UNIVERSITÉ DE RENNES 1

Microlocal Analysis

CC2 on the *inversion of elliptic operators*, the 12/11/2021 (30mn)

Documents are not allowed

Surname : **First name :**

Let $n \in \mathbb{N}$ as well as m and m' in $\mathbb{R} \cup \{-\infty\}$. Given two symbols $a \in S^m \equiv S^m_{1,0}(\mathbb{R}^n)$ and $b \in S^{m'}$, we admit that the composition of $Op(a)$ with $Op(b)$ is a pseudo-differential operator whose symbol is in $S^{m+m'}$. More precisely

$$Op(a)Op(b) = Op(a\#b), \qquad a\#b = ab + r, \qquad ab \in S^{m+m'}, \qquad r \in S^{m+m'-1}.$$

1. Take $n = 1$, $a = i\xi$ and $b = x$. What is r ?

$r(x,\xi) =$

2. Take $n = 1$, $a = i\xi$ and $b = x$. What is the symbol of the adjoint of $Op(a)Op(b)$?

$(a\#b)^*(x,\xi) =$

3. Let $a \in S^m$. We assume here that we can find some $b \in S^{-m}$ which is such that $a\#b - 1$ is in the class $S^{-\infty}$.

3.1. Prove that : $\exists\, R \in \mathbb{R}^*_+ ; \quad |\xi| \geq R \implies |(ab)(x,\xi)| \geq 1/2.$

T.S.V.P. $\Longrightarrow$

3.2. Prove that :

$$\exists (R,c) \in \mathbb{R}_+^* \times \mathbb{R}_+^*; \qquad |\xi| \geq R \quad \Longrightarrow \quad c\,(1+ \parallel \xi \parallel)^m \leq |a(x,\xi)|. \tag{1}$$

4. Let $a \in S^m$. We assume that we have the property (**??**) for some $R \in \mathbb{R}_+^*$.

4.1. Find $b_0 \in S^{-m}$ which is such that $Op(a)\,Op(b_0) = Id + \mathcal{R}_0$ with $\mathcal{R}_0 \in Op(S^{-1})$. Justify the answer.

4.2. Find $b_1 \in S^{-m-1}$ which is such that $Op(a)\,Op(b_0+b_1) = Id+\mathcal{R}_1$ with $\mathcal{R}_1 \in Op(S^{-2})$.