UNIVERSITÉ DE RENNES 1

Microlocal Analysis

CC5 on quantization, the 03/12/2021 (30mn)

Documents are not allowed

Surname : **First name :**

We work on $L^2 \equiv L^2(\mathbb{R};\mathbb{C})$ with the two (unbounded essentially) self-adjoint operators

$$X : L^2 \longrightarrow L^2 \qquad P : L^2 \longrightarrow L^2$$
$$f \longmapsto x\,f, \qquad f \longmapsto -i\partial_x f.$$

1. Compute the commutator $[X, P]$.

$[X, P] =$

2. Recall (in terms of X and P) the definition of the Weyl quantization of $x^2 p$.

$Q_{Weyl}(x^2 p) =$

3. Express the above expression only in terms of XPX.

$Q_{Weyl}(x^2 p) =$

4. Let $f(x, p)$ be a function in the Schwartz space, that is in $\mathcal{S}(\mathbb{R}^n \times \mathbb{R}^n; \mathbb{C})$. We denote by $\hat{f}(a, b)$ its Fourier transform (in both variables x and p).

4.1. Complete the two formulas below for the Weyl quantization of the symbol f :

$$Q_{Weyl}(f) = (2\pi)^{-n} \int\!\!\int \qquad\qquad \hat{f}(a, b)\ dadb$$
$$= (2\pi\hbar)^{-n} \int\!\!\int e^{-i \qquad \cdot\xi/\hbar}\ f\Big(\qquad , \xi\Big)\ dyd\xi.$$

4.2. What can be said about the action on $L^2(\mathbb{R}^n \times \mathbb{R}^n)$ of Q_{Weyl} ?

T.S.V.P. $\Longrightarrow$

4.3. Complete the following formula : $Q_{Weyl}(f)^* = Q_{Weyl}(\quad)$.

5. We recall that the Wick-ordered quantization Q_{Wick} of a polynomial in $z = x - ip$ and $\bar{z} = x + ip$ is obtained by putting all operators (coming from $\bar{z}$) to the right (acting first) and all operators (coming from z) to the left (acting second).

5.1. What is the value of $Q_{Wick}(\bar{z})(e^{-x^2/2})$ and the name of the operator $Q_{Wick}(\bar{z})$?

$Q_{Wick}(\bar{z})(e^{-x^2/2}) =$ Name :

5.2. Compute

$Q_{Wick}(\bar{z}z^3 + z)(e^{-x^2/2}) =$

6. Compute $Q_{Wick}(x^2)$ in terms of X^2 and Id.

$Q_{Wick}(x^2) =$