, which would lead to an excessive increase in the number of points. stop\_zone is a user-provided subroutine, having the following interface:

```
logical function stop_zone( x, y )
   real(kind=MF_DOUBLE), intent(in) :: x, y
end function
```

This function is called for each new point and the integration process will stop as soon as a TRUE value is encountered.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

#### Remarks:

- The drawing is always clipped at the viewport.
- The function mfStreamline has the same argument list; it returns the vector of the (integer) handles of the created graphic objects. Be aware that even for a single streamline, the returned handle is a vector (of size 1).
- For drawing 2D streamlines on a triangular mesh, use the msTriStreamline routine.
- NaN values are not allowed: an error is returned by the routine. If such values are present in the vector field (i. e. in u or v) you can try to define a stop\_zone user-function to avoid an error.

See also: mf/msQuiver

mf/msPatch graphic patch

 $First\ interface:$ 

```
call msPatch( x, y, c [, opacity] [, grid, grid_color] )
```

## Description:

This routine draws one colored polygonal shape from the mfArrays x, y and c. These vectors contain respectively the coordinates of each vertex and the attached color. The polygonal shape doesn't need to be closed, e. q. a triangle may be defined only using three points.

The drawing is clipped at the viewport.

Before use, the color axis must be explicitly set by the user via the msCAxis routine. Theoretically, data should be ranged entirely inside the color axis; on the contrary, the resulting behavior depends on the shading value (cf. msShading). When shading is equal to "flat", a quiet behavior is obtained and the colormap overflow can be monitored by use of the msSetColorOverflowPolicy routine; when shading is equal to "interp", a warning is emitted by the library because strange colors can be drawn. This latter case should be avoided if possible.

If the optional argument opacity is present, transparency can be used: opacity must be a real number between 0 (full transparency, invisible object) and 1 (full opacity).

A thin line may be drawn to show the boundary of the polygonal cell by using the optional argument grid equal to *TRUE*. The default is to not show the boundary. The color of this thin line may be specified in the optional argument grid\_color; by default, it is the foreground color.

A colormap must have been defined before the call, (via msColormap); otherwise, a warning is emitted and strange or unexpected results can occur.

 $Second\ interface:$ 

```
call msPatch( x, y [, color] [, opacity] [, grid, grid_color] [, clipping] )
```

# Description:

This second form fills the shape with a flat color (if specified), avoiding the use of an array for specifying the color. The color argument has the same meaning as in the routine msPlot.

grid and grid\_color are described above for the first interface.

The optional argument clipping is a boolean (default value is TRUE). When set to FALSE, it allows the drawing of a polygonal colored shape (only using flat color) outside the viewport. This facility is especially useful when the user wants to update a text string outside the viewport: due to the use of antialiased text drawing, he must erase before the ancient text with a rectangular patch of appropriate color.

# Remarks:

- the function mfPatch has the same argument list; it returns the (integer) handle of the created graphic object.
- this routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects of the figure, the Hold property must be set to "on" via the msHold routine.

See also: mf/msPColor

mf/msPlotHist data histogram

Calling syntax:

returns the (integer) handle of the newly created graphic object.

The vector mfArray x contains the data. The histogram is computed for data ranged from  $x_min$  to  $x_max$  using a number of bins equal to  $n_bin$ .

If present, color specifies the color for the bar plot. It may be a character string containing the usual color code (see msPlot), a real triplet containing the RGB values or a string containing a color name from the RGB database. If the color is left unspecified, then it is taken from the current colortable (for more information about colortables – or color schemes – see the Muesli User's Guide).

If present, the logical argument filled specifies that bars are filled with the color used (default is FALSE).

The subroutine form:

allows the user to keep some internal data.

The optional arguments color (& Co.) and filled have the same meaning as above.

The output mfArray num is the vector of the bins values.

If present, the mfArray x\_bin is the vector containing the abscissas of the bins.

Remarks:

- The drawing is always clipped at the viewport.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- This routine may be called many times in the same figure with different data: histograms will be superposed one above the other, showing cumulative statistics. When not specified, colors will be recycled from the current colortable. Note that if your data is stored inside a two-dimensional array, then it may be easier to call the msCumulHist routine instead.

See also: msHist, mfOut

#### msCumulHist

# cumulative data histogram

Interface:

```
subroutine msCumulHist( X, x_min, x_max, n_bin, x_nb )

type(mfArray), intent(in) :: X
real(kind=MF_DOUBLE), intent(in) :: x_min, x_max
integer, intent(in) :: n_bin
type(mfArray), optional :: x_nb
```

## Description:

Cumulative Histogram. Works on columns of the mfArray X.

If X has NCOL columns, draws first an histogram (via msHist) from the data of all columns, then draw a second histogram from the data of the NCOL-1 first columns, and so on. Each histogram is drawn in a different color, via the colors of the current colortable (for more information about colortables – or color schemes – see the *Muesli User's Guide*); if NCOL is greater than the maximum, following colors are cycled in the same set.

