SOME CORRECTIONS IN THE ARTICLE ANOMALOUS TRANSPORT

CHRISTOPHE CHEVERRY

Below, we list and correct a few errata in the article *Anomalous transport* [1]. These mistakes have been reported after the publication of the work.

• Correction in the article [1] of Lemmas 2.1 and 2.2 following a comment by *L. Miguel Rodrigues*. Recall that $R = R_0 + r \cos\theta$ and that:

$$a(r,\theta) := \big[r^{-2} \, (\partial_\theta \psi)^2 + (\partial_r \psi)^2 + I(\psi)^2 \big]^{1/2} \,. \tag{0.1}$$

First, Lemma 2.1 must be replaced by:

Lemma 0.1. *[computation of $\mathcal{C}$] With $a(\cdot)$ as in (2.14), we have:*

$$\mathcal{C} = \frac{1}{2} \, \operatorname{div} e_\parallel = \frac{1}{2\, r\, R} \, \Big[- \partial_r \Big(\frac{R\, \partial_\theta \psi}{a} \Big) + \partial_\theta \Big(\frac{R\, \partial_r \psi}{a} \Big) \Big] \,. \tag{0.2}$$

Proof. The first equality is explained in [1]. The second identity (on the right) differs from the calculation in [1], due to the introduction of R. It comes from a general formula to compute the divergence in general coordinates (see for instance Y. Hu, paragraph 4.4). □

Subsequently, Lemma 2.2 must be replaced by:

Lemma 0.2. *[circulation around $\mathscr{C}_\psi$] For all $\psi \in \mathbb{R}_+^*$, we have:*

$$\int_0^{2\pi} \Big[\frac{r\; \mathcal{C}\; a}{\partial_r \psi} \Big] \big(\gamma_\psi(\theta) \big) \; d\theta = 0 \,. \tag{0.3}$$

Proof. Observe that:

$$2 \; \frac{r\; \mathcal{C}\; a}{\partial_r \psi} = \frac{a}{\partial_r \psi} \; \Big[- \partial_r \Big(\frac{\partial_\theta \psi}{a} \Big) + \partial_\theta \Big(\frac{\partial_r \psi}{a} \Big) \Big] - \frac{a}{\partial_r \psi} \; \Big[\frac{\cos\theta \;\, \partial_\theta \psi}{R\; a} + \frac{r \;\, \sin\theta \;\, \partial_\theta \psi}{R\; a} \Big] \,. \tag{0.4}$$

As explained in the proof of Lemma 2.2 of [1], the contribution issued from the first part gives zero. Concerning the second part (on the right), it suffices to remark that:

$$\begin{aligned} 0 &= \int_{\mathscr{C}_\psi} \nabla (\ln R) \cdot dl = \int_0^{2\pi} \partial_\theta \big[\ln R \big(\gamma_\psi(\theta) \big) \big] \; d\theta \\ &= - \int_0^{2\pi} \Big[\frac{a}{\partial_r \psi} \; \Big(\frac{\cos\theta \;\, \partial_\theta \psi}{R\, a} + \frac{r \;\, \sin\theta \;\, \partial_r \psi}{R\, a} \Big) \Big] \big(\gamma_\psi(\theta) \big) \; d\theta \,. \end{aligned} \tag{0.5}$$

□

• A few comments are in order.

Remark 1. The coefficient $\mathcal{C}$ is zero if and only if:

$$- \; \partial_\theta \psi \; \partial_r \Big(\frac{R}{a} \Big) + \partial_r \psi \; \partial_\theta \Big(\frac{R}{a} \Big) = 0 \,. \tag{0.6}$$

In other words, the expression $R^{-2} \, \big(|\nabla \psi|^2 + I(\psi)^2 \big)$ must be a function of ψ. The analysis of the toy model of [1] must be amended accordingly.

Remark 2. In [1], Lemma 2.2 is used at the level of Paragraph 3.2.2, see line (3.42), to show the periodic behavior of the first potential function. Lemma 0.2 works also to this end. The general analysis of [1] and the conclusion of the main Theorem 1 are correct.

Date: March 15, 2017.

References

[1] Christophe Cheverry. Anomalous transport. *J. Differential Equations*, 262(3):2987–3033, 2017.

Université de Rennes 1, IRMAR, UMR CNRS 6625, 263 avenue du General Leclerc; F-35042 Rennes Cedex, FRANCE
E-mail address: christophe.cheverry@univ-rennes1.fr