
CC3, the 12/10/2018 (10mn)

Documents are not allowed

Surname :

First name :

Question. Define what is a Fredholm operator $T \in \mathcal{L}(E, F)$.

Exercise 1. Let $E = \mathbb{R}^2$. Given $\lambda \in \mathbb{C}$, define $T \in \mathcal{L}(E)$ and $P_\lambda \in \mathcal{L}(E)$ by

$$T := \begin{pmatrix} \lambda & 1 \\ 0 & \lambda \end{pmatrix}, \quad P_\lambda := \frac{1}{2i\pi} \int_{\Gamma_\lambda} (z - T)^{-1} dz,$$

where Γ_λ is the circle of center λ and radius 1. Determine P_λ by a direct computation.

T.S.V.P \Rightarrow

Exercise 2. Let E be the Banach space $C^0([0, 1]; \mathbb{R})$ equipped with the sup norm. Select a strictly increasing function $\varphi \in E$, and consider the operator $T : X \rightarrow X$ which is given by the multiplication $T(f) = \varphi f$. Determine the essential spectrum $\text{sp}_{ess}(T)$ of T .