Columns of X may have been initialized from data which contain different number of element. In such a case, the optional argument  $x\_nb$  is a vector which contains the useful number of elements in each column.

### Remarks:

- The drawing is always clipped at the viewport.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

See also: msHist, mf/msPlotHist

msImRead image read

# Interface:

```
call msImRead( mfOut(A,cmap), filename [, fmt, indexed] )
```

### Description:

Reads the image file filename: the colormap is extracted from the image and stored in the mfArray cmap and the pixels values are stored in the mfArray A.

The image can be displayed by using first msColormap and then msImage.

The optional argument fmt, if present, is a string which must contain the image format. Common accepted values for fmt are: "TIFF", "JPEG", "PNG", "XPM", ... . Actually, in the current version of MUESLI (but this should change in future), the file image is always converted in the XPM format via the 'convert' command of 'ImageMagick'. So, a great number of image formats is supported and, moreover, the argument fmt is not yet used for reading.

There are two options to fill the mfArray A, i. e. to give numerical values to pixels (the optional logical argument indexed is by default false):

- 1. real valued pixels each pixel of the array contains a numerical real value (ranged in [0.,1.]), and a linear mapping is used between the whole range of these values and the index range of the colormap. This first way is rather oriented to the numerical processing of the pixels value, and is well adapted to image obtained via the pseudo colors from any numerical matrix.
  - For example, you may apply some filters or compute the gradient of the pixels values and you can visualize the image in different ways by changing the colormap, which is always a continuum of colors.
- 2. indexed pixels each pixel of the array contains the index of the colormap.
  - This second way is more adapted to digital photos, or to color bitmaps which contain transparency (as for some PNG images see below).
  - You should not modify the whole colormap, else you may obtain some strange results when you visualize the image. In contrast, you can easily remove or add some colors in the colormap.

### Remarks about transparent pixels:

If the original image contains transparent pixels ("NONE" color in an XPM file, or transparency in PNG files), then it cannot be opened using the default indexing scheme: the current routine will give an error. You can open it only via the "indexed pixels" scheme. Moreover, these transparent pixels will be colored in medium gray by the msImage routine.

## Limitations:

Currently, these limitations are linked to the XPM format:

- original XPM images having more than two characters per color are not supported (to check yourself, open the XPM file with any text editor: the fourth integer number of the third line must be equal to 1 or 2). Moreover, they must have less than 8281 colors.
- for original XPM images, they must have maximum 12000 pixels in width if colors are stored in one char (*i. e.* when the number of colors is small, less than 91), or 6000 pixels in width if colors are stored in two chars (*i. e.* when the number of colors is greater than 91).
- for images converted in XPM by the 'convert' tool of 'ImageMagick', the number of colors is always less than 256. So, the quality of true-color images is pretty degraded and, at least up to the version 6.9.3 of 'ImageMagick', the picture is darker.

 $Other\ interface:$ 

```
call msImRead( mfOut(infos), filename [, fmt], only_infos=.true. )
```

allows the user to obtain some information about the image (currently, only the width and the height in pixels unit), which are returned in the mfArray infos.

See also: msImWrite, mfOut

msImWrite image write

# ${\it Interface:}$

```
call msImWrite( A, cmap, filename [, fmt, indexed] )
```

# Description:

Writes the mfArray A to the image file filename, using the colormap cmap.

The optional argument fmt, if present, is a string which must contain the image format. Common accepted values for fmt are: "TIFF", "JPEG", "PNG", "XPM", ... . Actually, in the current version of MUESLI (but this should change in future), the file image is always converted in the XPM format via the 'convert' command of 'ImageMagick'. So, a great number of image formats is supported.

The optional logical argument indexed (its default value is false) has the same meaning as those explained in the msImRead routine. It concerns obviously the mfArray A, not the image stored in the image file filename.

# Remarks about transparency:

- if the mfArray image contains transparent pixels, consider using an appropriate image format (XPM or PNG) with supports this transparency.
- transparency can be set in your image by two ways:
  - 1. for the indexed pixels indexing scheme, set a color (in cmap) to the triplet (NaN, NaN, NaN).
  - 2. for the <u>real valued pixels</u> indexing scheme, set an element of the image mfArray A to NaN.

#### Limitations:

- XPM images must have less than 8281 colors in their colormap.
- XPM images must have maximum 12000 pixels in width if colors are stored in one char (*i. e.* when the number of colors is small, less than 91), or 6000 pixels in width if colors are stored in two chars (*i. e.* when the number of colors is greater than 91).

 $See~also: {\tt msImRead}, {\tt mf/msImage}$ 

mf/msImage display

# Interface:

```
call msImage( image [, angle] [, flip] [, indexed] )
```

# Description:

Displays in the current figure the mfArray image, associated with the appropriate colormap previously set by msColormap.

