A SEISMIC TOMOGRAPHY USING GENETIC ALGORITHM

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Abstract. In this paper, an interest is taken in determining the wave velocity and the shape of an interface of a geological layer from travel time measures. We start with the forward problem represented by the Eikonal equation and its resolution using the method of characteristics. Next follows a description of the inverse problem formulated as a constrained nonlinear least squares problem. The optimization method us the genetic algorithm. Results corresponding to a seismic survey along the Moroccan coast are presented in this paper.

Key words and phrases. Geophysical Inverse Problem, Ray Tracing, Genetic Algorithm.

1 Introduction

Engineering investigations are usually based on seismic surveys to have an image of the geological subsurface. One of the aims of these surveys is to carry out an estimation of the depth of the bedrock and the mechanical properties of the rocks. This domain of geophysics is also referred to seismic traveltime tomography. Knowing its industrial applications, this geophysical domain has become in the recent years a very powerful tool to have a high resolution image of a complex geological structure.

This paper is based on a seismic survey done along the Moroccan coast in 2002 by the IFREMER\(^1\). The underground is supposed to be of two layers separated by an interface, and the goal of this study is to find the velocity field in both layers and the shape of the interface. This hypothesis is called wide angle tomography. If these unknowns come from a physical model \( m \) and \( d \) is a set of traveltime data, \( d \) can be computed from \( m \) by an integration along the wave path. The relationship \( d = g(m) \) forms the basis of the tomography study. The absence of a priori information which constrains the input data \( d \) makes the inverse problem harder and need to be solved using a global optimization technique. We need to handle the forward computation of the travel time of a seismic wave in a specified model and the inverse computation to find one or more models, for which computed travel times fit the observed ones. In the inverse problem we seek the best possible model within some specifications, and with a definition of what we mean by best possible. When solving such a seismic inverse problem we face the task of searching through a solution/model space of high dimension. This task is done by coupling genetic algorithm with ray tracing technique and using a multi-scale approach.

Three steps are required to solve this inverse problem:

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1. **Model parametrization**: giving a mathematical expression to describe the velocity of the two layers and the interface. The degrees of freedom of the model are the unknowns of the inverse problem.

2. **Forward calculations**: mapping the geological model into traveltimes. It consists in solving the Eikonal equation using the ray tracing method.

3. **Inversion**: definition of an automated procedure that finds a model which best fits the measured travelt ime data.

## 2 Parametrization Model

The geological domain is divided into two horizontally superposed layers. A rectangular grid mesh is considered in each layer, and the velocity is defined at the vertices (see [7] and [6]). Inside each cell of the grid, the velocity $v$ is taken as the bilinear interpolation of $\frac{1}{v^2}$ at the nodes of the current cell. $\frac{1}{v^2} = a_0 + a_1 x_1 + a_2 x_2 + a_3 x_1 x_2$. The interface between the two layers is a broken line having its nodes on some edges of the grid as it shows Figure 1. Taking only linear functions in the parametrization model is crucial for the computational cost of the inversion procedure because genetic algorithm requires running the forward calculations thousands of times. This choice of parametrization simplifies the equations of the forward calculations.

## 3 Eikonal Equation and Method of characteristics

The forward problem aims to compute the traveltime $T$ of a seismic wave from its propagation velocity field $v(x)$. The PDE that relates these two quantities is the Eikonal equation.

\[
\left(\frac{\partial T}{\partial x_1}\right)^2 + \left(\frac{\partial T}{\partial x_2}\right)^2 = \frac{1}{v(x)^2}
\]

We denote now by $p_i = \frac{\partial T}{\partial x_i}$, $i = 1, 2$ the components of the slowness vector and by $H(x, p, T) = n^{-1} \{(p_i p_i)^{n/2} - 1/e^n\}$ the Hamiltonian function where $n$ is an arbitrary positive integer. The Eikonal equation reads:

\[
H(x, p, T) = 0
\]
More details about the Eikonal equation or the seismic ray theory in general can be found in [1].

The method of characteristics seeks to replace the nonlinear first order partial differential equation (2) with an equivalent system of ordinary differential equations that exists along a special curve called characteristic. The unknown traveltime can be then expressed as an integral of the velocity along the characteristic.

