MASS FORMULAE FOR SUPERSINGULAR ABELIAN VARIETIES

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ABSTRACT: The moduli space $\mathcal{A}_g \otimes \mathbb{F}_p$ of $g$-dimensional principally polarised abelian varieties admits a number of stratifications. The Newton stratification for instance, classifying $p$-divisible groups up to isogeny, contains the stratum $\mathcal{I}_g$, the (Siegel) supersingular locus. The $p$-rank is constant on Newton strata, and known to be zero on $\mathcal{I}_g$.

The Ekedahl-Oort stratification and the stratification by $a$-number induce well-studied stratifications on $\mathcal{I}_g$ that depend on the isomorphism class of the $p$-torsion of the variety; every $a$-number stratum is a disjoint union of EO-strata.

We study a yet further refinement of these stratifications by considering the isomorphism classes of $p$-divisible groups. All abelian varieties with isomorphic $p$-divisible group are contained in a so-called central leaf in the foliation of the moduli space contains. In $\mathcal{I}_g$, every central leaf is finite. Computing their cardinalities is not yet within reach for higher dimensions, but it is feasible to compute their masses, i.e., cardinalities weighted by the automorphism groups.

In this talk, we will give mass formulae for $\mathcal{I}_3$, using the theory of polarised flag type quotients. In doing so, we find a mass stratification, namely, the coarsest stratification such that the mass function is constant on strata. We moreover combine the formulae with computations of the automorphism groups to study Oort's conjecture; we prove that every generic principally polarised supersingular abelian threefold over an algebraically closed field of characteristic $> 2$ has automorphism group $\mathbb{Z}/2\mathbb{Z}$. 