(i, j) indices of the image are mapped to the axis (-Y, X), so data visualisation is coherent with Contour and PColor.

The optional real argument angle is used to define the orientation in degrees of the image (default is 0 degree; any real values are accepted).

The optional argument flip is a string can take the following values: "NONE", "HORIZ" (or mirror, left/right swap), "VERT" (or upside/downside swap); default value is "NONE".

The optional logical argument indexed (its default value is FALSE) has the same meaning as those explained in the msImRead routine. It tells the current routine that the image mfArray contains integer indices of colors and not real values.

This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

#### Remarks:

- The drawing is always clipped at the viewport.
- The function mfImage has the same argument list; it returns the (integer) handle of the created graphic object.

See also: mf/msContour, mf/msPColor

### mf/msPlotPSLG

# plot a PSLG domain

# Description:

This routine plots a PSLG domain and, optionally, shows the numbering of nodes, edges and holes.

Calling syntax:

PSLG\_domain, of type mfPSLG, represents the 2D domain definition. It must be built by the user.

The optional color argument specifies the color used for drawing the cells: it may be a character string or a vector of type real and length 3 (see mf/msPlot).

The optional linewidth argument (real) specifies the line width, as a relative factor; by default its value is 1.

The three last optional arguments nod\_num, edg\_num and hol\_num, all booleans, are used to display respectively nodes', edges' and holes' numbering of the PSLG. By default, numbering is not displayed.

Be aware that only nodes involved in segments are drawn; unused nodes are therefore ignored. To see all registered nodes, use msPrintPSLG.

# Remarks:

- The drawing is always clipped at the viewport.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- The function mfPlotPSLG has the same argument list; it returns the (integer) handle of the created graphic object.

## mf/msTriMesh

# plot a triangulation

Description:

This routine plots a triangular mesh and, optionally, shows the numbering of triangles, nodes and faces.

First calling syntax:

The mfArrays x and y are vectors of same length which describe the position (x, y) of the nodes. The mfArray tri contains the triangulation (i. e. the triangles' indices, as returned by, e. g., the routine mfDelaunay).

The optional color argument specifies the color used for drawing the cells: it has the same meaning as in other plotting routine involving colors (see the msSetGrObj routine).

The optional linewidth argument (real) specifies the line width, as a relative factor; by default its value is 1.

The optional height argument (real) specifies, for the numbering of triangles, nodes and faces, the character height, as a relative factor; by default its value is 1.

The two last optional arguments tri\_num and nod\_num, both booleans, are used to display triangles' and nodes' numbering of the triangular mesh. By default, numbering is not displayed.

When the optional boolean argument boundary\_only is used with the value TRUE then only the boundary of the mesh is drawn. In this case, the triangles' numbers are never displayed, whatever the optional argument tri\_num is.

Second calling syntax:

```
call msTriMesh( tri_connect, ... [, fac_num] )
```

The first argument is tri\_connect of type mfTriConnect, the mesh connectivity (see msBuildTriConnect); it replaces the three first mfArrays in the first calling syntax.

Same optional arguments as in the first calling syntax may be added. Note however that an additional optional argument is possible in this second case: the boolean fac\_num concerns the numbering of the triangle faces.

This second use of msTriMesh uses a fast algorithm to draw the mesh; the printed files (both EPS and PDF) are also smaller in size.

# Remarks:

- The drawing is always clipped at the viewport.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- The function mfTriMesh has the same argument list; it returns the (integer) handle of the created graphic object.

See also: msPrintTriConnect

### mf/msPlotVoronoi

# plot a Voronoi diagram

# Description:

This routine plots a Voronoi diagram and, optionally, shows the numbering of points and vertices.

Calling syntax:

The only mandatory argument is voronoi, a structure of type mfVoronoiStruct, created by the mfVoronoi routine.

The optional color argument specifies the color used for drawing the cells: it has the same meaning as in other plotting routine involving colors (see the msSetGrObj routine).

The optional linewidth argument (real) specifies the line width, as a relative factor; by default its value is 1.

The optional height argument (real) specifies, for the numbering of nodes and vertices, the character height, as a relative factor; by default its value is 1.

The two last optional arguments nod\_num and vert\_num, both booleans, are used to display nodes' and vertices' numbering. By default, numbering is not displayed.

#### Remarks:

- The drawing is always clipped at the viewport.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- The function mfPlotVoronoi has the same argument list; it returns the (integer) handle of the created graphic object.

See also: msPrintVoronoi

# mf/msTriPColor

# pseudo-color on a triangulation

 $Calling\ syntax:$ 

```
call msTriPColor( x, y, z, tri )
```

# Description:

This routine is equivalent to the routine msPColor, but for data which are spread on a triangulation.

The mfArrays x, y and z are vectors of same length which describes position (x, y) of the vertices and the corresponding function value.

The mfArray tri contains the triangulation (i. e. the triangle's indices, as returned by, e. g. the routine mfDelaunay).

