It is not true that a separable locally compact group which has Kazhdan’s Property (T) necessarily has Serre’s Property (FA) as a discrete group. Indeed, a (discrete) group with Property (FA) cannot be the union of an increasing sequence of proper subgroups (see [Serre–77], Remarque 1 in Chap.I, §6, 6.1). However, there are examples of compact groups (which have therefore Property (T) as topological groups) which can be written as union of a strictly increasing sequence of subgroups. For a concrete example, let $G = \prod_{\mathbb{N}} \mathbb{Z}/2\mathbb{Z}$ be the direct product of countably many copies of the two-element group. Since $G$ can be viewed as a vector space over $\mathbb{Z}/2\mathbb{Z}$, there exists a surjective homomorphism $\varphi : G \to H$, where $H = \bigoplus_{\mathbb{N}} \mathbb{Z}/2\mathbb{Z}$ is the direct sum of countably many copies of $\mathbb{Z}/2\mathbb{Z}$. We can write $H = \bigcup_{n \in \mathbb{N}} H_n$ for an increasing sequence of proper subgroups $H_n$. So, $G$ is the union of the increasing sequence of proper subgroups $\varphi^{-1}(H_n)$.

The statement in Remark 2.3.7.ii has to be corrected as follows (and this is indeed what Alperin shows in [Alper–82]): let $G$ be a separable locally compact group with Property (T). Then $G$, viewed as a discrete group, has Property (FA'), that is, whenever $G$ acts on a tree without inversion of edges then every element in $G$ has a fixed vertex.

2. Page 91, correction in the definition of $\xi_0$

As defined, the function $\xi_0$ is not in $\mathcal{H}$, because the origin 0 is not fixed by $\alpha$. A way out is to introduce representatives $(s_i)_{i \in I}$ for the left cosets of $H$ modulo $G$, so that $G = \bigcup_{i \in I} s_i H$, and to define $\xi$ by

$$\xi_0(s_i h) = \alpha(h^{-1}) \eta_0 \quad \text{for all } i \in I \text{ and } h \in H.$$
3. Page 92, correction in the proof of Proposition 2.5.9

One should have “Let $H$ be a maximal torsion–free abelian subgroup of $G$” instead of “Let $H$ be a maximal free abelian subgroup of $G$”.

Indeed, the existence of a maximal torsion–free subgroup is a straightforward consequence of the Zorn Lemma, whereas a discrete abelian group $G$ need not have any maximal free abelian subgroup.

4. Page 94, on the hyperbolic distance

Proof that $d([x], [y])$ is well defined (lines 8-11): in order for the last inequality (line 9) to hold, one has to assume from the beginning that $x_{n+1}y_{n+1} \geq 0$; this can be achieved by replacing $x$ by $-x$, if necessary.

5. Page 210, on the group $G^X$ of Proposition 4.3.10

The topology of uniform convergence does not make sense here, unless $X$ is assumed to be compact.

More precisely, a topological group has two canonical uniform structures, the left-invariant one and the right-invariant one (Bourbaki, Topologie générale, chapitre III, § 3). Very often, they are distinct, and they don’t make $G^X$ a topological group.

However, if $X$ is compact, which is the case of interest in this part of our book, the compact-open topology on $G^X$ has all desired properties (Bourbaki, Topologie générale, chapitre X, § 3), and this is the topology which has to be considered here.

6. Pages 185 and 215, on elementary matrices over a field

Remark 4.1.2.i and Exercise 4.4.2: read $\nu_n(K) \leq n^2$ instead of $\nu_n(K) \leq n(n - 1)$.

7. Page 403, on the proof of Proposition F.2.2

It is indeed clear (by Definitions F.1.1 and F.2.1) that, if $(\pi_i)_{i \in I}$ converges to $\pi$, then $\pi$ is weakly contained in $\bigoplus_{j \in J} \pi_j$ for every subnet $(\pi_j)_{j \in J}$ of $(\pi_i)_{i \in I}$. Since the proof of the converse is less clear, we give more details.

Assume, by contradiction, that $\pi$ is weakly contained in $\bigoplus_{j \in J} \pi_j$ for every subnet $(\pi_j)_{j \in J}$ of $(\pi_i)_{i \in I}$ and that $(\pi_i)_{i \in I}$ does not converge to $\pi$.

On the one hand, there exists a neighbourhood $W = W(\pi, \varphi_1, \ldots, \varphi_n, \epsilon)$ of $\pi$ (as in Definition F.2.1) and a subnet $(\pi_j)_{j \in J}$ of $(\pi_i)_{i \in I}$ such that no
\( \pi_j \) belongs to \( W \) (take \( J = \{ i \in I : \pi_i \not\in W \} \)). Upon passing to a subnet of \( (\pi_j)_{j \in J} \), we can even assume that \( n = 1 \), that is, \( W = W(\pi, \varphi, \epsilon) \) for a function \( \varphi \) of positive type associated to \( \pi \).

