ANALYSIS OF A TWO-STRAIN TRANSMISSION MODEL WITH VACCINATION USING COMPUTER ALGEBRA

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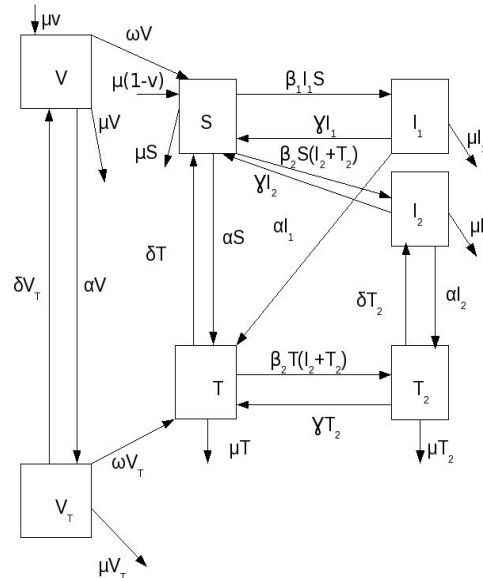[a]IREM de l'Université Abdou Moumouni, BP 10896 Niamey, Niger
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We present a typical example of a compartmental transmission model that can be dealt with algebraically. We use exact methods from real algebraic geometry and computer algebra to find all the equilibria of the ODE system describing the model and to study their stability as well as their bifurcations.

The model concerns a host population, a part of its individuals are under antibiotic (Ab) treatment against a two-strain bacterial pathogen. Individuals who are not under Ab treatment can be colonized by an antibiotic-susceptible (Ab-S) strain or by an antibiotic-resistant (Ab-R) strain of a bacterial pathogen, but not by both at the same time (i.e., maximal competition), while those under antibiotic treatment can only be colonized by the Ab-R strain. We assume that there is a fitness cost for resistance such that the Ab-R strain is somewhat less transmissible than the Ab-S strain. The host population is sub-divided into 7 compartments representing the fractions of the population in each state, 4 states for individuals not under Ab treatment: susceptible (S), colonized by the Ab-S strain (I_1) and colonized by the Ab-R strain (I_2), the individuals who are in the vaccinated state not under antibiotic treatment. those individuals are assumed to have a temporary complete immunity to infection by the 2 strains (V), and 3 states for individuals under Ab treatment: susceptible (T), and colonized by the Ab-R strain (T_2), the individuals who are in the vaccinated state currently under antibiotic treatment. Those individuals are assumed to have a temporary complete immunity to infection by the 2 strains (V_T) . We give below the transfer diagram of the model.

The model has a unique disease free equilibria and three other equilibrias. We give a complete characterization of their existence and stability. We also show that all codimension-one bifurcations are transcritical.