#### Remarks:

- The drawing is always clipped at the viewport.
- Use msTriMesh to show the grid or the boundary of the triangular mesh.
- The function mfTriPColor has the same argument list; it returns the (integer) handle of the created graphic object.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- NaN values are not (yet) allowed: an error is returned by the routine.

See also: mf/msTriContour, mf/msTriFill, msPatch

# mf/msTriFill

# coloring on a triangulation

 $Calling\ syntax:$ 

```
call msTriFill( x, y, val, tri )
```

## Description:

This routine is equivalent to the routine msPatch (when using flat colors), but for data which are spread on a triangulation.

The mfArrays x and y are vectors of same length which describes position (x, y) of the nodes.

The value to be colored is stored inside the vector mfArray val. This latter vector must have a length equal to the number of triangles. The mfArray val may contain NaN values: in such a case the corresponding triangles are not drawn.

The mfArray tri contains the triangulation (i. e. the triangle's indices, as returned by, e. g. the routine mfDelaunay).

#### Remarks:

- The drawing is always clipped at the viewport.
- Use msTriMesh to show the grid or the boundary of the triangular mesh.
- The function mfTriFill has the same argument list; it returns the (integer) handle of the created graphic object.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

See also: mf/msTriContour, mf/msTriPColor

### mf/msTriContour

# data contouring on a triangulation

Description:

This routine is equivalent to the routine msContour, but for data which are spread on a triangular grid.

It doesn't fill the regions between the contours with colors; for such a feature, see the mf/msTriContourF routine

First calling syntax:

The mfArrays X, Y and Z are vectors of same length which describes position (x, y) of the nodes and the corresponding function value z.

The mfArray tri is the triangulation (e.g. returned by the routine mfDelaunay).

If the optional C mfArray is given in front of (X,Y) coordinate matrices, then it will contain on output a two-row matrix storing the contour lines. Each contiguous drawing segment contains the value of the contour, the number of (x,y) drawing pairs, and the pairs themselves. The segment are appended end-to-end as

```
C = [ level_1, x_1, x_2, ..., level_2, x_1, x_2, ...; pairs_1, y_1, y_2, ..., pairs_2, y_1, y_2, ...]
```

By default, the number of level curves is equal to 9, and the level values are equally spaced within the *min* and *max* values of the Color Axis (see msCAxis); this can be changed by using the optional arguments nb\_levels and levels (see below).

The optional (integer) argument nb\_levels specifies the number of contours to be drawned; it cannot be used together with levels.

The optional argument levels, which must be a rank-1 mfArray, is used to specify the level values.

The optional arguments linespec (character string) and linewidth allow the user to change the appearance of the contour curves. The possible values for these arguments are described in msSetGrObj, under the "color", "linestyle" and "linewidth" entries.

By default, the level curves are labelled (use labels=.false. to remove the labels) and the contour curves are colored using the current colormap (use a color inside linespec to specify the color of all curves). Labels are displayed in the foreground color (e. g. black on white) by default; the user may choose another color via the labelscolor optional argument. Moreover the label size may be changed by using the optional labelsize real argument, which lets the user to specify a percentage of the standard size (default is labelsize=1.0d0).

Second calling syntax:

```
call msTriContour( [ mfOut(C),] Z, tri_connect, [ ... ] )
```

Here, tri\_connect (of type mfTriConnect) is the mesh connectivity (see msBuildTriConnect); it replaces the three mfArrays X, Y and tri in the first calling syntax. The other optional arguments are the same.

 $\cdots/\dots$ 

#### Remarks:

- The triangular mesh used doesn't need to be convex, and may also have holes. See mfPSLG to create triangulation with holes.
- The drawing is always clipped at the viewport.
- Use msTriMesh to show the grid or the boundary of the triangular mesh.
- The function mfTriContour has the same argument list; it returns the (integer) handle of the created graphic object.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- NaN values are allowed: all triangles involved by these special non finite values are simply ignored.

 $See~also: {\tt mf/msContourF}, {\tt mf/msTriPColor}, {\tt mf/msTriFill}$ 

### mf/msTriContourF

# data contouring on triangles with filled regions

First calling syntax:

Second calling syntax:

```
call msTriContourF( [ mfOut(C),] Z, tri_connect, [ ... ] )
```

# Description:

Similar to the msTriContour routine, except that the regions between the contours are filled with appropriate colors.

The whole region whose values are greater than a specified level is filled with the corresponding color. As a consequence, the region less than all specified levels is left in white. To avoid white regions, it is sufficient to add a level less than (or equal to) the smaller value in the Z array.

Be aware however that some arguments of msTriContour are not available here.

Lastly, be warned that the function version (i. e. mfTriContourF) returns a vector of handle(s), even if the number of handles is one. Indeed, under some circumstances, the whole triangular mesh may be splitted in many independant zones when the number of NaN values is great enough.