Let \( x(s) \) be a parametric representation of the characteristic curve with a parameter \( s \), \( T(x(s)) = T(s) \) and \( p(s) = \nabla T(x(s)) \). If \( T \) is a smooth solution of equation (2), we have:

\[
\frac{\partial p_i}{\partial s} = \frac{\partial}{\partial s} \left( \frac{\partial T}{\partial x_i} \right) = \sum_{j=1}^{3} \frac{\partial^2 T}{\partial x_i \partial x_j} \frac{\partial x_j}{\partial s} \tag{3}
\]

We have to replace the second order derivative \( \frac{\partial^2 T}{\partial x_i \partial x_j} \) with an expression with first order derivatives.

\[
H(x, p, T) = 0 \quad (4)
\]

\[
dH \Big/ dx_i = 0 \quad (5)
\]

\[
= \sum_{j=1}^{3} \frac{\partial H}{\partial p_i} \frac{\partial^2 T}{\partial x_i \partial x_j} + \frac{\partial H}{\partial T} \frac{\partial T}{\partial x_i} + \frac{\partial H}{\partial x_i} \quad (6)
\]

By imposing \( \frac{\partial x_i}{\partial s} = \frac{\partial H}{\partial p_i} \) and coupling equation (3) with equation (6) we have

\[
\frac{\partial p_i}{\partial s} = -\frac{\partial H}{\partial T} p_i - \frac{\partial H}{\partial x_i} \quad (7)
\]

Now all the unknowns \( x, p \) and \( T \) are only function of \( s \), and we can replace the partial derivatives with total derivatives. Equation (2) is transformed into the following system of five ODEs \((i = 1, 2)\):

\[
\begin{cases} 
\frac{dx_i}{ds} &= H \\
\frac{dp_i}{ds} &= \frac{dH}{dp_i} \\
\frac{ds}{dt} &= \frac{dH}{dp_i} - \frac{dH}{dx_i} \\
\frac{ds}{dT} &= \frac{dH}{dT} - p \\
\end{cases} \tag{8}
\]

Replacing \( H \) with \( n^{-1} \{(p_i p_i)^{n/2} - 1/v^n\} \), system (8) reads:

\[
\begin{cases} 
\frac{dx_i}{ds} &= (p)^{n/2-1} p_i \\
\frac{ds}{dp_i} &= \frac{1}{n} \frac{\partial}{\partial x_i} \left( \frac{1}{v^n} \right) \\
\frac{ds}{dT} &= \frac{1}{v^n} \\
\end{cases} \tag{9}
\]

System (9) is called the characteristic system and the characteristic curve is referred to the seismic ray. The precedent mathematical development is taken from [1] and [2]. The first four equations of (9) are independent from \( T \). They determine the seismic ray connecting the source to the receiver. These
equations need to be solved first, then the traveltime is evaluated by integrating the fifth equation over the ray (Γ):
\[ t = \int_{\Gamma} \frac{ds}{v_2} \]. Inside each layer of the elastic medium the velocity field has a continuous expression. A special process for the ray has to be done when the ray hits an interface separating two layers where the velocity field is discontinuous. The ray can be either reflected or transmitted (but physically both arise). The change of the direction of the slowness vector is given by the following formula:
\[
\tilde{p} = \tilde{p} - \{(\tilde{p} \cdot \tilde{n}) \mp \epsilon[1/\tilde{V}^2 - 1/V^2 + (\tilde{p} \cdot \tilde{n})^2]^{1/2}\} \tilde{n} \tag{10}
\]
where \( \tilde{p} \) and \( \tilde{V} \) are the slowness and the velocity of the transmitted or the reflected wave and \( \tilde{p} \) and \( V \) are the slowness and the velocity of the incident wave.

With the specific parametrization model described above and taking \( n = 2 \), system (9) becomes:
\[
\begin{align*}
\frac{dx_i}{ds} &= p_i \\
\frac{dp_i}{ds} &= a_{3-i} + a_3 x_{3-i} \\
\frac{dT}{ds} &= a_0 + a_1 x_1 + a_2 x_2 + a_3 x_1 x_2
\end{align*}
\tag{11}
\]

4 The Inversion Procedure

The solution of the inverse problem has to verify two conditions:

1. the root mean square error (RMSE) between the measured traveltimes and the simulated traveltimes corresponding to the solution has to be less than 100 ms.

2. With the solution model, the seismic wave has to reach all the receivers from all the source positions taken in the survey.

We denote by \( m \) the geological model grouping the velocity field and the interface shape, \( N_R \) the number of receivers reached by the wave and \( N_T \) the total number of receivers considered in the survey. The inverse problem can be formulated as an optimization problem in two equivalent ways:

1. \( \min \text{RMSE}(m) \) s.t. \( N_R(m) = N_T \)

2. \( \max N_R(m) \) s.t. \( \text{RMSE}(m) < 100 \text{ ms} \)

Formulation (2) will be retained because it is found more efficient with genetic algorithm with is used in this work.