Now, \( \varphi \) can be approximated, uniformly on compact subsets of \( G \), by sums of functions of positive type associated to irreducible unitary representations which are weakly contained in \( \pi \) (see Proposition F.2.7, which is independent of Proposition F.2.2). So, there exist irreducible unitary representations \( \sigma_1, \ldots, \sigma_n \) which are weakly contained in \( \pi \) and functions of positive type \( \psi_1, \ldots, \psi_n \) associated to \( \sigma_1, \ldots, \sigma_n \) such that no \( \pi_j \) belongs to \( W(\sigma, \psi, \epsilon/2) \), where \( \sigma = \bigoplus_{i=1}^n \sigma_i \) and \( \psi = \sum_{i=1}^n \psi_i \).

On the other hand, \( \pi \) and hence every \( \sigma_i \) is weakly contained in \( \bigoplus_{j \in J} \pi_j \). Since \( \sigma_i \) is irreducible, it follows that every \( \psi_i \) can be approximated, uniformly on compact subsets of \( G \), by functions of positive type associated to the \( \pi_j \)'s (and not just sums of such functions; see Proposition F.1.4). Upon passing to a subnet of \( (\pi_j)_{j \in J} \), we can approximate \( \psi_1, \ldots, \psi_n \), simultaneously and uniformly on compact subsets of \( G \), by functions of positive type associated to the \( \pi_j \)'s. It follows that \( \pi_j \in W(\sigma, \psi, \epsilon/2) \) for appropriate \( j \) and this is a contradiction.

**8. Page 405, correction in the proof of Proposition F.2.7**

The proof (lines 10-19) that \( \text{ext}(C) \) is contained in \( \text{ext}(P_{\leq 1}(G)) \) is incorrect. This has to be corrected as follows:

Let \( \varphi \in \text{ext}(C) \); decompose \( \varphi \) as \( \varphi = t\varphi_1 + (1-t)\varphi_2 \) for \( 0 < t < 1 \) and \( \varphi_1, \varphi_2 \in P_{\leq 1}(G) \). There exists a (cyclic) representation \( \rho \) with cyclic vector \( \eta \) such that \( \varphi(x) = \langle \rho(x)\eta, \eta \rangle \) for all \( x \in G \). As in the proof of Lemma F.1.3, \( \varphi_1 \) can be approximated by positive definite functions associated to \( \rho \) of the form

\[
\left\langle \rho(x) \left( \sum_{i=1}^n \lambda_i \rho(x_i) \eta \right), \sum_{i=1}^n \lambda_i \rho(x_i) \eta \right\rangle,
\]

that is, by sums of the form \( \sum_{i,j=1}^n \lambda_i \lambda_j \varphi(x_j^{-1}xx_i) \). Such a sum \( \psi \) lies in \( C \), since \( \varphi \) belongs to \( C \) and so every \( \psi \) can be approximated by convex combinations of corresponding functions of positive type associated to \( \pi \). This shows that \( \varphi_1 \) is in \( C \). Similarly, \( \varphi_2 \) is in \( C \). Since \( \varphi \) is an extreme point in \( C \), it follows that \( \varphi = \varphi_1 = \varphi_2 \), showing that \( \text{ext}(C) \) is contained in \( \text{ext}(P_{\leq 1}(G)) \).
9. Page 428, on the proof of Theorem G.3.1

Page 428, Line -6: In order to make the proof that (ii) implies (iii) more transparent, one should mention that the following formula holds:

\[ \langle f \ast \varphi, g \rangle = \langle \varphi, f^* \ast g \rangle \]

for all \( f, g \in L^1(G), \varphi \in L^\infty(G) \).

Here \( \langle \varphi, f \rangle = \int_G \varphi(x) f(x) dx \) denotes the duality between \( L^\infty(G) \) and \( L^1(G) \); recall that \( f^*(x) = \overline{f(x^{-1})} \Delta(x^{-1}) \) and observe that \( f^* \in L^1(G)_{1,+} \) if \( f \in L^1(G)_{1,+} \).

10. Some typos

Page 47, Line 11: read “with \( g_\alpha(e_i) = \lambda_\alpha e_i \)”.

Page 204, Line 4 of Lemma 4.3.4: \( \text{Sign} := \) is missing in \( \gamma \in F := \{ E_{1,2}(\pm 1), E_{2,1}(\pm 1), E_{1,2}(\pm t), E_{2,1}(\pm t) \} \).

Page 313, Line 8: read “each \( X \in K \) is either a constant or a centred Gaussian random variable” instead of “each \( X \in K \) is a centred Gaussian random variable”.

Page 333, Line 3, Definition B.2.3: read “\( h_1, h_2 \in H \)” instead of “\( h_1, h_2 \in G \)”.

Page 393 (Exercise E.4.1): read \( \pi_{it}^\pm \) instead of \( \pi_{i}^\pm \).

Page 399, Line 3 of Proposition F.1.4: read “a unit vector in \( \mathcal{H} \)” instead of “a unit vector \( \mathcal{H} \)”.

Page 407, Line 1: read “a linear functional” instead of “a positive linear functional”.

Page 410 (Example F.3.6): read \( \pi_{it}^- \) instead of \( \pi_{i}^- \).

Page 455, for [Fell–62] the pages should read 237–268.

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