See also: mf/msContourF

## mf/msTriQuiver

# vector field plot on a triangulation

Calling syntax:

This routine is equivalent to the routine msQuiver, but for data which are spread on a triangulation.

x, y, u and v must be rank-1 (i. e. vectors) mfArrays having the same number of rows. They represent respectively the X and Y coordinates of the points where the components (u, v) of the vector field have to be drawn.

The optional color argument specifies the color used for drawing the arrows: it is a one-letter color symbol or an escaped sequence, as described in the msSetGrObj routine.

The optional arrow\_length argument (real) specifies the length of the arrow as a multiplicative factor; by default its value is 1 so that the biggest arrow has approximately a length equal to 1/10th the full axis range.

The optional arrow\_head argument (real) specifies the size of the arrow head; by default its value is 1.

## Remarks:

- The drawing is always clipped at the viewport.
- An arrow is not drawn if any component of the corresponding vector  $(\mathbf{u}, \mathbf{v})$  is equal to NaN.
- The function mfTriQuiver has the same argument list; it returns the (integer) handle of the created graphic object.
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.

See also: mf/msTriPColor, mf/msTriStreamline

### mf/msTriStreamline

# vector field streamline on a triangulation

Calling syntax:

This routine is similar to the routine msStreamline, but for data which are spread on a triangulation. In particular, the arguments direction, n\_arrow, color, linewidth, arrow\_head, npt\_max and curv\_tol have exactly the same meanings.

The optional arguments tri and tri\_connect (see msBuildTriConnect) describe respectively the triangulation of the mesh and its connectivity. If they are both absent, then the triangulation and its connectivity are done by the routine itself. If the user already knows the connectivity, it is preferable to pass it to the current routine, to avoid a redundant computation. On the other hand, if the user knows only the triangulation, it should pass it as argument, and the current routine will compute only the connectivity.

#### Remarks:

- The drawing is always clipped at the viewport.
- The function mfTriStreamline has the same argument list; it returns the vector of the (integer) handles of the created graphic objects. Be aware that even for a single streamline, the returned handle is a vector (of size 1).
- This routine, as most of high level ones, first erase the current plot; in order to plot multiple graphics objects on the figure, the Hold property must be set to "on" via the msHold routine.
- NaN values are not allowed: an error is returned by the routine. If such values are present in the vector field (i. e. in u or v) you can try to define a stop\_zone user-function to avoid an error.

See also: mf/msTriQuiver

# mf/msSpy

# sparsity pattern visualization

Calling syntax:

draws non-zero elements of the mfArray A (usually sparse).

If the optional argument symbolspec (string) is present, it must contain the specification for a symbol and/or a color, as described in msSetGrObj at marker and color entries. By default, a star ("\*") is used, but note that it is well adapted only for small size matrices: for big matrices, a dot (".") may be prefered.

If the optional argument continuous (logical) is present, the matrix A is drawn using a color/grey scale for the absolute value of each element, as in msPColor. If you intend to spy a dense matrix, the continuous argument must be used. Default value is .false.

In the continuous case:

- if the optional argument color\_scale (string) is present, it must be equal to "lin" or "log". The latter case specifies the use of a logarithmic color/grey scale in the mapping between the value of the matrix elements and the color/grey index (default mapping is linear);
- if the optional argument nz\_threshold (real) is present, it specifies the threshold value under which an element is considered as zero and, thus, not displayed. The default value is zero for the linear scale and  $\max(|A_{i,j}|) \times \epsilon$  for the logarithmic scale. The threshold for small values is used for defining the color map range.
  - By setting nz\_threshold to the special value -1, the threshold will be set to the minimum value of the elements of |A|.

If the optional argument show\_nnz (logical) is present, the effective number of non-zero element is shown in the X-label. It's default value is '.true.'.

If the optional argument bitmap (character string) is present, it must contains the bitmap size (in pixels) which will be used for printing. The string reads under the form "WIDTHXHEIGHT" (e.g. "3000x3000"). This option is useful only for large, sparse matrices.

# Remarks:

- The drawing is always clipped at the viewport.
- The function mfSpy has the same argument list; it returns the (integer) handle of the created graphic object.
- The continuous and symbolspec options are exclusive.
- The grey scale mentionned above is the default colormap when none was selected by the user. Please use the msColormap routine to select another one, prior to the msSpy call.

# 2.6 Interactive routines

Caution: the routines available in this chapter require the selection of the X11 driver (which is usually selected by default). In batch mode, where the user selects the NULL driver (see msSetX11Device), these routines will not be available, and an error will occur if the user tries to call any of them.

A figure must be also opened (see mf/msFigure).

If you have multiple workspaces, switching from one workspace to another may breaks the interaction of the mouse with your Muesli program; therefore, it is recommended to stay in the same workspace during all the duration of the interactive process.

