Maximal curves over a finite field. A geometric approach

Gábor Korchmáros

A (projective, geometrically irreducible, non-singular algebraic) curve of genus $\mathfrak{g}$ defined over a finite field $\mathbb{F}_{q^2}$ is called $\mathbb{F}_{q^2}$-maximal if the number N_{q^2} of its points defined over $\mathbb{F}_{q^2}$ attains the Hasse-Weil bound, i.e. $N_{q^2} = q^2+1+2\mathfrak{g}q$. The starting point of a systematic study of maximal curves (more generally, curves with many points over $\mathbb{F}_{q^2}$) was a series of lectures delivered by J.P. Serre in 1985; see [1]. The main questions about maximal curves over a given field $\mathbb{F}_{q^2}$ have been the description of the spectrum of their genera, the classification of maximal curves having a fixed genus, and the determination of explicit equations and applications to Coding theory. The achievements in the subsequent years were the subject of the lectures of A. García and G. van der Geer at the European Congress of Mathematics, 2001, and were treated in details in some monographs. In the last decade it has been a renewed interest to maximal curves. Investigations have also concerned maximal curves not covered by the Hermitian curve, maximal curves Galois-covered by the Hermitian curve, and automorphism groups of maximal curves. Our survey aims to present the recent developments from a geometric point of view.

References

[1] J.P. Serre, Rational points on curves over finite fields, handwritten notes of Serre's Harward course 1985 by F. Gouvea, published in Documents Mathématiques (Paris) **18**. Société MathÃ©matique de France, Paris, 2020.