5 Genetic Algorithms

A genetic algorithm (GA) is an optimization technique that is based on the biological principles of evolution. Given an objective function to globally optimize, the input to the GA is a set of potential solutions to this problem. The quality of each candidate solution is evaluated quantitatively by metric a called fitness function which is the function to optimize.

In a pool of randomly generated candidates, most will not work at all, and these will be deleted.
However, purely by chance, a few may hold promise and evolve toward solving the problem. These promising candidate solutions are kept and allowed to reproduce. Multiple copies are made of them, but the copies are not perfect; random changes are introduced during the copying process. These digital offspring then go on to the next generation, forming a new pool of candidate solutions, and are subjected to a second round of fitness evaluation. Those candidate solutions which were worsened, or made no better, by the changes to their code are again deleted; but again, purely by chance, the random variations introduced into the population may have improved some individuals, making them into better, more complete or more efficient solutions to the problem at hand. Again these winning individuals are selected and copied over into the next generation with random changes, and the process repeats. The expectation is that the average fitness of the population will increase each round, and so by repeating this process for hundreds or thousands of rounds, very good solutions to the problem can be discovered.

Numerically, the vector of decimal numbers representing a candidate solution is mapped into a binary code. By analogy with evolution theory, a candidate solution is called chromosome and the binary encoding of each coordinate is called gene.

**Methods of Change** Two processes can take place in the generation of offsprings: mutation and crossover. Mutation can occur with a very small probability. A bit of the binary code of a gene is chosen randomly and then inverted: \((0 \rightarrow 1)\) or \((1 \rightarrow 0)\).

Crossover considers two parent chromosomes and swaps segments of their code, producing artificial ”offspring” that are combinations of their parents. The occurrence of this event and the length of the segments are taken randomly. The selection of the parent chromosomes used here is the rank selection. Each individual in the population is assigned a numerical rank based on its fitness, and selection is based on this ranking rather than absolute differences in fitness. The advantage of this method is that it can prevent very fit individuals from gaining dominance early at the expense of less fit ones, which would reduce the population’s genetic diversity and might hinder attempts to find an acceptable solution.

\[
\text{parents} \begin{cases} 010101000111 \\ 001100110011 \end{cases} \quad \text{offsprings} \begin{cases} 010101110011 \\ 001100000011 \end{cases}
\]

**The Algorithm** GA is summarized in the following (see [3], [4] and [5]):

1. Generate random population of \(n\) chromosomes and evaluate the fitness \(f(x)\) of each chromosome \(x\) in the population

2. Create a new population by repeating following steps until the new population is complete
   
   (a) Select two parent chromosomes from a population according to their fitness and with a crossover probability cross over the parents to form new offspring (children). If no crossover was performed, offspring is the exact copy of parents.
   
   (b) With a mutation probability mutate new offspring at each locus (position in chromosome).

3. Use new generated population for a further run of the algorithm

4. If the maximal number of evaluations of \(f(x)\) is reached, stop, and return the best solution in current population otherwise go to step 2
6 Numerical Results

In 2002, IFREMER has done a seismic survey in Morocco. The goal of this survey was to study the structure of the margin of the Moroccan coast in the sea and the ground. For the ground part of the study, the seismic source used in the experience is a vibrating truck which is more adequate than an explosive source for exploring shallow layers. Along of a profile of fifty kilometers long, 28 shoots were made at different locations and 25 receivers captured the shaking signals. In total 102 travelt ime picks with an error about 0.1s were used to constrain the Moroccan coast model.

To invert the traveltimes GA were run with a population of 80 chromosomes, taking crossover and mutation probabilities equal to 0.72 and 0.005 respectively and 50 iterations. The computational time is on a single machine is about 70 min. The preceding parameters of GA need to be tuned correctly to have good results so GA has to be run many times. To reduce the runtime, computations were distributed on a cluster of 8 machines which gives a speedup equal to 8 knowing that the evaluations of the cost function at each chromosomes are independent.

The solution model (see Figures 2 and 3) is obtained with 25 parameters and gives only 76 simulated traveltimes instead of 102 with an $RMES < 0.1s$. This is because some seismic ray does not verify the hypothesize of wide angle tomography considered in this paper. Some seismic rays can reach the receiver far from the source in a short time without having to be close the interface. We did not take care of finding such rays because it will be very costly in computations and the precision of the solution will not be increased.

The difficulty of this inverse problem is the need to apply a global optimization method given that the problem has many local solutions. Among different non-linear optimization techniques (simulates annealing, Monte Carlo), GA has shown to be efficient and fast for a multi-parametric model and finding the region of the global optimum. It can be the method of choice of a 3D inversion.

![Figure 2: Velocity Field of the profile considered in the survey of the Moroccan coast](image)

References

Figure 3: Ray tracing with the solution model


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