Some routines (msPan, msZoom and msPanAndZoom) accept the use of an additional keyboard key (usually the CTRL one) to modify the action. In such a case, it is required that the figure's window have the focus; on the contrary, the use of the key do nothing.

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See also:

Global graphic settings

Window's and figure's management

Figure properties

Figure annotation – Low level graphic object's manipulation

High level plotting routines

mfGinput graphic input

# ${\it Interface:}$

```
function mfGinput( event, key ) result ( out )
  logical, intent(in), optional :: event, key
  type(mfArray) :: out
```

### Description:

This function is an interactive facility to get the pointer coordinates when it is inside the current figure.

Usually, this function is called without the optional event and key argument (by default, event is TRUE): the library waits for a mouse click or a key pressed, and the cursor show a cross shape in the current active window.

The returned mfArray contains the coordinates (x, y) of the selected point. The point can be chosen by any mouse button (which can be also emulated via the L, M or R key). Be aware that the current routine implements a two-event procedure and therefore the location returned is that of the pointer for the second event! (mouse up in the usual case).

For other key pressed, the routine mfGinput terminates and leaves the returned mfArray empty.

#### Remarks:

- this routine is usually employed for graphic interaction. If the user doesn't have a mouse, the pointer can be moved via the keyboard's arrows (the shift key accelerates the move by a factor of 10); moreover each mouse button can be emulated by using an equivalent sequence (L, M or R followed by any other key).
- see the msGinput routine for more features.

When called with the **event** argument equal to *FALSE*, the library doesn't wait for a user event: it returns immediately the pointer coordinates in the **mfArray** out. A third element is added to out, indicating whether the pointer is inside the window (value 1) or outside (value 0); the user must use this flag to take any decision in its own program.

Moreover, if the key is present and equal to *TRUE*, then a fourth element is returned in out: the ASCII code of the key pressed recently.

See also: mfGinputRect

msGinput graphic input

Calling syntax:

# Description:

This function is an interactive facility to get the pointer position inside the current figure. See mfGinput for a simpler use.

In case of multiple opened graphic windows, the cursor has a cross shape in the current active one instead of the classical pointer.

The returned mfArray coords contains the coordinates (x, y) of the selected point. The point can be chosen either by the mouse (default behavior), or by a key pressed (see below).

The returned mfArray keycode contains an equivalent code for the event detected:

- If the optional argument whole\_keyboard is FALSE (default behavior), then keycode contains the button number (left=1, middle=2, right=3), even if the equivalent keys (L, M or R) have been used; Note that mfArray coords and keycode will be empty if any key different from L, M or R is pressed.
- On the contrary, keycode contains the value of the key pressed (actually ichar(key\_pressed), where ichar is the standard Fortran function which returns the ASCII code of the letter). coords will be empty only if the ESCAPE key is pressed. Only when using keyboard, the mouse wheel can be detected. The values returned are ichar(8)=56 for wheel up, and ichar(2)=50 for wheel down (think that on a numeric keypad, 8 is for arrow up, and 2 is for arrow down).

If present, the returned mfArray color contains the RGB triple of the colored pixel just under the pointer.

Usually (i. e. when the optional argument rect\_size is not present), the cursor shown on the screen has the shape of a big crosshair, whose length is that of the selected window; on the contrary, if the mfArray rect\_size is present and not empty, it must contain the size (both width and height) of a rectangle which follows the pointer during its move. This rectangle is always centered around the pointer position. Moreover, if the logical optional argument rect\_inside\_axes is present and equal to TRUE, the rectangle drawn is constrained to be entirely inside the axes.

See also: mfGinputRect

#### mfGinputCustom

# graphic input using custom cursors with alternative

Description:

The first Calling syntax has no argument:

```
out = mfGinputCustom( )
```

Three elements are returned in the mfArray out: the coordinates (x, y) of the pointer when the mouse has been clicked, and the case used (1 or 2)

- 1. without the Control key pressed, displaying the first custom cursor;
- 2. with the Control key pressed, displaying the second custom cursor.

The other Calling syntax implements a magnetic grid and has one argument:

```
out = mfGinputCustom( magnetic_grid_rule )
```

The magnetic\_grid\_rule subroutine, provided by the user, must have the following interface:

where x1, y1, x2, y2 are world-coordinates. The (x1,y1) is the pointer position in the figure, and the (x2,y2), computed by the user, is where the user-defined cursor is displayed (icase is 1 or 2, according the status of the Control Key, pressed or not, as mentioned above). If, for some reasons, no (x2,y2) point can be computed, then the routine must set the last argument (valid) to FALSE and in this case the cursor will show its standard shape (i. e. the left arrow).

In this second calling syntax, four elements are returned in the mfArray out:

- 1. x-coordinate of the clicked location.
- 2. y-coordinate of the clicked location.
- 3. an integer indicating whether the Control Key has been pressed or not.
- 4. an integer giving the validity of the position; if the validity is zero, the position should be ignored.

