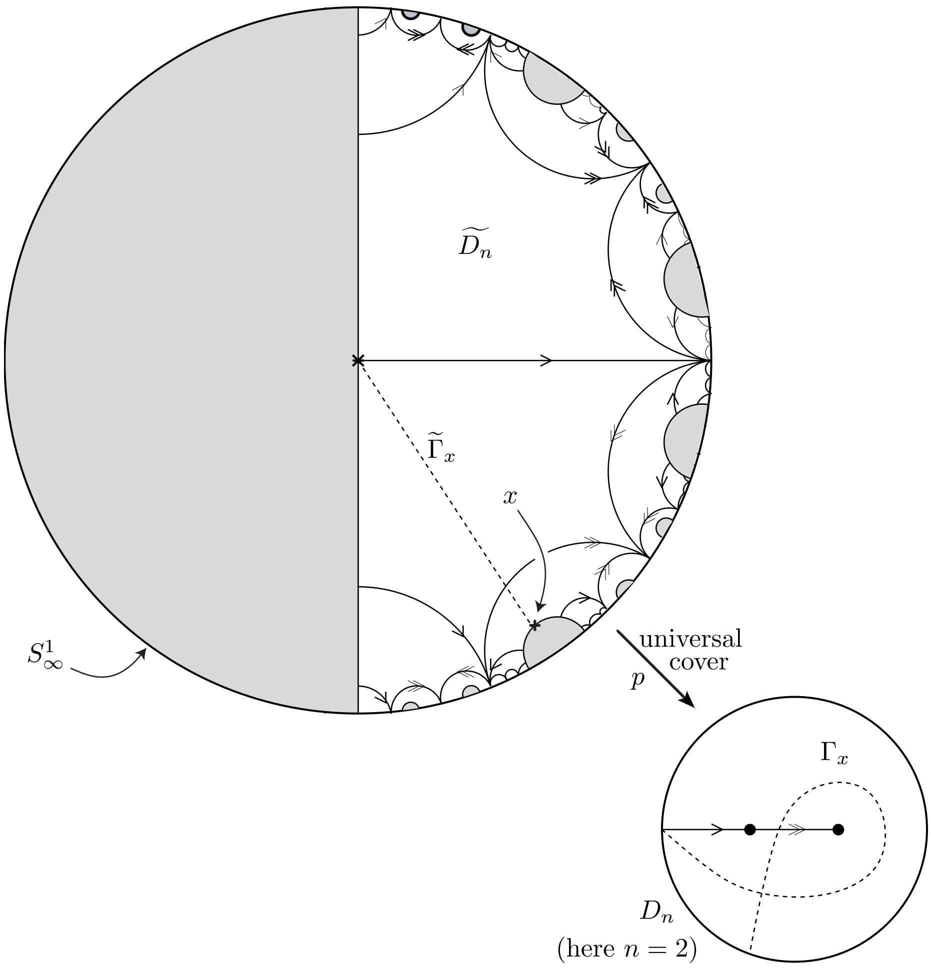


FIGURE 1. The disk $D_n$, and its universal cover $\widetilde{D_n}$ embedded in $\mathbb{H}^2$—indicated as the unshaded part.

on curve diagrams—and the lifting of $\varphi(\Gamma_x)$ is a path in $\widetilde{D_n}$ from $*$ to some point in $\partial\widetilde{D_n}$. This is the point that we define to be $\varphi(x)$. Note that, if $x$ was a point in $S^1_\infty$, then all the paths mentioned were necessarily infinite. Note also that the path $\varphi(\Gamma_x)$ is, in general, not a geodesic even if $\Gamma_x$ is geodesic.

Let us summarize what we have achieved so far. We have described an action by homeomorphisms of $B_n$ on a topological space $\partial\widetilde{D_n} \setminus \{*\}$, which we managed to identify with $\mathbb{R}$. We shall say that an ordering of $B_n$ is of Nielsen–Thurston type if it arises from a point $x$ in $\mathbb{R}$ with $\mathrm{Stab}(x) = \{1\}$ in this action of $B_n$ on $\mathbb{R}$. More precisely:

DEFINITION 1.4. We say that a left-ordering $\prec$ of $B_n$ is of *Nielsen–Thurston type* if there exists an element $x$ of $\mathbb{R}$ such that, for all $\beta, \beta'$ in $B_n$, the relation $\beta \prec \beta'$ is equivalent to $\beta(x) <_{\mathbb{R}} \beta'(x)$.

Our aim will now be to classify the order types of $B_n$ of Nielsen–Thurston type. For instance, we shall prove that for $n \geqslant 3$ there are uncountably many