Note: Before use, the msDefineCustomCursors routine must be called to define the two cursor shapes.

# mfGinputRect

# rectangular graphic selection

 ${\it Interface:}$ 

```
function mfGinputRect() result ( out )
    type(mfArray) :: out
```

#### Description:

This function is an interactive facility to get the size and position of a rectangle drawn inside the current figure.

The returned mfArray contains the bounding box of the rectangle. This rectangle must be selected by the left mouse button only; for other key pressed, the routine mfGinputRect terminates and leaves the returned mfArray empty.

In case of multiple opened graphic windows, the cursor has a cross shape in the current active one, instead of the classical pointer.

See also: mfGinput

# mfGetModKeys

modifier keys state

 ${\it Interface:}$ 

```
function mfGetModKeys( ) result ( out )
    logical :: out(2)
```

#### Description:

This function checks the state of the keyboard for the **Shift** and the **Control** keys.

It returns an array of two booleans: the **first** one (resp. the **second** one) tells if one of the **Shift** keys (resp. the **Control** keys) was down when this routine was called.

# msPan manual scrolling

 $Calling\ syntax:$ 

call msPan( )

#### Description:

Activates the 'pan' mode for scrolling in the current figure. It ends when the [ESCAPE] key is pressed. Scrolling is monitored via the mouse (or the equivalent keyboard key):

- click-and-drag with the *left* button ('L' key) scrolls through the figure to a new view;
- middle-click does nothing;
- right-click ('R' key) resets the view to the initial one; if the [CTRL] key is pressed during the click (or the '6' key is pressed) then it centers the view with respect to the bounding box of all graphic objects.

#### Remark:

Scrolling in the current figure should be used before putting annotations on the figure, otherwise, parts of graphic objects drawn outside the axes will not be moved with this routine; however, a call to msRedrawFigure will fix this problem.

See also: msZoom, msPanAndZoom

msZoom figure zoom

 $Calling\ syntax:$ 

call msZoom( )

# Description:

Activates the 'zoom' mode in the current figure. It ends when the [ESCAPE] key is pressed.

The zoom is monitored via the mouse:

- the *left* button is used to "zoom in" at the pointer location:
  - \* a single click is used to enlarge the current view with a factor of 1.41 (or 1.19 if the [CTRL] key is pressed during the click);
  - \* a click-and-drag zooms to the new selected area;
- a *middle*-click is used to "zoom out" at the pointer location (by a factor of 1.41 or 1.19 if the [CTRL] key is pressed during the click);
- a *right*-click resets the view to the initial one; if the [CTRL] key is pressed during the click, then it sets the view to an area which shows all the graphic objects.

See also: msPan, msPanAndZoom

#### msPanAndZoom

# combine scrolling and zooming

Calling syntax:

call msPanAndZoom( )

#### Description:

Activates both the 'pan' mode for scrolling and the 'zoom' mode for zooming in the current figure. It ends when the [ESCAPE] key is pressed.

Scrolling is monitored via the three mouse buttons:

- click-and-drag with the left button scrolls the figure to a new view;
- middle-click does nothing;
- right-click resets the view to the initial one; if the [CTRL] key is pressed during the click, then it sets the view to an area which shows all the graphic objects.

Zooming is monitored via the mouse wheel by a factor of 1.41 (or 1.19 if the [CTRL] key is pressed during the click). Note that the figure remains centered during this zooming process.

#### Remark:

Scrolling in the current figure should be used before putting annotations on the figure, otherwise, parts of graphic objects drawn outside the axes will not be moved with this routine; however, a call to msRedrawFigure will fix this problem.

See also: msPan, msZoom

## msMoveLegend

# adjust legends frame in axes

 $Calling\ syntax:$ 

```
call msMoveLegend( [ handle ] )
```

# Description:

This interactive routine allows the user to move the legend frame inside the figure via the mouse.

When a unique legends frame has been created by msLegend, the routine has no argument.

Use the optional argument to move a legends frame created by mfLegend and referenced by its handle.

Note that the legend frame may be located outside the axes; this may be useful in the case where a great number of curves are plotted in the figure. During the interactive move on the screen, the legend frame may appear truncated by the window boundary under X11; however, printing in EPS or PDF should give correct results.

The use of this routine could follow msPan, msZoom or msPanAndZoom in order to optimize its position.

## msMoveGrObj

## move graphic object

 ${\it Interface:}$ 

#### Description:

Allows the user to move some types of graphic object, by interaction with the mouse (left button click only). It ends when the [ESCAPE] key is pressed. Supported graphic object types are: text (generated by mfText), arrow (generated by mfArrow) and polygon (generated by mfPatch using a flat color).

When this routine is called, it waits for a *mouse down* event (first part of a click) while the pointer have the shape of an opened hand. To grab a *text* or *polygon* object, you can click elsewhere inside its bounding box (displayed using a gray animated dashed line); for an *arrow*, you have three hotspots (sensitive area): one is the arrow base, the second is the middle and the third is the arrow head.

When a mouse down event is detected, the shape of the pointer becomes a closed hand and the graphic object (referenced by its handle) can be moved in the figure. Note that a text or polygon object can be only translated, without changing its orientation; however, for an arrow and according to the selected hotspot, you can change both its direction (when you have clicked on its base or its head) and its location (when you have clicked in its middle).

When a mouse up event is detected (end of a click), the pointer shape changes back to an opened hand.

The prompt argument lets you chosing an appropriate sentence on the screen. By default, the prompt is adapted to the type of graphic object:

```
text: -> Grab and move the text string surrounded by a dashed line... arrow: -> Grab and move the arrow wearing three hotspots... polygon: -> Grab and move the polygon surrounded by a dashed line...
```

#### Remarks:

- Note that the graphic objects may be clipped at the axes framebox. If you move a graphic object out of the axes, it may no longer be visible; however, the dashed line(s) around it will be always visible and will be an appreciable help.
- An example of use of the msMoveGrObj routine can be found in the test program named MoveGrObj\_test inside the tests/fgl folder.

See also: msPan, msZoom, msMoveLegend

#### msAnimation

#### screen animation mode

 $Calling\ syntax:$ 

```
call msAnimation( "on" | "off" )
```

#### Description:

The aim of this routine is to avoid the flickering between frames when you want to show quickly (usually in a do loop) a succession of graphic objects.

Note that, at the end of the animation, you will not be able to print the figure in EPS or PDF files (graphic objects are not saved into memory), unless you run yourself the same graphic commands out of the animation.

Before setting animation to "on", a window must be opened (with msFigure). Moreover axis must be defined as constant, by calling the msAxis routine before.

After drawing each frame, the only way to show the display of the graphic object in the figure is to call the msShowNow routine. The MUESLI User Guide presents a detailed example of a typical sequence of such an animation.

At the end of the animation sequence, the user should set animation to "off".

#### Remarks:

During an animation:

- the "hold" property cannot be changed by the user. Therefore, the msHold routine shouldn't be used
- all graphic objects are volatile; as a consequence the handle returned by all plotting functions beginning by mf is equal to zero. Therefore, you cannot use any routine requiring a valid handle.
- don't use msClf (which does a complete erasing of the figure) between frames: in such a case, this will annihilate a large part of the benefit of the current routine. Use msCla instead.
- a varying text string may be displayed outside the axes, and erased between frames with the use of a white rectangle over it (see mf/msPatch).
- if the msPColor routine is used, then the optional arguments X and Y must be used.

#### msShowNow

# screen update during an animation

 $Calling\ syntax:$ 

call msShowNow( )

# Description:

This routine may be called only during an animation (see msAnimation). It forces the update of the screen to show the new display of the current frame.

msCla clear current axes

 $Calling\ syntax:$ 

call msCla()

# Description:

Clears the current axes, i. e. fills the rectangle inside the axes using the background color.

A typical usage concerns the animation of any graphic objects (curve, symbol, text, ...): at the beginning of a loop, the axes should be erased with msCla (quick way) and then graphic objects are drawn. Avoid to use msClf (heavy way) because this latter routine do a complete reset of the figure (among other things, the axes range is lost).

See also: msFigure, msClose

# ${\tt msSetGBuffer}$

# graphic buffer policy

```
{\it Interface:}
```

# Description:

Activates the graphic buffer (default is "on"). Usually increases the display speed.

The graphic buffer may be turned "off" for debugging purpose, for example.

See also: msBBuf, msEBuf

#### msBBuf

# begin of graphic buffering

 $Calling\ syntax:$ 

call msBBuf( )

# Description:

Begins to draw in the graphic buffer, instead of directly on the screen.

# Remarks:

- this routine is active only if the graphic buffer is not turned off by the msSetGBuffer routine;
- actually, many internal low-level graphic routines are already buffered.

See also: msSetGBuffer, msEBuf

# ${\tt msEBuf}$

# end of graphic buffering

 $Calling\ syntax:$ 

call msEBuf( )

Description:

Ends to draw in the graphics buffer.

 ${\tt msBBuf}$  and  ${\tt msEBuf}$  calls should always be paired.

 $See \ also: {\tt msSetGBuffer}, {\tt msBBuf}$ 

# msDefineCustomCursors define custom cursors for use with mfGinputCustom

#### Interface:

## Description:

Register two customized cursors, provided by the user in the XBM format. The arguments <code>cursor\_\*</code> are the filenames of these images. Each cursor is defined by two bitmap images: one for the cursor shape itself and the other for the mask. A color must be also specified for each cursor.

A size of  $16 \times 16$  pixels is recommended. The cursor main image must define the Hot Spot position, for example

```
#define _x_hot 8
#define _y_hot 8
```

Last, each color is a colorname from the RGB X11 database.

See also: mfGinputCustom

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